The Pearson Guide to

Quantitative Aptitude and Data Interpretation for the CAT

The Pearson Guide to
Quantitative Aptitude and
Data Interpretation
for the CAT
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Nishit K. Sinha

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THE PEARSON GUIDE TO

Quantitative Aptitude and Data Interpretation for the CAT

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THE PEARSON GUIDE TO

Quantitative Aptitude and Data Interpretation for the CAT

NISHIT K. SINHA
CAT CONSULTANT

PEARSON

Chandigarh • Delhi • Chennai

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To
Papa and Ma
Mr Kumar Kalyan Pd Sinha
and
Mrs Sanjila Sinha

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Preface

The fact that there was no surprise element in CAT 2010 with respect to CAT 2009 was itself a surprise. Probably IIMs/CAT committee wanted to regain the ground that they might have lost during CAT 2009 online test glitches.

So everybody was expecting some changes in CAT pattern in 2011, and CAT committee obliged. The CAT changed again in 2011—with the introduction of (a) 2-Section format instead of 3-section format and (b) sectional time limit. This is a welcome change as now the CAT is akin to other global exams like the GMAT and GRE. They also comprise only two sections and have sectional time limit.

This book covers everything that you need to prepare for the Quantitative Aptitude and Data Interpretation Section as per CAT 2011 pattern. Before we move ahead, let us see the cut-off percentile of different IIMs for CAT 2011 (for general category):

S. No	*IIM's*	*Overall score*	*QA/DI*	*VA/LR*
1	IIM-A	99	94	94
2	IIM-B	90	80	90
3	IIM-C	99.55	94.24	93.73
4	IIM-L	90	85	85
5	IIM-I	90	85	85
6	IIM Trichy	80	70	70
7	IIM Kashipur	75	70	70
8	IIM Udaipur	80	70	70
9	IIM Ranchi	90	70	70
10	IIM Shillong	***	65	65
11	IIM Kozhikode	85	55	55

*** Not given.

This book is divided into four parts: Part 1—Quantitative Aptitude; Part 2—Data Interpretation; Part 3—CAT Papers; and Part 4—Section Tests. Each of the first two parts is further distributed among modules to facilitate the learning of students. Once a student is through with chapters, there are benchmarking tests with calibrated percentile. Moving on further, there are CAT papers to know the level of your preparedness. Finally there are four section tests to help you assess your preparation level.

Further, to facilitate the online testing, a CD containing Section Tests and Full Length Tests is appended.

Though I have taken utmost care in preparing the manuscript and going through the subsequent proofs, some errors may have crept in the book. Kindly email me your valuable constructive advice on my email id: nsinha.alexander@gmail.com

NISHIT K. SINHA

Acknowledgements

This book bears the imprint of many people— my colleagues, my students and my teachers who have had a significant impact on my thought process and have generously extended help whenever I needed.

I am thankful to my present organization for giving me enough time to work on this project.

I would like to thank my teachers Mr Anoop Singhania, Mr Vinay Singh, Mr M. K. Alam Bhutto, Mr Jairam Singh and Mr Arun Sharma.

I would also like to thank Mr Anil Kumar, Mr Saral Choudhary, Mr Sachin Goyal, Mr Shayank Bariar, Mr Manoj Kumar, Mr Parichay Shrestha, Mr Mohit Berry, Mr Sandeep Singh, Mr Siddharth Tripathi, Mr Tanzar Aftab, Mr Anup Singh, Mr Chandrabhal Rathod, Abhishek Anand and of course Ma'am Parul Jaiswal for their valuable inputs.

My special thanks to my brothers Ravi Shankar Prasad, Sharat Chandra Mayank, Amit Kumar and Vinit Kumar.

I extend heartfelt thanks to all my students, past and present, who have helped me improve the content and the presentation of the book.

I would like to thank Mrs Preeta Priyamvada for her painstaking efforts in going through the manuscript. A special mention of thanks to Mr Raza Khan, Sr Managing Editor, Pearson, who played a pivotal role in getting this book printed in such a short span of time.

I may have forgotten some names here. I wish to express my gratitude towards all who have contributed in the making of this book.

Nishit K. Sinha

CAT Demystified

CAT stands for the Common Admission Test. It is a test conducted by IIMs for admission into several programs offered by them. Besides IIMs, there are a good number of colleges which accept CAT score in their first round of selection process. As of now, there are 13 IIMs offering PGP at following places: Ahmedabad, Bangalore, Calcutta, Lucknow, Indore, Kozhikode, Shillong, Ranchi, Rohtak, Raipur, Udaipur, Trichy, Kashipur.

History of the CAT

For last almost three decades, since the CAT has been started, it has changed its colours many a time in terms of number of questions, sections asked and orientation of those questions. Here we will discuss the pattern of CAT 2000 onwards.

Chart 1

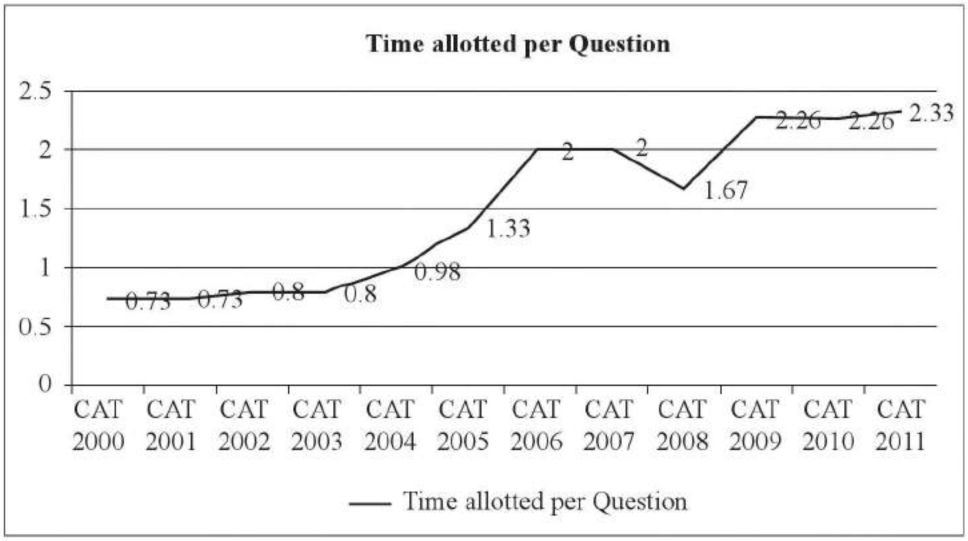
	Number of sections	*Total number of questions*	*Total Marks*	*Time allowed*
CAT 2000	3	165	N.A.	120 minutes
CAT 2001	3	165	N.A.	120 minutes
CAT 2002	3	150	N.A.	120 minutes
CAT 2003	3	150	N.A.	120 minutes
CAT 2004	3	123	150	120 minutes
CAT 2005	3	90	150	120 minutes
CAT 2006	3	75	300	150 minutes
CAT 2007	3	75	300	150 minutes
CAT 2008	3	90	360	150 minutes
CAT 2009	3	60	450 (scaled score)	135 minutes
CAT 2010	3	60	450 (scaled score)	135 minutes
CAT 2011	2	60	450 (scaled score)	140 minutes

CAT 2011 has two sections: (a) Quantitative Ability & Data Interpretation (b) Verbal Ability & Logical Reasoning with 30 questions in each section. CAT 2011 also has sectional time limit of 70 minutes for each section.

Before CAT 2004, the CAT did not mention how many marks one question carried? Marks carried per question were announced for the first time in CAT 2004.

Quite obvious from the above table that time allotted per question has risen sharply from CAT 2000 to CAT 2011. One possible conclusion drawn from here is that the CAT is focussing more on accuracy than speed, and secondly, it expects students to gain a certain level of competence across all the areas in a particular section. With the number of questions going down and time going up, students do not have much choice of questions to choose from.

Chart 2



Sectional Breakups and Getting an IIM Call

One thing that has remained constant during this period of CAT 2000 - CAT 2010 is the number of sections and the way these sections have been joined—Quantitative Aptitude (QA), Logical Reasoning and Data Interpretation (LR/DI), and English Usage/Reading Comprehension (EU/RC).

Though CAT 2011 changed it all:

Chart 3

Year	*QA*	*LR DI*	*EU RC*	*Total no. of questions*
CAT 2000	55	55	55	165
CAT 2001	50	50	50	150
CAT 2002	50	50	50	150
CAT 2003	50	50	50	150
CAT 2004	35	38	50	123
CAT 2005	30	30	30	90
CAT 2006	25	25	25	75
CAT 2007	25	25	25	75
CAT 2008	25	25	40	90
CAT 2009	20	20	20	60
CAT 2010	20	20	20	60
CAT 2011	30 (QA+DI)		30 (Verbal + LR)	60

One possible reason predicted for the 2-section format is the fact that the CAT is trying to emulate the standard global papers like the GRE/GMAT.

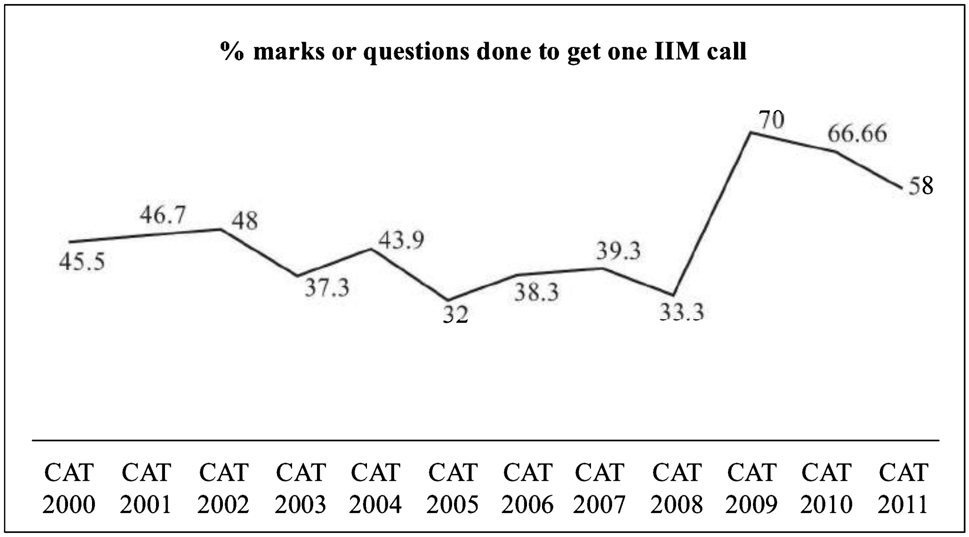
However, past CAT trends show that a student is required to get around 70% marks to get at least one IIM call (with clearing the sectional cut-off). The following table and bar chart give us some clarity regarding the same:

Chart 4

Year	*Total marks or questions*	*Marks/Qs required to get at least one IIM call*
CAT 2000	165	75
CAT 2001	150	70
CAT 2002	150	72
CAT 2003	150	56
CAT 2004	123	54
CAT 2005	150	48
CAT 2006	300	115
CAT 2007	300	118
CAT 2008	360	120
CAT 2009	60	42
CAT 2010	60	40
CAT 2011	60	35

Following line chart gives questions solved or marks required as a percentage of total marks or total questions (as applicable):

Chart 5



Note: Above calculation is based upon the data collected from the students who got IIM calls in that particular year.

So, to get at least one IIM call in CAT 2008, a student was required to get 33.33% marks out of the total with clearing the cut-off across the sections. Though in the online format of the CAT (since 2009), percentage questions to be done to get at least one IIM call has gone up, it is primarily because the exam is perceived to be easier in its totality than pen and paper based exams. A student might have got 10 easy questions out of 20 questions in a section.

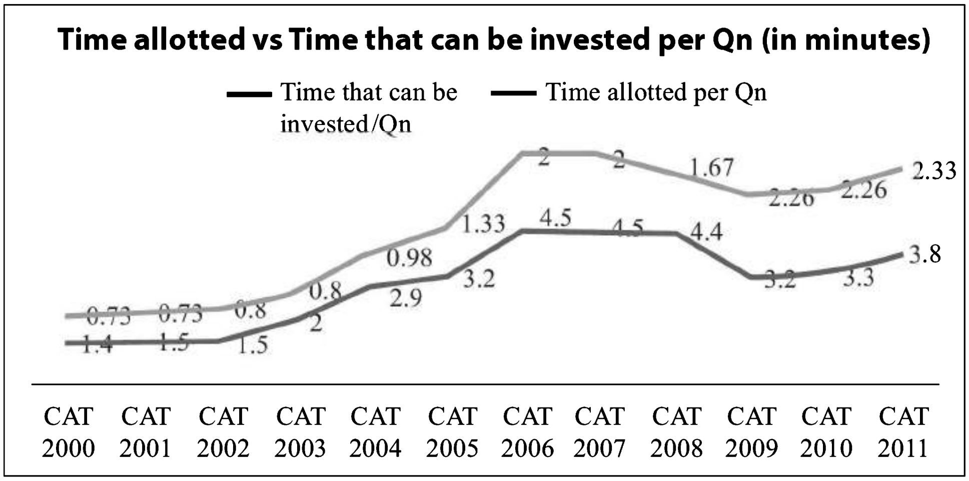
If we convert the requirement of marks to be obtained from the above line chart into questions to be done, we get the following table:

Year	*No. of questions to be solved*	*Time allotted*	*Time invested per qn*
CAT 2000	85	120 minutes	1.4
CAT 2001	80	120 minutes	1.5
CAT 2002	81	120 minutes	1.5
CAT 2003	60	120 minutes	2.0

Year	*No. of questions to be solved*	*Time allotted*	*Time invested per qn*
CAT 2004	42	120 minutes	2.9
CAT 2005	38	120 minutes	3.2
CAT 2006	33	150 minutes	4.5
CAT 2007	33	150 minutes	4.5
CAT 2008	34	150 minutes	4.4
CAT 2009	42	135 minutes	3.2
CAT 2010	40	135 minutes	3.3
CAT 2011	35	135 minutes	3.8

Chart 6—For CAT 2000 to CAT 2008, at 90% accuracy with 1/4 negative marking, these are the approximate number of questions to be done. For CAT 2009 to CAT 2011, net of these many questions is to be done.

To summarize this whole discussion till now, we will compare chart 2 with chart 6 and present them in a unified line chart given below (chart 7).



This line brings to us an important information—For CAT 2006, CAT 2007 or CAT 2008, even if a student has taken approximately 4.5 minutes to solve a question with 90% accuracy, s/he has got enough marks to get at least one IIM call (provided s/he clears the sectional cut-off too). For online formats for CAT, it is around 3.3 minutes per question.

So, it's more about accuracy now than speed.

How to Prepare for QA

Purpose of QA Being Included in the CAT

Have you ever thought why quantitative-aptitude questions are asked in the CAT? Is it to check your math ability with questions from algebra, or permutation and combination? If that were the case, they would have asked questions also from group theory and applied mathematics, which they don't. The questions asked in quantitative aptitude help test the student's reasoning ability based on mathematical skills. They also test the student's comprehension of basic mathematical concepts. And even if somebody has not studied mathematics after class 10, they can solve these problems with a little practice. What is required is an understanding of the basic concepts. It is, therefore, clear that through the test of QA, the examiners intend to check functional ability pertaining to basic mathematical operations.

Five-point Action Plan for QA

1. Address your math-phobia first You will not be in the best state of mind to crack the CAT if past failures or inabilities in mathematics haunt you. As it is said, it's okay to lose a couple of battles. Keep your eyes set on the war!

2. Make a roadmap 'Divide and win' is the key. Your preparation should be divided into topics and every topic should be further divided into sub-topics. Once you have a topic-centric roadmap of the entire QA syllabus, carefully distribute the time you want to devote to each section on a daily basis.

3. Strengthen your fundamentals Till you have understood the concepts and their various applications thoroughly, do not start solving the problems. Let this process take some time. Now, before you start solving questions, go over the fundamental concepts once again and see which concept should be applied to get the right answer.

4. Adopt a systematic practice technique Don't jump to solving problems immediately. Make an effort to understand the basic theories behind the mathematical concepts, however trivial they may seem. And only then solve the problems.

5. Focus on weaknesses If students are weak in a particular area (say, geometry or permutations and combinations), they tend to neglect these questions and leave the solutions to choice. This is not only unproductive but also damaging. Identify the areas with which you are not comfortable. Numbers, geometry and algebra account for a major portion of the questions in the CAT; hence, they deserve due attention.

Consistency with Perseverance is the Key

If you do not find ways to solve a problem on the first day, sleep on it and tackle it the next day. Sometimes even the simplest of answers elude you and lead to frustration. Even the best of mathematicians face this problem; therefore, leave it for another day. And if you are unable to solve this problem even on the second day, try it on the third day or the third time. If you are still not able to solve it, check the solution provided at the end of the book or take help. The bottom line is that only consistent efforts and practice can bring in positive results. If you want to go through this book in 10 days' time, you can do that also. And you will be able to learn the fundamentals, but remember it will be only short-term. You will be required to go through the same chore again after a few days and repeat this exercise.

How to Use This Book

Let me share with you what my feelings were before I started working on this book. I thought of the kind of book I would prefer if I had to learn the theories of public administration when I do not have any background of this subject. I realized I would want a book which is strong in fundamental concepts but lucid in language; I would need a book which is self-explanatory but not verbose; I would need a book which is all-encompassing but not irrelevant; I would need a book with a good number of practice questions but no repetitive questions (if I cannot learn a concept with 200 relevant questions, chances are low that I will be able to learn that concept with 250 questions having the same level of relevance); I would need a book which can help me judge my progress from time to time. And this is what I have sincerely tried to provide in this book.

How to Go Ahead with This Book

Step 1 Go through the learning objectives and remain focused on them. After completing the chapter, check if these objectives have been fulfilled.

Step 2 Start with the concepts. Before moving on to the next concept, go through the worked-out examples related to that concept. Move ahead only when you have internalized them. Sometimes this might appear to be drudgery, but you must do it.

Step 3 Do the warm-up exercise (in QA part only). It is a precursor to the problems. Most of the questions in the warm-up exercise will help you check only your understanding of the concept and not the application of it. Ideally, you should not give more than 2 minutes to any question in the warm-up exercise. If you get less than 75 per cent questions correct, go over the concepts for which you got the answers wrong.

Step 4 Do the foundation exercise. Solve the problems in this set without any time constraint. This level tests your comprehension of the concept, and is a precursor to the application-based questions. Try to solve all the questions in one sitting, whatever time it takes (one or two breaks of 10 minutes can be taken). If you don't get the correct answers, attempt the same question once again in the next sitting (preferably the next day). This will help you develop:

- A thorough understanding of the concepts
- An experience of the problems being asked at the basic level
- Confidence building

Step 5 Next is the moderate exercise. This level tests your ability to apply a particular concept, or a combination of concepts, to a problem. You might find that concepts of geometry are being used in problems on time, speed and distance. This will help you develop:

- The ability to identify easy and difficult questions
- Mental images of the problems
- A logical connection between concepts and their application.

Step 6 Do not go for the advanced level immediately. Relax. Do not solve any QA question for one day. The following day, take the benchmarking test. Your performance in this test can tell you a lot about your progress. Ideally, you should aim for 85 percentile or more.

Step 7 After you have done all the moderate exercises and benchmarking tests, go over the topics again. Only then attempt the advanced questions. Most of the questions here are above the regular CAT level. The idea is to prepare you for a higher level. If you excel at this level, CAT will be a cakewalk for you.

Step 8 Take the Review Tests. Target anything above 98 percentile. If you get it consistently in all the RTs, you are prepared for the CAT. If not, repeat the process from Step 5. If in any one of the RTs, you get less than 85 percentile, repeat the process from Step 4.

All the best.

About the Author

Nishit Sinha is a postgraduate from IIM Lucknow. For the last ten years, he has been training students for the CAT. Many of his students have now graduated from premier B-Schools across the globe. He is also the author of widely praised books *The Pearson Guide to Quantitative Aptitude and Data Interpretation for the CAT, The Pearson Guide to Verbal Ability and Logical Reasoning for the CAT* and *Demystifying Number System*.

He is currently running the test preparation organization dueNorth Academics, based in Dehradun.

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PART

1

Quantitative Aptitude

Module 1

- Number and Its Properties

Module 2

- Arithmetic

Module 3

- X+2 Maths

Module 4

- Measurement

Module 5

- Miscellaneous

Module 1

Numbers and Their Properties

1

CHAPTER

Vedic Mathematics

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- How to do Faster Calculations?
- Multiplications
- Squares
- Cubes
- Properties of Squares and Cubes

Introduction to the topic

Vedic mathematics is the ancient system of mathematics drawn from the Vedas. The Vedas are ancient texts that encompass a broad spectrum of knowledge, covering all aspects of life. These include the sutras (verses) pertaining to mathematics. In the early 20th century, Swami Shri Bharati Krishna Tirthaji Maharaja claimed to have rediscovered a collection of 16 ancient mathematical sutras from the Vedas and published it in a book titled *Vedic Mathematics*. Historians, however, do not agree on whether or not these were truly a part of the Vedic tradition. If these sutras dated back to the Vedic era, they would be a part of an oral rather than a written tradition. Despite controversies, they are a novel and useful approach to computation; they are flexible in application and easy to remember. They can often be applied in the algebraic contexts and in simple arithmetic as well.

TYPES OF CALCULATIONS

The different types of calculations that form the basis of mathematics are:

1. Addition
2. Subtraction
3. Multiplication
4. Division
5. Ratio comparison
6. Percentage calculations

When we talk about the techniques of calculations, addition and subtraction can simply not have any short-cuts. Since addition and subtraction are the basic units, we can at best only approximate the values.

In case of multiplication, the techniques of vedic maths can be used.

Ratio comparison techniques are discussed in the chapter on ratio, proportion and variation and the percentage calculations in the chapter on percentage.

VEDIC MATH TECHNIQUES IN MULTIPLICATION

There are several techniques of multiplication. We will discuss them one by one.

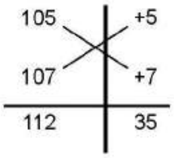
Method 1: Base Method

In this method, one number is used as a base; for example, 10, 50, 100 etc. The number that is closer to both the numbers should be taken as the base.

Example 1 105×107

Solution In this case, both the numbers are close to 100, so 100 is taken as the base. We will now find the deficit/surplus from the base.

Base = 100, Surplus = 5 and 7



The right part (after slash) ⇒ this is the product of the surplus. Since the base = 100 and the surpluses are 5 and 7, the product would be $5 \times 7 = 35$.

The left part (before slash) ⇒ It could be either of the numbers plus the surplus of the other multiplicand. Hence, the left part would be either (105 + 7) or (107 + 5) = 112 (both will always be the same) i.e., 112.

The left part would be equivalent to the number × v100. In this case, $112 \times 100 = 11200$

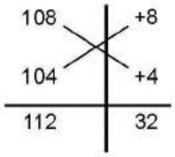
Now, we add both the right part and the left part $= 11200 + 35 = 11235$

Hence, the result of the multiplication would be 11235.

To know more about Vedic maths, go to *www.pims.math.ca/pi/issue6/page15-16.pdf*

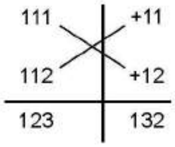
Example 2 108×104

Solution



Example 3 111×112

Solution

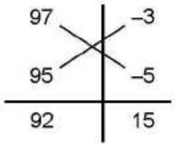


Here, it is $11 \times 12 = 132$. But it can have only two digits. Thus, 1 will be carried over to the left part and the right part will be only 32. Left part will be either 111 + 12 + 1 (1 for the carry over) or (112 + 11 + 1), i.e., 124. So, the result will be 12432.

For 102×104, the answer will be 10608. Please note that the right part will be 08 and not simply 8.

Example 4 97×95

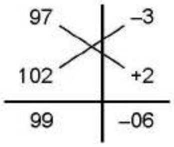
Solution



Base = 100, Deficit = $97 - 100 = -3$ and $95 - 100 =- 5$

Example 5 97×102

Solution



96×108

Base = 100, Deficit = $97 - 100 = -3$,

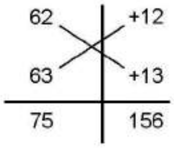
Surplus = $102 - 100 = 2$

The right part will now be $(-3) \times 2$, i.e., −06. To take care of the negative, we will borrow 1 from the left part, which is equivalent to borrowing 100 (because we are borrowing from the hundred digits of the left part). Thus, this part will be $100 - 06 = 94$.

So, the answer = 98 94

Example 6 62×63

Solution



We will assume here the base as 50 owing to the fact that the numbers are close to 50.

Base = 50, Surplus = 62 – 50 = 12,

Surplus = 63 – 50 = 13

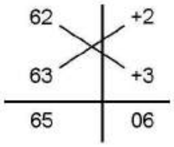
The left hand side = 156 and the right hand side = 75. Since the base is assumed to be equal to 50, so the value of the right hand side = 75 × 50 = 3750. Besides, only two digits can be there on the right hand side, so 1(100) is transfered to the left hand side leaving 56 only on the left hand side.

So, the value on the right hand side = 3750 + 100 = 3850

Value on the left hand side = 56

Net value = 3850 + 56 = 3906

Let us do the same multiplication by assuming 60 as the base.


62
+2
63
+3
65
06

Base = 60, Surplus = 62 – 60 = 2, Surplus = 63 – 60 = 3

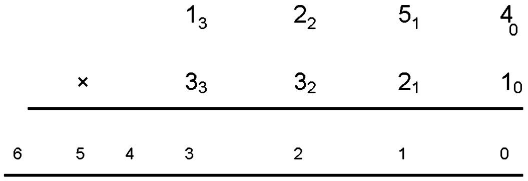
Since the base is assumed to be equal to 60, the value of the right hand side = 65 × 60 = 130 × 30 = 3900

So, net value = 3906

Method 2: Place Value Method

In this method of multiplication, every digit is assigned a place value and the multiplication is done by equating the place values of multiplicands with the place value of the product.

Let us see this with some examples:

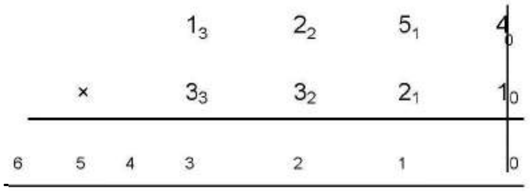

1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

Conventionally, the unit digit is assigned a place value 0, the tens place digit is assigned a place value 1, the hundreds place digit is assigned a place value 2, the thousands place digits is assigned a place value 4 and so on.

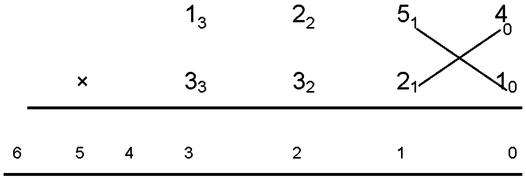
This multiplication is a two-step process.

Step 1 Add the place values of the digits of the numbers given (1254 × 3321) to obtain the place value of the digits of the product.

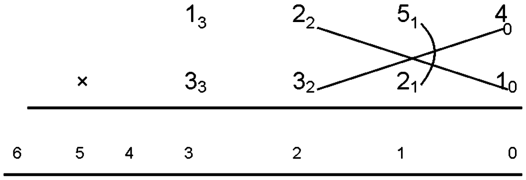
For example, using the place values of the multiplicands, i.e., using 0, 1, 2 and 3 of the number 1254 and the same place values 0, 1, 2 and 3 of the another multiplicand 3321, we can get 0 place value in the product in just one way, i.e., adding 0 and 0.


1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

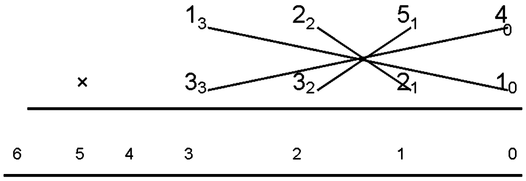
Place value 1 in the product can be obtained in two ways.


1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

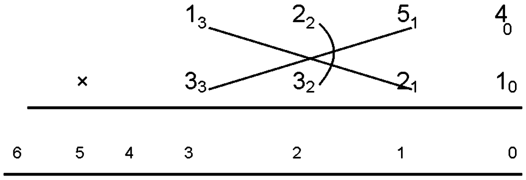
Place value 2 can be obtained in three ways.


1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

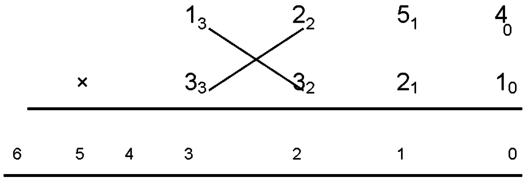
Place value 3 can be obtained in four ways.


1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

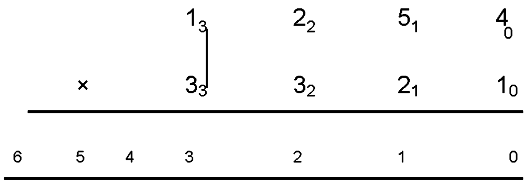
Place value 4 can be obtained in three ways.


1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

Place value 5 can be obtained in two ways.

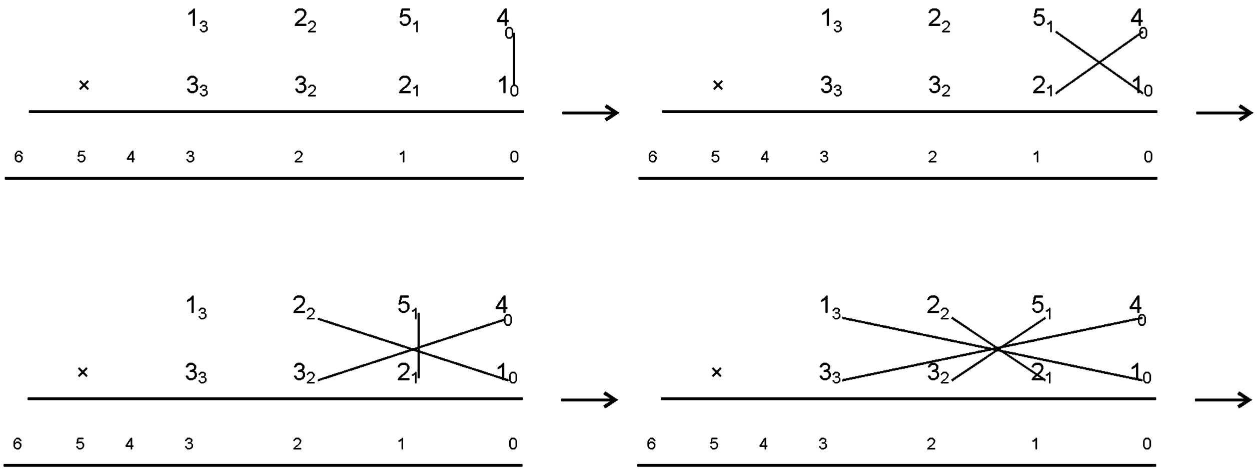

1₃ 2₂ 5₁ 4₀
× 3₃ 3₂ 2₁ 1₀
6 5 4 3 2 1 0

Place value 6 can be obtained in one way.



And this is the maximum place value that can be obtained.

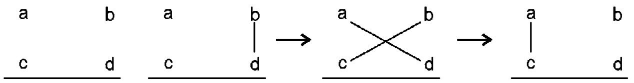
Step 2 Multiply the corresponding numbers one by one.



In this manner, we can find the product = 4164534

This method is most useful in case of the multiplications of 2 digits × 2 digits or 2 digits × 3 digits or 3 digits × 3 digits multiplication.

E.g., *ab* × *cd*



Similarly, we can have a proper mechanism of multiplication of 2 digits × 3 digits or 3 digits × 3 digits also using the place value method.

Method 3: Units Digit Method

This method of multiplication uses the sum of the units digit, provided all the other digits on the left hand side of the unit digit are the same.

Example 7 75 × 75

Solution

1.0+7	5
7	5
56	25

The sum of the units digit = 10, so we add 1.0 in one of the digits on the left hand side.

Example 8 62 × 63

Solution

0.5+6	3
6	2
39	06

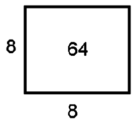
The sum of the units digit = 5, so we add 0.5 in one of the digits on the left hand side.

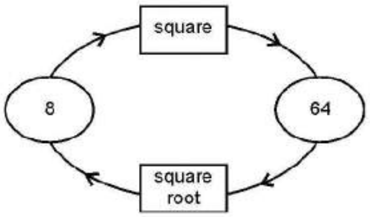
SQUARING

A **square number**, also called a **perfect square**, is an integer that can be written as the square of some other integer. In other words, a number whose square root is an integer is known as the square number of a perfect square.

For example, 9 is a square number since it can be written as 3 × 3.

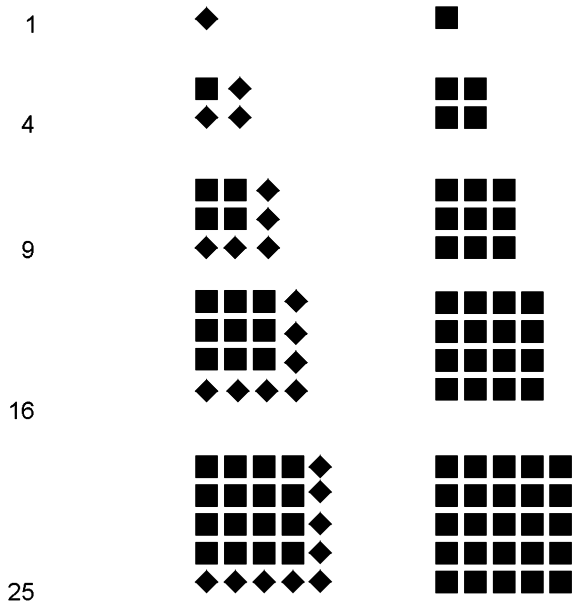
This can be seen through the following flow-chart also.





Properties of a Square Number

1. The number N is a square number if it can be arranged as N points in a square:



Thus, it can be deduced that the formula for the nth square number is n^2. This is also equal to the sum of the first n odd numbers $n^2 = \sum_{k=1}^{n}(2k-1)$, as can be seen in the above figure, where a square results from the previous one by adding an odd number of points (marked as '◆'). For example, $5^2 = 25 = 1+3+5+7+9$.

It should be noted that the square of any number can be represented as the sum $1 + 1 + 2 + 2 + \ldots + n - 1 + n - 1 + n$. For instance, the square of 4 or 4^2 is equal to $1 + 1 + 2 + 2 + 3 + 3 + 4 = 16$. This is the result of adding a column and row of thickness 1 to the square graph of three. This can also be useful for finding the square of a big number quickly. For instance, the square of $52 = 50^2 + 50 + 51 + 51 + 52 = 2500 + 204 = 2704$.

2. A square number can only end with digits 00, 1, 4, 6, 9, or 25 in base 10, as follows:
3. If the last digit of a number is 0, its square ends in 00 and the preceding digits must also form a square.
4. If the last digit of a number is 1 or 9, its square ends in 1 and the number formed by its preceding digits must be divisible by four.
5. If the last digit of a number is 2 or 8, its square ends in 4 and the preceding digit must be even.
6. If the last digit of a number is 3 or 7, its square ends in 9 and the number formed by its preceding digits must be divisible by four.
7. If the last digit of a number is 4 or 6, its square ends in 6 and the preceding digit must be odd.
8. If the last digit of a number is 5, its square ends in 25 and the preceding digits (other than 25) must be 0, 2, 06, or 56.
9. A square number cannot be a perfect number. (If the sum of all the factors of a number excluidng the number itself is equal to the number, then the number is known to be a perfect number.)
10. The digital sum of any perfect square can be only 0, 1, 4, 9, 7. (Digital sum of any number is obtained by adding the digits of the number until we get a single digit. Digital sum of $385 = 3 + 8 + 5 = 1 + 6 = 7$)

An easy way to find the squares is to find two numbers which have a mean of it. This can be seen through the following example:

To find the square of 21, take 20 and 22, then multiply the two numbers together and add the square of the distance from the mean: $22 \times 20 = 440 + 1^2 = 441$. Here, we have used the following formula

$$(x - y)(x + y) = x^2 - y^2$$

known as the difference of two squares. Thus,
$(21-1)(21 + 1) = 21^2 - 1^2 = 440$.

Odd and Even Square Numbers

Squares of even numbers are even, since $(2n)^2 = 4n^2$.

Squares of odd numbers are odd, since $(2n + 1)^2 = 4(n^2 + n) + 1$.

Hence, we can infer that the square roots of even square numbers are even, and square roots of odd square numbers are odd.

Methods of Squaring

As we have seen in the case of multiplication, there are several methods for squaring also. Let us see the methods one by one.

Method 1: Base 10 Method

Understand it by taking few examples:

- Let us find out the square of 9. Since 9 is 1 less than 10, decrease it still further to 8. This is the left side of our answer.
- On the right hand side put the square of the deficiency that is 1^2. Hence, the answer is 81.
- Similarly, $8^2 = 64$, $7^2 = 49$.
- For numbers above 10, instead of looking at the deficit we look at the surplus. For example,
  $11^2 = (11 + 1).\ 10 + 1^2 = 121$
  $12^2 = (12 + 2).\ 10 + 2^2 = 144$
  $14^2 = (14 + 4).\ 10 + 4^2 = 18\ .\ 10 + 16 = 196$
  and so on.

This is based on the identities $(a + b)(a - b) = a^2 - b^2$ and $(a + b)^2 = a^2 + 2ab + b^2$.

We can be use this method to find the squares of any number, but after a certain stage, this method loses its efficiency.

Method 2: Base 50*n* Method here, (*n* is any natural number)

This method is nothing but the application of $(a + b)^2 = a^2 + 2ab + b^2$.

This can be seen in the following example:

Example 9 Find the square of 62.

Solution Because this number is close to 50, we will assume 50 as the base.

$(62)^2 = (50 + 12)^2 = (50)^2 + 2 \times 50 \times 12 + (12)^2 = 2500 + 1200 + 144$

To make it self explanatory a special method of writring is used.

$(62)^2$ = [100's in (Base)]^2 + Surplus | Surplus^2

= 25 + 12 | 144 = 38 | 44 [Number before the bar on its left hand side is number of hundreds and on its right hand side are last two digits of the number.]

$(68)^2 = 25 + 18 \mid 324 = 46 \mid 24$

$(76)^2 = 25 + 26 \mid 676 = 57 \mid 76$

$(42)^2 = 25 - 8 \mid 64 = 17 \mid 24$ $[(a - b)^2 = a^2 - 2ab + b^2]$

Example 10 Find the square of 113.

Solution Since this number is closer to 100, we will take 100 as the base.

$(112)^2 = (100 + 12)^2 = (100)^2 + 2 \times 100 \times 12 + (12)^2 = 10000 + 2 \times 1200 + 144$

$(112)^2$ = [100's in (Base)]^2 + 2 × Surplus | Surplus^2

$= 100 + 2 \times 12 \mid 12^2 = 125 \mid 44$

Alternatively, we can multiply it directly using base value method.

Had this been 162, we would have multiplied 3 in surplus before adding it into [100's in (Base)]^2 because assumed base here is 150.

$(162)^2$ = [100's in (Base)]^2 + 3 × Surplus | Surplus^2

$= 225 + 3 \times 12 \mid 12^2 = 262 \mid 44$

Method 3 : 10^n Method

This method is applied when the number is close to 10^n.

With base as 10^n, find the surplus or deficit (×)

Again answer can be arrived at in two parts

$(B + 2x) \mid x^2$

The right-hand part will consist of *n* digits. Add leading zeros or carry forward the extra to satisfy this condition.

$108^2 = (100 + 2 \times 8) \mid 8^2 = 116 \mid 64 = 11664$

$102^2 = (100 + 12 \times 2) \mid 2^2 = 104 \mid 04 = 10404$

$93^2 = (100 - 2 \times 7) \mid (-7)^2 = 86 \mid 49 \Rightarrow 8649$

$1006^2 = (1000 + 2 \times 6) \mid 6^2 = 10|12 \mid 036 = 1012036$

The right-hand part will consist of 2 digits. Add leading zeros or carry forward the extra to satisfy this condition.

$63^2 = (25 + 13) \mid 13^2 = 38 \mid 169 = 3969$

$38^2 = (25 - 12) + (-12)^2 = 13 \mid 144 = 1444$

Square Mirrors

$14^2 + 87^2 = 41^2 + 78^2$

$15^2 + 75^2 = 51^2 + 57^2$

$17^2 + 84^2 = 71^2 + 48^2$

$26^2 + 97^2 = 62^2 + 79^2$

$27^2 + 96^2 = 72^2 + 69^2$

Some Special Cases

1. Numbers ending with 5

If a number is in the form of n5, the square of it is n (n+1) | 25

E.g., $45^2 = 4 \times 5 \mid 25 = 2025$

$135^2 = 13 \times 14 \mid 25 = 18225$

This is nothing but the application of the multiplication method using the sum of units digits.

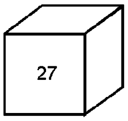
We can use this method to find out the squares fractions like $1\frac{1}{2}, 2\frac{1}{2}, 3\frac{1}{2}$ ... also.

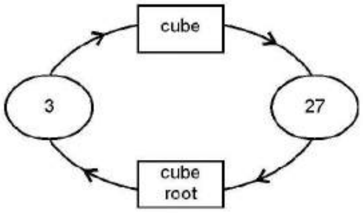
Process: Multiply the integral portion by the next higher integer and add $\frac{1}{4}$.

For example, $\left(6\frac{1}{2}\right) = 6\times7+\frac{1}{4} = 42\frac{1}{4}$

CUBING

A number whose cube root is an integer is called a perfect cube.





Properties of a Cube

1. The sum of the cubes of any number of consecutive integers starting with 1 is the square of some integer. (For example, $1^3 + 2^3 = 9 = 3^2$, $1^3 + 2^3 + 3^3 = 36 = 6^2$, etc.)
2. Unit digit of any cube can be any digit from 0-9.

Methods of Cubing

We can find the cube of any number close to a power of 10 say 10^n with base = 10^n by finding the surplus or the defict (x). The answer will be obtained in three parts. $B + 3x \mid 3 . x^2 \mid x^3$

The left two parts will have n digits.

104^3

Base B = 100 and surplus = $x = 4$

$(100 + 3 \times 4)|3 \times 4^2|4^3 = 112|48|64 = 1124864$

109^3

Base B = 100 and $x = 9$

$(100 + 3 \times 9)|3 \times 9^2|9^3 = 127|243|729 = 1295029$

98^3

Base B = 100 and $x = -2$

$(100 - 3 \times 2) \mid 3 \times (-2)^2 \mid (-2)^3 = 94 \mid 12 \mid -8 = 94 \mid 11 \mid 100 - 8 = 941192$

VEDIC MATHS TECHNIQUES IN ALGEBRA

1. If one is in ratio, the other one is zero

This formula is often used to solve simple simultaneous equations which may involve big numbers. But these equations in special cases can be visually solved because of a certain ratio between the coefficients. Consider the following example:

$6x + 7y = 8$

$19x + 14y = 16$

Here, the ratio of coefficients of y is the same as that of the constant terms. Therefore, the "other" is zero, i.e., $x = 0$. Hence, the solution of the equations is $x = 0$ and $y = 8/7$.

Alternatively,

$19x + 14y = 16$ is equivalent to. $(19/2)x + 7y = 8$.

Thus, x has to be zero and no ratio is needed, just divide by 2!

Note that it would not work if both had been "in ratio":

$6x + 7y = 8$

$12x + 14y = 16$

This formula is easily applicable to more general cases with any number of variables. For instance,

$ax + by + cz = a$

$bx + cy + az = b$

$cx + ay + bz = c$

which yields $x = 1, y = 0, z = 0$.

2. When samuccaya is the same, that samuccaya is zero

Consider the following symbols: N_1 – Numerator 1, N_2 – Numerator 2, D_1 – Denominator 1, D_2 – Denominator 2 and so on.

This formula is useful for solving equations that can be solved visually. The word "samuccaya" has various meanings in different applications. For instance, it may mean a term, which occurs as a common factor in all the terms concerned. For example, an equation "$12x + 3x = 4x + 5x$". Since "x" occurs as a common factor in all the terms, therefore, $x = 0$ is the solution. Alternatively, samuccaya is the product of independent terms. For instance, in $(x + 7)(x + 9) = (x + 3)(x + 21)$, the samuccaya is $7 \times 9 = 3 \times 21$, therefore, $x = 0$ is the solution. It is also the sum of the denominators of two fractions having the same numerical numerator, for example:

$1/(2x - 1) + 1/(3x - 1) = 0$ means $5x - 2 = 0$.

The more commonly used meaning is "combination" or total. For instance, if the sum of the numerators and the sum of denominators are the same then that sum is zero. Therefore,

$$\frac{2x+9}{2x+7} = \frac{2x+7}{2x+9}$$

Therefore, $4x + 16 = 0$ or $x = -4$.

This meaning ("total") can also be applied in solving the quadratic equations. The total meaning not only imply sum but also subtraction. For instance, when given $N_1D_1 = N_2/D_2$, if $N_1 + N_2 = D_1 + D_2$ (as shown earlier) then this sum is zero. Mental cross multiplication reveals that the resulting equation is quadratic (the coefficients of x^2 are different on the two sides). So, if $N_1 - D_1 = N_2 - D_2$ then that samuccaya is also zero. This yields the other root of a quadratic equation.

The interpretation of "total" is also applied in multi-term RHS and LHS. For instance, consider

$$\frac{1}{x-7} + \frac{1}{x-9} = \frac{1}{x-6} + \frac{1}{x-10}$$

Here, $D_1 + D_2 = D_3 + D_4 = 2x - 16$. Thus $x = 8$.

There are several other cases where samuccaya can be applied with great versatility. For instance, "apparently cubic" or "biquadratic" equations can be easily solved as shown below:

$$(x - 3)^2 + (x - 9)^3 = 2(x - 6)^3.$$

Note that $x - 3 + x - 9 = 2(x - 6)$. Therefore, $(x - 6) = 0$ or $x = 6$.

Consider

$$\frac{(x+3)^3}{(x+5)^3} = \frac{x+1}{x+7}$$

Observe: $N_1 + D_1 = N_2 + D_2 = 2x + 8$.

Therefore, $x = -4$.

2

CHAPTER

Number System

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Numbers and their different types
- Definitions and properties of those numbers
- Concepts attached to these numbers
- Kinds of questions which are asked in the CAT
- Methods of solving

Introduction to the topic

With respect to preparation for the CAT, number system has been seen to be one of the most important topics in the quantitative appitude (QA) paper. During the last 15 years, it has been observed that the QA paper has had 20%–30% questions from the chapter on number system. This chapter will give you a clear understanding of the definitions and the concepts, and help you develop a keen insight about numbers and their properties.

TYPES OF QUESTIONS

Questions asked from this topic are of the following two kinds:

1. Based on definitions or properties of properties of numbers
2. Based on concepts.

1. Based on definitions Questions of this kind are based on definitions and properties of different kinds of numbers, involving formulae and some very basic calculations.

2. Based on concepts The other kind of questions are those that are based on the following concepts:

- i. LCM and HCF
- ii. Divisibility
- iii. Divisibility rules (for base other than 10)
- iv. Number of divisors
- v. Number of exponents
- vi. Remainders
- vii. Base system
- viii. Units digit
- ix. Tens digit
- x. Pigeon-hole principle

CLASSIFICATION OF NUMBERS/ INTEGERS

Natural Numbers

Natural numbers are counting numbers, i.e., the numbers which we use to count. For example, 1, 2, 3,... The lowest natural number is 1.

Whole Numbers

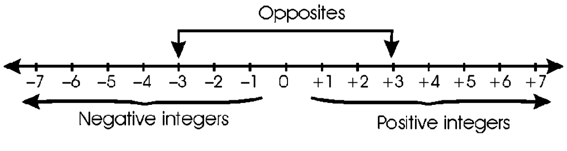
When zero is included in the list of natural numbers, the numbers are known as whole numbers. For example, 0, 1, 2,... The lowest whole number is 0.

Integers

Integers are whole numbers, the negative of whole numbers and zero. For example, 43434235, 28, 2, 0, −28, and −3030 are integers, but numbers like 1/2, 4.00032, 2.5, Pi, and −9.90 are not whole numbers.

Number Line

The number line is used to represent the set of real numbers. A representation of the number line is given below:



Properties of Number Line

- The number line goes on till infinity in both directions. This is indicated by the arrows.
- Integers greater than zero are called positive integers. These numbers are to the right of zero on the number line.
- Integers less than zero are called negative integers. These numbers are to the left of zero on the number line.
- The integer zero is neutral. It is neither positive nor negative.
- The sign of an integer is either positive (+) or negative (−), except zero, which has no sign.
- Two integers are the opposites if each of the two are at the same distance from zero, but on the opposite side of the number line. One will have a positive sign while the other a negative sign. In the number line above, +3 and −3 are labelled as opposites. In other words, the whole negative number scale looks like a mirror image of the positive number scale, with a number like −15 being at the same distance away from 0 as the number 15 is.
- The number half-way between −1 and −2 is −1.5; just as the number halfway between 1 and 2 is 1.5.
- We represent positive numbers without using a +ve sign. For example, we would write 29.1 instead of +29.1. But when we talk of negative numbers, the sign must be there.

Prime Numbers and Composite Numbers

There are two types of natural numbers, prime and composite, except 1.

Prime numbers

The numbers which are divisible by 1 and itself only are known as prime numbers.

In other words, we can say that all the numbers which have only two factors are known as prime numbers.

Prime numbers can also be seen as the building blocks. We combine two or more than two prime numbers, same or distinct, to create numbers bigger than these prime numbers. For example, 3 × 2 = 6.

To know more about primes, go to *www.primes.utm.edu/largest.html*

List of all the prime numbers less than 1000

2	3	5	7	11	13	17	19
23	29	31	37	43	47	53	59
61	67	73	79	83	89	97	101
103	107	109	113	127	131	137	139
149	151	157	163	167	173	179	181
193	197	199	211	223	227	229	233
239	241	251	257	263	269	271	277
281	283	293	307	311	313	317	331
337	347	349	353	359	367	373	379
383	389	397	401	409	419	421	431
433	439	443	449	457	461	463	467
479	487	491	499	503	509	521	523
541	547	557	563	569	571	577	587
593	599	601	607	613	617	619	631
641	643	647	653	659	661	673	677
683	691	701	709	719	727	733	739
743	751	757	761	769	773	787	797
809	811	821	823	827	829	839	853
857	859	863	877	881	883	887	907
911	919	929	937	941	947	953	967
971	977	983	991	997			

Number of prime numbers for every 100 numbers

Nos. from-to	1-100	101-200	201-300	301-400	401-500	501-600	601-700	701-800	801-900	901-1000
No. of primes	25	21	16	16	17	14	16	14	15	14

Largest prime number till date and the history of prime number The largest known prime number, today, is 7,816,230-digit prime number, $2^{25964951}-1$, found in early 2005. But how big have the "largest known primes" been historically and when can we see the first billion-digit prime number?

Table 1.1 **Records before the advent of electronic computers**

Number	*Digits*	*Year*	*Prover*	*Method*
$2^{17}-1$	6	1588	Cataldi	Trial division
$2^{19}-1$	6	1588	Cataldi	Trial division
$2^{31}-1$	10	1772	Euler	Trial division
$(2^{59}-1)/179951$	13	1867	Landry	Trial division
$2^{127}-1$	39	1876	Lucas	Lucas sequences
$(2^{148}+1)/17$	44	1951	Ferrier	Proth's theorem

The prime number found by Lucas in 1876 was accepted as the largest prime number till 1951. In 1951, Ferrier used a mechanical desk calculator and techniques based on partial inverses of Fermat's little theorem (see the pages on remainder theorem) to slightly better this record by finding a 44-digit prime.

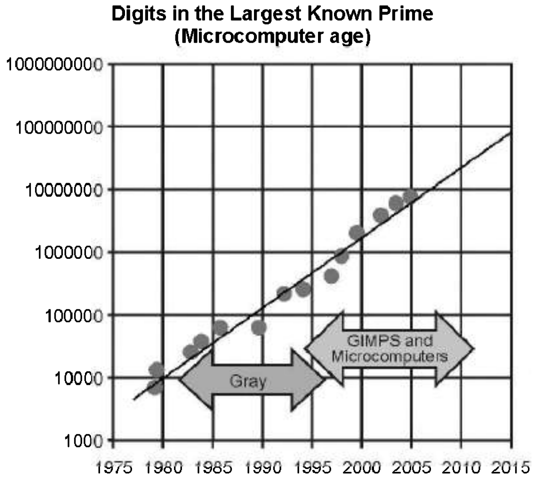
In 1951, Ferrier found the prime $(2^{148} + 1)/17 = 20988936657440586486151264256610222593863921$. However, this record was very short-lived. In the same year, the advent of electronic computers helped in finding a bigger prime number. Miller and Wheeler began the electronic computing age by finding several primes as well as the new 79-digit record: $2^{127}-1$.

During this period, everybody was working hard to find out the primes with the help of computers. Records were broken at a never-before pace. Sometimes, within a day.

When will we have a one-billion digit prime?

Using the regression line given above, we can expect

- 10,000,000-digit prime by 2007,
- a 100,000,000-digit prime by 2016, and
- a 1,000,000,000-digit prime by early 2027.



Formula for prime numbers

Till now, all attempts made to arrive at a single formula to represent all prime numbers have proved to be fruitless. It is all because there is no symmetricity between the differences among the prime numbers. Sometimes, two consecutive prime numbers differ by 2, sometimes by 4, and sometimes even by 10,000 or more.

However, there are some standard notations, which give us a limited number of prime numbers.

$N^2 + N + 41 \rightarrow$ For all the values of N from –39 to +39, this expression gives us a prime number.

$N^2 + N + 17$ is again a similar example.

Properties of Prime Numbers

1. All prime numbers end in 1, 3, 7, or 9, except for 2 and 5. (Numbers ending in 0, 2, 4, 6 or 8 can be divided by 2 and numbers ending in 5 can be divided by 5)
2. If p is a prime number and p divides a product ab of integers, then p divides a or p divides b.
3. If p is prime and a is any integer, then $a^p - a$ is divisible by p. This is knows as **Fermat's Little theorem.**
4. If p is a prime number other than 2 and 5, $1/p$ is always a recurring decimal, with a period of $p-1$ or a divisor of $p-1$.
5. An integer $p > 1$ is prime if and only if the factorial $(p-1)! + 1$ is divisible by p. This is known as **Wilson's theorem.**
   Conversely, an integer $n > 4$ is composite if and only if $(n-1)!$ is divisible by n.
6. If n is a positive integer greater than 1, then there is always a prime number p with $n < p < 2n$.
7. All the prime numbers $P > 3$ give a remainder 1 or 5 when divided by 6; that is, all the prime numbers > 3 will be of the format $6N \pm 1$. But vice versa is not always true.
8. There is only one set of three prime numbers with a gap of 2 between two prime numbers and that set is 3, 5 and 7.

Composite numbers

A number is composite if it is the product of two or more than two distinct or same prime numbers. For example, → 4, 6, 8,…

$4 = 2^2$
$6 = 2^1 \times 3^1$
Lowest composite number is 4.

Even and Odd Numbers

Suppose N is an integer. If there exists an integer P such that $N = 2P + 1$, then N is an *odd number*. If there exists an integer P such that $N = 2P$, then N is an *even number*.

Putting in a simple language, even numbers are those integers which are divisible by 2 and odd numbers are those integers which are not divisible by 2.

Even and odd numbers can be positive as well as negative also.

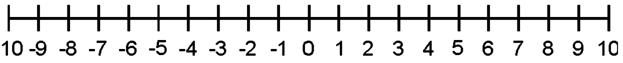
In other words, if x is any integer (even or not), then $2x$ will be an even integer, because it is a multiple of 2. Also x raised to any positive integer power will be an even number, so x^2, x^3, x^4 etc. will be even numbers.

Any integer that is not a multiple of 2 is called an *odd* number. For instance, −1, 3, 6883 and −8147 are all odd numbers. Any odd number raised to a positive integer power will also be an odd number, so if x is an odd number, then so will x^2, x^3, x^4 etc. be odd numbers.

The concept of even and odd numbers are most easily understood in the binary base. The Above definition simply states that even numbers end with a 0, and odd numbers end with a 1.

Comparing Integers

We can compare two different integers by looking at their positions on the number line. For any two different places on the number line, the integer on the right is greater than the integer on the left. Note that every positive integer is greater than any negative integer.



E.g., $9 > 4$, $6 > -9$, $-2 > -8$, and $0 > -5$
$-2 < 1$, $8 < 11$, $-7 < -5$, and $-10 < 0$

Points to Remember

- 1 is neither prime nor composite.
- 0 is neither positive nor negative.

Example 1 Two of a, b, c and d are even and two are odd, not necessarily in order. Which of the following is definitely even? **(CAT 1997)**

(a) $a + b + c - 2d$
(b) $a + 2b - c$
(c) $a + b - c + d$
(d) $2a + b + c - d$

Solution Since we do not know which two are even and which two are odd, we will have to do a bit of hit-and- trial to solve this problem, with the help of options.

In option (a), if a and b are even, and c and d are odd, then this will lead us to odd number.

In option (b), if a and b are even, and c is odd, then this will lead us to odd number.

In option (c), whatever is the value of a, b, c and d, it is always going to be an even number.

In option (d), if a and b are odd, and c and d are even, then this will lead us to odd number.

Explanation Whatever kind of calculation we do with two even and two odd numbers, we will always get an even result.

So, the answer is option (c).

Example 2 If N, N + 2 and N + 4 are prime numbers, then the number of possible solutions for N is are: **(CAT 2003)**

(a) 1 (b) 2 (c) 3 (d) None of these

Solution There is only one triplet of prime numbers where difference between any two prime number is 2, that is 3, 5 and 7. So, N = 3 is the only solution.

Hence, the answer is (a).

Proof of above example We know that prime numbers are of the form 6M ± 1 (except 2 and 3). Now if N is of the format 6M + 1, then N + 2 will of 6M + 3 format and N + 4 will be of 6M + 5 format. Out of these three numbers, since N + 2 is of 6M + 3 format, it will be divisible by 3.

Similarly, if N is of the format 6M – 1, then N + 2 will of 6M + 1 format and N + 4 will be of 6m + 3 format. Out of these three numbers, since N + 4 is of 6M + 3 format, it will be divisible by 3.

In both the cases, we find that one number out of given three number is divisible by 3. In the example given above (3, 5 and 7), one of the given three numbers is divisible by 3.

Example 3 Let x and y be positive integers such that x is prime and y is composite. Then which of the following is true? **(CAT 2003)**

(a) $y - x$ cannot be an even integer
(b) xy cannot be an even integer
(c) $\frac{x+y}{x}$ cannot be an even integer
(d) None of these

Solution Eliminating the options,

To eliminate option (a), If $y = 4$ and $x = 2$, then $y - x$ can be even.

To eliminate option (b), If $y = 4$ and $x = 2$, then yx can be even.

To eliminate option (c), If $y = 6$ and $x = 2$, then it can be even.

So, the answer is option (d).

QUESTIONS BASED ON CONCEPTS

In this section, we will discuss the concepts and questions based upon them.

LCM and HCF

Meaning of LCM

A common multiple is a number that is a multiple of two or more than two numbers. The common multiples of 3 and 4 are 12, 24,…

The least common multiple (LCM) of two numbers is the smallest positive number that is a multiple of both.

Multiples of 3 → 3, 6, 9, 12, 15, 18, 21, 24,…

Multiples of 4 → 4, 8, 12, 16, 20, 24, 28,…

So, the LCM of 3 and 4 will be 12, which is the lowest common multiple of 3 and 4.

First of all, the basic question is what kind of numbers we can use for LCM.

Let us see this through an example— LCM of 10, 20 and 25 is 100. It means that 100 is the lowest number which is divisible by all these three numbers.

But cannot the LCM be (–100)? Since (–100) is lower than 100 and divisible by each of 10, 20 and 25. Or, it can be zero also.

Or, What will be the LCM of (–10) and 20?

Will it be (–20) or (–200) or (–2000) or the smallest of all the numbers, i.e., $-\propto$

Answer to all these questions is very simple. LCM is a concept defined only for positive numbers, be it an integer or a fraction i.e., LCM is not defined for negative numbers or zero.

Now, we will define a different method of finding out the LCM of two or more than two positive integers.

Process to find out LCM

Step 1 Factorize all the numbers into their prime factors.

Step 2 Collect all the distinct factors.

Step 3 Raise each factor to its maximum available power and multiply.

Example 4 Find LCM of 10, 20, 25.

Solution *Step 1* $10 = 2^1 \times 5^1$
$20 = 2^2 \times 5^1$
$25 = 5^2$

Step 2 2, 5

Step 3 $2^2 \times 5^2 = 100$

The biggest advantage of using this method lies in the fact that we can find out the LCM of any number of numbers in a straight line without using the conventional method. It can be understood in the following way with the previous example.

First of all, find out the LCM of 10, 20 = 20, and now the LCM of 20 and 25 = 100 (For this you will have to check which factor of 25 is not present in 20 and then multiply by this factor. Since 25 has 5^2 and 20 has 5^1 only, so we will multiply 20 by 5.)

Example 5 Find LCM of 35, 45, 55.

Solution First of all, find out the LCM of 35 and 45.

Now $35 = 5^1 \times 7^1$ and $45 = 3^2 \times 5^1$.

So, it can be observed here that 35 does not have 3^2 in it, so we will multiply 35 by 3^2.

So, LCM of 35 and 45 = 35×3^2. (You can start with 45 also to find out about the missing factors of 35 in 45.)

Now, we will find out the LCM of 35×3^2 and 55

$55 = 5^1 \times 11^1$

Now, 11^1 is not there with 35×3^2. So, we will multiply 35×3^2 with 11^1.

So, finally the LCM = $35 \times 3^2 \times 11^1 = 3465$.

Relationship between factors of a number and its LCM We can use the factors of a number in several ways to produce that under as the LCM.

Example 6 N = 72. How many sets of two values (a, b) are there for which the LCM (a, b) is 72?

Solution N = 72 = $2^3 \times 3^2$

Values of a and b have to be chosen in such a way that both 2^3 and 3^2 are contained in either a or b.

Possible sets are (72 ,1) (72, 2) (72, 3) (72, 4) (72, 6) … (72, 36), (72, 72). These are a total of 12 sets.

The other possible sets are (9, 8), (9, 24), (8, 18), (8, 36),…

These are a total of six sets.

So, the total number of possible sets are 12 + 6 = 18 sets

> In general, if $N = a^p \times b^q \times c^r \times \ldots$, then the set of values (x, y) such that LCM (x, y) is N can be summed up as:
> Number of sets (x, y) =
> $$\frac{(2p+1)(2q+1)(2r+1)\ldots-1}{2}+1$$

Meaning of HCF

Factors are those positive integral values of a number, which can divide that number. HCF, which is known as GCD (Greatest Common Divisor) also, is the highest value which can divide the given numbers.

Factors of 20: 1, 2, 4, 5, 10, 20.
Factors of 30: 1, 2, 3, 5, 6, 10, 15, 30.
So, 10 will be the HCF of 20 and 30.

Process to find out HCF

Step 1 Factorize all the numbers into their prime factors.

Step 2 Collect all the common factors.

Step 3 Raise each factor to its minimum available power and multiply.

Example 7 Find HCF of 100, 200 and 250.

Solution *Step 1* $100 = 2^2 \times 5^2$
$200 = 2^3 \times 5^2$
$250 = 5^3 \times 2^1$

Step 2 2, 5

Step 3 $2^1 \times 5^2 = 50$

Alternatively, to find out HCF of numbers like 100, 200 and 250, it is required to observe the quantity which can be taken out common from these numbers. By writing these numbers as $(100x + 200y + 250z)$, it can be very easily observed that 50 is common out of these numbers.

Summarizing LCM and HCF

It is very essential to understand the mechanism of finding out LCM and HCF. We can simply understand the mechanism to find out lowest common multiple and highest common factor through this example.

Example 8 Find out LCM and HCF of 16, 12, 24.

Solution

No.	Multiples	Factors
16	16, 32, 48, 64, 80, 96, 112, 128,…	1, 2, 4, 8, 16
12	12, 24, 36, 48, 60, 72, 84, 96, 108,…	1, 2, 3, 4, 6, 12
24	24, 48, 72, 96, 120, 144, 168, 192,…	1, 2, 3, 4, 6, 8, 12, 24
	Common Multiple	Common Factor
	48, 96, 144	1, 2, 4
	Lowest Common Multiple	Highest Common Factor
	48	4

Standard Formulae

1. LCM × HCF = Product of two numbers.
   This formula can be applied only in case of two numbers. However, if the numbers are relatively prime to each other (i.e., HCF of numbers = 1), then this formula can be applied for any number of numbers.
2. LCM of fractions = LCM of numerator of all the fractions/HCF of denominator of fractions.
3. HCF of fractions = HCF of numerator of all the fractions/LCM of denominator of fractions.
4. HCF of (sum of two numbers and their LCM) = HCF of numbers.

Example 9 HCF of two natural numbers A and B is 120 and their product = 10,00 0. How many set of values of A and B is/are possible?

Solution HCF (A, B) = 120 ⇒ 120 is a common factor of both the numbers (120 being the HCF). Hence 120 is present in both the numbers. So the minimum product of A and B = 120 × 120 = 14400. Hence no set of A and B are possible satisfying the conditions.

QUESTIONS BASED ON STANDARD APPLICATION OF LCM AND HCF

Case 1 Time and work

Example 10 Tatto can do a work in 10 days and Tappo can do the same work in 12 days. How many days will it take if both of them start working together?

Solution Let us assume total work = LCM of (10, 12) units = 60 units. Now, since 60 units of work is being done by Tatto in 10 days, so Tatto is doing 6 units of work per day and similarly, Tappo is doing 5 units of work per day. Hence, together they are doing 11 units of work in one day. So, finally they will take $\frac{60}{11} = 5\frac{5}{11}$ days to complete the work.

Case 2 Time, speed and distance: Circular motion

Example 11 The speed of A is 15 m/s and the speed of B is 20 m/s. They are running around a circular track of length

1000 m in the same direction. After how much time will they meet at the starting point if they start running at same time?

Solution Time taken by A in taking one round = 66.66

Time taken by B in taking one round = 50

LCM (66.66, 50) = 200

Case 3 Number system: Tolling the bell

Example 12 There are two bells in a temple. Both the bells toll at a regular interval of 66.66 sec and 50 sec respectively. After how much time will they toll together for the first time?

Solution Time taken by the 1st bell to toll = 66.66

Time taken by the 2nd bell to toll = 50

LCM (66.66, 50) = 200

It can be observed here that mathematical interpretation of both the questions are same, just the language has been changed.

Case 4 Number system: Number of rows

Example 13 There are 24 peaches, 36 apricots and 60 bananas and they have to be arranged in several rows in such a way that every row contains the same number of fruits of one type. What is the minimum number of rows required for this to happen?

Solution We can put one fruit in one row, and in (24 + 36 + 60) 120 rows, we can arrange all the fruits. Or, we can put two fruits in one row and can arrange all the fruits in 60 rows. But for the rows to be minimum, the number of fruits should be maximum in one row.

HCF of 24, 36, 60 = 12, so 12 fruits should be there in one row.

Hence, the number of rows = 10

Case 5 Number system: Finding the remainder

Example 14 There is a number which when divided by 4 and 5 gives 3 as the remainder. What is the lowest three-digit number which satisfy this condition?

Solution Let us assume that there is no remainder. So, the number has to be a multiple of LCM of 4 and 5. Now, LCM (4, 5) = 20

But there is a remainder of 3 when divided by 4 and 5. So, the number will be of the form (20N + 3).

Hence, the numbers are 23, 43, 63, 83, 103 and so on…

So, 103 is the answer.

DIVISIBILITY RULES (FOR DECIMAL SYSTEM)

Divisibility rules are quite imperative, because with the help of these rules we can infer if a particular number is divisible by another number or not, without actually dividing it. Divisibility rules of numbers are specific to that particular number only. It simply means that divisibility rules of different numbers will be different. We shall now see a list of divisibility rules for some of the natural numbers.

Divisibility Rules

For 2 If unit digit of any number is 0, 2, 4, 6 or 8, then that number will be divisible by 2.

For 3 If sum total of all the digits of any number is divisible by 3, then the number will be divisible by 3. (e.g., 123, 456 etc.)

Example 15 How many values of A are possible if 3245684A is divisible by 3?

Solution Sum total of the number = 32 + A

For this number to be divisible by 3, A can take three values, namely, 1 or 4 or 7. (No other values are possible since A is the unit digit of the number.)

For 4 If last two digits of a number are divisible by 4, then that number will be divisible by 4 (e.g., 3796, 248, 1256).

For 5 If last digit of the number is 5 or 0, then that number will be divisible by 5.

For 6 If last digit of the number is divisible by two and sum total of all the digits of the number is divisible by 3, then that number will be divisible by 6.

For 7 The integers are divisible by 7 if and only if, the difference of the number of its thousands and the remainder of its divisible by 1000, is divisible by 7.

For 7 If the difference between the numbers of tens in the number and twice the unit digits is divisible by 7, then the given number is divisible by 7.

E.g., Let us take the number 795. The units digit is 5 and when it is doubled it, we get 10. The remaining part of the number (i.e., the tens) is 79. If 10 is subtracted from 79 we get 69. Since this result is not divisible by 7, the original number 795 is not divisible by 7.

For 8 If last 3 digits of the number are divisible by 8, then the number itself will be divisible by 8 e.g., 128, 34568, 76232 etc.

For 9 If sum of the digits of the number is divisible by 9, then the number will be divisible by 9 e.g., 129835782.

1 + 2 + 9 + 8 + 3 + 5 + 7 + 8 + 2 = 45. Since 45 is divisible by 9, then the number will be divisible by 9.

Example 16 How many pairs of A and B are possible in number 89765A4B if the number is divisible by 9, given that last digit of the number is even?

Solution Sum of digits of number is 8 + 9 + 7 + 6 + 5 + A + 4 + B = 39 + A + B.

So, A + B should be 6 or 15. Next value should be 24 but since A and B are digits so it can't be more than 18. Possible pairs of A and B are

A	B
0	6

1	5
2	4
3	3
4	2
5	1
6	0
7	8
8	7
9	6
6	9

Since B is even, six possible set of values of A and B are there.

For 11 A number is divisible by 11, if the difference between the sum of the digits at the even places and the sum of the digits at the odd places is divisible by 11 (zero is divisible by 11) e.g., 6595149 is divisible by 11 as the difference of $6 + 9 + 1 + 9 = 25$ and $5 + 5 + 4 = 14$ is 11.

For 12 If the number is divisible by 3 and 4, then the number will be divisible by 12, e.g., 144, 348.

For 13 **(A + 4B)**, where B is the units place digit and A is all the remaining digits.
E.g., Checking the divisibility of 1404 by 13: Here, A=140 and B = 4, then $A+4B=140+4\times4=156$. 156 is divisible by 13, so 1404 will be divisible by 13.

For 14 If the number is divisible by both 2 and 7, then the number will be divisible by 14.

For 15 A number is divisible by 15, if the sum of the digits is divisible by 3 and unit digit of the number is 0 or 5, e.g., 225, 450, 375 etc.

For 16 A number is divisible by 16, if the number formed by the last 4 digits of the given number is divisible by 16 e.g., 12578320 is divisible by 16, since last 4 digits of the number, 8320 is divisible by 16.

For 17 **(A – 5B),** where B is the units place digit and A is all the remaining digits.

For 18 Number should be divisible by both 9 and 2.

For 19 **(A + 2B),** where B is the units place digit and A is all the remaining digits.

If the sum of the number of tens in the number and twice the unit digit is divisible by 19, then the number is divisible by 19.

For example, let us take the number 665. The units digit is 5 and when it is doubled, we get 10. The remaining part of the number is 66. If 10 (which is the unit digit doubled) is added to 66 we get 76. Since, 76 is divisible by 19, it means the original number 665 is also divisible by 19.

For 20 The number should be divisible by 4 and 5.

Process to find out the divisibility rule for prime numbers

Process for the divisibility rule for P, a prime number is as follows:

Step 1 Find the multiple of P, closest to any multiple of 10. (This essentially will be of the form 10K + 1 or 10K –1.)

Step 2 If it is 10 K – 1, then the divisibility rule will be A + KB, and if it is 10K + 1, then the divisibility rule will be A – KB, where B is the units place digit and A is all the remaining digits.
E.g., Finding out the divisibility rule of 23: Lowest Multiple of 23, which is closest to any multiple of 10 $= 69 = 7 \times 10 - 1$
So, rule is A + 7 B.

Divisibility rule for any number of the format $10^n \pm 1$, where *n* is any natural number

In this method, we will be considering two factors, the first one being the value of n and the second one being the sign '+' or '–'. The value of '*n*' signifies how many digits will be taken one at a time and the sign signifies in what manner these digits will be taken. Let us see this with the help of some examples.

When *n* = 1

Case 1 $10^n + 1 = 11$

This method tells us that now we will be considering one digit at a time of the number which is to be divided by 11 and since the sign is '+' between 10^n and 1, we will alternately subtract and add starting from the right hand side.
E.g., To check if 123345 is divisible by 11, we will start from the right hand side, by subtracting the first number and then adding the third and then subtracting the fourth and so on…

Case 2 $10^n - 1 = 9$

Since the value of *n* is one, we will take one digit at a time and due to the '–' sign between 10^n and 1, we will keep on adding the digits from the right hand side (or, can be done from the left hand side also in this case).

When *n* = 2

Case 3 $10^n + 1 = 101$

Since the value of *n* here is two, we will make the pairs of digits from right hand side, and then we will alternately subtract and add each of the pairs.
E.g., Checking if 12345678 is divisible by 101: Make the pairs first – 12 34 56 78. Now we will do this calculation – (78) – (56) + (34) – (12) = 44, which is not a multiple of 101, so this number 12345678 is not divisible by 101.

Case 4 $10^n - 1 = 99$

Since the value of *n* is two, we will take two digit at a time after making the pairs and due to the '–' sign between 10^n and

1, we will keep on adding the pairs from right hand side (or, can be done from left hand side also in this case).

E.g., Checking if 122166 is divisible by 99: Let us make the pairs first: 12 21 66. Now add the pairs 12 + 21 + 66 = 99, so this number 122166 is divisible by 99.

And so on, we extend it for any natural number value of *n*.

NUMBER OF DIVISORS/FACTORS

If one integer can be divided by another integer an exact number of times, then the first number is said to be a *multiple* of the second, and the second number is said to be a *factor* of the first.

For example, 48 is a *multiple* of 6 because 6 goes into 48 an exact number of times (8 times in this case). Similarly, 6 is a *factor* of 48. On the other hand, 48 is not a multiple of 5, because 5 does not go into 48 an exact number of times. So, 5 is not a factor of 48.

When we talk about the number of divisors of any number, we are basically talking about positive integral divisor of that number. Likewise, it can be observed that 20 is having six divisors namely 1, 2, 4, 5, 10 and 20.

Its very essential here to understand the mechanism of formation of divisors.

Since $20 = 2^2 \times 5^1$

We will take three power of 2 viz., 2^0, 2^1 and 2^2 and two powers of 5 viz., 5^0 and 5^1.

Divisors will come from all the possible arrangements of powers of 2 and powers of 5.

$2^0 \times 5^0 = 1$
$2^0 \times 5^1 = 5$
$2^1 \times 5^0 = 2$
$2^1 \times 5^1 = 10$
$2^2 \times 5^0 = 4$
$2^2 \times 5^1 = 20$

Now, to find out the number of divisors of any number, we can use the following formula

If N is any number which can be factorised like $N = a^p \times b^q \times c^r \times \dots$, where a, b and c are prime numbers.

Number of divisors = $(p + 1)(q + 1)(r + 1)\dots$

Example 17 Find the number of divisors of $N = 420$.

Solution $N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

So, the number of divisors =
$(2 + 1)(1 + 1)(1 + 1)(1 + 1) = 24$

Example 18 Find the total number of even and prime divisors of $N = 420$.

Solution $N = 420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

Odd divisors will come only if we take zero power of 2 (since, any number multiplied by any power (≥1) of 2 will give us an even number)

So, odd divisors will come if we take $N_1 = 2^0 \times 3^1 \times 7^1 \times 5^1$

So, number of odd divisors = $(0 + 1)(1 + 1)(1 + 1)(1 + 1) = 8$

So, total number of even divisors = total number of divisors – number of odd divisors = 24 – 8 = 16

Alternatively, we can also find out the number of even divisors of $N = 420$ directly (or, in general for any number).

$420 = 2^2 \times 3^1 \times 7^1 \times 5^1$

To obtain the factors of 420 which are even, we will not consider 2^0, since $2^0 = 1$

So, the number of even divisors of 420 = $(2)(1 + 1)(1 + 1)(1 + 1) = 16$

(We are not adding 1 in the power of 2, since we are not taking 2^0 here, i.e., we are not taking one power of 2.)

Prime divisor = 4 (namely 2, 3, 5 and 7 only)

Example 19 $N = 2^7 \times 3^5 \times 5^6 \times 7^8$. How many factors of N are divisible by 50 but not by 100?

Solution All the factors which are divisible by 50 but not divisible by 100 will have atleast two powers of 5, and one power of 2.

And its format will be $2^1 \times 5^{2+y}$.

So, number of divisors = $1 \times 6 \times 5 \times 9 = 270$

Alternatively, this is equal to (Number of factors divisible by 50) – (Number of factors divisible by 100).

Condition for two divisors of any number *n* to be co-prime to each other

Let us see it for $N = 12$

Total number of factors of 12 = 6 (namely 1, 2, 3, 4, 6, 12)

Now, if we have to find out the set of factors of this number which are co-prime to each other, we can start with 1.

Number of factors which are co-prime to 1 = 5 (namely 2, 3, 4, 6, 12)

Next in line is the number of factors which are co-prime to 2 = 1 (namely 3)

Number of factors co-prime to 3 = 1 (namely 4)

So, the total number of set of factors for 12 which are co-prime to each other = 6

So, we can induce that if we have to find out the set of factors which are co-prime to each other for $N = a^p \times b^q$, will be equal to $[(p + 1)(q + 1) - 1 + pq]$.

If there are three prime factors of the number, i.e., $N = a^p \times b^q \times c^r$, then set of co-prime factors can be given by $[(p + 1)(q + 1)(r + 1) - 1 + pq + qr + pr + 3\,pqr]$

Alternatively, we can find out the set of co-prime factors of this number by pairing it up first and then finding it out with the third factor.

Example 20 Find the set of co-prime factors of the number N = 720.

Solution $720 = 2^4 \times 3^2 \times 5^1$

Using the formula for three prime factors $[(p + 1)(q + 1)(r + 1) - 1 + pq + qr + pr + 3\,pqr]$

We get, $[(4 + 1)(2 + 1)(1 + 1) - 1 + 4.2 + 2.1 + 4.1 + 3.4.2.1] = 67$

Alternatively, let us find out first for $2^4 \times 3^2 = [(4 + 1)(2 + 1) - 1 + 4.2] = 22$

Now $p^{22} \times 5^1$ will give us $[(22 + 1)(1 + 1) - 1 + 22.1] = 67$

Sum of Divisors

Like the number of divisors of any number, we can find out the sum of divisors also.

If N is any number which can be factorised like $N = a^p \times b^q \times c^r \times \ldots$, where a, b and c are prime numbers. Then sum of the divisors

$$= \frac{(a^{p+1} - 1)(b^{q+1} - 1)\ (c^{r+1} - 1)}{(a-1)(b-1)(c-1)}$$

REMAINDERS

Dividend = Quotient × Divisor + Remainder

Basics of Remainder

1. When any positive number A is divided by any other positive number B, and if B > A, then the remainder will be A itself.
   For example remainder of 5/12 = 5
   Remainder of 212/678 = 212
2. Remainder should always be calculated in its actual form i.e., you can not reduce the fraction to its lower ratio.
   For example, remainder of 1/2 = 1
   Remainder of 2/4 = 2
   Remainder of 3/6 = 3
   It can be observed that despite all the fractions being equal, remainders are different in each case.

Example 21 What is the remainder when 5×10^5 is divided by 6×10^6?

Solution As we know that we cannot reduce the fractions to its lower terms, the remainder obtained will be equal to 5×10^5.

3. Concept of negative remainder: As obvious from the name, remainder implies that something has been left out. So, remainder simply can never be negative. Its minimum value can be zero only and not negative.

Example 22 What is the remainder when 50 is divided by 7?

Solution $-50/7 = (-56 + 6)/7$; which gives a remainder of 6.

Or, when we divide -50 by 7, we get -1 as the remainder. Now, since remainder has to be non-negative, so we add 7 (quotient) to it which makes final remainder as→ $-1 + 7 = 6$.

There are two methods to find out the remainder of any expression:

- Cyclicity Method
- Theorem Method

Cyclicity method

For every expression of a remainder, there comes attached a specific cyclicity of remainders.

Example 23 What is the remainder when 4^{1000} is divided by 7?

Solution To find the cyclicity, we keep finding the remainders until some remainder repeats itself. It can be understood with the following example:

Number/7 →	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder →	4	2	1	4	2	1	4	2

Now, 4^4 gives us the same remainder as 4^1, so the cyclicity is of 3 (Because remainders start repeating themselves after 4^3.)

So, any power of 3 or a multiple of 3 will give a remainder of 1. So, 4^{999} will give 1 as the remainder.

Final remainder = 4

Example 24 What is the remainder when 4^{96} is divided by 6? **(CAT 2003)**

Solution Finding out the cyclicity,

Number/6 →	4^1	4^2	4^3	4^4	4^5	4^6	4^7	4^8
Remainder →	4	4	4	4	4	4	4	4

The remainder in all the cases is 4, so the final remainder will be 4. Actually, there is no need to find remainders upto 4^8 or even 4^3. 4^2 itself gives us a remainder of 4 when divided by 6, which is the same as the remainder obtained when 4^1 is divided by 6. So, we have got the cyclicity here, which is of 1.

Hence, final remainder = 4

It also can be observed here that if we write $4^{100}/6 = 2^{200}/6 = 2^{199}/3$, then remainder obtained will be 2, which is not the right answer (as given in CAT brochure of next year i.e., CAT 2004)

Remainder Theorem

The product of any two or more than two natural numbers has the same remainder when divided by any natural numbers, as the product of their remainders.

Let us understand this through an example.

Example 25 Remainder $\frac{12\times13}{7}$ = Remainder $\frac{156}{7} = 2$

Solution Normal way of doing this is – Product →→→ Remainder

Theorem method – Remainder →→→ Product →→→ Remainder

So, first of all we will find the remainders of each number individually and then multiply these individual remainders to find out the final remainder.

Remainder 12/7 = 5

Remainder 13/7 = 6

Remainder $\frac{12\times13}{7}$ = Remainder $(5 \times 6)/7$

= Remainder $30/7 = 2$

Example 26 What is the remainder obtained when $(1421 \times 1423 \times 1425)$ is divided by 12? **(CAT 2000)**

Solution Remainder of $1421/12 = 5$

Remainder of $1423/12 = 7$

Remainder of $1425/12 = 9$

Remainder $(1421 \times 1423 \times 1425)/12$

= Remainder $(5 \times 7 \times 9)/12$

= Remainder $(5 \times 63)/12$

= Remainder $(5 \times 3)/12 = 3$

Concept of Successive Division

Suppose N is any number which is divided successively by 3 and 5, which means – first, we divide N by 3 and then the quotient obtained is divided by 5.

E.g., Let us see the case when 50 is divided by 5 and 3 successively.

50 divided by 5 gives 10 as the quotient. Now, we will divide 10 by 3. It gives finally a quotient of 3 and remainder of 1.

Example 27 When a number N is divided successively by 3 and 5, remainder obtained are 1 and 2 respectively. What is the remainder when N is divided by 15?

Solution It can be seen that we are required to calculate it from back-end.

Family of numbers which when divided by 5 gives remainder $2 = 5\,S + 2$

So, $N = 3\,(5\,S + 2) + 1 = 15\,S + 7$

Now, when N is divided by 15, remainder = 7

Fermat's remainder therorem

Let P be a prime number and N be a number not divisible by P. Then the remainder obtained when A^{P-1} is divided by P is 1.

Example 28 What is the remainder when 2^{100} is divided by 101?

Solution Since it satisfies the Fermat's theorem format, remainder = 1

Euler's theorem of remainder

Let $f(n)$ be the number of integers less than n and co-prime with n, then the remainder obtained when $m^{f(n)}$ is divided by n is 1, where m and n are co-prime to each other.

As we can see:

$f(2) = 1, f(3) = 2, f(4) = 2, f(5) = 4$

So, the remainder obtained when $8^{f(5)=4}$ is divided by 5 is 1. It can also be seen that the remainder obtained when any power of 8 divisible by 4 such as 8^4 or 8^8 or 8^{12} and so on will give the same remainder when divided by 5.

Similarly, the remainder obtained when $12^{f(5)=4}$ is divided by 5 is 1;

or, the remainder obtained when $8^{f(7)=6}$ is divided by 7 is 1.

Derivations

1. $\frac{(A+1)^N}{A}$ will always give 1 as the remainder.

   (For all natural values of A and N)

Example 29 What is the remainder when 9^{100} is divided by 8?

Solution For A = 8, it satisfies the above condition. So, remainder = 1

Alternatively, we can apply either cyclicity or theorem method to find the remainder. (Do this yourself).

2. $\frac{A^N}{A+1}$ When N is even, remainder is 1 and when N is odd, remainder is A itself.

Example 30 What is the remainder when 2^{10} is divided by 3?

Solution Since here N is even, so remainder = 1

3. i. $(a^n + b^n)$ is divisible by $(a + b)$, if n is odd.
   Extension of the above formula – $(a^n + b^n + c^n)$ is divisible by $(a + b + c)$, if n is odd and a, b, c, are in arithmetic progression.
   Similarly, the above situation can be extended for any number of terms.
   ii. $(a^n - b^n)$ is divisible by $(a + b)$, if n is even.
   iii. $(a^n - b^n)$ is divisible by $(a - b)$, if n is either odd or even.

Example 31 What is the remainder when $(15^{23} + 23^{23})$ is divided by 19? **(CAT 2004, 2 marks)**

Solution It can be observed that $(15^{23} + 23^{23})$ is divisible by 38, so it will be divisible by 19 also. Hence, remainder = 0.

Alternatively, this problem can be done either by cyclicity method or theorem method.

Example 32 What is the remainder when $(16^3 + 17^3 + 18^3 + 19^3)$ is divided by 70? **(CAT 2005, 1 mark)**

Solution We know, this is a basic multiplication and division question. But using the above approach makes it a lot simple.

We know that $(a^n + b^n)$ is divisible by $(a + b)$, if n is odd. Taking cue from this, we can say that $(a^n + b^n + c^n)$ is divisible by $(a + b + c)$, if n is odd and similarly $(a^n + b^n + c^n + d^n)$ is divisible by $(a + b + c + d)$. Now $16 + 17 + 18 + 19 = 70$, so remainder is zero. (As a, b, c, d are in AP)

UNIT DIGIT

As we have seen the cyclicity of remainders above, cyclicity exists for unit digit of the numbers also. (But there is no relation between the cyclicity of remainders and unit digit.)

Taking a very simple example– $2^5 = 32$, and so we know that unit digit of 2^5 is 2. But problem occurs when we start getting big numbers like 25678^{2345} etc. To find out the unit digit of these kinds of numbers, we have some standard results, which we use as formulae.

(Any even number)^{4n} = … 6

It means that any even number raised to any power, which is a multiple of 4, will give us 6 as the unit digit.

(Any odd number)^{4n} = … 1

It means that any odd number raised to any power, which is a multiple of 4, will give us 1 as the unit digit.

Exception 0, 1, 5, 6 [these are independent of power, and unit digit will be the same respectively]

Example 33 Find the unit digit of $25678^{2345} \times 3485^{4857}$.

Solution Unit digit of 25678^{2345} = Unit digit of 8^{45}

(To find out unit digit, we need to have unit digits only. And similarly, to find out tens digit we need to have the tens and units digit only. In the present case, we are considering only last two digits of the power because divisibility rule of 4 needs only the last two digits of the number.)

$8^{45} = 8^{44+1} = 8^{44} \times 8^1 = (\ldots 6) \times 8 = \ldots 8$

Example 34 What is the unit digit of ${}_{32}32^{32}$?

Solution 32 is an even number which is having a power of the form 4*n*. So, it will give 6 as the unit digit.

Example 35 When 3^{32} is divided by 50, it gives a number of the format (asdf… ·*xy*) (*xy* being the last two digits after decimal). Find *y*.

Solution It can be observed that unit digit of $3^{32} = 1$. Now any number having 1 as the unit digit will always give 2 at the unit place when divided by 50.

So, answer is 2.

Example 36 What is the last non-zero digit of the number 30^{2720}? **(CAT 2005, 2 marks)**

Solution $30^{2720} = [30^4]^{680} = \ldots 10000 \ldots. 00$

Unit digit can also be found out by cyclicity method as well.

It can be seen that

Unit digit of $2^1 = 2$

Unit digit of $2^2 = 4$

Unit digit of $2^3 = 8$

Unit digit of $2^4 = 6$

Unit digit of $2^5 = 2$

So, it can be inferred that unit digit of 2^1 = unit digit of 2^5 = unit digit of 2^9

Hence, cyclicity of 2 = 4, i.e., every fourth power of 2 will give the same unit digit.

Similarly,

Cyclicity of 3 = 4

Cyclicity of 4 = 2

Cyclicity of 7 = 4

Cyclicity of 8 = 4

Cyclicity of 9 = 2

Cyclicity of 0 or cyclicity of 1 or cyclicity of 5 or cyclicity of 6 = 1. To know more about Unit Digid, refer to the book. Demystifying Number System.

TENS' DIGIT

There lies the cyclicity of tens' place digit of all the digits. This is given below:

Digits	*Cyclicity*
2, 3, 8–	20
4, 9 –	10
5–	1
6–	5
7–	4

Example 37 What is the tens' place digit of 12^{42}?

Solution For this, we need to break 12^{42} first by using binomial theorem as $(10 + 2)^{42}$. Obviously this expression will have 43 terms, and out of these 43 terms first 41 terms will have both of their tens and units place digit as 0.

Last two terms will be → ${}^{42}C_{41} \times 10^1 \times 2^{41} + {}^{42}C_{42} \times 10^0 \times 2^{42}$

Now, we will find the tens' place digit of all these terms individually.

Tens digit of ${}^{42}C_{41} \times 10^1 \times 2^{41} = 42 \times 10 \times (02)$ [Cyclicity of 2 is 20, so 2^{41} will have same tens digits as 2^1] = 840, so 40 are the last two digits.

Similarly, ${}^{42}C_{42} \times 10^0 \times 2^{42} = 1 \times 1 \times 04 = 04$

So, finally last two digits are → 40 + 04 = 44, so 4 is the tens' place digit.

Note: $(25)^n$ and $(76)^n$ will always give 25 and 76 as the units and tens digit for any natural number value of *n*.

NUMBER OF EXPONENTS

Let us take a simple number – 10^5

This is read as – 10 to the power 5, or we say that exponent of 10 is 5.

In simple terms, exponents are also known as Power.

Example 38 What is the maximum value of *s* if $N = (35 \times 45 \times 55 \times 60 \times 124 \times 75)$ is divisible by 5^s?

Solution If we factorize $N = (35 \times 45 \times 55 \times 60 \times 124 \times 75)$, then we can see that 5 appears 6 times, it means *N* is divisible by 5^6.

So, maximum value of *s* = 6

Exponent of any prime number P in *n*!

$= \left[\frac{n}{\text{p}}\right] + \left[\frac{n}{\text{p}^2}\right] + \left[\frac{n}{\text{p}^3}\right] + \ldots + \left[\frac{n}{\text{p}^x}\right]$, where $n \geq \text{p}^x$ and [.] denotes the greatest integer value i.e., we have to consider only the integral value.

Let us find the exponent of 5 in 1000! = $1000/5 + 1000/5^2 + 1000/5^3 + 1000/5^4 = 200 + 40 + 8 + 1 = 249$

Example 39 What is the highest power of 5 which can divide *N* = (22! + 17894!)?

Solution Number of times this number is divisible by 5 is same as the number of zeroes at the end of this number. Since 22! have 4 zeroes at its end, so N will also be having only four zeroes at its end. Hence, highest power of 5, which can divide N is 4.

Process to find out the exponent of any composite number in *n*!

There are three different kinds of composite numbers.

1. Product of two or more than two prime numbers with unit power of all the prime numbers
   For example, 15 (5×3), 30 $(2 \times 3 \times 5)$ etc.
2. (Any prime number)^n, where $n > 1$
   For example, 4 (2^2), 27 (3^3)
3. Product of two or more than two prime numbers with power of any one prime number more than 1.
   For example, 12 $(2^2 \times 3)$, 72 $(2^3 \times 3^2)$ etc.

Let us find out the exponents of the above-written composite numbers one by one:

- Find out the exponent of 15 in 100!

15 is the product of two distinct prime numbers 5 and 3. So, to find out the exponents of 15, we need to find out the exponents of 5 and 3 individually.

So, by applying the same formula of finding out the exponents for any prime number in both of these cases individually, minimum of those two will be the answer.

$100/5^x = [100/5] + [100/5^2] = 20 + 4 = 24$

$100/3^x = [100/3] + [100/3^2] + [100/3^3] + [100/3^4]$
$= 33 + 11 + 3 + 1 = 48$

Obviously, 24 is going to be the answer.

- Find out the exponent of 25 in 100

$25 = 5^2$

In this case, we first find out the exponents of 5 and then divide it by 2 (actually the power) to find the exponents of 25.

$100/5^x = [100/5] + [100/5^2] = 20 + 4 = 24$

So, $100/25^x = 24/2 = 12$

- Similarly, we can find out for the third category of numbers also.

BASE SYSTEM

In our decimal system of writing the numbers, we use 10 digits (0 – 9). In this system, largest number of single digit = 9, and the moment we have to form a number bigger than this number we need two-digit numbers starting from 10. Similarly, largest number of two digits = 99 and after this we have 100, a number of three digits.

Lets assume a system of writing where we use only 6 digits (0 – 5). Largest single digit number in this system will be 5 and next to this will be 10. Similarly, largest two digit number will be 55 and next to this is 100.

This whole procedure can be summed up in the following table:

$(0\text{-}9)_{10}$	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
$(0\text{-}8)_9$	0	1	2	3	4	5	6	7	8	10	11	12	13	14	15	16	17	18
$(0\text{-}7)_8$	0	1	2	3	4	5	6	7	10	11	12	13	14	15	16	17	20	21
$(0\text{-}6)_7$	0	1	2	3	4	5	6	10	11	12	13	14	15	16	20	21	22	23
$(0\text{-}5)_6$	0	1	2	3	4	5	10	11	12	13	14	15	20	21	22	23	24	25
$(0\text{-}3)_4$	0	1	2	3	10	11	12	13	20	21	22	23	30	31	32	33	100	101
$(0\text{-}2)_3$	0	1	2	10	11	12	20	21	22	100	101	102	110	111	112	120	121	122

Types of Questions

Questions from this concept are asked in three different ways:

1. $(\text{Base})_{10}$ to any other base and vice-versa
2. $(\text{Base})_x$ to $(\text{Base})_y$ and vice-versa; none of x and y being equal to 10 but x and y will be given.
3. $(\text{Base})_x$ to $(\text{Base})_y$, value of x and y will not be given.

1. (Base)$_{10}$ to any other base and vice-versa

Method 1

Let us see in case of $(74)_{10}$.

$(74)_{10} = 7 \times 10^1 + 4 \times 10^0$, since the base is 10.

Now, if we have to convert this number in 9 base, then we will try to write it in terms of powers of 9.

$(74)_{10} = 8 \times 9^1 + 2 \times 9^0 = (82)_9$

$(74)_{10} = 1 \times 8^2 + 1 \times 8^1 + 2 \times 8^0 = (112)_8$

$(74)_{10} = 1 \times 7^2 + 3 \times 7^1 + 4 \times 7^0 = (134)_7$
$(74)_{10} = 2 \times 6^2 + 0 \times 6^1 + 2 \times 6^0 = (202)_6$

While converting the numbers from decimal system to any other system of writing the numbers, we should be concerned with the following two rules

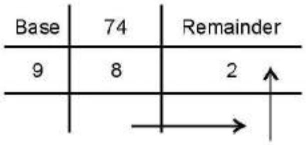
i. Take maximum possible power of the base and then keep writing rest of the number with the help of lesser power of base (as illustrated in the above example).
ii. Once we have used $(\text{base})^n$; where n is the maximum power, we will be required to

E.g., Write the co-efficients of all the powers of base from 0 to $(n - 1)$ as in the case of $(74)_{10} = (202)_6$.

Now, suppose we have to convert $(356)_7$ in the base of 10.
$(356)_7 = 3 \times 7^2 + 5 \times 7^1 + 6 \times 7^0 = (188)_{10}$

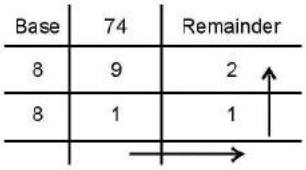
Method 2

Converting $(74)_{10}$ to the base of $(\ )_9$



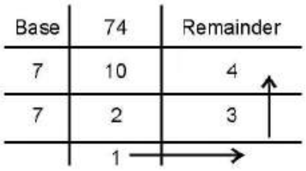
So, $(74)_{10} = (82)_9$

Converting $(74)_{10}$ to the base of $(\ )_8$



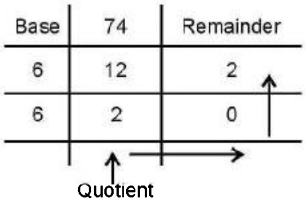
So, $(74)_{10} = (112)_8$

Converting $(74)_{10}$ to the base of $(\ )_7$



So, $(74)_{10} = (134)_7$

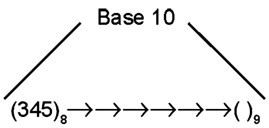
Converting $(74)_{10}$ to the base of $(\ )_6$



So, $(74)_{10} = (202)_6$

2. $(\text{Base})_x$ to $(\text{Base})_y$ and vice-versa; none of *x* and *y* being equal to 10 but *x* and *y* will be given

Converting $(345)_8$ to the base of $(\ )_9$



We will do this problem with the help of creating a bridge of base 10 between base 8 and base 9.

Step 1 Convert $(345)_8$ into base 10.
$345 = 3 \times 8^2 + 4 \times 8^1 + 5 \times 8^0 = (229)_{10}$

Step 2 Now convert this number in base 10 into base 9.
$(229)_{10} = 2 \times 9^2 + 7 \times 9^1 + 4 \times 9^0 = (274)_9$

However, if new base is a power of old base and vice-versa, then it can be converted directly also in the new base, i.e., we are not needed to go to base 10 for these kind of conversions. E.g., $(\text{Base})_2$ to $(\text{Base})_4$ or $(\text{Base})_2$ to $(\text{Base})_8$ conversion does not require a bridge of base 10.

Converting $(101110010)_2$ to Octal $(\ )_8$ system

At first, we will club three digits of binary number into a single block and then write the decimal equivalent of each group (left to right).

So, $(101110010)_2$ is now $(101)_2(110)_2(010)_2$
Now, $(101)_2 = 1 \times 2^2 + 0 + 1 \times 2^0 = 5$
$(110)_2 = 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 6$
$(010)_2 = 0 \times 2^2 + 1 \times 2^1 + 0 \times 2^0 = 2$
So, $(101110010)_2 = (562)_8$

Converting $(101110010)_2$ to Hexa-decimal $(\ )_{16}$ system

At first, we will club four digits of binary number into a single block and then write the decimal equivalent of each group (left to right).

So, $(101110010)_2$ is now $(0001)_2(0111)_2(0010)_2$
Now, decimal equivalent of $(0001)_2 = 1$
Decimal equivalent of $(0111)_2 = 7$
Decimal equivalent of $(0010)_2 = 2$
$(101110010)_2 = (172)_{16}$

3. $(\text{Base})_x$ to $(\text{Base})_y$, value of x and y will not be given

In these type of questions, normally some calculation is given in some unknown system of writing numbers and on the basis of that we will be required to solve the questions.

Example 40 In a system of writing of *N* digits,
$4 \times 6 = 30$ and $5 \times 6 = 36$. What will be the value of $N = 3 \times 4 \times 5$ in the same system of writing?

Solution Let us assume that there are *N* digits in this system of writing.

So, $(30)_N = 3 \times N^1 + 0 \times N^0 = 24$
$\Rightarrow 3N = 24$
$\Rightarrow N = 8$
So, this system of writing has 8 digits.

In this system $3 \times 4 \times 5 = 60$ will be written as 74. $(60 = 7 \times 8^1 + 4 \times 8^0)$

Alternatively, since this system is having 6 as one of its digits, minimum value of *N* will be 7. Again, 24 is written as 30 in this system, so *N* is less than 10. Now use hit and trial for $N = 7$ or 8 or 9 to find out *N* in $24 = (30)_N$.

DECIMAL CALCULATION

So far we have seen the calculations involving natural numbers only. Let's work now with decimals.

Converting decimal system numbers to any other system

Suppose (12.725) is a number in decimal system which is required to be converted into octal system (8 digits)

We will first convert 12 into octal system.

$(12)_{10} = (14)_8$

Now to convert $(0.725)_{10}$ into $(\ )_8$, we will apply the following method:

$0.725 \times 8 = 5.8$ Take out integral part from here.
$0.8 \times 8 = 6.4$ Take out integral part from here.
$0.4 \times 8 = 3.2$ Take out integral part from here.
$0.2 \times 8 = 1.6$ Take out integral part from here.

And keep doing this till we get decimal part as zero, i.e., the product should be an integer.

$(0.725)_{10} = (0.5632\ldots)_8$

So, $(12.725) = (14.5632\ldots)_8$

Converting any other system numbers to decimal system

Now suppose if $(15.453)_7$ is to be converted into decimal system, then the process is as follows:

We will first convert $(15)_7$ into decimal system.

$(15)_7 = 1 \times 7^1 + 5 \times 7^0 = (12)_{10}$

Now $(0.453)_7 = 4 \times 7^{-1} + 5 \times 7^{-2} + 3 \times 7^{-3} = 0.6821$

So, $(15.453)_7 = (12.6821)_{10}$

Basic Algebraic Calculations Involving Base Systems

Addition

$$\begin{array}{r} 325_7 \\ +\ 456_7 \\ \hline \end{array}$$

Start with the units place digit, 5 + 6 = 11 which is 14_7. So, unit digit is 4 and carry over is 1.

Next is tens place digit, 2 + 5 + 1 (carry over) = 8 which is 11_7. So, tens digit is 1 and carry over is again 1.

Next is 3 + 4 + 1 (carry over) = 8 which is 11_7.

$$\begin{array}{r} 325_7 \\ +\ 456_7 \\ \hline 1114_7 \\ \hline \end{array}$$

Subtraction

$456_8 - 367_8$

Starting with the units digit, since 6 is smaller than 7, we will borrow 1 from the tens place digit. So, now it is 14 (when the base is 10, we get 10 but here base is 8, so we get 8) and 7 subtracted from it = 14 – 7 = 7, which is the units digit.

Next, tens digit is now 4 and we have to subtract 5 from it. We again borrow 1 from hundred's place digit. So, now it is 12 and 12 – 6 = 6, which is the tens place digit.

Now, hundred's place digit is 3 (4 – 1), so 3 – 3 = 0

$$\begin{array}{r} 456_8 \\ -\ 367_8 \\ \hline 67_8 \end{array}$$

Note: Another method of doing these kinds of calculations is to convert these values (in whatever base) into decimal system, then do the actual calculation in decimal system itself and finally converting the numbers into the required or given system.

Some standard systems of writing

1. Decimal system
   Digits used: 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9.
   Total digits used = 10 digits
2. Hexa-decimal system
   Digits used: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E and F.
   Total digits used = 16
3. Octal system
   Digits used: 0, 1, 2, 3, 4, 5, 6 and 7.
   Total digits used = 8
4. Binary system
   Digits used: 0, 1
   Total Digits used = 2

Comparison of Different Base Systems

It should be understood here that using different bases creates a difference only in the numbers (more than single digit) and not in the digit. This means in that $(5)_{10}$ is same as $(5)_9$ or any other system of writing the numbers.

Example 41 How many different values of N are possible in the following calculation

$(2)_N \times (4)_N = (8)_N$?

Solution Any value of N which has 8 as a digit in that system will satisfy this calculation. So, any value of $N \geq 9$ is possible. Hence infinite values of N are possible.

This concept can be further understood through the mechanism of multiplication. Multiplication is basically the condensed form of addition. So, 2 added up four times will give 8, and hence all the base systems which have the following can be one of the values of N:

0 1 2 3 4 5 6 7 8.

Example 42 How many different values of N are possible in the following calculation

$(4)_N + (5)_N = (9)_N$?

Solution In this case also, the digits used can be of any system that has the digit 9. So, all the inegral values $N \geq 10$ are possible.

Example 43 How many different values of N are possible in the following calculation.

$(4)_N \times (5)_N = (24)_N$?

Solution Obviously, there can be either just one possible value of N or no possible value of N.

$(4)_{10} \times (5)_{10} = (20)_{10}$ [We know $(4)_N = (4)_{10}$ and $(5)_N = (5)_{10}$]

Now, $(24)_N = 2 \times N^1 + 4 \times N^0 = 20$

Hence, $N = 8$. And this will be the only value of N.

Divisibility Rules For Systems Other Than Decimal System

I would like to emphasize that different number systems are just different ways to write numbers. Thus the divisibility of one number by another does not depend on the particular system in which they are written.

At the same time, in each system there are some tricks to determine divisibility by certain specific numbers. These are the divisibility tests.

Let us investigate the tests for divisibility by 3 and 9. We will try to generalize these tests for any number base system.

Is 1234565642317 divisible by 6?

We know the divisibility rule for 9 – Sum of digits of the number should be divisible by 9.

Sum of the digits of this number is 42.

Now, we can answer this question easily, since the sum of the digits (which is 4210) is divisible by 6, the number itself is also divisible by 6.

In general, the sum of the digits of a number written in the base n system is divisible by $(n - 1)$ if and only if the number itself is divisible by $(n - 1)$.

So, divisibility rule for 4 in a base system of 5 in that the sum of the digits of the number should be divisible by 4. For example, 31_5 is divisible by 4.

Similarly, if we have to find out the divisibility rule of 12 in the base of 11, it will be nothing but same as the divisibility rule of 11 in the base of 10. Generalizing this whole concept, divisibility rule of any natural number N in the base of $(N - 1)$ will be same as divisibility rule of 11 on base 10.

Before you proceed, you should have a clear understanding of how to find the divisibility rule for any prime number in base 10.

The only difference is that in case of base 10, we find the multiples of prime numbers in the format $10\,n \pm 1$, and in case of base B, we find the multiples of prime numbers in the format $\text{B}\,n \pm 1$.

Let us find out the divisibility rule of $(5)_8$.

Multiples of 5 in base 10

Multiples of 5 in base 10	5	10	15	20	25	30
Multiples of 5 in base 8	5	12	17	24	31	26

Obviously, 17 is the first number in the format of $8\,n \pm 1$.

$17 = 8 \times 2 - 1$ [Yes, it is correct. 17_8 and RHS is in base 10]

So, divisibility rule of 5 on base 8 = A + 2B, where A and B have the same meaning as that in divisibility rules of 10.

We can check this by the following method:

First few numbers in base 8							
0	1	2	3	4	5	6	7
10	11	12	13	14	15	16	17
20	21	22	23	24	25	26	27
30	31	32	33	34	35	36	37

Every 5th number in this series will be divisible by 5. So, 5 is the first number divisible by 8. The next few numbers are 12, 17, 24, 31, 36.

We can use base to generate good concepts in remainders also.

Example 44 What is the remainder when $(1234)_6$ is divided by 5?

Solution Divisibility rule of $(5)_6$ will be similar to that of the divisibility rule of $(9)_{10}$. So, if the sum of digits of the number in base 6 is divisible by 5, then it will be divisible by 5.

Adding the digits of $(1234)_6 = 10$, which is divisible by 5, the remainder obtained when $(1234)_6$ is divided by 5 is 0.

Example 45 What is the remainder when $(12341)_6$ is divided by 5?

Solution The sum of the digits of $(12341)_6 = 11$, so the remainder obtained will be 1.

Pigeon-hole Principle

Although only a few questions have been asked from this concept so far in CAT, which is also true for some of the arithmetic topics like Profit and Loss; Calendar etc., the importance of this topic lies in the fact that this concept is purely logical and having a good practice over this concept gives a logical edge over others.

The concept is that if we put $(N + 1)$ or more pigeons in N holes (nests), then at least one hole will be there which will have 2 or more pigeons.

Example 46 What is the minimum number of people in any group of five people who have an identical number of friends within the group, provided if A is a friend of B, then B is also a friend of A?

Solution Since there are five persons in the group, so possible number of friends is 0, 1, 2, 3, 4. It seems here that everybody is having a different number of friends, so the answer is zero. But anybody having four friends ensures that nobody is having 0 friends. So, at least two persons must have the same number of friends.

PRACTICE EXERCISES

■■■

WARM UP

Q.1. Which of the following is the smallest?
(a) $5^{1/2}$ (b) $6^{1/3}$ (c) $8^{1/4}$ (d) $12^{1/5}$

Q.2. A number N is divisible by 6 but not divisible by 4. Which of the following will not be an integer?
(a) N/3 (b) N/2 (c) N/6 (d) N/12

Q.3. If a, b and c are consecutive positive integers, then the largest number which always divides $(a^2 + b^2 + c^2)$
(a) 14 (b) 55
(c) 3 (d) None of these

Q.4. $\dfrac{(3.134)^3 + (1.866)^3}{(3.134)^2 - 3.134 \times 1.866 + (1.866)^2} = ?$
(a) 25 (b) 2.68 (c) 1.038 (d) 5

Q.5. If n^2 is a perfect cube, then which of the following statements is always true?
(a) n is odd
(b) n is even
(c) n^3 is a perfect square
(d) n is a perfect cube

Q.6. If $(5x + 11y)$ is a prime number for natural number values of x and y, then what is the minimum value of $(x + y)$?
(a) 2 (b) 3 (c) 4 (d) 5

Q.7. For what values of x is $25^x + 1$ divisible by 13?
(a) All real values of x
(b) Odd natural values of x
(c) Even values of x
(d) All the integral values of x

Q.8. Which of the following numbers lies between 5/6 and 6/7?
(a) 71/84 (b) 31/42 (c) 129/168 (d) 157/339

Q.9. By multiplying with which of the following numbers does the product of $8 \times 9 \times 10 \times 11 \times 12$ become a perfect square?
(a) 55 (b) 11 (c) 165 (d) 310

Q.10. What is the difference between the sum of the cubes and that of squares of the first ten natural numbers?
(a) 5,280 (b) 2,640 (c) 3,820 (d) 4,130

Q.11. If $3 - 9 + 15 - 21 + \ldots$ up to 19 terms $= x$, then x is a/an
(a) odd number (b) even number
(c) prime number (d) irrational number

Q.12. What is the unit's digit of $21^3 \times 21^2 \times 34^7 \times 46^8 \times 77^8$?
(a) 4 (b) 8 (c) 6 (d) 2

Q.13. If the unit's digit in the product $(47n \times 729 \times 345 \times 343)$ is 5, what is the maximum number of values that n may take?
(a) 9 (b) 3 (c) 7 (d) 5

Q.14. In how many ways can 846 be resolved into two factors?
(a) 9 (b) 11
(c) 6 (d) None of these

Q.15. If a number is divided by 15, it leaves a remainder of 7. If thrice the number is divided by 5, then what is the remainder?
(a) 5 (b) 6 (c) 7 (d) 1

Q.16. A number when divided by 391 gives a remainder of 49. Find the remainder when it is divided by 39.
(a) 10 (b) 9
(c) 11 (d) Cannot be determined

Q.17. p and q are two prime numbers such that $p < q < 50$. In how many cases would $(q + p)$ be also a prime number?
(a) 5 (b) 6
(c) 7 (d) None of these

Q.18. How many distinct factors of 1,600 are perfect cubes?
(a) 3 (b) 4 (c) 6 (d) 2

Q.19. The LCM of 96,144 and N is 576. If their HCF is 48, then which of the following can be one of the values of N?
(a) 168 (b) 192 (c) 144 (d) 244

Q.20. If p and q are consecutive natural numbers (in increasing order), then which of the following is true?
(a) $q^2 < p$ (b) $2p > p$
(c) $(q + 1)^2 > p^2$ (d) $(p + 2)^3 < q^3$

Q.21. $(17^{21} + 19^{21})$ is not divisible by
(a) 36 (b) 8 (c) 9 (d) 18

Q.22. Which of the following will divide $11^{12296} - 1$?
(a) 11 and 12 (b) 11 and 10
(c) 10 and 12 (d) 11 only

Q.23. If *a, b, c* and *d* are consecutive odd numbers, then $(a^2 + b^2 + c^2 + d^2)$ is always divisible by
(a) 5 (b) 7 (c) 3 (d) 4

Q.24. Four bells toll at intervals of 14, 21 and 42 min respectively. If they toll together at 11.22 a.m., when will they toll together for the first time after that?
(a) 11.56 a.m. (b) 12.04 p.m.
(c) 12.06 p.m. (d) 11.48 a.m.

Q.25. When x is divided by 6, remainder obtained is 3. Find the remainder when $x^4 + x^3 + x^2 + x + 1$ is divided by 6.
(a) 3 (b) 4 (c) 1 (d) 5

Q.26. I have 7^7 sweets and I want to distribute them equally among 2^4 students. After each of the student got maximum integral sweets, how many sweets are left with me?
(a) 8 (b) 5
(c) 1 (d) None of these

Q.27. When I distribute some chocolates to my 40 students, three chocolates will be left. If I distribute the same number of chocolates to my students and my colleague Manoj Dawrani, seven chocolates are left. Find the minimum number of chocolates I have.
(a) 1,443 (b) 1,476
(c) 1,480, (d) None of these

Q.28. The LCM of two numbers is 40 times their HCF. The sum of the LCM and HCF is 1,476. If one of the numbers is 288, find the other numbers?
(a) 169 (b) 180 (c) 240 (d) 260

Q.29. 1010101 ... 94 digits is a 94 digits number. What will be the remainder obtained when this number is divided by 375?
(a) 10 (b) 320
(c) 40 (d) None of these

Q.30. Chandrabhal adds first N natural numbers and finds the sum to be 1,850. But actually one numbers was added twice by mistake. Find the difference between N and that number.
(a) 40 (b) 33 (c) 60 (d) 17

Q.31. When I distribute a packet of chocolate to 7 students, I am left with 4 chocolates. When I distribute the same packet of chocolate to 11 students, I am left with 6 chocolates. How many chocolates will be left with me if I distribute the same packet of chocolate among 13 students (a packet of chocolate contains total number of chocolates N, $1000 < N < 1050$)?
(a) 2 (b) 0 (c) 6 (d) 7

Q.32. How many prime numbers are there between 80 and 105?
(a) 3 (b) 4 (c) 5 (d) 8

Q.33. If x and y are consecutive natural numbers in an increasing order, then which of the following is always true?
(a) $x^y > y^x$ (b) $y^x > x^y$
(c) $x^x > y^y$ (d) $y^y > x^x$

Q.34. What is the remainder when 5^{79} is divided by 7?
(a) 1 (b) 0 (c) 5 (d) 4

FOUNDATION

Q.1. LCM of two natural numbers is 590 and their HCF is 59. How many sets of values are possible?
(a) 1 (b) 2 (c) 5 (d) 10

Q.2. MUL has a waiting list of 5005 applicants. The list shows that there are at least 5 males between any two females. The largest number of females in the list could be
(a) 920 (b) 835 (c) 721 (d) 1005

Q.3. HCF of two numbers A and B is 24. HCF of two other numbers C and D is 36. What will be the HCF of A, B, C and D?
(a) 12 (b) 24
(c) 36 (d) 6

Q.4. How many zeroes will be there at the end of $25 \times 35 \times 40 \times 50 \times 60 \times 65$?
(a) 6 (b) 8 (c) 5 (d) 7

Q.5. What is the unit digit of $576847 \times 564068 \times 96467 \times 458576$?
(a) 2 (b) 4 (c) 6 (d) 8

Q.6. What is the unit digit of $1! + 2! + 3! + \dots + 99! + 100!$?
(a) 3 (b) 1 (c) 5 (d) 6

Q.7. How many divisors will be there of the number 1020?
(a) 12 (b) 20
(c) 24 (d) 36

Q.8. In question 7, what is the difference between the number of even divisors and number of prime divisors?
(a) 13 (b) 12
(c) 11 (d) None of these

Q.9. $N = 7!^3$. How many factors of N are multiples of 10?
(a) 736 (b) 1008 (c) 1352 (d) 894

Q.10. A number N has odd number of divisors. Which of the following is true about N?
(a) All the divisors of this number will be odd.
(b) There will be atleast $(N - 11)$ prime divisors.
(c) N will be a perfect square.
(d) Atleast one divisor of the number should be odd.

Q.11. How many zeroes will be there at the end of the expression $N = 2 \times 4 \times 6 \times 8 \times \ldots \times 100$?
(a) 10 (b) 12
(c) 14 (d) None of these

Q.12. How many zeroes will be there at the end of the expression $N = 10 \times 20 \times 30 \ldots \times 1000$?
(a) 1280 (b) 1300
(c) 1320 (d) None of these

Q.13. How many zeroes will be there at the end of the expression $N = 7 \times 14 \times 21 \times \ldots \times 777$?
(a) 24 (b) 25
(c) 26 (d) None of these

Q.14. The number from 1 to 33 are written side by side as follows: 123456 … 33. What is the remainder when this number is divided by 9?
(a) 0 (b) 1 (c) 3 (d) 6

Q.15. The number 444444 … (999 times) is definitely divisible by
(a) 22 (b) 44
(c) 222 (d) All of these

Q.16. Find the unit digit of $N = {}_{17}27!^{37!}$
(a) 1 (b) 3 (c) 7 (d) 9

Q.17. How many divisors of $N = 420$ will be of the form $4n + 1$, where n is a whole number?
(a) 3 (b) 4 (c) 5 (d) 8

Q.18. $N = 2^3 \times 5^3 \times 7^2$. How many sets of two factors of N are co-prime?
(a) 72 (b) 64
(c) 36 (d) None of these

Q.19. What is the unit digit of $2^{3^{4^5}}$?
(a) 2 (b) 4 (c) 8 (d) 6

Q.20. How many zeroes will be there at the end of $1003 \times 1001 \times 999 \times \ldots \times 123$?
(a) 224 (b) 217
(c) 0 (d) None of these

Q.21. How many zeroes will be there at the end of $36!^{36!}$?
(a) $7 \times 6!$ (b) $8 \times 6!$ (c) $7 \times 36!$ (d) $8 \times 36!$

Q.22. The number formed by writing any digit 6 times (e.g., 111111, 444444, etc) is always divisible by
(i) 7 (ii) 11 (iii)13
(a) i and ii (b) ii and iii
(c) i and iii (d) i, ii and iii

Q.23. What is the maximum value of HCF of $[n^2 + 17]$ and $(n + 1)^2 + 17]$?
(a) 69 (b) 85
(c) 170 (d) None of these

Q.24. What is the number of pairs of values of (x, y), which will satisfy $2x - 5y = 1$, where $x \leq 200$, and x and y are positive integers?
(a) 38 (b) 39 (c) 40 (d) 41

Q.25. $N = 2^3 \times 5^3$. How many sets of two factors of N are co-prime to each other?
(a) 12 (b) 24 (c) 23 (d) 11

Q.26. What is the sum of digits of the least multiple of 13, which when divided by 6, 8 and 12 leave 5, 7 and 11 as the remainder?
(a) 5 (b) 6 (c) 7 (d) 8

Q.27. What is the unit digit of $7^{11^{22^{33}}}$?
(a) 1 (b) 3 (c) 7 (d) 9

Q.28. What is the remainder when $(1! + 2! + 3! + \ldots 1000!)$ is divided by 5?
(a) 1 (b) 2 (c) 3 (d) 4

Q.29. $A = 3^{150} \times 5^{76} \times 7^{140}$, $B = 3^{148} \times 5^{76} \times 7^{141}$, $C = 3^{148} \times 5^{80} \times 7^{139}$, $D = 3^{151} \times 5^{80} \times 7^{142}$, then the order of A, B, C and D from largest to smallest is
(a) DACB (b) CDBA
(c) CDAB (d) DCAB

Q.30. The HCF of 0.3, 0.15, 0.225, 0.0003 is
(a) 0.0003 (b) 0.3 (c) 0.15 (d) 0.0015

Q.31. How many numbers between 1 to 550 are divisible by 5 but not by 9?
(a) 98 (b) 97
(c) 101 (d) None of these

Q.32. A and B are two distinct digits. If the sum of the two-digit numbers formed by using both the digits is a perfect square, what is the value of (A + B)?
(a) 9 (b) 11 (c) 13 (d) 17

Q.33. A number N = 897324P64Q is divisible by both 8 and 9. Which of the following is the value of P + Q?
i. 2 ii. 11 iii. 9
(a) either i or ii (b) either ii or iii
(c) either i or ii or iii (d) None of these

Direction for questions 34 and 35: Read the passage below and solve the questions based on it.

A = Set of first N positive numbers. There are 16 numbers in A which are divisible by both *X* and *Y*. There are 50 numbers in A which are divisible by *X* but not by *Y* and 34 numbers in A divisible Y but not by *X*.

Q.34. How many numbers in A are divisible by any of the two numbers?
(a) 100 (b) 50
(c) 200 (d) None of these

Q.35. How many numbers in *N* are divisible by *X*?
(a) 42 (b) 56
(c) 66 (d) None of these

Q.36. Nitin had forgotten his 6 digit bank account number but only remembered that it was of the form X515X0 and was divisible by 36. What was the value of X?
(a) 4 (b) 7 (c) 8 (d) 9

Q.37. Students from the Delhi Public School are writing their exams in Kendriya Vidyalaya. There are 60 students writing their Hindi exams, 72 students writing there French exam and 96 students writing their English exam. The authorities of the Kendriya Vidyalaya have to make arrangements such that each classroom contains equal number of students. What is the minimum number of classrooms required to accommodate all students of Delhi Pubic School?
(a) 19 (b) 38 (c) 13 (d) 6

Q.38. In the Jyotirmayi school, all classes started at 9.00 am. The school has three sections: primary, middle and secondary. Each class for the primary section lasts for half an hour, for the middle section for forty five minutes and for the secondary section for half an hour. A lunch break has to be given for the entire school when each of three sections have just finished a respective class and are free. What is the earliest time for the lunch break?
(a) 11.00 am (b) 10.30 am
(c) 12.00 pm (d) 12.30 pm

Q.39. In the firing range, 4 shooters are firing at their respective targets. The first, the second, the third and the fourth schooter hits the target once every 5s, 6s, 7s, 8s respectively. If all of them hit their target at 10.00 am, when will they hit their target together again?
(a) 10.14 am (b) 10.28 am
(c) 10.30 am (d) 10.31 am

Q.40. Two friends Harry and Jayesh were discussing about 2 numbers. They found the two numbers to be such that one was twice the other. However, both had the same number of prime factors while the larger one had 4 more factors than the smaller one. What are the numbers?
(a) 40,80 (b) 20,40
(c) 30,60 (d) 50,100

Q.41. To celebrate their victory in the World Cup, the Sri Lankans distributed sweets. If the sweets were distributed among 11 players, 2 sweets were left. When the sweets were distributed among 11 players, three extra's and 1 coach, even then 2 sweets were left. What is the minimum number of sweets in the box?
(a) 167 (b) 334 (c) 332 (d) 165

Q.42. The first 20 natural numbers from 1 to 20 are written next to each other to form a 31 digit number *N* = 12345 67891011121314151617181920. What is the remainder when this number is divided by 16?
(a) 0 (b) 4 (c) 7 (d) 9

Q.43. Two friends Kanti and Sridhar were trying to find the HCF of fifty distinct numbers. If they were finding the HCF of two numbers at a time, how many times this operation should be repeated to find the HCF of 50 numbers?
(a) 20 (b) 25 (c) 49 (d) 50

Q.44. How many zeroes will be there at the end of *N* = 18! + 19!?
(a) 3 (b) 4
(c) 5 (d) Cannot be determined

Q.45. Manish was dividing 2 numbers by a certain divisor and obtained remainders of 437 and 298 respectively. When he divides the sum of the two numbers by the same divisor, the remainder is 236. What is the divisor?
(a) 499 (b) 735
(c) 971 (d) None of these

Q.46. I purchased a ticket for the football match between France and Italy in the World Cup. The number on the ticket was a 5 digit perfect square such that the first and the last digit were the same and the 2nd and 4th digit were the same. If the 3rd digit was 3, then what was the ticket number?
(a) 24,342 (b) 12,321
(c) 21,312 (d) None of these

Q.47. How many integers *N* in the set of integers {1, 2, 3, … , 100} are there such that $N^2 + N^3$ is a perfect square?
(a) 5 (b) 7 (c) 9 (d) 11

Q.48. In a birthday party, all the children were given candy bars. All the children got three candy bars each except the child sitting at the end who got only 2 candy bars. If each child had been given only 2 candy bars there would have been 8 candy bars remaining. How many children were there and how many candy bars were distributed?
(a) 9, 26 (b) 6, 26 (c) 9, 18 (d) 6, 15

Q.49. A natural number N satisfies following conditions.
(A) Number is having all the 9's
(B) It is divisible by 13.
How many digits are there in N?
(a) 5 (b) 6 (c) 7 (d) 8

Q.50. What is the minimum number of identical square tiles required to cover a floor of dimension 3.78 m × 4.8 m?
(a) 2,520 (b) 3,780
(c) 5,040 (d) 6,480

Q.51. What is the smallest five-digit number which when divided by 7, 11 and 21 leaves a remainder of 3 in each case?
(a) 10,019 (b) 10,001
(c) 10,111 (d) 10,167

Q.52. A milkman has 3 jars containing 57 litres, 129 litres and 177 litres of pure milk respectively. A measuring can leaves the same amount of milk unmeasured in each jar after a different number of exact measurements of milk in each jar. What is the volume of largest such can?
(a) 12 litres (b) 16 litres
(c) 24 litres (d) 48 litres

Q.53. A boy was carrying a basket of eggs. He fell down and some of the eggs were broken. The boy has 10 eggs left with him. When asked by his mother how many eggs were broken, the boy could not recall. However he recalled that when he counted the total number of eggs 3 at a time 1 egg was left. When counted 4 at a time, 1 egg was left and when counted 5 at time a no egg was left. How many eggs were broken?
(a) 15 (b) 25 (c) 30 (d) 35

Q.54. How many prime numbers are there which, when divided by another prime number, gives a quotient which is same as the remainder?
(a) 0 (b) 1
(c) 2 (d) More than 2

Q.55. The number of natural numbers n such that $\frac{(n+1)^2}{n+7}$ is an integer, is
(a) 4 (b) 5
(c) 6 (d) None of these

Q.56. Find the product of all the factors of 3^{16}.
(a) 3^{33} (b) 3^{68} (c) 3^{136} (d) 3^{128}

Q.57. What is the remainder when 90^{91} is divided by 13?
(a) 0 (b) 7 (c) 12 (d) 1

Q.58. Find the remainder when the product of 10 consecutive natural numbers starting from 8641 is divided by 8640.
(a) 1 (b) 55 (c) 10 (d) 0

Q.59. Palindromes are numbers which when read from both the ends will give us the same number, for example, 123321 etc. In $(P)_N$, P is a palindrome of S digits, where S is even, written in base N. Which of the following number will definitely divide P?
(a) N – 1 (b) N + 1
(c) 2 N + 1 (d) None of these

Q.60. How many numbers are there between 400 and 600 in which 8 occurs only once?
(a) 36 (b) 18 (c) 19 (d) 38

Q.61. If $n^2 = 123.45654321$, which of the following is the exact value of n.
(a) 11.1001 (b) 11.1101
(c) 11.1111 (d) 11.1011

Q.62. A mule said to a horse, "If I take one sack off your back, my load will be double of yours and if you take one off my back our loads will be the same." How many sacks in all were they carrying?
(a) 5 (b) 7 (c) 12 (d) 14

Q.63. Divide 45 into 4 parts such that if the first is increased by 2, the second is decreased by 2, the third multiplied by 2 and the fourth divided by 2, the result is the same.
(a) 20, 8, 5, 12 (b) 12, 5, 20, 8
(c) 5, 8, 12, 20 (d) 8, 12, 5, 20

Q.64. Find the remainder when $3x^2 – x^6 + 31x^4 + 21x + 5$ is divided by $x + 2$.
(a) 10 (b) 12
(c) 11 (d) None of these

Q.65. Four prime numbers are in ascending order of their magnitudes. The product of the first three is 385 and that of last three is 1001. The largest given prime number is
(a) 11 (b) 13 (c) 17 (d) 19

Q.66. What is the remainder when 4^{44} is divided by 15?
(a) 1 (b) 2 (c) 3 (d) 4

Q.67. LCM of two integers P and Q is 211. What is the HCF of P and Q?
(a) 37 (b) 1
(c) 3 (d) Cannot be determined

Q.68. How many times does the digit 6 appear when we count from 11 to 400?
(a) 34 (b) 74 (c) 39 (d) 79

Q.69. In Q.68, how many numbers will be having 8 as its digit?
(a) 74 (b) 75 (c) 76 (d) 77

Q.70. S is a number formed by writing 8 for 88 times. What will be the remainder of this number when divided by 7?
(a) 4 (b) 5 (c) 8 (d) 1

Q.71. We are writing all the multiples of 3 from 111 to 324. How many times will we write digit 3?
(a) 18 (b) 19 (c) 21 (d) 22

Q.72. What is the remainder when 7 + 77 + 777 + 7777 + … (till 100 terms) is divided by 8?
(a) 0 (b) 2 (c) 4 (d) 6

Q.73. A number has exactly 15 composite factors. What can be the maximum number of prime factors of this number?
(a) 2 (b) 3 (c) 4 (d) 5

Q.74. $N = 204 \times 221 \times 238 \times 255 \times \ldots \times 850$. How many consecutive zeroes will be there at the end of this number N?
(a) 8 (b) 10 (c) 11 (d) 12

Q.75. 1st 126 natural numbers are put side by side in the ascending order to create a large number $N = 123456\ldots125126$. What will be the remainder when N is divided by 5625?
(a) 5126 (b) 26 (c) 126 (d) 156

Q.76. When a number S is divided by 3, 4 and 7 successively, remainders obtained are 2, 1 and 4 respectively. What will be the remainder when the same number is divided by 84?
(a) 43 (b) 53 (c) 63 (d) 73

Q.77. What is the remainder when $1714 \times 1715 \times 1717$ is divided by 12?
(a) 3 (b) 8 (c) 2 (d) 9

Q.78. $N^2 = 12345678987654321$. Find N.
(a) 101010101 (b) 11111
(c) 111111111 (d) 1000000001

Q.79. If a, b, c and d are distinct integers in the range 10 to 15 (both inclusive), the greatest value of $(a + b)(c + d)$ is—
(a) 750 (b) 731 (c) 700 (d) 729

Q.80. The smallest natural number which is a perfect square and is of the form abbb lies in between
(a) 1,000 to 2,000 (b) 2,000 to 3,000
(c) 3,000 to 4,000 (d) 4,000 to 5,000

MODERATE

Q.1. How many number of zeroes will be there at the end of 12! expressed in base 6?
(a) 4 (b) 5 (c) 6 (d) 7

Q.2. Find the remainder when $2222^{5555} + 5555^{2222}$ is divided by 7.
(a) 1 (b) 3 (c) 0 (d) 5

Q.3 LCM of first 100 natural numbers is N. What is the LCM of first 105 natural numbers?
(a) $5! \times N$ (b) 10403 N
(c) 105N/103 (d) 4N

Q.4. How many divisors of 10^5 end with a zero?
(a) 1 (b) 3 (c) 9 (d) 16

Q.5. Following expression holds true if we replace some of '+' signs by '×' signs.
$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 100$
How many '+' signs are needed to be replaced by '×'?
(a) 2 (b) 3 (c) 4 (d) 1

Q.6. In a particular country, all the numbers are expressed with the help of three alphabets a, b and c.
15 is written as abc.
6 is written as bc.
60 is written as bcbc.
How would one write 17 in that country?
(a) abb (b) bab (c) baa (d) aba

Q.7. When a certain two-digit number is added to another two digit number having the same digits in reverse order, the sum is a perfect square. How many such two-digit numbers are there?
(a) 4 (b) 6 (c) 8 (d) 10

Q.8. What is the remainder when ${}_{32}32^{32}$ is divided by 7?
(a) 2 (b) 3 (c) 4 (d) 6

Q.9. N is a 1001 digit number consisting of 1001 sevens. What is the remainder when N is divided by 1001?
(a) 7 (b) 700
(c) 777 (d) None of these

Q.10. Find four positive numbers such that the sum of the first, third and fourth exceeds the second by 8; the sum of the squares of the first and second exceeds the sum of the squares of the third and fourth by 36; the sum of the products of the first and second, and of the third and fourth is 42; the cube of the first is equal to the sum of the cubes of the second, third, and fourth.
(a) 2, 1, 9,3 (b) 2, 4, 6, 8
(c) 6, 5, 4, 3 (d) None of these

Q.11. Digital sum of a number is obtained by adding all the digits of a number until a single digit is obtained. Find the digital sum of 19^{100}.
(a) 1 (b) 4 (c) 7 (d) 9

Q.12. Find the HCF of $(2^{100} – 1)$ and $(2^{120} – 1)$.
(a) $2^{10} – 1$ (b) $2^{20} – 1$
(c) 1 (d) None of these

Q.13. What is the remainder when $22^{33} + 10^{35}$ is divided by 45?
(a) 2 (b) 11
(c) 8 (d) None of these

Q.14. An intelligence agency forms a code of two distinct digits selected from 0, 1,… ,9 such that the first digit of the code is non-zero. The code, handwritten on a slip, however, can potentially create confusion when read upside down, e.g., the code 91 may appear as 16. How many codes are there for which no such confusion can arise?
(a) 80 (b) 63
(c) 71 (d) None of these

Q.15. If p, p + 2, p + 4 are prime numbers, then the number of possible solutions for p is
(a) 0 (b) 1
(c) 2 (d) None of these

Q.16. Suppose N is an integer such that the sum of the digits of N is 2, and $10^9 < N < 10^{10}$. How many values of N are possible?
(a) 11 (b) 10 (c) 9 (d) 8

Q.17. Ten students solved a total of 35 questions in a maths Olympiad. Each question was solved by exactly one student. There is at least one student who solved exactly one problem, at least one student who solved exactly two problems and at least one student who solved exactly three problems. What is the minimum number of students who has/have solved at least five problems?
(a) 1 (b) 2
(c) 3 (d) None of these

Q.18. N! is having 37 zeroes at its end. How many values of N is/are possible?
(a) 0 (b) 1 (c) 5 (d) Infinite

Q.19. In the above question, how many values of N will be even?
(a) 0 (b) 2 (c) 3 (d) Infinite

Q.20. N! is having 30 zeroes at its end. How many values of N is/are possible?
(a) 0 (b) 1 (c) 5 (d) Infinite

Q.21. What is the remainder when $(1^1 + 2^2 + 3^3 + … + 100^{100})$ is divided by 4?
(a) 0 (b) 1 (c) 2 (d) 3

Q.22. A 3-digit number in which all the 3-digits are odd is such that if the cubes of the digit are added, the sum would be equal to the number itself. If one of the digit is 7, find the number.
(a) 171 (b) 371 (c) 575 (d) 775

Q.23. A teacher said that there were 100 students in his class, 24 of whom were boys and 32 were girls. Which base system did the teacher use in this statement?
(a) 9 (b) 5 (c) 6 (d) 8

Q.24. What is the remainder when 3^{450} is divided by 108?
(a) 3 (b) 1 (c) 27 (d) 81

Q.25. P is a natural number. 2P has 28 divisors and 3P has 30 divisors. How many divisors of 6P will be there?
(a) 35 (b) 40 (c) 45 (d) 48

Q.26. pqr is a three digit natural number such that $pqr = p^3 + q^3 + r^3$. What is the value of r?
(a) 0 (b) 1
(c) 3 (d) Cannot be determined

Q.27. There are two three-digit numbers. When one number is divided by another number, quotient obtained is 6 and remainder is 0. Sum total of both the numbers is a multiple of 504. What is the difference between the numbers?
(a) 720 (b) 360 (c) 120 (d) 420

Q.28. LCM of 12^{24}, 16^{18} and N is 24^{24}. Number of all the possible values of N = S. What is the value of S?
(a) 25 (b) 1,800
(c) 1,825 (d) None of these

Q.29. Each of P, Q, R and S equals either 0 or 1. It is given that
If Q = 0, then R = 1
If R = 0, then P = S
If S = 0, then P = 1
Assume R = 0, find the value of (P + Q + R + S)?
(a) 0 (b) 1 (c) 2 (d) 3

Q.30. When asked about his date of birth in 1996, Mayank replied that "last two digits of my birth year stands for my age." When Siddharth was asked about his age, he also replied the same. But Siddharth is older to Mayank. What is the difference in their age?
(a) 46 (b) 50
(c) 0 (d) Cannot be determined

Direction for questions 31 to 35: Read the passage below and solve the questions based on it.

There is a prison with 100 cells inside it. Cells are numbered from 1 to 100 and every cell is occupied by one prisoner only. One day jailer decides to release some of the prisoners and for this he defines an algorithm of 100 steps which follows:

Step 1 Reverse the position of all the cells which are divisible by 1.

Step 2 Reverse the position of all the cells which are divisible by 2.

Step 3 Reverse the position of all the cells which are divisible by 3.

………

……………….

………………………………..

Step 99 Reverse the position of all the cells which are divisible by 99.

Step 100 Reverse the position of all the cells which are divisible by 100.

Initially all the cells are closed. After executing all these steps, prisoners of all the cells which remain open are released.

Q.31. How many prisoners are released?
(a) 25 (b) 10
(c) 5 (d) None of these

Q.32. Which of the following cell number will be open at the end?
(a) Cell number 56 (b) Cell number 64
(c) Cell number 72 (d) Cell number 84

Q.33. Which of the following is true about the family of cell numbers N that will be open at the end?
(a) All the elements of N will be having only two factors including the number itself.
(b) All the elements of N will be having odd number of factors excluding the number itself.
(c) All the elements of N will be having odd number of factors including the number itself.
(d) None of these

Q.34. If number of prisoners in each cell is equal to the cell number, then how many prisoners will be released?
(a) 285 (b) 385
(c) 485 (d) None of these

Q.35. There are many islands on earth for which we do not have much information, or sometime we do not have any information about those islands. On such an island, which uses a system of S digits ($S \neq 10$) to write the numbers, selling price of a goat is Rs 1,143. Parul bought a goat and paid the shopkeeper Rs 1,150. Now shopkeeper returned Parul Rs 5. How many digits are used to write the numbers in that system?
(a) 9 (b) 8 (c) 7 (d) 6

Q.36. What is the value of N in the following expression $345_6 + 632_7 + 487_9 = (N)_5$?
(a) 11,412 (b) 11,214
(c) 10,412 (d) 21,412

Q.37. In a certain system of writing of numbers we write $(12\Phi)_4 = (124)_\Phi$, where Φ is the unit digit in the first place and number of digits in that system in the second place.
How many values of Φ are possible?
(a) 0 (b) 1 (c) 3 (d) Infinite

Q.38. Digital sum of a number is obtained by adding all the digits of a number until a single digit is obtained. What is the digital sum of 4444^{4444}?
(a) 2 (b) 6 (c) 7 (d) 3

Direction for questions 39 to 41: Read the passage below and solve the questions based on it.

$N = A^2 \times B^3 \times C^4$, where A, B and C are prime numbers.

Q.39. How many factors of N are perfect squares?
(a) 6 (b) 12
(c) 8 (d) None of these

Q.40. How many factors of N are cube of any natural number?
(a) 2 (b) 3 (c) 4 (d) 6

Q.41. How many factors of N will be having exactly 3 factors?
(a) 2 (b) 3
(c) 4 (d) None of these

Direction for questions 42 and 43: Read the passage below and solve the questions based on it.

M and N are two natural numbers such that M + N = 949. LCM of M and N is 2628.

Q.42. What is the HCF of M and N?
(a) 23 (b) 73
(c) 69 (d) None of these

Q.43. When M and N are divided by P, remainder obtained is same. How many values of P are possible?
(a) 0 (b) 1 (c) 2 (d) 3

Q.44. What is the difference between the largest four digit number and the smallest four digit number, both written in hexadecimal system?
(a) FFFF (b) EFFF
(c) EEEE (d) EFEF

Q.45. A ten digit number is said to be interesting if its digits are all distinct and it is a multiple of 11111. How many interesting integers are there?
(a) 1,001 (b) 101
(c) 10,001 (d) None of these

Q.46. Sum of a three-digit number and its mirror image is a multiple of 111. What is this sum (provided 6 is one of the digits of this three-digit number)?
(a) 999 (b) 666 (c) 888 (d) 777

Q.47. Consider the following statements:

(i) If $(2)_N \times (4)_N = (8)_N$, where $(X)_N$ is a number written in a system of writing having N digits, then N can have infinite values.

(ii) If $(4)_N \times (5)_N = (24)_N$, where $(X)_N$ is a number written in a system of writing having N digits, then N will have more than 1, but finite values.

(iii) If $(5)_N \times (6)_N = (3A)_N$, where $(X)_N$ is a number written in a system of writing having N digits and A is the unit digit of this number (3A) written in a system of writing the numbers having N digits, then A can have four values.

How many of the above written statements are true?

(a) 1 (b) 2
(c) 3 (d) None of these

Q.48. How many numbers between 1 to 250 can be represented in the form x^y, where $y>x>1$?

(a) 6 (b) 7 (c) 8 (d) 9

Q.49. What will be the ten's place digit in the following expression:

$1! + 2! + 3! + \ldots + 70!$?

(a) 1 (b) 3 (c) 5 (d) 6

Q.50. A two-digit number is K times the sum of its digits. When these numbers are interchanged, it equals N multiplied by the sum of digits. Which of the following is equal to N?

(a) K – 11 (b) 99 – K
(c) 11 – K (d) K – 99

Q.51. What is the remainder when $_{32}33^{34}$ is divided by 7?

(a) 4 (b) 2 (c) 3 (d) 1

Q.52. 1's are given 100 times, 2's are given 100 times and 3's are given 100 times. Now numbers are made by arranging these 300 digits in all possible ways. How many of these numbers will be perfect squares?

(a) 10101 (b) 1001001
(c) 100010001 (d) None of these

Q.53. Which of the following number is the greatest number dividing a family of numbers $(N^5 - N)$, where N is any natural number?

(a) 0 (b) 15
(c) 30 (d) 120

Q.54. When Rahul Ghosh distributes Rs 5 to each of his employees, he is left with Re 1. And when he distributes Rs 6 to each of his employees, he is left with Rs 5. What is the sum of digits of the minimum possible sum with Rahul Ghosh if he is having more than Rs 100?

(a) 1 (b) 2 (c) 3 (d) 4

Q.55. The integers 34041 and 32506,when divided by a three digit integer N, leave the same remainder. What can be the value of N?

(a) 289 (b) 307 (c) 317 (d) 319

Q.56. In the above question, how many values of N is/are possible?

(a) 2 (b) 1
(c) 4 (d) None of these

Q.57. What is the remainder when 77777... upto 56 digits is divided by 19?

(a) 1 (b) 7 (c) 9 (d) 13

Q.58. There are 3 consecutives odd integers such that the product is a prime number. Find the largest of the prime number.

(a) 1 (b) 3 (c) 5 (d) 7

Q.59. A square floor of the dimensions 72 cm × 72 cm has to be laid with rectangular tiles whose length and breadth are in the ratio 3 : 2. What is the difference between the maximum number of tiles and minimum numbers of tiles, given that the length and the breath are integers?

(a) 858 (b) 864 (c) 868 (d) 872

Q.60. A man took a 5 digit number ending in 9 and raised it to an even power greater than 50. He then multiplied it with 17 raised to a multiple of 4. What is the last digit of the resulting number?

(a) 1 (b) 9 (c) 3 (d) 7

Q.61. Rahul took an odd number of prime numbers and added them. He found the result to be even. Which of the following could be concluded?

(a) The numbers include an even number.
(b) The numbers do not include an even number.
(c) There are even number of even numbers.
(d) None of the above.

Q.62. When asked as to how many guavas Lovely has, she replies, "if you add three quarters of the number of guavas that I have to three quarters of one, guava, you will get the numbers of guavas I have". How many guavas does she have?

(a) 2 (b) 5
(c) 7 (d) None of these

Direction for questions 63 to 65: Read the passage below and solve the questions based on it.

Rohan purchased some pens, pencils and erasers for his young brothers and sisters for the ensuing examinations. He had to buy at least 11 pieces of each items in a manner that the number of pens purchased be more than the number of pencils, which is more than the number of erasers. He purchased a total of 38 pieces.

Q.63. How many erasers did Rohan purchase?

(a) 11 (b) 10
(c) 8 (d) Cannot be determined

Q.64. If each eraser costs Rs 3, each pencil Rs 2 each pen Rs 10, what is the maximum amount that Rohan could have spent?
(a) 207 (b) 255 (c) 288 (d) 300

Q.65. If the number of pencils cannot be equally divided among his 4 brothers and sisters, how many pens did he purchase?
(a) 11 (b) 12 (c) 13 (d) 14

Q.66. What is the remainder when [100! + 1] is divided by 101?
(a) 100 (b) 1 (c) 0 (d) 2

Q.67. A is a set of first ten natural numbers. B is a subset of A such that B consists of exactly 2 numbers, both co-primes to each other. Find the maximum number of such possible subsets.
(a) 29 (b) 28 (c) 33 (d) 31

Q.68. When written in the lowest term, rational numbers A and B, have denominators 60 and 70 respectively. Now A and B have been added. What is the smallest possible denominator of (A + B)?
(a) 72 (b) 84 (c) 105 (d) 148

Direction for questions 69 and 70: Read the passage below and solve the questions based on it.

The multiplication of two numbers is shown below.

$$\begin{array}{r} AD4 \\ \underline{\times E} \\ \underline{A206} \end{array}$$

where A, D, and E are all distinct digits.

Q.69. The value of D is
(a) 3 (b) 9 (c) 2 (d) 7

Q.70. The value of A + E is
(a) 6 (b) 9 (c) 11 (d) 10

Q.71. P, Q, R, S and T are five prime numbers, where P < Q < R < S < T. It is also given that P + Q + R + S + T = 482. What is the value of P^5?
(a) 243 (b) 32
(c) 16807 (d) More than one value

Q.72. If n is a natural number greater than 1, then $1075^n - 1075^{n-1}$ is not divisible by
(a) 15 (b) 23 (c) 43 (d) 179

Direction for questions 73 and 74: Read the passage below and solve the questions based on it.

(a, b) is an ordered pair such that a and b are two-digit numbers a is the number obtained by interchanging the digits of b.

Q.73. How many ordered pairs are there such that $(a+b)$ is a perfect square?
(a) 4 (b) 8
(c) 3 (d) None of these

Q.74. How many ordered pairs are there such that $(a - b)$ is a perfect square greater than zero?
(a) 3 (b) 5
(c) 13 (d) None of these

Q.75. N is an even number 8 when written in the decimal system. If S is the sum of the digits of N, when it is written in base 7, we can surely say that S would always be
(a) Even (b) Odd
(c) Composite (d) Both (a) and (c)

ADVANCED

Q.1. Find the last non zero digit of 96!.
(a) 2 (b) 4 (c) 6 (d) 8

Q.2. What will be remainder when 1212121212… 300 times, is being divided by 99?
(a) 18 (b) 81 (c) 54 (d) 36

Q.3. Two players A and B are playing a game of putting '+' and '–' signs in between any two integers written from 1 to 100. A starts the game by putting a plus sign anywhere between any two integers. Once all the signs have been put, the result is calculated. If it is even then A wins and if it is odd then B wins, provided they are putting signs by taking turns one by one and either of them can put any sign anywhere between any two integers. Who will win at the end?
(a) A (b) B
(c) Either A or B (d) Cannot be determined

Q.4. What is the remainder when 2^{100} is divided by 101?
(a) 1 (b) 100
(c) 99 (d) None of these

Q.5. If S = $\{X_1, X_2, X_3 \dots X_n\}$, where $n > 0$, $X_1 = 1$ is a set of positive numbers where $X_n = X_{n-1} + 2n - 1$. Which of the following sets of numbers does S resemble?
(a) Fibonacci (b) Cube
(c) Square (d) Prime

Q.6. A three-digit number ABC is a perfect square and the number of factors of this number is also a perfect square. If (A + B + C) is also a perfect square, then what is the number of factors of the 6-digit number ABCABC?

(a) 32 (b) 52
(c) 72 (d) Cannot be determined

Q.7. How many divisors of 10^5 will have atleast one zero at its end?

(a) 9 (b) 12 (c) 15 (d) 25

Q.8. Let $V_1, V_2, V_3, \ldots V_{100}$ be hundred positive integers such that $V_i + V_{i+1} + V_{i+2} + V_{i+3} = K$, where K is a constant and $i = 1, 2, 3, \ldots 97$. If $V_3 = 9$, then what is the value of V_{99}?

(a) 9 (b) K – 9
(c) (K/2 – 9) (d) Cannot be determined

Q.9. In the above question, if $V_5 = 7$, then what is the value of V_{90}?

(a) 7 (b) K – 7
(c) (K – 7)/2 (d) Cannot be determined

Q.10. *abcd* is a four-digit number in base 7 such that 2 (*abcd*) = *bcda*. (*a, b* ≠ 0) Find the value of *a*.

(a) 1 (b) 2
(c) 3 (d) Cannot be determined

Q.11. If K is any natural number, such that $100 \le K \le 200$, how many values of K exist such that K! has 'z' zeroes at its end and (K + 2)! has 'z + 2' zeroes at its end?

(a) 2 (b) 4
(c) 6 (d) None of these

Q.12. Tatto bought a notebook containing 96 pages leaves and numbered them which came to 192 pages. Tappo tore out the latter 25 leaves of the notebook and added the 50 numbers she found on those pages. Which of the following is not true?

(a) She could have found the sum of pages as 1990
(b) She could have found sum of pages as 1275
(c) She could have got sum of pages as 1375
(d) None of these

Direction for questions 13 to 15: Read the passage below and solve the questions based on it.

There are 50 integers a1, a2, a3, … ,a50; not all of them necessarily different. Let the greatest integer of these 50 integers be referred to as G and smallest integer be referred to as L. The integers a1 to a24 form a sequence S1 and the rest form a sequence S2. Each member of S1 is less than or equal to each member of S2.

Q.13. All values in S1 are changed in sign, while those in S2 remain unchanged. Which of the following statements is true?

(a) Every member of S1 is greater than or equal to every member of S2
(b) G is in S1
(c) If all the numbers originally in S1 and S2 had the same sign, then after the change of sign, the largest number of S1 and S2 is in S1
(d) None of these

Q.14. Elements of S1 are in ascending order and those of S2 are in descending order. a24 and a25 are interchanged then which of the following is true?

(a) S1 continues to be in ascending order
(b) S2 continues to be in descending order
(c) Both (a) and (b)
(d) Cannot be determined

Q.15. Every element of S1 is made greater than or equal to every element of S2 by adding to each element of S1 an integer *x*. Then, *x* cannot be less than

(a) 2^{10}
(b) The smallest value of S2
(c) The largest value of S2
(d) (G-L)

Q.16. Twenty-five boxes of sweets are delivered to Mr Roy's home. Mr Roy had ordered sweets of three different types. What is the minimum number of boxes of sweets which are having sweets of same type?

(a) 1 (b) 8
(c) 9 (d) Cannot be determined

Q.17. A warehouse contains 200 shoes of size 8, 200 shoes of size 9 and 200 shoes of size 10. Of these 600 shoes, there are 300 left shoes and 300 right shoes. What is the minimum number of usable shoes?

(a) 50 (b) 100
(c) 200 (d) None of these

Q.18. A teacher was doing some calculation exercise on the blackboard. When the teacher went out, a naughty student Chunmun erased some of the numbers written on the blackboard. Now it appeared like this

$$\begin{array}{r} 2\,3\,\_\,5\,\_ \\ +\,1\,\_\,6\,4\,2 \\ \hline 4\,2\,4\,2\,3 \end{array}$$

When teacher entered the room, he realized that still this calculation was right, but in some other system of writing (i.e., not 10). How many digits are there in that system?

(a) 11 (b) 9 (c) 7 (d) 8

Q.19. Tatto, Tappo and Bubbly were solving problems from a problem book. Each solved exactly 60 problems, but they solved only 100 problems altogether. Any problem is known as "easy" if it was solved by all of them, and "difficult" if it was solved by only one of them. What is the difference between the number of "difficult" problems and number of "easy" problems?

(a) 10 (b) 20
(c) 30 (d) 40

Q.20. LCM of two numbers A and B = $P^x \times Q^y$, where P and Q are prime numbers and x and y are positive whole numbers. How many set of values are possible for A and B?

(a) $xy\,(x+y)$ (b) $xy\,(x-y)$

(c) $x^2y^2\,(x+y)$ (d) None of these

Q.21. When 7179 and 9699 are divided by another natural number N, remainder obtained is same. How many values of N will be ending with one or more than one zeroes?

(a) 24 (b) 124

(c) 46 (d) None of these

Q.22. There exists a 5 digit number N with distinct and non-zero digits such that it equals the sum of all distinct three digit numbers whose digits are all different and are all digits of N. Then the sum of the digits of N is a necessarily.

(a) Perfect square (b) Cube

(c) Even (d) None of these

Q.23. Starting with 1, positive integers are written one after the other. What is the 40,0000th digit that will be written?

(a) 3 (b) 6

(c) 8 (d) None of these

Q.24. Which of the following would always divide a six-digit number of the form ababab?

(a) 10,101 (b) 11,111

(c) 10,001 (d) None of these

Q.25. If in the number system of a particular country, 25 means 5 tens and 2 units, 467 means 7 hundreds, 6 tens and 4 units. Then find the value of 173 × 425?

(a) 4,04,491 (b) 7,35,255

(c) 6,22,744 (d) 5,25,376

Q.26. Let A be the set of integers N such that

i. $100 \le N \le 500$

ii. N is even

iii. N is divisible by either 2 or 3 or 4 but not by 7.

How many elements are there in set A?

(a) 171 (b) 172 (c) 170 (d) 173

Q.27. Three distinct prime numbers, less than 10 are taken and all the numbers that can be formed by arranging all the digits are taken. Now, difference between the largest and the smallest number formed is equal to 495. It is also given that sum of the digits is more than 13. What is the product of the numbers?

(a) 30 (b) 70 (c) 105 (d) 315

Q.28. What is the remainder when ${}_{55}56^{57}$ is divided by 17?

(a) 1 (b) 4 (c) 13 (d) 17

Q.29. What is the remainder when ${}_{20}51^{97}$ is divided by 17?

(a) 10 (b) 9 (c) 15 (d) 7

Q.30. $[111\ldots111(200\text{ digits}) - 222\ldots22(100\text{ digits})]^{1/2}$ is equal to

(a) 1313 … 1313(100 digits)

(b) 2121…2121(100 digits)

(c) 1111…1111(100 digits)

(d) 3333…333(100 digits)

Q.31. N is a number which when divided by 10 gives 9 as the remainder,
when divided by 9 gives 8 as the remainder,
when divided by 8 gives 7 as the remainder,
when divided by 7 gives 6 as the remainder,
when divided by 6 gives 5 as the remainder,
when divided by 5 gives 4 as the remainder,
when divided by 4 gives 3 as the remainder,
when divided by 3 gives 2 as the remainder,
when divided by 2 gives 1 as the remainder.
What is N?

(a) 2519 (b) 841 (c) 839 (d) 2521

Q.32. What is the remainder when $(10^3 + 9^3)^{752}$ is divided by 12^3?

(a) 1 (b) 729 (c) 752 (d) 1000

Q.33. $f(x, x) = x^x$ and $f(f(x, x) = {}_{x}x^{x}$ and so on its keeps on going. What is the value of $f(f(f(f(7,7))))$ is divided by 5?

(a) 1 (b) 2 (c) 3 (d) 4

Q.34. The students of class 10th of Morgan High School took a test which had a maximum of 50 marks. The teacher misplaced the text notebooks of two of the students— Robin and Garry, but remembered that Garry had scored something between 10 and 15 and Robin something between 32 to 40. She also remembered that the product of the marks obtained by the two students was also equal to 10 times the marks obtained by two of them. How many marks did Garry scored?

(a) 11 (b) 12 (c) 13 (d) 14

Q.35. The History teacher was referring to a year in the 19th century. Rohan found an easy way to remember the year. He found that the number, when viewed in a mirror, increased 4.5 times. Which year was the teacher referring to?

(a) 1,801 (b) 1,810

(c) 1,818 (d) More than one value

Q.36. Srini wrote his class 10th board examination this year. When the result came out he searched for his hall ticket to see his roll number but could not trace it. He could remember only the first three digits of the 6 digit number as 267. His father, however, remembered that the number was divisible by 11. His mother gave the information that the number was also divisible by 13. They tried to recollect the number when all of a sudden Srini told that the number was a multiple of 7. What was the unit digits of the number?

(a) 5 (b) 7

(c) 2 (d) Cannot be determined

Q.37. Prof. Mathur and Prof. Singh attended the All India Historian's meet last week. Prof. Mathur told Prof. Singh, "I found out that your teaching experience is twice that of mine". Prof. Singh replied in the affirmative. Prof. Mathur continued, "But last time when both of us came for the same meet, I remember that your teaching experiencr was thrice that of mine". "That was 2 years ago." Prof. Singh said. How many years has Prof. Singh been working?
(a) 8 yrs (b) 10 yrs (c) 12 yrs (d) 16 yrs

Direction for questions 38 and 39: Read the passage below and solve the questions based on it.

ABCDEF is a 6-digit number with distinct digits. Further, the number is divisible by 11 and the sum of its digits is 24. Further A > C > E and B > D > F.

Q.38. The sum A + C + E is equal to
(a) 12 (b) 6
(c) 8 (d) Cannot be determined

Q.39. A + B is always
(a) 10 (b) 9
(c) 6 (d) Cannot be determined

Q.40. Raju had to divide 1080 by N, a two-digit number. Instead, he performed the division using M which is obtained by reversing the digits of N and ended up with a quotient which was 25 less than what he should have obtained otherwise. If 1080 is exactly divisible both by N and M, find the sum of the digits of N.
(a) 6 (b) 8
(c) 9 (d) None of these

Q.41. Let S = {1, 2, 3,… n} be a set of N natural numbers. Let T be a subset of S such that the sum of any three elements of T is not less than N. Find the maximum number of elements in any such subset T for N = 40?
(a) 26 (b) 27
(c) 28 (d) None of these

Q.42. The last digit of the LCM of $(3^{2003} - 1)$ and $(3^{2003} + 1)$ is
(a) 8 (b) 2 (c) 4 (d) 6

Q.43. a, b and c are positive integers such that, $a + b + c = 2003$. Let $E = (-1)^a + (-1)^b + (-1)^c$. Find the number of possible values of E.
(a) 2004 (b) 3 (c) 1003 (d) 2

Q.44. Ajay took a 4-digit number in base 5 notation. He subtracted the sum of the digits of the numbers from the number. From the result, he struck off one of the digits. The remaining 3 digits were 1, 0 and 2. Then the digit struck off by Ajay was:
(a) 2 (b) 1
(c) 4 (d) Cannot be determined

Direction for questions 45 and 46: Read the passage below and solve the questions based on it.

N is a single digit integer satisfying the following two conditions.
i. N is non-zero.
ii. N is the right most digit of the number $(n!)^4$, where n is a natural number greater than 1.

Q.45. What is the number of possible values of N?
(a) 1 (b) 2
(c) 0 (d) None of these

Q.46. If condition (a) is relaxed, the number of possible values of N is.
(a) 1 (b) 2
(c) 0 (d) More than 2

Q.47. A teacher wrote a number on the blackboard and the following observations were made by the students
The number is a four-digit number.
The sum of the digits equals the product of the digits.
The number is divisible by the sum of the digits.
The sum of the digits of the number is
(a) 8 (b) 10 (c) 12 (d) 14

Q.48. The N of odd numbers are taken. Product of these odd numbers is of the form $(4n + 1)$, where n is any natural number Which of the following is true regarding the number of numbers?
(a) There must have been an odd number of numbers of the form $4n + 1$
(b) There must have been an even number of numbers of the form $4n + 1$
(c) There must have been an even number of numbers of the form $4n + 3$
(d) None of these

Q.49. 16 students were writing a test in a class. Rahul made 14 mistakes in the paper, which was the highest number of mistakes made by any student. Which of the following statements is definitely true?
(a) At least two students made the same number of mistakes
(b) Exactly two students made the same number of mistakes
(c) At most two students made the same number of mistakes
(d) All students made different number of mistakes.

Q.50. The sum of the factorials of the three-digits of a 3-digit number is equal to the three-digit number formed by these three digits, taken in the same order. Which of the following is true of the number of such three-digit numbers, if no digit occurs more than once?
(a) No such number exists
(b) Exactly one such number exists
(c) There is more than one such number, but they are finite in number
(d) There are infinite such numbers

Direction for questions 51 and 52: Read the passage below and solve the questions based on it.

The inhabitants of planet Romeo, a remote planet in the milky way, use a number system which is similar to the decimal system used here on the earth, except that it has eleven distinct digits instead of the usual ten digits. The extra digit is an alien digit called x, which is inserted between the digits 7 and 8. Now $8 - 7 = 2$ and x is the digit, which is equidistance from 7 and 8, i.e., $x - 7 = 8 - x = 1$. All the algebric signs and operations carry the same meaning as in the usual sense.

Q.51. Romeo, an inhabitant of planet Romeo, evaluated an expression and arrived at an answer of '$6x$'. Then which of the following can be the expression that Romeo evaluated?
(a) $x^2 + x - 1$ (b) $x^2 + x + 1$
(c) $x^2 + x + 2$ (d) None of these

Q.52. What is the decimal equivalent of the two digit number '$9x$'?
(a) 98 (b) 107
(c) 118 (d) None of these

Q.53. If P is the product of four consecutive positive integers, then which of the following statements is not true?
(a) P is perfectly divisible by 24
(b) P is not a perfect square
(c) (P + 1) is a perfect square
(d) (P – 2) is a perfect square

Q.54. Let S be a two-digit number such that both S and S^2 end with the same digit and none of the digits in S equals zero. When the digits of S are written in the reverse order, the square of the new number so obtained has the last digit as 6 and is less than 3000. How many values of S are possible?
(a) 3 (b) 4 (c) 5 (d) 6

Q.55. How many different four digit numbers are there in the octal (Base 8) system, expressed in that system?
(a) 3584 (b) 2058 (c) 6000 (d) 7000

Q.56. How many numbers less than or equal to 500 are there, each of which is the product of more than three distinct prime numbers?
(a) 3 (b) 4 (c) 5 (d) 6

Q.57. How many numbers below 100 can be expressed as a difference of two perfect squares in only one way?
(a) 15 (b) 25 (c) 35 (d) 45

Q.58. The LCM of the two quotients, obtained when two numbers a and b, are divided by their HCF is 77. LCM of another pair of quotients, when other two numbers c and d have the same HCF as that of a and b, and when divided by it, is 221. Which of the following could be, ratio of the sum of a and b to that of the difference between c and d?
(a) 4 : 17 (b) 11:4 (c) 2:9 (d) 39:2

Q.59. Let p, q and s be integers such that $p^2 = sq^2$. Then it follows that
(a) p is an even number
(b) if s divides p, then s is a perfect square
(c) s divides p
(d) q^2 divides p

Q.60. Let N be a positive integer not equal to 1. Then none of the numbers 2, 3,…., N is a divisor of (N! – 1). Thus, we can conclude that
(a) (N! – 1) is a prime number
(b) At least one of the numbers $N + 1, N + 2, \dots, N! - 2$ is a divisor of (N! – 1)
(c) The smallest number between N and N! which is a divisor of (N! + 1), is a prime number
(d) None of the foregoing statement is necessarily correct

TRUE/FALSE

---

1. A natural number M is divided by another natural number N and the remainder obtained = R ≠ 0 and M > N. It is always possible to find out the exact remainder when M is divided by P (where P is a factor of N).
State whether True or False.

2. A natural number M is divided by another natural number N and the remainder obtained = R “‘ 0 and M > N. It is always possible to find out the exact remainder when M is divided by P (where P is a multiple of N).
State whether True or False.

3. A, B, C are three natural numbers. If we know the LCM of A, B and LCM of B, C separately, then it is always possible to find out the LCM of A, B, C.
State whether True or False.

4. A, B, C are three natural numbers. If we know the HCF of A, B and HCF of B, C separately, then it is always possible to find out the HCF of A, B, C.
State whether True or False.

5. A, B, C, D are four natural numbers. If we know the LCM of A, B and LCM of C, D separately, then it is always possible to find out the LCM of A, B, C, D.

   State whether True or False.

6. A, B, C, D are four natural numbers. If we know the HCF of A, B and HCF of C, D separately, then it is always possible to find out the HCF of A, B, C, D.

   State whether True or False.

7. If we know the total number of odd factors of a number, then we can always find out the total number of factors of that number.

   State whether True or False.

8. If we know the total number of even factors of a number, then we can always find out the total number of factors of that number.

   State whether True or False.

9. If a number is odd, then it cannot have total number of factors as an even number.

   State whether True or False.

ANSWERS

Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(d)	(d)	(d)	(d)	(b)	(a)	(c)	(b)	(a)	(a)	(d)	(c)	(d)	(d)	(b)	(a)	(b)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Answer	(b)	(c)	(d)	(b)	(c)	(d)	(d)	(b)	(c)	(a)	(b)	(c)	(d)	(c)						

Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(a)	(a)	(a)	(a)	(c)	(b)	(b)	(c)	(b)	(d)	(c)	(c)	(c)	(a)	(b)	(d)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(a)	(c)	(b)	(d)	(c)	(c)	(d)	(a)	(b)	(b)	(a)	(a)	(c)	(c)	(a)	(b)	(a)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(a)	(a)	(c)	(b)	(a)	(b)	(c)	(a)	(b)	(c)	(d)	(c)	(a)	(b)	(a)	(c)	(c)	(d)	(b)	(a)
Q. No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
Answer	(c)	(c)	(d)	(d)	(b)	(a)	(b)	(d)	(b)	(b)	(d)	(d)	(a)	(b)	(c)	(b)	(c)	(c)	(d)	(a)

Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(b)	(c)	(b)	(a)	(c)	(c)	(b)	(c)	(a)	(b)	(c)	(c)	(b)	(b)	(a)	(c)	(c)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(b)	(c)	(d)	(a)	(d)	(a)	(c)	(c)	(b)	(b)	(b)	(c)	(b)	(b)	(a)	(a)	(c)	(b)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(d)	(b)	(c)	(b)	(d)	(c)	(a)	(b)	(a)	(c)	(d)	(d)	(c)	(b)	(b)	(c)	(a)	(b)	(a)	(a)
Q. No.	61	62	63	64	65	66	67	68	69	70	71	72	73	74	75					
Answer	(a)	(d)	(a)	(a)	(c)	(c)	(d)	(b)	(a)	(d)	(b)	(b)	(b)	(b)	(d)					

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(a)	(a)	(c)	(d)	(d)	(a)	(d)	(d)	(c)	(a)	(d)	(a)	(d)	(c)	(b)	(c)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(a)	(d)	(a)	(a)	(b)	(b)	(a)	(a)	(d)	(a)	(a)	(c)	(d)	(c)	(b)	(a)	(a)	(c)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60
Answer	(d)	(c)	(d)	(b)	(a)	(b)	(a)	(c)	(a)	(b)	(c)	(c)	(d)	(a)	(d)	(b)	(c)	(d)	(b)	(d)

■■■ True/False

1. True

   To find out the remainder when M divided by P, we simply need to divide R by P.

2. False

   If we divide M by P, we would get a range of remainders in terms of R and not the exact value of remainder in all the cases.

3. True

   LCM, by its meaning, is the lowest number divisible by all the numbers constituting it. Final LCM will be the LCM of the pairs of numbers.

4. True

   HCF, by its meaning, is the highest number that can divide the numbers constituting it. Final HCF will be the HCF of the pairs of numbers.

5. True

   LCM, by its meaning, is the lowest number divisible by all the numbers constituting it. Final LCM will be the LCM of the pairs of numbers.

6. True

   HCF, by its meaning, is the highest number that can divide the numbers constituting it. Final HCF will be the HCF of the pairs of numbers.

7. False

8. False

9. False

   Total number of factors do not have any relationship with the number being odd or even. For example, all the perfect squares (irrespective of being odd or even) have total number of factors = Odd number.

HINTS AND SOLUTIONS

Moderate

1. In case of decimal system, we obtain 10 by multiplying 5 and 2, and then to find the number of zeroes, we search the exponents of 5. In case of base 6, 10 will be obtained by multiplying 3 and 2. So, here we will check for the exponents of 3 to know about the number of zeroes. And obviously it is 5 [12/3 + 12/9].

2. The remainder obtained when $2222^{5555} + 5555^{2222}$ is divided by 7 will be the same as the remainder when $3^{5555} + 4^{2222}$ is divided by 7. Now find the individual remainder and solve it.

3. If we look at the numbers $100 < N \leq 105$, we see only 101 and 103 do not have their factors in N (because these are primes). So, obviously the new LCM will be $101 \times 103 \times N$.

4. $10^5 = 2^5 \times 5^5$

   Now all the factors of 10^5 which will end in one zero will be zero power of 2 and 1-5 powers of 5 and vice-versa. This will be equal to 9.

5. $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = 55$

   So, by replacing the signs we need to make 45 extra. This is possible only if we write in this way:

   $1 \times 2 + 3 \times 4 + 5 + 6 + 7 \times 8 + 9 + 10 = 55 + 45 = 100$

6. The key is the fact that in this country only three symbols are used to write numbers.

   So, $6 = (20)_3 = (bc)_3$

   So, $b = 2$, $c = 0$ and $a = 1$

   $17 = (122)_3 = abb$

8. Remainder of ($_{32}32^{32}$ divided by 7) = Remainder of ($_{4}32^{32}$ divided by 7)
   Now find cyclicity of remainder of ($_{4}32^{n}$ divided by 7).
   Remainder when $_{4}32^{1}$ divided by 7 = 2
   Remainder when $_{4}32^{2}$ divided by 7 = 4
   Remainder when $_{4}32^{3}$ divided by 7 = 2
   So, the cyclicity is 2, 4, 2, 4 and so on.
   For every even value of n, remainder = 4
   So, answer = (d)

9. $1001 = 7 \times 11 \times 13$
   We know that any digit written 6 times consecutively (like 111111, or 222222 etc.) will be divisible by 3, 7, 11, 13, 37. So, this question is– what is the remainder when 11111 is divided by 1001. Find it out by actual division method.

10. Go through options.

12. Use the formula given in the concepts.

14. Digits which can create confusion = 1, 6, 8, 9 (0 cannot create confusion because passwords has to be two-digit numbers).
    Total two digit numbers with distinct digit = 81
    Two digit numbers created by 1, 6, 8, 9 = 12
    So, total numbers left = 69
    But 69 and 96 wont create confusion (it looks same upside down), so total numbers = 71

16. See the solution of CAT 04 given at the end of this book.

18. N! is having 37 zeroes at its end, so N = 150 (can be arrived at by a guess).
    Obviously, $150 \leq N < 155$ is the answer.

19. Use the data of the Q.18.

23. The question is: In which system of writing, 24 + 32 = 100. Go through options.

25. 2P is having 28 (7×4) divisors but 3P is not having a total divisor which is divisible by 7. So, the first part of the number P will be 2^5.
    Similarly, 3P is having 30 (3×10) divisors but 2P does not have a total divisor which is divisible by 3. So, 2nd part of the number P will be 3^3.
    So, $P = 2^5 \times 3^3$.

26. *pqr* can be 370 or 371. So, it is not possible to arrive at a unique answer.

30. Mayank DOB = 1948 and Siddharth DOB = 1898

Questions 31 to 35

All the numbers which will have odd number of factors will be open at the end. Now all the perfect squares only have the total number of divisors = Odd factors

Now solve the questions.

36. Convert all the numbers to base 10 and then find the value.

37. $(124)_\Phi$ shows that value of Φ should be 5 or more than 5 (since 4 is a digit used in this system).
    $(12\Phi)_4$ shows that value of Φ can be at max 3 (since the system = 4).
    Now this is a contradiction. So, no such value exists.
    $N = A^2 \times B^3 \times C^4$, where A, B and C are prime numbers.

39. For the factors to be perfect squares, assume $A^2 = X$, $B^2 = Y$ and $C^2 = Z$

So, for the factors to be perfect square, we can write $N = XYZ^2$
So, total number of factors = (1 + 1) (1 + 1) (2 + 1) = 12

41. All the squares of prime numbers give 3 factors. Use this to solve the question.

44. Largest number = FFFF and smallest number = 1000

52. The digital sum of any perfect square can be 1, 4, 9, 7, 0 only. 100 times 1's are given, 100 times 2's are given and 100 times 3's are given. Digital sum of all these digits taken all at a time will be 6. So, this will not give any perfect square.

55. The difference will be = 34041 – 32506 = 1535

56. Factorize $1535 = 307^1 \times 5^1$

[illegible]

63. Number of pen > number of pencils > number of erasers
Minimum number of each of the items = 11. So, number of erasers = 11 [If we take it 12, then minimum value of pencils = 13, and minimum value of pens = 14, sum of these three exceeds 38].

64. Number of pens = 15, number of pencils = 12, and number of erasers = 11.

65. Different possibilities for the number of pencils = 12 or 13. Since it cannot be divided into his 4 brothers and sisters, it has to be 13. In that case, number of pens = 14

66. Use Wilson's Theorem.

71. P + Q + R + S + T = 482. Sum of five prime numbers is even possible only if four of these are odd and one is even (all these cannot be odd). So, P = 2.

75. Assume some values of N and check it out.

Advanced

1. Factorize 96! into prime factors. Find the unit digit of all the factors individually and multiply to get the unit digit of 96!.
$96! = 2^{92} \times 3^{46} \times 5^{22} \times \ldots$
Now 5^{22} and 2^{22} can be eliminated, since these will result in zeroes. Find the unit digit of all the remaining.

2. This number 1212121212... 300 times is divisible by 9. So, we can write 1212121212...300 times = 9 N, where N is the quotient obtained when divided by 9. Now this question is like –

$$\frac{1212121212\ldots300 \text{ times}}{99}$$

$$= \text{Remainder } \frac{9}{9} \times \frac{134680\ldots\text{written } 50 \text{ times}}{11}$$

$[121212 = 13468 \times 9]$
Now we will have to find the reminder obtained when 134680134680... 50 times is divided by 11.
For this, we are supposed to use the divisibility rule of 11 from right hand side.
[Using the divisibility rule from left hand side might give us the wrong remainder, like if we find out the remainder obtained when 12 is divided by 11, remainder = 1 = (2 –1) ≠ (1 – 2)]

$$\text{Remainder } \frac{134680\ldots\text{written } 50 \text{ times}}{11} = 2$$

So, the total remainder = 18
Alternatively, divisibility rule of $10^n - 1$, $n = 2$ can be used to find the remainder in this case.

3. Whatever is the sign between two consecutive integers starting from 1 to 100, it will be odd. So, we are getting 50 sets of odd numbers. Now, whatever calculation we do among 50 odd numbers, result will always be even. So, A will win always.

4. Using Fermat's Theorem, the remainder will be 1.

6. More than one value is possible, so we cannot determine any unique number.

7. Total number of factors of $10^5 = 36$
Total number of factors of 10^5 ending with no zero = 11
Hence, the total number of factors having at least one zero at its end = 25

8. It can be seen that $V_1 = V_5 = V_9 = \ldots$ and so on
$V_2 = V_6 = V_{10}$ and so on.
So, $V_3 = V_7 = \ldots = V_{99} = 9$

9. $V_5 = V_9 = \ldots = V_{89}$, so we cannot find the value of V_{90}.

10. 2 values of a are possible, so cannot be determined.

12. When we are adding the sum of page numbers of 25 pages, it will always be an odd number. So, she could not have found the sum of pages as 1990.

[illegible]

13. Assume S1 = {1, 2, 3, ..., 24} and S2 = {25, 26, 27, 28, ..., 50} and now check the options. None of the first three options are correct.

14. Assume S1 = {1, 2, 3, ... , 24} and S2 = { 50, 49, 48, 47, ... , 25}. Now even if we have interchanged 24

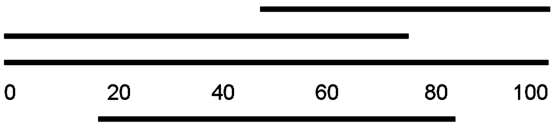
and 25, S1 continues to be in ascending order and S2 continues to be in descending order. However, by choosing negative values of a24 and a25, we can show that S1 continues to be in ascending order, but S2 is no longer in descending order.

15. Do it by the method of elimination.

16. This is one classic example of pigeon-hole principle. Since Mr Roy has ordered for 25 boxes and three different types of sweets, so minimum 9 boxes of sweets will have the same type of sweets.

18.
$$\begin{array}{cccccc} & 2 & 3 & \_ & 5 & \_ \\ + & 1 & \_ & 6 & 4 & 2 \\ & 4 & 2 & 4 & 2 & 3 \end{array}$$
The key here is the numbers written in bold letters. When 5 and 4 are added in decimal system, the value is equal to 9. But its unit digit here is represented by 2. Going through the options, only option (c) is satisfying this equation.

19.



Easy problems = Q.40 to Q.60 [not both including] = 20
Difficult problem = Q.1 to Q.20 + Q.80 to Q.100 [not both including] = 40
So, the answer = 40 – 20 = 20

21. 9699 – 7179 = 2520 and all the factors of this number will be the value of N.

24. 10101 = 3 × 7 × 13 × 37.
ababab = 10101 × (10a + b)

27. Prime numbers less than 10 = 2, 3, 5, 7
So, option (d) cannot be the answer.
Sum of digits is more than 13 – so, set of [2, 3, 5] is not possible.
So, option (a) cannot be the answer.
Now check for options (b) and (c) by taking the values.

30. Go through options.
Checking option (a) 1313 … 1313 (100 digits). So for 1 digit, it should be equal to 1 only.
Now, taking 1st two digits of [111 … 111 (200 digits)] and 1 digit from [222 … 22 (100 digits)] → $[11-2]^{1/2}$ = 3, so option (a) is not correct.
Similarly, we will check all the options.

31. Check it through the options.
Alternatively, answer will be LCM (2,3,4,…, 9) – 1 = 2520 – 1 = 2519

32. There is no rocket science involved in it. Simply write $(10^3 + 9^3)^{752} = (1729)^{752}$ and now find the remainder from the given method of calculations.

35. Going through the options, 8181/1818 = 4.5

36. His roll number is divisible by 1001 and the first three digits are 267. Hence the last three digits will also be 267.

37. Go through the options.

[illegible]

The number is 125367.

40. $1080 = 2^3 \times 3^2 \times 5^1$.
Now, N × Q = 1080 and M × (Q – 25) = 1080
Do a bit of hit and trial.
N = 27

42. The given numbers are two consecutive even numbers, so their HCF = 2
Now, using LCM × HCF = Product of two numbers
LCM × 2 = (… 6) × (… 8)
IT can be seen now that the unit digit of LCM = 4

43. $a + b + c = 2003$
It is possible only if all the three a, b and c are odd, or two of the three are even and one is odd.
Hence, E can have only two values.

[illegible]

45. In this question, we have to find out the possible unit digits of $(n!)^4$. This can be either 1 or 5 or 6.

47. Using options, the only possible value is 4112. The key here is:
The sum of the digits equals the product of the digits.

49. The number of mistakes made by all the students will be between 0 and 14, i.e., students are having a total of 15 options to make mistakes. Since the number of students = 16, at least two students will have the same number of mistakes (that can be zero also, i.e., two students are making no mistakes). Hence, option 1 is the answer.

50. There is only one number, 145, which exhibits this property.

52. Actually the value of 9 here would become 10 and x represents 8.
So, $(9x)_{11} = 10 \times 11^1 + 8 \times 11^0 = 118$

54. In this question, several restrictions are operating:
If S and S^2 are ending with the same unit digit, then it can be 0, 1, 5, 6, but it is given that none of the digits

is equal to zero, so the unit digit can be only 1, 5, 6.
Next, unit digit of the square of the number written in reverse order is 6, so tens place digit of the actual number should be either 4 or 6.
So, the actual numbers could be 41, 45, 46, 61, 65, 66.
Now, this square is less than 3000, so the only possibilities are 41, 45, 46.

55. The total number of numbers of four digits in octal system = $7 \times 8 \times 8 \times 8 = 3584$
When we convert this number into octal system, this is equal to 7000.

60. Eliminate the options.
For example, option (1) can be eliminated by assuming N = 2

BENCHMARKING TEST 1

This test paper contains 20 questions of 4 marks each. One-third of the mark allotted to a particular question will be deducted in case of wrong answer.

Time Given: 45 Minutes **Total Marks: 80** **Qualifying Marks: 24**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
42+	99+
36	98.5+
30	98
22	92
16	85

Q.1. The remainder when 25! is divided by 10^7 is
(a) 2 (b) 4×10^6
(c) 6×10^6 (d) 2×10^6

Q.2. A box contains 90 balls of different colours: 13 yellow, 19 green, 27 red, 10 black, 7 brown and 14 white. The smallest number N such that any N ball drawn from the box will contain at least 14 balls of the same colour is
(a) 79 (b) 69
(c) 70 (d) 72

Q.3. At a bookstore, 'Modern Book Store', is flashed using neon lights. The words are individually flashed at the intervals of $2\frac{1}{2}$ s, $4\frac{1}{4}$ s and $5\frac{1}{8}$ s, respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again is
(a) 49.5 s (b) 73.5 s
(c) 1744.5 s (d) 85.5 s

Q.4. A number has exactly 32 factors out of which 4 are not composite. Product of these 4 factors is 30. How many such numbers are possible?
(a) 2 (b) 6
(c) 3 (d) Not possible

Q.5. 1st 126 natural numbers are put side by side in the ascending order to form a large number: 123456… 125126. What will be the remainder when this large number is divided by 5625?
(a) 5126 (b) 126
(c) 26 (c) None of these

Q.6. P and Q are two distinct whole numbers and P + 1, P + 2, P + 3,… , P + 7 are integral multiples of Q + 1, Q + 2, Q + 3,…, Q + 7 respectively. What is the minimum value of P?
(a) 0 (b) 240
(c) 1080 (d) None of these

Q.7. A child was asked to add first few natural numbers (that is 1 + 2 + 3 +…) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong, the child discovered that he had missed one number in the sequence during addition. The number he missed was
(a) Less than 10 (b) 10
(c) 15 (d) More than 15

Q.8. $1000^7 - 17$ is written in decimal notation. What is the sum of its digits?
(a) 181 (b) 182
(c) 183 (d) None of these

Q.9. N is an integer. For how many distinct values of N, $N^4 - 20N^2 + 4$ is a prime number?
(a) 1 (b) 3
(c) 3 (d) None of these

Q.10. Of 128 boxes of oranges, each box contains atleast 120 and at most 144 oranges. The number of boxes containing the same number of oranges is at least:
(a) 5 (b) 6
(c) 103 (d) None of these

Q.11. What is the value of $\sum_{N=1}^{99} \frac{1}{\sqrt{N} + \sqrt{N+1}}$?
(a) 9 (b) $\sqrt{99}$
(c) $\sqrt{100} - \sqrt{99}$ (d) None of these

Q.12. a, b, c, d and e are prime numbers, not necessarily consecutive. If $a + b + c + d + e = 350$, and $a < b < c < d < e$, how many values of a^5 is/are possible?
(a) 1 (b) More than 1, but finite
(c) Infinite (d) Nothing can be said

Q.13. The ten-digit number $9793x6160y$ is divisible by 11. If $0 < x < y$, find the sum of remainders when the number is divided by $x + y$ and x successively.
(a) 4 (b) 6 (c) 7 (d) 9

Q.14. What is the remainder when 128^{500} is divided by 153?
(a) 63 (b) 67
(c) 123 (d) None of these

Q.15. What is the minimum number of weights which enable us to weigh any integer number of grams of gold from 1 gm to 100 gms on a standard balance with two pans, weights can be put on either pan of balance?
(a) 4 (b) 5
(c) 6 (d) None of these

Q.16. $N = 2^8 \times 3^{10} \times 5^8 \times 7^2$. What is the number of factors of this number N which are factors of 360 but not the factors of 540?
(a) 120 (b) 210
(c) 114 (d) 144

Q.17. What is the remainder when 2^{104} is divided by 101?
(a) 1 (b) 2 (c) 16 (d) 64

Q.18. Given that:
$X_1 = 5, X_2 = 25$ and $X_{n+2} = \text{GCD}(X_{n+1}, X_n) + X_n$.
What is the LCM of (X_{19} and X_{20})?
(a) 1840 (b) 2560
(c) 2160 (d) 2440

Q.19. $N_1 + N_2 + N_3 + \ldots + N_N = 100$, where $N_1, N_2, N_3 \ldots$ and N_N are $n(n>1)$ consecutive natural numbers such that $N_1 < N_2 < N_3 < N_4 \ldots < N_N$. If $N_1 > 8$, then how many values of n is/are possible?
(a) 4 (b) 8 (c) 2 (d) Infinite

Q.20. A teacher writes 101 numbers on the blackboard, of which 50 are zeros, and 51 are ones. A student is asked to perform the following operation 100 times on the board: strike out any two numbers. If they are equal, write another zero. If they are unequal, write a one. What are the numbers left on the board?
(a) Single One (b) Single Zero
(c) Two Ones (d) None of these

ANSWERS

Benchmarking Test 1

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(b)	(b)	(b)	(d)	(d)	(b)	(d)	(b)	(a)	(a)	(d)	(b)	(b)	(d)	(c)	(b)	(c)	(a)

Module 2

Arithmetic

- Percentage
- Profit Loss and Discount
- Average and its Applications
- Ratio Proportion and Variation
- Time and Work
- Time Speed and Distance

3

CHAPTER

Percentage

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- What is percentage
- Definitions and properties
- Percentage–ratio equivalence
- Usage of percentage as a calculation device
- Methods of solving questions
- Concept of SI and CI

Introduction to the topic

Percentage is of immense importance when it comes to problem solving in quantitative aptitude (QA) and data interpretation (DI). Despite the fact that the number of questions asked from this concept is not very high in the CAT, a sound understanding and good practice of percentage calculations comes handy in almost all the topics of QA and DI. However, percentage is an important concept for all the other B-school and aptitude examinations.

WHAT IS PERCENTAGE?

Before we talk of the utility of the concept 'percentage', we should be clear regarding when should we read (%) as per cent and when as percentage. Whenever (%) symbol is attached to any value, known or unknown, the word which we use for symbol (%) is per cent, e.g., $x\%$ is read as x per cent or 20% is read as 20 per cent, not 20 percentage. The word percentage is used whenever (%) is not attached to any value.

This whole nomenclature can be understood with the help of the following example:

The salary of Abhishek is 20% more than the salary of Abhinav. By how much percentage is the salary of Abhinav less than the salary of Abhishek?

Let us first have some standard meaning of percentage:

- Percentage as a common platform
- Percentage as a rate measurer

Percentage as a Common Platform

The concept of percentage can be further understood with the help of the following table, which gives us the marks obtained by different students in their Class 10 exam:

Student in country	*Marks obtained*
America	100
India	25
China	45
France	50

Right now, by just having this piece of information, we cannot decide that students of which country have performed the best and the data required here is the total marks obtained in each country.

Now, suppose if we add the following data to this question, then the whole situation looks like:

Actually, here we are using percentage as a common platform to compare all the given values.

Student in country	*Marks obtained*	*Total marks*	*Marks obtained/ 100 marks*
America	100	1000	10%
India	25	25	100%
China	45	300	15%
France	50	100	50%

To know more about percentage, go to *www.gomath.com/exercises/percentage.php*

Percentage as a Rate Measurer

The concept of percentage is best-suited to find out the rate of change when two or more than two quantities are attached to each other by any common relationship. In this case, we get the respective change which should be brought in the value of quantities, when the value of other quantities are changing. We will see the application of percentage as a rate measurer while doing product stability ratio.

Before we move ahead, it is important to understand the basic statements used in percentage.

Basic statement 1

What is $x\%$ of y?

$$\Rightarrow \frac{x \times y}{100}$$

It can also be seen that $x\%$ of $y = y\%$ of x

For example 4.5% of 20,000 = 20,000% of 4.5

This one simple fact can be used to divide or multiply any number by 50 or 25 or so. Let us see this with the help of an example: We are trying to find out the value of 25×32, which is nothing but $32 \times 100/4 = 800$. Similarly if we have to divide any number by 50, we should be multiplying the number by 100 and dividing it by 2 finally.

Using this, we can see that if we have to calculate 24% of 25 (or any other calculation of similar nature), it is better to find out 25% of 24 ($= 24 \times 100/4$) = 600.

Example 1 What is 20% of 50% of 40% of 20?

Solution Percentage means 'per hundred'.

So, 20% of 50% of 40% of 20 =

$(20/100) \times (50/100) \times (40/100) \times 20 = 0.8$

What we can observe here is that even if we change the order of values here, the final result will be the same.

Example 2 Statistics show that 20% of smokers get lung cancer and 80% of lung cancer patients are smokers. If 30% of the population smokes, then the percentage of population having lung cancer is

(a) 4 (b) 3 (c) 8 (d) 7.5

Solution Assume that the total population is 100.

30 people smoke, and out of them 6 people will be having lung cancer. This 6 represents 80% of lung cancer patients (because they smoke). Hence, total percentage of population having lung cancer = $\frac{6}{80} \times 100 = 7.5\%$

Basic statement 2

What percentage of x is y?

$$\Rightarrow \frac{y \times 100}{x}$$

(This can be easily deduced by assuming Z% of x is y and then proceeding as given in statement 1.)

Some more example of the same kind are given below:

1. What is the percentage growth in the sales value of the year 2002-03 over the sales value of 2003-04?

2. What is the percentage hike in the salary of Manoj this year compared to his salary hike last year?

Basic statement 3

Change in value

(a) Percentage change = $\dfrac{\text{Change}}{\text{Initial value}} \times 100$

(b) Percentage point change – It is the numerical difference between the values for which we have to calculate change.

Let us assume some values to understand the above written concept

Market share	*2002-03*	*2003-04*
Maruti	40%	48%
Honda	30%	26%

Percentage change in the market share of Maruti over the years $= \dfrac{48-40}{40} \times 100 = 20\%$

Percentage point change in the market share of Maruti over the years = 48% – 40% = 8%

Similarly, if we have to increase any quantity N by S%, then it is equal to $N\left(1+\dfrac{S}{100}\right)$ and when the same quantity N is to be decreased by S%, then final quantity $= N\left(1-\dfrac{S}{100}\right)$

It is worth mentioning here that

$100 \to 20\%\uparrow \to 100 \times 1.2 = 120$	$100 \to 20\%\downarrow \to 100 \times 0.8 = 80$
$150 \to 30\%\uparrow \to 150 \times 1.3 = 195$	$150 \to 30\%\downarrow \to 150 \times 0.7 = 105$
$210 \to 27\%\uparrow \to 210 \times 1.27 = 266.7$	$210 \to 27\%\downarrow \to 210 \times 0.73 = 153.3$

So, if the final value and percentage increase or percentage decrease is given and we have to find out the initial value, then it can be done in similar way.

Using $S \to 30\%\uparrow \to S \times 1.3 = 195$

So, if the final value 195 and 30%↑ is given, then initial value $S = \dfrac{195}{1.3} = 150$

Example 3 My Reliance India phone bill for the month of May is Rs B. Moreover, there is a service tax of S% which is to be levied upon this value. But since they are overcharging their customers they offer a discount of D% on it. So, now I have two options to make the payment

Rs B → S%↑→ D% ↓→ Final bill

Rs B → D% ↓→ S%↑→ Final bill

Which option is beneficial for me if S > D?

Solution Prima facie, it might appear that the 1st one is better than the 2nd one or the 2nd one is better than 1st one, but a close and deep inspection will reveal that final bill is same in both the cases.

It can be checked with the help of assuming values also.

B = Rs 100, S% = 20% and D% = 10%

Rs 100 – (20%↑) – Rs 120→ (10%↓) – Rs 108 (Final bill)

Rs 100 – (10%↓) – Rs 90→ (20%↑) – Rs 108 (Final bill)

So, both the values are same at the end.

Importance of base Whenever we are talking about percentage, it is important to specify what it is relative to, i.e., what is that total which corresponds to 100%. The following situation illustrates this point:

In a certain school 60% of all students are male, and 10% of all students are computer science majors. If 5% of males are computer science majors, what percentage of computer science majors are male?

Here, we are asked to calculate the ratio of male computer science majors to all computer science majors. We know that 60% of all students are male, and among hese 5% are computer science majors, so we conclude that $.6 \times .05 = .03$ or 3% of all students are male computer science majors. Dividing this by the 10% of all students that are computer science majors, we arrive at the answer: 3%/10% = .3 or 30% of all computer science majors are male.

While in QA, choosing the right denominator is often not a problem but sometimes it becomes very problematic in Data Inerpretation (DI), because we are unsure as to which value should be taken as denominator or base. However, if we go by some keywords viz., of/than/over/to, the quantity, which is attached to these keywords will be the denominator.

It can be seen through the examples also.

Example 4 A student multiplies a number x by 5 instead of dividing it by 5. What is the percentage change in the result due to this mistake?

Solution Let us assume $x = 5$

So, actual result should have been $100 \div 5 = 20$

But the result obtained $= 100 \times 5 = 500$

So, percentage change $= (500 - 20) \times 100/20 = 2400\%$

(It can be very well observed that if we take base as 500, percentage change = 96%. What we should primarily take care of, with comparison to which number we are calculating percentage change?)

SUCCESSIVE PERCENTAGE CHANGE

Suppose we have to increase a quantity successively by 20% and 30%. Ideally this should be done by taking 100 as the initial value and then changing this initial value first by 20% and then by 30%. It can be seen below:

$100 \to 20\%\uparrow \to 120 \to 30\%\uparrow \to 156$

So, net percentage increase = 56%

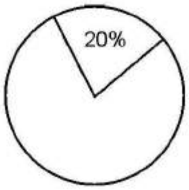
This is known as the **straight line method** of solving the problems.

Alternatively, we can use successive change formula also: $\left(a+b+\frac{ab}{100}\right)\%$

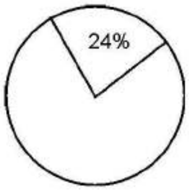
Using this formula, $20 + 30 + \frac{600}{100} = 56\%$

This kind of calculation is very much important in DI questions, especially in pie chart which can be seen with the help of following example

The following figure shows the sales figure of Due North Inc. for the year 2002-03 and 03-04. It is also given that the share of north India in total sales figure is 20% and 24% in respective years.



2002-03



2003-04

Total sales for the year 2002-03 is Rs 1202 crores and for the year 2003-04, it is Rs 1381 crores.

Now the question is: What is the percentage growth in the sales value of north India in 2003-04 over the sales value of 2002-03?

Normal way of doing this question is:

Sales value of north India in 2002-03 = Rs 240.4 crores

Sales value of north India in 2003-04 = Rs 331.4 crores

So, growth = Rs 91 crore

So, the percentage growth $= \frac{\text{Growth}}{\text{Initial value}}\times 100 = 37.93\%$

Alternatively, percentage increases in two factors in the given pie-charts viz., share is increasing from 20% to 24% and then total sales value is increasing from Rs 1202 crores to Rs 1381 crores.

Rather than calculating the sales value of individual years, we can directly calculate the net percentage change by calculating percentage changes independently in the above-mentioned two factors.

Percentage growth in share = 20%

$\left(\frac{\text{Growth}\times 100}{\text{Initial value}};\text{and remember that is not }4\%\right)$

Percentage growth in total value ≅ Approx. 15%

So, 100 → 20%↑ → 120 → 15%↑→ 138, or
100 → 15%↑→ 115 → 20%↑→ 138

So, net percentage growth = 38%

Example 5 The price of petrol is increased by 20%. But expenses increase only by 10%. What is the percentage increase or decrease in consumption?

Solution If consumption remains constant, then the expenses should have also increased by 20%. But since expenses increase by only 10%, so consumption has been reduced.

These kinds of problems can be done in three ways:

i. Straight line method
100 (Initial expenditure)→ 20%↑ in price→120→ x%↓ in consumption→110 (Final expenditure),
$x\% = \frac{10}{120}\times 100 = 8.33\%$

ii. 10 × 10 method
We know, Price × Consumption = Expenditure
Assume Initial price = Initial consumption = Rs 10
So, Initial expenditure = Rs 100

	Price		Consumption		Expenditure
Initially ... →	10	×	10	=	100
Finally ... →	12	×	C	=	110

So, C = 110/12 = Rs 9.16
So, percentage reduction = 8.33%

iii. Formula method
Here a = 20%, b is not known and final percentage change = 10%
20 + b + 20b/100 = 10
⇒ 20 + b + b/5 = 10
⇒ b = –8.33%
So, there is a decrease of 8.33% in the consumption.

PERCENTAGE–RATIO EQUIVALENCE

The essence of percentage–ratio equivalence lies in the fact that most of the percentage calculations like 25%, 37.5%, 33.33% etc., hover around some particular ratios only. And having a good command over these ratios is definitely going to give a good percentage calculation speed which is very essential for DI calculations as well as QA calculations. (See more such calculation in ratio, proportion and variation chapter.)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	100%	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600
2	50	100	150	200	250											800
3	33.33	66.66	100													533.33
4	25	50	75	100												400
5	20	40	60	80	100											
6	16.66	33.33	50	66.66	83.33	100										
7	14.28	28.56	42.85	57.14	71.42	85.71	100									
8	12.5	25	37.5	50	62.5	75	87.5	100								
9	11.11	22.22	33.33	44.44	55.55	66.66	77.77	88.88	100							
10	10	20	30	40	50	60	70	80	90	100						
11	9.09	18.18	27.27	36.36	45.45	54.54	63.63	72.72	81.81	90.90	100					
12	8.33	16.66	25	33.33	41.66	50	58.33	66.66	75	83.33	91.66	100				
13	7.69												100			
14	7.14	14.28	21.42	28.57	35.71	42.85								100		
15	6.66	13.33	20	26.66	33.33	40	46.66	53.33	60	66.66	73.33	80	86.66	93.33	100	
16	6.25	12.5	18.75	25	31.25	37.5	43.75	50	56.25	62.5	68.75	75	81.25	87.5	93.75	100

PRODUCT STABILITY RATIO

In a number of topics/concepts, we encounter the relationship where the product of two quantities equals the third quantity.

For example, Speed × Time = Distance
Price × Consumption = Expenditure
Number of persons × Days = Work done
Length × Breadth = Area of rectangle

Apart form these cases, a number of times we see cases where one quantity is increased to get another quantity, e.g., if we increase cost price to obtain a certain profit, we obtain selling price or if we increase principal, we obtain amount.

If we generalize product stability ratio, it can be written like

$$A \times B = P$$

Now, if A is increased by a certain percentage, then B is required to be decreased by certain percentage so that the product (P) remains stable. For example, if we increase A by 25% and P has to be constant, then B is required to be decreased by 20%.

This procedure can be summed up in the following way:

Change in A	*Change in B*	*Change in P*
100%↑	50%↓	0%
50%↑	33.33%↓	0%
33.33%↑	25%↓	0%
25%↑	20%↓	0%
20%↑	16.66%↓	0%
16.66%↑	14.28%↓	0%
14.28%↑	12.5%↓	0%
12.5%↑	11.11%↓	0%
11.11%↑	10%↓	0%
10%↑	9.09%↓	0%
9.09%↑	8.33%↓	0%
8.33%↑	7.69%↓	0%
And so on......		

So, if A is increased by 25%, then we need to decrease B by 20% to maintain the product stable.

This one mathematical information can be used in so many forms:

1. **Percentage** If A is 25% more than B, then by how much percentage B is less than A?
2. **Profit and Loss** An article is sold for Rs 125 at a profit of 25%. What is the cost price of the article?
3. **Time, Speed and Distance (TSD)** When speed of a car is increased by 25%, time taken reduces by 20 minutes in covering a certain distance. What is the actual time taken to cover the same distance by actual speed?
4. **TSD** Mayank goes to his office from his home at a speed of 20 kmph and gets late by 10 minutes. However, when he increases his speed to 25 kmph, he is 20 minutes early. What is the distance from his office to his home?
5. **Time and Work** Efficiency of Amit is 25% more than Vinit. Vinit takes 20 days to complete a work. How many days will Amit take to do the same work?
6. **Time and Work** 20 men can do some job in 50 days. In how many days will 25 men do the same job?
7. **Simple Interest (SI)** Rate of interest is 12.5% per annum SI. What is the principal if amount obtained after two years is Rs 1250?
8. **Percentage** Due to a price hike of 25%, 5 kg less sugar is available for Rs 100. What is the original price per kg?
9. **Mensuration** Length of a rectangle is increased by 25%. By what percentage should the breadth be decreased so that area remains constant?

In all the above written situations, just one mathematical information has been used, i.e., if A is increased by 25%, then B decreases by 20%. Let us see the solution of all the questions given above.

Solution 1 Normal Method

Let us assume B = 100, then A = 125

Now, B is 25 less than A.

Percentage of B is less than A = $25/125 \times 100 = 20\%$

Product Stability Ratio Method

Using product stability rule, since A is 25% more than B, so B is 20% less than A.

Solution 2 Normal Method

$CP \times 1.25 = SP$

So, $CP = SP/1.25 = 125/1.25 =$ Rs 100

Product Stability Ratio Method

If we increase CP by 25%, we will get SP.

So, if we decrease SP by 20%, we will get CP.

Hence CP = Rs 100

Solution 3 Normal Method_

Since we know $S = V \times T$ (Distance = Speed × Time)

New speed = 1.25 V, so new time = T/1.25

So, reduction in time = $T - T/1.25 = 0.25\,T/1.25 = T/5$

$T/5 = 20 \text{ min} \Rightarrow T = 100 \text{ min}$

Product Stability Ratio Method

Since speed has been increased by 25%, so time will reduce by 20%.

Now, 20% T (Time) = 20 min

So, Total time = 100 min

Solution 4 Normal Method

Let us assume that distance = D

So, $D/20 - D/25 = 30/60$ hr. = ½

So, D = 50 km

Product Stability Ratio Method

$S = V \times T$

$25\%\uparrow 20\%\downarrow$

So, 20% T = 30 min

$\Rightarrow$ T = 150 mins = 2½

So, total distance = 20 × 2½ = 50 km

Solution 5 Normal Method

Vinit is taking 20 days to complete the work i.e., Vinit is doing 100% work in 20 days. So, Vinit is doing 5% work in a day. Since efficiency of Amit is 25% more than Vinit, so Amit is doing 6.25% work per day.

So, number of days taken by Amit = 100/6.25 days = 16 days

Product Stability Ratio Method

Efficiency of Amit is 25% more than Vinit. So, Amit will take 20% less days than Vinit.

So, number of days taken by Amit = 16 days

Solution 6 Normal Method

Using Work = Number of persons × Number of days

Work = $20 \times 50 = 1000$

Now, $1000 = 25 \times D$

So, D = 40

Product Stability Ratio Method

Number of persons increases by 25%, so number of days will decrease by 20%.

So, number of days = 40 days

Solution 7 Normal Method

Using the formula for SI = PRT/100

$P = (SI \times 100)/RT$

Putting the values gives us P = Rs 1,000

Product Stability Ratio Method

Interest for two years = 25%

So, if we decrease the amount by 20%, then we will get the principal.

Hence, Principal = Rs 1,000

Solution 8 Normal Method

Let us assume that original price per kg = Rs P per kg

So, final price per kg = Rs 1.25 P

Hence, (120/P) – (120/1.25P) = 5
Solving this equation gives P = Rs 4 per kg

Product Stability Ratio Method

Since, the price hike is 25%, 20% less quantity of sugar will be available for Rs 100.

20% = 5 kg ⇒ 100% = 25 kg
So, 25 kgs were available for Rs 100 initially.
So, Price = Rs 4/kg

Solution 9 Normal Method

	Length		*Breadth*		*Area*
Initially…→	10	×	10	=	100
Finally… →	12.5	×	B	=	100

So, B = 8
Percentage decrease = 20%

Product Stability Ratio Method

Till now, it must have become very obvious that the breadth will decrease by 20% to keep the area constant.

Extension of Product Stability Ratio

This table is a two-way table, i.e., if we decrease A by 50%, then B is needed to be decreased by 100%.

If we express the percentage figures given in the product stability table in ratios, then it comes like the following:

Change in A	*Change in B*	*Change in P*
$\frac{1}{1}\uparrow$	$\frac{1}{2}\downarrow$	0 %
$\frac{1}{2}\uparrow$	$\frac{1}{3}\downarrow$	0 %
$\frac{1}{3}\uparrow$	$\frac{1}{4}\downarrow$	0 %

So, $\frac{1}{50}\uparrow$ corresponds to $\frac{1}{51}\downarrow$. It means that if we increase A by 2%, then B is needed to be decreased by 1.96% (approx.) so that P remains constant.

Similarly, it can be done with all the reciprocals.

But the problem which lies with this table is that it has values which are reciprocals only.

So, what are we required to do if we increase A by 15%?

Take this as

Change in A	*Change in B*	*Change in P*
$15\% = \frac{3}{20}\uparrow\Leftrightarrow$	$\frac{3}{23}\downarrow$	0%
$35\% = \frac{7}{20}\uparrow\Leftrightarrow$	$\frac{7}{27}\downarrow$	0%

APPLICATION OF PERCENTAGE

1. Simple Interest and Compound Interest

Interest is defined as the “Time value of money.” As the time passes on, value of money keeps on changing. And while factors like inflation or depreciation in money decreases the value of money with the passage of time, interest the counters all these.

The basic difference between simple interest and compound interest lies in the fact that while in case of simple interest, the principal as well as the interest remain same for the entire period given, in case of compound interest, after a certain period, both the principal and the compound interest keep on changing.

Simple interest

In case of simple interest, interest is reckoned at a flat rate for the entire period of time for the amount borrowed.

Simple interest can be calculated by finding the percentage of principal for that period of time.

If the principal is P, rate of interest per annum = R% and time = T years, then Simple Interest will be equal to RT % of P.

So, Simple Interest (SI) = $\frac{P\times(R\times T)}{100}$

Example 6 A sum of money becomes 3 times in 5 years. In how many years will the same sum become 6 times at the same rate of SI?

Solution The sum of money gets 3 times means that 200% is being added to the original sum (principal) in 5 years.

So, 500% will be added up in $12\frac{1}{2}$ years.

Compound interest

In case of compound interest, interest is reckoned on the interest of previous years also apart from reckoning it on the current year.

So, compound interest can be seen as the extension of simple interest in such a way that previous year interest also becomes principal for the next year.

If principal is P, rate of interest per annum = R% and time = n years, then

Compound Interest = $P\left(1+\frac{R}{100}\right)^n - P$

Amount = $P\left(1+\frac{R}{100}\right)^n$

It can be seen that $P\left(1+\frac{R}{100}\right)^n$ is nothing but $P\left(1+\frac{R}{100}\right)\left(1+\frac{R}{100}\right)$… *n* times, which is simply the application of successive percentage change.

Example 7 The difference between two years of compound interest and simple interest at 10% over Rs X is Rs 10. What is the value of x?

Solution

	SI	*CI*
At the end of 1st year	10%	10%
At the end of 2nd year	10%	10% + 10% of 10% =11%
	=20%	=21%

So, 1% = Rs 10

⇒ 100% = Rs 1000

It is pertinent to understand here that if the rate of Interest = R% per annum for both CI and SI, then the difference between CI and SI for 2 years will be equal to R% of R% = $R^2/100\%$

In the above case, R = 10%, so the difference between CI and SI for 2 years = 1%

Comparison between SI and CI

Assume that two different sums are getting doubled at their respective rates of SI and CI in 5 years. The following table gives us the mechanism of getting money n times in the above situation.

	After 5 years	*After 10 years*	*After 15 years*	*After 20 years*
At SI	2times	3 times	4 times	5 times
At CI	2 times	4 times	8 times	16 times

2. Simple Annual Growth Rate (SAGR) and Compounded Annual Growth Rate (CAGR)

Consider the following table pertaining to the sales value of Due North Inc. in different years:

Year	*2004-05*	*2005-06*	*2006-07*	*2007-08*
Sales Value (Rs in crores)	100	110	124	131

If we find out the growth over the given period, then it is equal to 31%.

However, there are two more growth models, namely, Simple Annual Growth Rate (SAGR) and Compounded Annual Growth Rate (CAGR).

$$\text{SAGR} = \frac{\text{Overall growth\%}}{\text{No. of years for which growth can be calculated}}$$

$$= \frac{31\%}{3} = 10.33\%$$

To find out CAGR, we are needed to use the approach of CI and we will consider 131 as the amount and 100 as the principal.

So, $131 = 100\left(1+\frac{R}{100}\right)^3$

To find out the value of R, we will apply the method of hit and trial, or otherwise find out the value of $\sqrt[3]{1.31}$.

Since we know, $(1.1)^3 = 1.331$, so value of R will be less than 10% and very close to 10%.

So, it will be around 9.7% – 9.8%.

Compound interest reckoned half-yearly or quarterly If the rate of interest is R% annually and CI is compounded half yearly for n years, then the expression for Compound

$$\text{Interest} = P\left(1+\frac{\frac{R}{2}}{100}\right)^{2n} - P$$

If the rate of interest is R% annually and CI is compounded quarterly, then the expression for Compound

$$\text{Interest} = P\left(1+\frac{\frac{R}{4}}{100}\right)^{4n} - P$$

3. Calculation

Percentage is mostly helpful in multiplication and division. Let us learn this with the help of examples.

Multiplication

To use this percentage method effectively for multiplying two values, we should be thorough with the equivalent ratio of the percentage figures. Besides, having a good addition speed will be an added advantage.

Example 8 63 × 72.

Solution The moment we see any number, we should start mental scanning of the percentage–ratio equivalence. Here,

$63 \times 72 = (62.5 + 0.5) \times 72$

$= 5/8 \times 100 \times 72 + 0.5 \times 72$ (5% of 72)

$= 4500 + 36$

$= 4536$

Example 9 76 × 24

Solution $= (75 + 1) \times 24 = ¾ \times 100 \times 24 + 1 \times 24 = 1800 + 24 = 1824$

Division

Example 10 Dividing 243 by 50.

Solution $\frac{243}{50} = \frac{243}{100} \times 2 = 2.43 \times 2 = 4.86$

To divide any number by 50, we will divide it by 100 and multiply by 2.

Example 11 Dividing 312 by 25.

Solution $\frac{312}{25} = \frac{312}{100} \times 4 = 3.12 \times 4 = 12.48$

To divide any number by 25, we will divide it by 100 and multiply by 4.

Similarly, while dividing any number by any such number for which we can find out a comparable value in terms of 100 $\left(\text{as } 25 = \frac{100}{4}\right)$ should be used.

PRACTICE EXERCISES

■■■

WARM UP

Q.1. What fraction is equal to 57.12% (approximately)?
(a) 4/8 (b) 5/8 (c) 5/7 (d) 4/7

Q.2. What is 50% of 40% of Rs 1,200?
(a) Rs 450 (b) Rs 100
(c) Rs 300 (d) Rs 240

Q.3. a% of a + b% of b = 2% of ab, then what percentage of a is b?
(a) 50% (b) 75%
(c) 100% (d) Cannot be determined

Q.4. Subtracting 34% of x from x is equivalent to multiplying x by N. Which of the following is the value of N?
(a) 0.66 (b) 66 (c) 0.34 (d) 6.6

Q.5. Subtracting 48% of N from N is equivalent to multiplying N by
(a) 0.48 (b) 1.48 (c) 0.52 (d) 0.42

Q.6. Tipu's salary has increased from Rs 7,200 to Rs 8,100. What is the percentage increase in Tipu's salary?
(a) $33\frac{1}{3}\%$ (b) 25% (c) $16\frac{2}{3}\%$ (d) $12\frac{1}{2}\%$

Q.7. 42% of a number is subtracted from 61% of itself and the resulting value is 38. What is the number?
(a) 100 (b) 250 (c) 200 (d) 300

Q.8. The length of rectangle is increased by 30% and the breadth is decreased by 25%. What is the percentage change in the area of the rectangle due to this?
(a) 2.5% increase (b) 25% decrease
(c) 25% increase (d) 2.5% decrease

Q.9. If a value is increased twice successively by 20%, then what is the net percentage increase in that value?
(a) 20% (b) 40% (c) 21% (d) 44%

Q.10. The numbers are respectively 30% and 40% of a third number. What percentage is the first number of the second number?
(a) 25% (b) 50% (c) 40% (d) 75%

Q.11. Two numbers are 15% and 20% less than a third number. What percentage is the first number as a percentage of the second number?
(a) $116\frac{2}{3}\%$ (b) $83\frac{1}{3}\%$
(c) $106\frac{1}{4}\%$ (d) $123\frac{2}{3}\%$

Q.12. 10 litres of water is added to 50 litres of a solution containing 20% of alcohol in water. What is the concentration of alcohol in the solution now?
(a) 20% (b) $16\frac{2}{3}\%$
(c) $12\frac{1}{2}\%$ (d) $33\frac{1}{3}\%$

Q.13. A man spends 25% of his money on food. After spending 50% of the remaining, he is left with Rs 375. How much money was with that man initially?
(a) Rs 5,000 (b) Rs 4,000
(c) Rs 5,800 (d) Rs 1,000

Q.14. A man earns Rs 4,000 in a month and saves 30% of his income. What is his expenditure in a year?
(a) Rs 28,600 (b) Rs 18,400
(c) Rs 32,000 (d) None of these

Q.15. After spending 20% on clothes, 10% on books, 9% on purchasing gift for girl friend and 7% on others, Chandra has a balance of Rs 2,700. How much money was there with him initially?
(a) Rs 5,000 (b) Rs 6,000
(c) Rs 7,000 (d) None of these

Q.16. A man spends 60% of his income. His income increases by 40% and due to this, he increases his expenditure also by 40%. Find the percentage increase in his savings.
(a) 30% (b) 4%
(c) 4.44% (d) None of these

Q.17. If the numerator of a fraction is increased by 20% and its denominator decreased by 10% the fraction becomes 3/2. What is the value of the original fraction?
(a) $\frac{2}{3}$ (b) $\frac{7}{8}$ (c) $\frac{9}{8}$ (d) $\frac{6}{7}$

Q.18. In an exam of two papers maths and chemistry, 60% of the students pass in maths and 70% pass in chemistry. What is minimum percentage of students who could have failed in both the subjects?
(a) 0% (b) 30%
(c) 40% (d) None of these

Q.19. Everyday a fruit seller sells 50% of his stock. 10% of the stock gets spoiled during the night. If a total of 1983 fruits got spoiled in the three days period, how many fruits did he start with?
(a) 20,000 (b) 25,000
(c) 24,000 (d) None of these

Q.20. A certain sum becomes 7/3 times itself in ten years under simple interest. Find the rate of interest.
(a) $7\frac{1}{2}$% p.a (b) $13\frac{1}{3}$% p.a
(c) 20% p.a. (d) 10% p.a.

Q.21. An amount is lent at x% p.a. simple interest for two years. Instead, had it been lent at $2x$% p.a. simple interest for y more years, then the interest would have been five times the earlier interest. Find the value of y.
(a) 2 (b) 3 (c) 4 (d) 5

Q.22. According to a new plan rolled out by HISP Bank, the rate of simple interest on a sum of money is 8% p.a. for the first two years, 10% p.a. for the next three years and 6% p.a. for the period beyond the first five years. Simple interest accrued on a sum for a period of eight years is Rs 12,800. Find the sum.
(a) Rs 24,000 (b) Rs 16,000
(c) Rs 15,000 (d) None of these

Q.23. Find the least number of integral years in which a sum of money invested at 20% compound interest per annum will become more than double itself.
(a) 1 (b) 4 (c) 2 (d) 3

FOUNDATION

Q.1. In the country of Tattoland, a new term tercentage has been introduced in the place of percentage. While percentage is per 100, tercentage is per 125. What will be the tercentage marks obtained by a student in Tattoland if his percentage of marks is 60?
(a) 48 tercentage (b) 75 tercentage
(c) 60 tercentage (d) Cannot be determined

Q.2. In the country of Tappoland, a new term hercentage has been introduced in the place of percentage. While percentage is per 100, hercentage is per 1000. What will be the hercentage marks obtained by a student in Tappoland if his percentage is 60?
(a) 6 hercentage
(b) 600 hercentage
(c) 166.66 hercentage
(d) Cannot be determined

Q.3. In a ΔABC measurement of $\angle ABC = 15°$. What will be the value of $\angle ABC$ when seen through a magnifying glass which can magnify any object by 20%?
(a) Value reduce by 20%
(b) Same
(c) Value increased by 20%
(d) Value decreased by 16.66%

Q.4. In a maternity centre, 5% of all the childbirth cases result in twins. What is the approximate percentage of twins out of total children born?
(a) 5% (b) 7.6% (c) 9.5% (d) 10.4%

Q.5. Due to a price hike of 20%, 4 kg less sugar are available for Rs 120. What is the initial price per kg of sugar?
(a) Rs 5/kg (b) Rs 4/kg
(c) Rs 6/kg (d) Rs 5.5/kg

Q.6. A person has to pay 20% tax on first Rs 4,500 earned, for amount earned between Rs 4,500 and Rs 7,500 the tax is 25% and for the amount earned more than Rs 7,500, the tax is 30%. If he paid 23.33% of his income as tax, then what is his income?
(a) Rs 9,000 (b) Rs 8,500
(c) Rs 10,000 (d) Rs 10,500

Q.7. A clothes-shop offers a discount to all its customers on the basis of the value of the purchase. If the value of purchase is less than Rs 3,200, the customer gets a discount of 14%. The customer gets 16% discount on

purchase between Rs 3,200 – Rs 6,300. The customer gets a discount of certain % on the purchase exceeding Rs 6,300. A customer gets a total discount of Rs 1,520 which is equal to 16% of the purchase value. How much discount does the customer get for amount exceeding Rs 6,300?
(a) 16% (b) 18% (b) 20% (b) 22%

Q.8. Three friends P, Q and R donate 8%, 7% and 9% of their salary to a charitable hospital in the given order. Salary of P and Q is same and the difference of their donations is Rs 74. The total donation by the P and Q is Rs 525 more than R's donation. What is the percentage of the total salary of the three donations?
(a) 7.95% (b) 8.3%
(c) 6.34% (d) None of these

Q.9. Fresh watermelons contain 90% water by weight whereas dry watermelons contain 20% water by weight. What is the weight of dry watermelon obtained from 20 kgs of fresh watermelon?
(a) 2.4 kg (b) 2 kg
(c) 2.5 kg (d) Cannot be determined

Q.10. In solution of sugar and water the ratio of sugar and water by weight is 1 : 4. This solution is heated and in the process it loses 50% weight. What is the ratio of weight of sugar and water in the final mixture?
(a) 4 : 1 (b) 2 : 1 (c) 2:3 (d) 3.5:2

Q.11. Rate of inflation is 1000% per annum. What is the value of an article two years from now if it costs Rs 6 today?
(a) Rs 66 (b) Rs 126
(c) Rs 660 (d) Rs 726

Q.12. Dataman Infosys Systems is a Lucknow-based software company which is growing tremendously. It doubled its turnover in 1998 from Rs 50 crores in 1997. Then it tripled its turnover in 1999 and grew by 50% in 2000. What is the turnover at the end of 2000?
(a) Rs 250 cr (b) Rs 450 cr
(c) Rs 300 cr (d) Rs 600 cr

Q.13. In the recently concluded CBSE board examination, a total of 6,00,000 students appeared. 40% of them were females while the rest were males. Pass percentage among males is 75% and the overall pass percentage is 70%. What is the pass percentage for females?
(a) 37.5% (b) 50% (c) 62.5% (d) 70%

Q.14. Because of the budget presented by Yashwant Sinha, the price of sugar increased by 40%. The Verma family reduced its consumption so that expenditure on sugar is up by 12%. If the total consumption of sugar before the rise in price was 50 kg. What is the present consumption of sugar (in kg)?
(a) 48 kg (b) 40 kg (c) 36 kg (d) 32 kg

Q.15. Tatto's working hours per day were increased by 25% and his wages per hour were increased by 20%. By how much per cent was his daily earnings increased?
(a) 20% (b) 25% (c) 50% (d) 45%

Q.16. A's income is reduced from Rs 75,000 to Rs 60,000, while B's income for the same period is increased from Rs 60,000 to Rs 75,000. What percentage of decrease in A's income is the increase in B's income?
(a) 125% (b) 75% (c) 133% (d) 100%

Q.17. 18% of A plus 15% of B plus 19% of C is equal to 17% of the sum of A, B, and C. If A – B = 500 and A – C = 3400, what is the value of A + B + C?
(a) 12,400 (b) 11,600
(c) 13,500 (d) None of these

Q.18. Out of 51,600 candidates who appeared in an examination, 35% failed, while 15% passed with honours. What is the number of candidates who passed the examination, but failed to obtain honours, assuming that no student who has failed can obtain honours?
(a) 25,000 (b) 30,000
(c) 27,309 (d) 25,800

Q.19. A and B have some guavas divided among themselves. A says to B "If I give you 25% of the guavas I have, I will still have 2 more guavas than you have." To this, B says "If you give me guavas equal to 70% of what I have now, I will have 4 more guavas than you have." What is the total number of guavas that they have?
(a) 80 (b) 64 (d) 36 (d) 88

Q.20. Raghupati Raghav was able to score a total of 600 in 12 tests. He scored less than or equal to 80% of his average score per test in four of these tests. If he did not score more than 60 in any of the tests, what is the minimum number of tests in which he should have scored more than 50?
(a) 8 (b) 4 (c) 3 (d) 2

Q.21. A locomotive engine runs at a speed of 50 km/h when no compartment is attached to it. For every new compartment that is attached to it, the speed of the train is reduced by 10% of the earlier speed. At most, how many compartments can be attached so that the train can cover a distance of 180 km in a maximum of 8 hours?
(a) 7 (b) 5 (d) 6 (d) 8

Q.22. Babloo organized a party by taking contributions from the participants. 60% of the participants contributed 80% of the funds. The average contribution of all the people who attended is Rs 50. What is the average contribution of the remaining 40% of the people?
(a) Rs 100 (b) Rs 150
(c) Rs 250 (d) Rs 25

Q.23. If the price of an article rose by 25% every odd year and fell by 20% every even year, what would be the percentage change after 180 years?
(a) 10% increase (b) 25% increase
(c) No change (d) 20% decrease

Q.24. Pranav saves 10% of his total salary. Next year he increases his expenses by 20% but his percentage savings remain the same. What is the percentage increase in his salary next year?
(a) 10% (b) 20% (c) 40% (d) 16.66%

Q.25. In an examination, 80% students passed in philosophy and 70% students passed in maths. At the same time 15% failed in both the subjects. If 390 students passed in both the subjects, how many students appeared in the examination?
(a) 500 (b) 400 (c) 800 (d) 600

Q.26. The salary of Anil and Vinit is 20% and 30% less than the salary of Dheeraj respectively. By what percentage is the salary of Anil more than the salary of Vinit?
(a) 33.33% (b) 50%
(c) 10% (d) 14.28%

Q.27. In the recent census report of Patna, it was found that 40% of total male adult population and 55% of total female adult population is married. What percentage of total adult population of Patna is married? (Polygamy is not in practice in Patna.)
(a) 46.32% (b) 43.6%
(c) 49.81% (d) 40%

Q.28. Sona and Ellaraputtu are very close friends. One day they were figuring out about their respective incomes and expenses. It was found that income of Sona and Ellaraputtu is 'S' and 'E' respectively. Sona spends 12% of her income and Ellaraputtu also spends the same amount. What percentage of her income is Ellaraputtu spending?
(a) E/(12S) (b) 12S/E
(c) 12 E/S (d) S/(12E)

Q.29. Sixty students appeared in the physics and math paper of an examination. Of these 60% students passed in physics and 70% passed in math. What is the maximum percentage of students who failed in both the subjects?
(a) 40% (b) 30% (c) 0% (d) 100%

Q.30. In an election, 10% of the voters on the voter's list did not cast their votes and 60 voters cast their ballot papers blank. There were only two candidates. The winner was supported by 47% of all voters in the list and he got 308 votes more than his rival. The number of voters on the list were
(a) 3,600 (b) 6,200
(c) 4,575 (d) 6,028

Q.31. A rainy day occurs once in every 10 days. Half of the rainy days produce rainbows. What in the percentage of all the days when we have no rainbows?
(a) 95% (b) 10% (d) 50% (d) 5%

Q.32. One-third of Mayank's marks in history equals his marks in geography. If he obtained 180 marks in two subjects together, how many marks did he get in geography?
(a) 40 (b) 45 (c) 60 (d) 36

Q.33. If the price of the sugar is increased by 20%, one can buy 2 kg less sugar in Rs 50. What is the quantity of sugar that could be bought before the price hike?
(a) 10 (b) 8 (c) 16 (d) 12

Q.34. The entry fee in an exhibition was Rs 10. Later this was reduced by 25%, which increased the sale of tickets by 20%. Find the percentage increase in the number of visitors.
(a) 54 (b) 57 (c) 60 (d) 66

Q.35. A car bought for Rs 13,350 depreciates each year by 12½%. In how many years will its value be reduced to Rs 6,000?
(a) 4 years (b) 6 years
(c) 8 years (d) 10 years

Q.36. An electrical contractor purchased a certain amount of wire, 10% of which was stolen. After using 85% of the remainder he had 47.25 m of wire left. How much wire did he purchase?
(a) 600 m (b) 103 m
(c) 325 m (d) 350 m

Q.37. Anoop got 273 marks in his graduation exam and got 5% more than the pass percentage. If Siddharth got 312 marks then by what percentage above the pass marks, did he pass the exam?
(a) 9% (b) 12.5%
(c) 20% (d) 25%

Q.38. There were only two contestants A and B in the recent assembly election in Bihar at Raghopur constituency. B got 50% of the votes that A got. Had A got 200 votes less, there would have been a tie. What is the total number of votes polled?
(a) 1,000 (b) 800
(c) 1,200 (d) Cannot be determined

Q.39. A city had a population of 30,00,000 in the beginning of 1999. Its average growth rate is 4% per year, but due to a massive earthquake in 2001, its population is reduced by 8% in that year. But it again maintained the same growth rate of 4% in following years. What will be the approx. population of the city at the end of 2003?
(a) 32,06,460 (b) 34,68,420
(c) 31,52,360 (d) 32,28,810

Q.40. A is an alloy of tin and copper and B is an alloy of copper and zinc. A has 30% copper while B has 50% copper. x gms of A is mixed with 30 gms of B to form another alloy which has 45% copper. What is the value of x?

(a) 5 (b) 10 (c) 15 (d) 20

Q.41. Rajesh Raj has Rs 90,000 with him. He purchases a Bike, a home theatre and a flat TV for Rs 15,000, Rs 13,000 and Rs 35,000 respectively and puts the remaining money in his bank account which pays 15 per cent per annum Compound Interest. After 2 years he sells off the three items at 80% of their original price and also withdraws his entire money from the bank by closing the account. What is the total change in his asset?

(a) −4.5% (b) +3.5%
(c) −4.32% (d) +5.5%

Q.42. After three successive raises, Aftab's salary became equal to $\frac{378}{125}$ of his initial salary. By what percentage was the salary raised the first time if the third rise was twice as high (in percentage) as the second rise was twice as high (in percentage) as the first rise?

(a) 10% (b) 15% (c) 20% (d) 25%

Q.43. One litre of water is evaporated from 6 litres of solution containing 4% of sugar. The percentage of sugar in the remaining solution is

(a) 1.4% (b) 5.2% (c) 4.8% (d) 3.4%

Q.44. A's income is 60% of B's income, and A's expenditure is 70% of B's expenditure. If A's income is 75% of B's expenditure, find the ratio of A's savings to B's savings.

(a) 5:1 (b) 1:5 (c) 3.5:1 (d) 2:7

Q.45. There are 1,10,000 books at Khuda Baksh Oriental Public Library, 40% of which are science books. It was decided to add 20,000 new books to the library. How many of the new books should be science books in order to bring the percentage of science books in the library equal to 45%?

(a) 15,000 (b) 1,500
(c) 1,450 (d) 14,500

Q.46. If 33 untrained labourers can do a piece of work in 15 days of 12 hours each, how many trained labourers can do 50% more work in 11 days of 9 hours each? (Assuming that it takes 2 trained labourers to do the work of 5 untrained labourers.)

(a) 42 (b) 36 (c) 90 (d) 100

Q.47. X's income is 75% of Y's income, and X's expenditure is 80% of Y's expenditure. If X's income is 90% of Y's expenditure, find the ratio of X's savings to Y's savings.

(a) 1:2 (b) 2:1 (c) 1:4 (d) 2:3

Q.48. Kajol went to a fruit market with a certain amount of money. With the money she has, she can buy either 40 apples or 70 mangoes. She retains 15% of her money for auto fare. If she buys 35 mangoes, how many more apples can she buy?

(a) 15 (b) 20
(c) 14 (d) None of these

Q.49. In an election for the post of class representative at Pitampura centre where two candidates were contesting, 10% of the students did not vote (because they were late, as usual) and 60 votes recorded were declared invalid (because stray marks were made on the ballot paper). Winning candidate won the election by 308 votes and it was found that he had been supported by 47% of the whole students of the class. Which of the following can be the number of valid votes recorded for each candidate?

(a) 2308, 2000 (b) 2904, 2596
(c) 2914, 2606 (d) 2464, 2772

Q.50. If the length and breadth of a rectangular plot are increased by 20% and 50% respectively, Then the new area is how many times the original area?

(a) 5/8 (b) 9/5 (c) 3/7 (d) 7/3

MODERATE

Q.1. There are fifty successive percentage discounts given in a series of 2%, 4%, 6%, 8%... and so on. What is the net discount?

(a) 98% (b) 2550%
(c) 100% (d) Infinite

Q.2. In Question 1, after how many such percentage discounts in succession will the effective discount be higher than 50%?

(a) 7th (b) 8th (c) 9th (d) 10th

Q.3. My Alliance India phone bill for the month of May is Rs B. On top of this, there is a service tax of S% which is to be levied upon this value. But since they are having a guilt feeling of overcharging their customers fraudulently, they offer a discount of D% also on top of this. So, my billing goes like this

Rs B→ S%↑→ D% ↓→ Final Bill

But surprisingly, my final bill is equal to Rs B itself.

Now, which of the following is true regarding S and D (assume all the values to be positive)?

(a) S > D (b) S = D
(c) D > S (d) Cannot be determined

Q.4. Assuming percentage to be true for all the real values, which of the following can be true?
40% of A = 60% of B
i. A > B ii. B > A iii. B = A
(a) i and ii only (b) ii and iii only
(c) i and iii only (d) None of these

Direction for questions 5 and 6: Read the passage below and solve the questions based on it.

Alok has certain number of Oranges with him. He sells each orange for Rs 2. Shazmi bought 25% of the oranges and Sandeep bought one-third of what was left. Mohit bought 50% of what was left and Siddharth bought 4 oranges. Total amount paid by Shazmi and Mohit is Rs 24.

Q.5. How many oranges did Sandeep purchase?
(a) 3 (b) 4 (c) 6 (d) 8

Q.6. How many oranges were left with Alok?
(a) 3 (b) 4
(c) 6 (d) None of these

Direction for questions 7 and 8: Read the passage below and solve the questions based on it.

All students of class X took the pre-board test. One third of the class received B, one fourth received B+, one sixth received B−, and one eighth of the class failed (grade C). Everyone else got A.

Q.7. What can be the minimum number of students in class *X*?
(a) 8 (b) 48
(c) 24 (d) Cannot be determined

Q.8. How many students in class X got A?
(a) 3 (b) 6
(c) 4 (d) Cannot be determined

Q.9. A manufacturer of detergent cakes increased the price of his product as a result of which there was a decrease of 20% in its sales. If the sales turnover increases by 20% due to this change what was the percentage increase in the price?
(a) 20% (b) 36% (c) 40% (d) 50%

Q.10. Ravi Shankar wishes to buy an air conditioner (AC) with the money in the bank, which currently is earning interest at the rate of 15 pcpa compounded annually. But Tanzar, his friend forecasts that the inflation rate applicable to AC is going to be 14%; 15% and 16% respectively for the next 3 consecutive years and advises Ravi Shankar to postpone the purchase by 3 years. Does Ravi Shankar gain monetarily, if he takes Tanzar's advice?
(a) Yes
(b) No
(c) He neither gains nor losses
(d) He gains only if the purchase is made in the second year.

Q.11. In the recently concluded referendum in New York, regarding calling back US troops from Iraq, 600 people voted on a referendum, but after some discussion the opponents increased by 150%. The motion was then rejected by a majority two times as great as that by, which it was formerly passed. How many people voted against the initial referendum?
(a) 400 (b) 300 (c) 200 (d) 500

Q.12. The number of students present in the class of Manoj Yadav is M and out of those M students, 22.1875% are stupids. If the number of stupids in the class is N, then what can be the minimum value of M+N?
(a) 1,955 (b) 391
(c) 16,200 (d) 12,21,785

Q.13. If 75% of a class of 40 students can sing and only 20% cannot dance, the maximum number of students who can neither sing nor dance is
(a) 0 (b) 6
(c) 8 (d) Cannot be determined

Q.14. There are five contestants A, B, C and D in the assembly election from the Bihar Sharif constituency. It is given that none of the contestants got less than 1% of the valid vote. Consider the following statements in isolation to each other:
i. A got 49% of the total valid votes
ii. B got 55% of the total votes
iii. C got 46% of the total valid votes
iv. D got 48% of the total valid votes
With how many of the following statements when used independently, it is possible to find out the winner of the election?
(a) 0 (b) 1 (c) 2 (d) 3

Q.15. For admission in a graduation program of Delhi University, 90% of the candidates who appeared for the written test were males and the rest were females, 60% of the males and 80% of the females passed in the written test. What is the total number of students who appeared for the written test, if the total number of passed candidates was 1240?
(a) 2,500 (b) 2,000 (c) 2,750 (d) 3,500

Q.16. In a laboratory experiment, a sample of air, which is a mixture of oxygen and water vapour is taken. Water vapour, contains hydrogen and oxygen gases. If air contains a total of 70% oxygen (including that contained in the water vapour) by weight while water

vapour contains 16·66% of oxygen by weight, how many kilograms of water vapour is present in 1 kilogram of air?
(a) 0.2 (b) 0.36 (c) 0.32 (d) 0.25

Q.17. In Patna University, out of every 7 students learning science, 3 take literature as well; for every student taking at least one of these two courses, there are 3 students who take up neither. If 10% of the students take only science, the percentages who have taken science is:
(a) $17^{1}/_{2}\%$ (b) $23^{1}/_{3}\%$
(c) $7^{1}/_{2}\%$ (d) Cannot be determined

Q.18. 60% of the students of a class are boys and in an examination, 80% of the girls scored more than 40 marks (out of a maximum possible 150 marks). If 60% of the total students scored more than 40 marks in the same examination, find the fraction of the boys who scored 40 marks or less.
(a) 8/15 (b) 7/15 (c) 4/5 (d) 1/5

Q.19. A thousand people voted on a resolution with 10% of the votes being invalid. After some discussion 1,000 people voted again. This time there were 20% invalid votes. The opponents were increased by 50% while the motion was now rejected by a majority, which is 300% more than it was formerly passed by. How many people voted against the resolution after the discussion?
(a) 400 (b) 200 (c) 300 (d) 500

Q.20. Recently I had gone to a locality called Shadigarh for conducting a survey about the number of married persons in the locality. The population of the locality is 7,200 and $^{11}/_{18}$th of those are males and the rest females. If 40% of the males are married, find percentage of married females in the locality.
(a) $48^{1}/_{7}\%$ (b) $52^{4}/_{7}\%$
(c) $62^{6}/_{7}\%$ (d) $71^{1}/_{7}\%$

Q.21. Nitish was to get a 50% hike in his pay but the computer operator wrongly typed the figure as 80% and printed the new pay slip. He received this revised salary for three months before the organization realized the mistake. What percentage of his correct new salary will he get in the fourth month, if the excess paid to him in the previous three months is to be deducted from his fourth month?
(a) 20% (b) 25% (c) 40% (d) 33 1/3%

Q.22. In the US opinion polls held during August, 60% of the respondents favoured Walter Mondale while the rest favoured George W. Bush. It was found in September polls that 10% of Mondale supporters switched their preference to Bush, while the same percentage of Bush's supporters also switched their preference to Mondale. What percentage of the electorate should now switch their preference from Mondale to Bush so that they are at par?
(a) 6% (b) 8% (c) 10% (d) 14%

Q.23. An index of 12 shares contains, among others, the shares of Reliance, HLL and Infosys with weightage of 7%, 13% and 15% respectively. What is the increase in the prices of other shares, if these three rise by 9%, 10% and 4% respectively, while the index rises by 6%?
(a) 5.34% (b) 4.91%
(c) 4.58% (d) Cannot be determined

Q.24. A community service organisation has raised 80% of the donation that it needed for a new building by receiving an average donation of Rs 400 per person from a group of people. To raise the remaining amount, the organisation now intends approaching another group of people which has two-thirds the number of people as the first group. What should be the average donation per person from the new group?
(a) Rs 100 (b) Rs 150
(c) Rs 200 (d) Rs 250

Q.25. 60% of the employees of Due North Inc. are female, and 75% of the men earn more than Rs 25,000 per year. If 45% of the company's employees earn more than Rs 25,000 per year, what fraction of the women employed by the company earn Rs 25,000 per year or less?
(a) 2/11 (b) 1/4 (c) 1/3 (d) 3/4

Q.26. The pollution in normal atmosphere is less than 0.01%. Due to leakage of methyl isocyanide (MIC) from a factory the pollution is increased to 20%. If every day 80% of the pollution in the atmosphere is neutralized, in how many days will the atmosphere become normal?
(a) 5 days (b) 3 days
(c) 4 days (d) None of these

Q.27. The marks obtained by Arpit in English, maths, science and Hindi in Class 10 are as follows (Maximum marks per subject = 100)
(a) The marks obtained in maths is 1.5 times the marks obtained in English
(b) Arpit got 64% in these 4 subjects.
(c) Arpit got the maximum and minimum marks in science and Hindi, respectively, with a difference of 48 marks between them
(d) An addition of 50% of the marks obtained in English to the final score gives an overall percentage of 70%

What would be his percentage of marks if only maths and science marks are counted?
(a) 14 (b) 82 (c) 84 (d) 86

Q.28. Amit puts some amount in bank and obtained an interest of Rs 15 at the end of one year. He added Rs 85 to this amount and put the whole amount again in the bank for another year. At the end of the 2nd year, Amit got Rs 420 as total (interest + principal). What amount did he put in the beginning and what is the rate of interest offered by the bank if the minimum amount that can be deposited in the bank is Rs 75?

(a) 250, 5% (b) 300, 5%
(c) 200, 8% (d) 350, 3.5%

Q.29. Amul, Binit and Chand start a business by investing Rs 70,000 that earns them a profit of Rs 42,000 at the end of the year. Amul invests his share in the profit in a scheme that gives him 10% interest compounded annually and Binit invests his share in a scheme that gives him 20% interest compounded annually. Amul gets Rs 2,520 as interest at the end of 2 years and Binit gets an interest of Rs 4,200 at the end of one year. Find Chand's investment in the business.

(a) Rs 1,60,000 (b) Rs 15,000
(c) Rs 17,520 (d) Cannot be determined

Q.30. A food processing unit received fresh consignment of berries from Himachal Berry Corporation containing 'a' kilogram of berries. After analysis these berries were found to contain 99% water. After 15 days, sample of berries from the same lot was found to contain 98% water. What must be the weight of same lot at that time?

(a) $\frac{a}{2}$ kg (b) $\frac{2a}{3}$ kg
(c) $\frac{4a}{5}$ kg (d) $\frac{a}{3}$ kg

Q.31. Kishore is an employee of a MNC firm. He needs Rs 10 lakhs for buying a new flat in Patel Nagar. His company is offering housing loan at an interest rate of 15% for 5 years. The two banks ICICI and HFDC are offering loan for 5 year period at an interest rate of 18% and 13% respectively. How much should Kishore borrow from the two banks so that he can pay the interest rate same as that of his own MNC firm?

(a) 5 lakhs from HDFC and 5 from ICICI
(b) 3 lakhs from HDFC and 7 from ICICI
(c) 6 lakhs from HDFC and 4 from ICICI
(d) 8 lakhs from HDFC and 2 from ICICI

Q.32. The population of towns A and B is in the ratio of 1:4. For the next 2 years, the population of A would increase and that of B would decrease by the same percentage every year. After 2 years, their population became equal. What is the percentage change in the population?

(a) 33.33% (b) 66.66%
(c) 25% (d) Not possible

Direction for questions 33 to 35: Read the passage below and solve the questions based on it.

Petroleum product sales (in million tones)

Petroleum product	*2002-03*	*2003-04*
Diesel	36.0	36.6
Petrol	7.6	7.9
LNG	8.3	9.3
Crude oil	11.9	11.7
ATF	2.3	2.5
Total	**104.2**	**107.4**

Choose (a) if only A is true.
Choose (b) if only B is true.
Choose (c) if both A and B are true.
Choose (d) if neither A nor B is true.

Q.33. A. The percentage share of ATF in petroleum product sales has increased in 2003-2004 over the previous year.
B. Percentage share of diesel in petroleum product sales has decreased in 2003-2004 over the previous year.

Q.34. A. LNG has experienced the maximum percentage increase in petroleum product sales in 2000-2004 over the previous year.
B. Change in percentage shares of crude oil in petroleum product sales in 2003-2004 over the previous years is approximately 0.2%.

Q.35. A. Percentage decrease in crude oil sales in 2003-2004 over the previous year is more than percentage increase in diesel sales during the same period.
B. LNG sales in 2003-2004 is less than 25% of diesel sales in 2003-2004.

Direction for questions 36 to 38: Read the passage below and solve the questions based on it.

In each question, there are two statements—A and B, either of which can be right or wrong on the basis of the information given below

Direct taxes collected in different years (Rs in crores)

Taxes	*1998-99*	*1999-00*	*2000-01*	*2001-02*	*2002-03*
Corporation tax	22,750	23,460	25,190	27,080	28,880
Income tax	18,210	20,460	23,790	26,110	29,080

Taxes	*1998-99*	*1999-00*	*2000-01*	*2001-02*	*2002-03*
Service tax	220	240	200	230	260
Expenditure tax	300	320	340	350	345
Interest tax	1,280	1,190	1,350	1,410	1,490
Water tax	8	15	20	22	25

Mark your answer as
- (a) if only A is right.
- (b) if only B is right.
- (c) if both A and B are right.
- (d) If both A and B are wrong.

Q.36. A. The growth rate for Service Tax from 1998-99 to 1999-2000 was higher than that for income tax for the same period.
B. The growth rate for Service Tax from 1998-99 to 1999-2000 was lower than that for the period 2001-02 to 2002-03.
(a) 4 (b) 2 (c) 3 (d) 1

Q.37. A. The ratio of income tax to corporation tax was higher in 1998-99 than in 1999-2000.
B. Service Tax, as a percentage of expenditure tax, was greater in 2000-01 than in 2001-02.
(a) 1 (b) 2 (c) 3 (d) 4

Q.38. A. The average annual growth rate for income tax during the period 1998-99 to 2002-03 was more than 1.5 times the same for corporation tax during the same period.
B. The average annual growth rate for interest tax was more than 5% during the given period.
(a) 1 (b) 3 (c) 2 (d) 4

Direction for questions 39 and 40: Read the passage below and solve the questions based on it.

Abhay, Bhaskar, Chitra and Danny work in the same office. Abhay's salary is 20% less than Bhaskar's salary. Chitra's salary is 25% more than Abhay's salary. Danny's salary is 20% more than Abhay's salary.

Q.39. If Abhay has a salary of Rs 10,000 per month, who has a salary of Rs 12,500 per month?
(a) Bhaskar (b) Chitra
(c) Danny (d) 1 and 2

Q.40. If Chitra received a raise and her new salary is 10% higher than Danny's, what is her new monthly salary? (use the data from the previous question).
(a) Rs 12,750 (b) Rs 13,200
(c) Rs 13,333 (d) Rs 14,225

Direction for questions 41 to 44: Read the passage below and solve the questions based on it.

There are two statements given in each question: A and B, either of which can be true or false on the basis of the information given below.

Following table gives us the detailed information regarding the admission process of a reputed B-School IIM-Jehanabad for the year 2003 and 2004.

Year	*Gender*	*Number of applicants*	*Number of candidates appeared for written test*	*Number of candidates called for an interview*	*Number of candidates selected for the course*
2003	Male	6,100	5,900	60	12
	Female	1,900	1,500	15	5
2004	Male	6,300	6,000	64	13
	Female	4,000	3,500	30	6

Choose (a) if only A is true
Choose (b) if only B is true
Choose (c) if both A and B are true
Choose (d) if neither A nor B is true

Q.41. A. The success rate of moving from the written test to interview stage for females was worse than for males in 2003.
B. The success rate of moving for the written test to interview stage for males was better in 2003 than in 2004.

Q.42. A. The percentage of absentees in the written test among males decreased from 2003 to 2004.
B. The percentage of absentees in the written test in 2003 among females was larger than that among females in 2004.

Q.43. A. The percentage of the female candidates selected in 2004 from female applicants is more than the percentage of male candidates selected for year 2004 out of total male applicants of that year.
B. In 2003, from among those called for an interview, the males had a greater success rate than females.

Q.44. A. Difference in the total number of male and female applicants is more than 50% of the total candidates who appeared for the written test in 2004.

B. The total number selected for the job for both the years is more than 70% of the total female candidates called for an interview in both the years.

Q.45. In a laboratory, equal number of rats were injected with solution A. In a week, 55% of rats injected with solution A got infected with a disease and of that, equal number of rats were treated with vaccine C and D. Now, percentage of rats treated with vaccine C who recovered was 70% and that with vaccine D was 50%, while others died. Calculate the number of rats on whom the experiment was carried out in the laboratory, if the number of rats who died even after being treated with vaccines due to the disease is 77.
(a) 600 (b) 700
(c) 750 (d) 800

ADVANCED

Direction for questions 1 to 3: Read the passage given below and solve the questions based on it.

The table gives the production of five agricultural crops between 1999-2000 and 2000-2001.

Crops	*Target for 2001-2002 (in MT)*	*Output for 2000-2001 (in MT)*	*% increase in output over 1999-2000*
Rice	162.7	160.4	25
Wheat	18.9	16.2	18
Tea	20.5	22.5	35
Cotton	14.7	18.9	32
Coffee	12.3	14.8	16

Q.1. The production of rice in 1999-2000 (in MT) is approximately
(a) 124 (b) 128
(c) 132 (d) 136

Q.2. Output of which crop in 2000-01 is showing maximum percentage difference from the target set in 2001-02?
(a) Wheat (b) Coffee
(c) Cotton (d) Tea

Q.3. A new parameter is invented to judge the target: output ratio. That parameter is known as efficacy ratio and is defined as the ratio of target for 2001-02 and output for 2000-01. For how many given crops the efficacy ratio is more than 1?
(a) 1 (b) 3 (c) 2 (d) 4

Q.4. Some people form a joint account for one year with a condition that "each person will deposit an amount equal to the number of people in the account that month. Also the person who withdraws from the account in between will get their amount at the end of the year." After 6 months, 25% of the people withdrew and 33.33% of the remaining withdrew after 3 months. At the end of the year, there was an amount of Rs 4,860 in their account before paying off the withdrawn amounts. Find the number of people in the beginning of the year.
(a) 12 (b) 24
(c) 36 (d) None of these

Direction for questions 5 to 9: Read the passage below and solve the questions based on it.

The number of taxi drivers and auto drivers working in three shifts are given below:

Vehicle	*Shift*	*Number of drivers*	*Working hours*	*Total collection (in 1,000 Rs)*
Taxi	M1	425	10	132.1
	M2	672	8	240.2
	M3	135	6	42.3
Auto	M1	350	9	57.3
	M2	425	8	100.8
	M3	95	7	18.9

A taxi driver charges Rs 13 for every 1.6 km, while an auto driver charges Rs 8 for every 1.3 km. For the night shift, charges are 1.5 times the normal rate.

Q.5. By what percentage is the average collection per taxi driver more during the M2 shift than during the M1 shift?
(a) 4% (b) 15% (c) 34% (d) 82%

Q.6. What is the average distance travelled by the auto drivers during the M3 shift?
(a) 1,356 km (b) 2,045 km
(c) 3,054 km (d) 2,672 km

Q.7. Who get the highest returns for their efforts with respect to work hours?
(a) Auto Drivers — M3 Shift
(b) Taxi drivers — M2 shift

(c) Taxi drivers — M3 shift
(d) Auto drivers — M1 shift

Q.8. What is the maximum difference between the average distances covered in an hour by drivers of any two shifts?
(a) 3,100 km (b) 3,300 km
(c) 3,500 km (d) 3,400 km

Q.9. Which of the following is false?
(a) Lowest contribution per driver is by auto drivers in the M1 shift
(b) Lowest returns for the efforts are for auto drivers in the M1 shift
(c) The percentage of drivers during the M3 shift is 21% of the total drivers
(d) None of these

Q.10. Consider a big cube formed of 64 smaller cubes of which 40 are coloured. The smaller cubes are arranged such that the exposure of the coloured cubes to the outside is minimized. What is the percentage of the exposed area that is coloured?
(a) 62.5% (b) 50%
(c) 41.6% (d) None of these

Q.11. On May 1, 2002, a person sold all his investments for Rs 10,8,000, incurring a loss of 10% with respect to the previous month. On April 1, 2002 he had invested all his money in shares, which represented a 20% increase over the sum he had on March 1, 2002 which represented a 20% decrease over the money he had on February 1, 2002 which represented a 25% increase over the money he had on January 1, 2002. The person had maximum money in
(a) February 2002 (b) January 2002
(c) April 2002 (d) None of these

Direction for questions 12 and 13: Read the passage below and solve the questions based on it.

Patna and Lucknow are the only cities of India which are having only adults. Female population in Patna grows by 50% to 60,000, but maintaining the same male to female ratio as previous year. Population in Lucknow increased by 12.5% to 90,000, but the ratio of male to female changed by 1:1 to 4:5. Male population in Patna was 10% more than the male population in Lucknow in the previous year.

Q.12. What is the female population in Lucknow in the second year?
(a) 55,000 (b) 45,000
(c) 4,00,000 (d) 50,000

Q.13. What is the total population in Patna after the increase (i.e., in the second year)?
(a) 80,000 (b) 1,00,000
(c) 20,000 (d) 26,000

Q.14. The owner of a flower shop follows a particular pattern for his business. During a period of inflation, he raises his price by P% and during a slowdown he decreases his existing prices by P%. After a year in which there was inflation first, followed by a slowdown, the cost of a red-rose bouquet decreases by Rs 162. After another year, in which there was inflation once more followed by a slowdown, the cost of this bouquet reduced by a further Rs 147.42. What was the original price of the red-rose bouquet?
(a) Rs 1,500 (b) Rs 1,000
(c) Rs 1,800 (d) Rs 1,111

Direction for questions 15 to 17: Read the passage below and solve the questions based on it.

Three leading drug manufacturers, Arpit Pharma, Bittu Pharma and Chintu Pharma each launch an antiseptic drug. Each company uses all or some of the 5 constituents A, B, C, D and E. The table below gives the percentage composition of these constituents in the respective drugs manufactured by different companies.

	A	*B*	*C*	*D*	*E*
Arpit Pharma	12	15	20	28	25
Bittu Pharma	37	15	13	15	20
Chintu Pharma	26	15	10	34	15

The above values are in percentage of composition. All manufacturers produce their drug as a capsule of 300 mg.

Q.15. The effectiveness of constituents A, B, C and D are in the ratio 5 : 6 : 4 : 3, while E is just a sweetener and does not have any curing effect. Which of the following company's capsule is most effective?
(a) Arpit Pharma (b) Bittu Pharma
(c) Chintu Pharma (d) Data Insufficient

Q.16. If the side effects caused by drugs are proportional to the ratio of C's composition to the sum of B's and E's composition, which drug has least side effect?
(a) Arpit's drug
(b) Bittu's drug
(c) Chintu's drug
(d) All of them have equal side effect

Q.17. Arpit Pharma finds that its sales are dipping because its drug tastes too sweet. So, it decides to decrease the composition of E and increase that of C, keeping other composition unchanged. It also wants to make sure that the side effect is not more than double its previous value. What could be the maximum percentage of C's

composition in Arpit's Drug? (Use data from the previous questions)
(a) 25% (b) 30% (c) 34% (d) 37%

Q.18. If 3000 capsules are produced by these 3 companies per annum, the consumption of component C annually would be
(a) 60 kg (b) 90 kg
(c) 105 kg (d) Data Insufficient

Q.19. In Ranchi, only two newspapers *Dainik Jagran* and *Prabhat Khabar* are published. It is known that 25% of the city population reads *Dainik Jagran* and 20% reads *Prabhat Khabar* while 8% reads both *Dainik Jagran* and *Prabhat Khabar*. It is also known that 30% of those who read *Dainik Jagran* but not *Prabhat Khabar* look into advertisement and 40% of those who read *Prabhat Khabar* but not *Dainik Jagran* look into advertisements while 50% of those who read both *Dainik Jagran* and *Prabhat Khabar* look into advertisements. What is the percentage of the population who read an advertisement?
(a) 10.8% (b) 11.3%
(c) 13.9% (d) 12.7%

Direction for questions 20 to 22: Read the passage below and solve the questions based on it.

The following table gives the distribution of number of B-schools out of a sample of 100 B-Schools based on the scores obtained by them on five different parameters—Infrastructure, Placements, Research, Industry Interface and Intellectual Capitals. The maximum possible score in any of these parameters is 100 points.

Score/ Parameter	*80 points and above*	*60 points and above*	*40 points and above*	*20 points and above*
Infrastructure	16	30	50	65
Placements	12	28	37	50
Research	30	50	70	92
Industry Interface	24	42	57	73
Intellectual Capitals	27	43	47	70

For example, 16 out of 100 schools surveyed have scored 80 points or above in the parameter Infrastructure.

Q.20. What is the maximum number of business schools which could have scored 40 points or more but less than 60 points in at least three of the five parameters?
(a) 20 (b) 18
(c) 12 (d) None of these

Q.21. What is the minimum number of business schools which could have scored 40 points or more but less than 80 points in at least two of the five parameters?
(a) 10 (b) 5
(c) 12 (d) None of these

Q.22. Which of the following is a possible figure for number of schools which scored 20 points or more but less than 80 points in each of the five parameters?
(a) 38 (b) 36
(c) 42 (d) None of these

Q.23. In D-segment car market, only three competitors (Honda, Toyota and Hyundai) exist. Last year the sales of firm Toyota were 10% more than Honda. In a year, both the firms Honda and Toyota increased their respective sales by 20%. This year, the sales of the firm Toyota are five times that of Hyundai. How much were the sales of the firm Hyundai last year, if the total sales remained constant over the two years period?
(a) 46% (b) 25%
(c) 40.33% (d) 38.67%

Q.24. Chintu is given a quadratic equation $ax^2 + bx + c = 0$ and is asked to make another quadratic equation from this with $a = 1$. Also one root of the second quadratic equation is same as one of the roots of the first equation but opposite in sign and the other root of the second equation is two times the second root of the first equation. Find the percentage change in the constant term of the second equation as compared to the first equation?
(a) 200% increase (b) 300% decrease
(c) 400% increase (d) 100% decrease

Q.25. There are two investment schemes offered by two different financial institutions X and Y as follows
Scheme X: Rate of interest = 10% per annum and period of maturity = 2 years.
Amount is returned with compound interest compounded annually.

Scheme Y: Rate of interest = 11% per annum amount is returned with simple interest.
For what duration, should I invest Rs 25,000 in scheme Y, so as to get same amount in return if I had invested in scheme X?

(a) $\frac{21}{11}$ Years (b) $\frac{31}{11}$ Years

(c) $\frac{19}{7}$ Years (d) $\frac{19}{7}$ Years

Direction for questions 26 to 28: Read the passage below and solve the questions based on it.

Given below is the conversion rate of some of the currencies with respect to rupees.

	USD	*Pound*	*Euro*	*Canadian Dollar*	*Australian Dollar*
Rs Price/ unit	Rs 44	Rs 66	Rs 54	Rs 32	Rs 36

Q.26. Mr Kumar Kalyan Prasad Sinha is having two Pounds with him. How many Euros can he purchase for two Pounds?
(a) 2.33 (b) 2.44 (c) 2.55 (d) 2.66

Q.27. Value of USD is expected to rise by 10% against Rupee next year whereas value of Rupee is expected to decline by 10% against Canadian Dollar next year. How many USD can be purchased for 5 Canadian Dollars?
(a) 3.67 (b) 2.97
(c) 4.27 (d) None of these

Q.28. Mrs Sanjila Sinha has Rs 1,000 with her and her husband Mr Kumar Kalyan Prasad Sinha has Rs 2,000 with him. They want to purchase maximum number of currency notes given above but they are not ready to combine their money. What will be the maximum number of currency notes, as given above other than rupees which they can have?
(a) 62 (b) 94 (c) 93 (d) 32

Direction for questions 29 and 30: Read the passage below and solve the questions based on it.

The table given below shows the break-up of the percentage of people of different age groups frequenting bars in 4 different metro cities viz., Delhi, Hyderabad, Bangalore and Patna in the year 2002.

Q.29. Which city has the maximum number of people, upto 25 years of age, frequenting bars?

Cities	*Percentage break-up for age groups (Years) in 2002*						
	Up to 15	*15-20*	*20-25*	*25-30*	*30-35*	*35-40*	*Above 40*
Delhi	8	13	24	21	11	17	6
Hyderabad	3	8	35	23	10	16	5
Bangalore	4	21	27	11	8	14	15
Patna	1	7	43	32	9	5	3

(a) Bangalore (b) Patna
(c) Both 1 and 2 (d) Cannot be determined

Q.30. Which age group contributes to the maximum number of people frequenting bars in given cities?
(a) 20-25 (b) 25-30
(c) 30-35 (d) Cannot be determined

Direction for questions 31 to 34: Read the passage below and solve the questions based on it.

Five friends– Akshay, Binod, Chandu, Dev and Eeshwar– went on a shopping trip. Before shopping, the first person had Rs 400, the second person had 25% less than the first person, the third person and the fourth person had 50% less than the first person and the fifth person had 66.66% less than the second person. While shopping they did not lend or borrow from each other. After the shopping was over, it was observed that they were left with Rs 165, Rs 95, Rs 70, Rs 40 and Rs 10, not necessarily in that order. We don't know who is the first or the second or onwards. Following is known about the money they started with, they spent, or they were left with:

i. Akshay started with more money than Dev.
ii. Binod spent Rs 15 more than Chandu.
iii. Eeshwar started with more money than just one another person of the group.
iv. Akshay spend the most but did not end with the least.
v. Chandu started with 66.66% of the money that Binod started with.
vi. Dev spend the least and ended with more than Akshay and Chandu.
vii. Eeshwar spent Rs 35.

Q.31. Who ended with the maximum amount of money?
(a) Akshay (b) Binod
(c) Chandu (d) Eeshwar

Q.32. How much money did Akshay spend?
(a) Rs 205 (b) Rs 190
(c) Rs 35 (d) Rs 360

Q.33. In ascending order of spending, Eeshwar would rank at which position?
(a) 1 (b) 2 (c) 4 (d) 5

Q.34. Who ended with Rs 40?
(a) Akshay (b) Binod
(c) Chandu (d) Dev

Q.35. In a certain town, at least 50% of the people read a newspaper. Among those who read a newspaper, at most 25% read more than one paper. Only one of the following statements follows from the statements given below. Which one is it?
(a) At the most 25% read exactly one newspaper
(b) At least 25% read all the newspaper
(c) At the most 37½% read exactly one newspaper
(d) At least 37½% read exactly one newspaper

Q.36. Two vessels contain equal quantities of 40% alcohol. Anil changed the concentration of the first vessels to 50% by adding extra quantity of pure alcohol. Balu changed the concentration of the second vessels to 50% replacing a certain quantity of the solution with pure alcohol. By what percentage is the quantity of alcohol added by Anil more than that replaced by Balu?

(a) 20% (b) 25%

(c) 40% (d) Cannot be determined

Direction for questions 37 to 40: Read the passage below and solve the questions based on it.

Six students Abhay, Bhupi, Chandi, Deb, Ellaraputtu and Fastinder appear in CAT 2006. Below is given the percentage marks obtained by them in different headings DS, DI, QA, EU, RC, GD and PI. (GD and PI are the part of CAT 06)

Its also known to us that each of DS, DI, QA, EU and RC carry same weightage. And the weightage given to PI is same as weightage given to GD.

Maximum marks in CAT 06 is 250, including GD and PI.

	Written Exam					*GD*	*PI*	*Total*
	DS	DI	QA	EU	RC			
A	45	50	40	90	80		75	62.4
B	55	60	65	70	75	80		66.4
C	75	72.5		65	82.5	88	84	
D	80	70	85	65	60	84	76	
E	90	60	80	60	50	88	72	70.4
F	85	80	75	45	70	72		

Q.37. What is the score obtained by Abhay in GD?

(a) 15 (b) 16 (c) 17 (d) 18

Q.38. If Deb has scored the maximum among the six students, what can be the maximum score that Chandi can get in QA?

(a) 20 (b) 21 (c) 22 (d) 23

Q.39. What is Bhupi's percentage score in PI?

(a) 42 (b) 56 (c) 64 (d) 72

Q.40. If Fastinder and Chandi end up with the same score, what is the difference between Chandi' QA score and Fastinder's PI score?

(a) 0 (b) 1 (c) 4 (d) 7

TRUE/FALSE

Q.1. Length of a rectangle is increased by 30% and its breadth is increased by 10%. Perimeter of this rectangle can increase by 25%.

State whether True or False.

Q.2. Length of a rectangle is increased by 30% and its breadth is increased by 10%. Perimeter of this rectangle can increase by 32%.

State whether True or False.

Q.3. Length of a rectangle is increased by 40% and its breadth is increased by 20%. Percentage increase in the perimeter of the rectangle over the previous perimeter depends upon the dimension of the rectangle.

State whether True or False.

Q.4. Length of a rectangle is increased by 20% and its breadth is increased by 10%. Percentage increase in the area of the rectangle over the previous area depends upon the dimension of the rectangle.

State whether True or False.

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(c)	(a)	(c)	(d)	(c)	(d)	(d)	(d)	(c)	(b)	(d)	(d)	(a)	(d)	(c)	(a)	(c)	(b)
Q. No.	21	22	23																	
Answer	(b)	(d)	(b)																	

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(c)	(a)	(a)	(b)	(a)	(c)	(c)	(d)	(b)	(c)	(b)	(c)	(d)	(c)	(d)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(d)	(c)	(b)	(d)	(d)	(a)	(b)	(b)	(b)	(a)	(b)	(d)	(c)	(b)	(d)	(c)	(d)	(d)	(b)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(d)	(c)	(c)	(b)	(d)	(b)	(a)	(c)	(c)	(b)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(a)	(d)	(c)	(d)	(c)	(d)	(d)	(a)	(c)	(b)	(c)	(b)	(b)	(d)	(a)	(b)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(d)	(a)	(b)	(b)	(a)	(b)	(b)	(b)	(a)	(c)	(a)	(c)	(a)	(a)	(c)	(b)	(b)	(d)	(b)
Q. No.	41	42	43	44	45															
Answer	(a)	(b)	(d)	(b)	(b)															

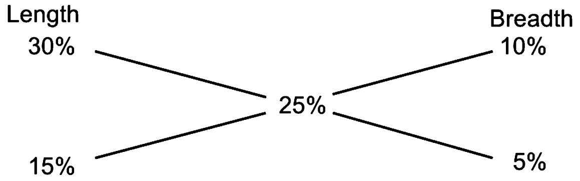
■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(c)	(b)	(b)	(b)	(b)	(d)	(c)	(c)	(a)	(d)	(d)	(c)	(b)	(c)	(b)	(d)	(c)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(b)	(b)	(a)	(b)	(a)	(c)	(d)	(a)	(d)	(d)	(b)	(a)	(d)	(a)	(b)	(c)	(c)	(b)

■■■ True/False

1. True

Increase in perimeter has to be in between 10% and 30%. Hence, 25% increase in perimeter is possible. Actually it's a simple question of alligation. Ratio between length and breadth of rectangle should be 3:1.



Hence, the ratio between length and breadth = 15 : 5 = 3 : 1

2. False

Taking a cue from the above problem, 32% increase in perimeter is not possible.

3. True

Percentage increase in the perimeter of rectangle can be any value between 20% – 40%, and it depends upon the dimension of the rectangle (i.e., the length and breadth).

4. False

Percentage increase in the area of rectangle does not depend upon the dimension of the rectangle and it remaims constant for all the values of length and breadth. In this case, percentage increase in the area = 32%.

HINTS AND SOLUTIONS

Warm up

1. $\frac{4}{7} = 4 \times \frac{1}{7} = 4 \times 14.28 = 57.12\%$

2. $\frac{50}{100} \times \frac{40}{100} \times 1200 = 240$

3. $\frac{a}{100} \times a + \frac{b}{100} \times b = \frac{2}{100} ab$

   $a^2 + b^2 = 2ab$ and $(a-b)^2 = 0$

   a= b

4. x – 0.34 x = Nx

   or, 0.66x = Nx

   N = 0.66

5. N – 0.48 N = 0.52 N

6. $\frac{900}{7200} = \frac{1}{8} = 12.5\%$

7. 0.61x – 0.42x = 38

   or, 0.19x = 38

   $x = 200$

8. Let Length = 100, Breadth = 100

   Hence, Area = 10000

   Now, Length = 130, Breadth = 75

   Hence, Area = 130 × 75 = 9750

   Required Percentage $= \frac{250}{10000} \times 100$

   2.5 % decrease

9. Let number = 100

   1st 20% increment, No becomes = 120

   2nd 20% increment, No becomes = 144

10. Let 3rd No = 100

    1st No. = 30 and 2nd No. = 40

    Hence, 1st is 75% of 2nd No.

11. Let 3rd No. = 100

    1st No. = 85

    2nd No. = 80

    Required Percentage $= \frac{85}{80} \times 100 = \frac{425}{5} = 106\frac{1}{4}\%$

12. Alcohol in solution = 20% of 50 l = 10 l

    Concentration of alcohol in new solution

    $= \frac{16}{40} \times 100 = 16\frac{2}{3}\%$

13. Assume that he had initially Rs 100.

    After spending on food, money left = Rs 75

    After spending 50% of Rs 75, he had Rs 37.5

    Money he had initially = $\frac{100}{37.5} \times 37.5 \times 10 =$ Rs 1000

14. Monthly expenditure = 70% of 4000 = Rs 2800

    Expenditure in a year = Rs 2800× 12 = Rs 33600

15. Let Initial Amount = Rs 100

    Money left = 100 – (20+10+9+7)

    = Rs (100–46)

    = Rs 54

    Money he had $= \frac{100}{54} \times 2700 =$ Rs 5000

16. Let Income = Rs 100

    Expenditure = Rs 60

    Savings = Rs 40

    Net Income = Rs 140

    New Expenditure = Rs 84

    New Savings = Rs 56

    Require increment = $\frac{16}{40} \times 100 = 40\%$

17. Require Increment

    $$\frac{3-\frac{1}{2}}{\frac{18+2}{9}} = \frac{\frac{5}{2}}{\frac{20}{9}} = \frac{5}{2} \cdot \frac{9}{20} = \frac{9}{8}$$

18. Since, we have to find the minimum percentage, it can be possible that no one has failed in both the subjects.

19. The best way of solving this question would be the method of elimination, i.e., going through the options.

    Obviously, option (d) cannot be the answer, because if 500 people are voting in against the resolution, then motion cannot be passed.

    Checking option (a): If 400 people are voting against the resolution for the first time, then 500 are voting in favour. Now, in the second voting, persons voting against the resolution would be = 600, so persons voting in favour = 200

In the first voting, resolution was passed by 100 votes and in the second voting, resolution was defeated by 400 votes. And it is 300% more than 100.

So, the answer is option (a).

20. $S.I. = \left(\frac{7}{3} - 1\right) = \frac{4}{3}$

$S.I. = \left(\frac{7}{3} - 1\right) = \frac{4}{3}$

$R = \frac{40}{3} = 13\frac{1}{3}\%$

21. $S\left(\frac{P \times x \times 2}{100}\right) = \frac{P \times 2x \times (y+2)}{100}$

$y = 3$

22. $\frac{x \times 8 \times 2}{100} + \frac{x + 10 + 3}{100} + \frac{x \times 6 \times 3}{100} = 12800$

$0.64\,x = 12800$

$x = 20000$

23. $\text{CI} = P\left(1 + \frac{20}{100}\right)^n = p\left(\frac{6}{5}\right)^n$

When, n = 4, only then condition given is satisfied.

Foundation

1. The numerator has been increased by 25%. Hence, denominator will also increase by 25%.

   Value in tercentage = 60 × 1.25 = 75 tercentage

2. Numerator has become 10 times, hence denominator will also become 10 times.

3. There will be no change in the measurement of angles. It's the size that will increase, and not the slope or angle.

4. No. of twins cases out of 100 = 5

   No. of twins out of 100 children = 10

   Percentage of twins = $\frac{10}{100} \times 100 = \frac{100}{11} = 9.5\%$

5. Price Quantity

   $\uparrow \frac{1}{5}$ $\frac{1}{6} \downarrow$

   Or, $\frac{1}{6}x = 4$

   $X = 24$

   Initially price per kg = $\frac{120}{24}$ Rs 5/kg

6. Going through choices is the best way. (a) is the answer.

8. Salary of P and Q = x and salary of R = y then

   $\frac{8x}{100} - \frac{7x}{100} = 74$ and $x = 7400$

   then, donation by R = Rs 585

   then $\frac{9}{100}y = 585$

   $y = \frac{585 \times 100}{9}$ = Rs 6500

   Required percentage = $\frac{1695}{21800} \times 100 = 7.95\%$

10. Weight of solution = (1+4) = 5 units

    weight of sugar = 1 unit

    weight of water after heating = 1.5 units

    Required ration = 1 : 1.5 = 2 : 3

11. It can be done by either of the methods of compound interest or straight line method of solving questions.

    6 gets increased by 1000% = Final Value = 6 + 60 = 66

    60 gets increased by 1000% = Final Value = 66 + 660 = 726

12. Turnover in 1998 = Rs 100 cr

    Turnover in 1999 = Rs 300 cr

    Turnover in 2000 = Rs 450 cr

13. $\frac{40}{60} = \frac{75-70}{70-x}$ and $\frac{2}{3} = \frac{5}{70-x}$

    and 140 – 2x = 15

    2x = 125

    x = 62.5%

14. Original price = Rs 100

    Expenditure = Rs 50×100= Rs 5000

    Expenditure after rise = Rs 5600

    Then, 140 × x = 5600

    $X = 40$

15. Hours × wage = earnings

    Previously, $x \times y = xy$

    $\frac{5}{4}x \times \frac{6}{5}y = \frac{3}{2}xy$

    Increment = $\frac{\frac{3}{2}xy}{xy} \times 100 = \frac{\frac{1}{2}xy}{xy} \times 100 = 50\%$

16. Decrease in A's income = Rs 15000

    Increase in B's income = Rs 15000

    Required ratio $= \frac{1500}{15000} \times 100 = 100\%$

17. 0.18A + 0.15B +0.19C = 0.17A+0.17B+0.17C

    0.01A – 0.02B+0.02C=0 (1)

    A – B = 500 (2)

    A – C = 3400 (3)

Solving 1, 2 and 3 we get

A = 5800, B = 5300 and C = 2400

A + B + C = 13500

18. No. of candidates failed = $\frac{35}{100}\times 51600 = 18060$

No. of candidates passed with honour

$= \frac{15}{100}\times 33540 = 5031$

Note: 15% of the remaining

No. of students failed to obtain honour but got passed = 28509

19. Let A has x guavas and B has y guavas, the

$x-\frac{1}{4}x=\left(y+\frac{1}{4}x\right)+2$ (1)

and $y-\frac{7}{10}y=x+\frac{1}{10}y-4$ (2)

Solving 1 and 2, we get

$x = 44 \quad y = 20$

21. Let *n* = number of components attached, V_n = speed of train when 'n' compartments are attached

$V_0 = 50$

$V_1 = 45$

$V_2 = 40.5$ and so on

At *n* = 7, we get the answer.

22. *x* = number of participants

total fund = 50*x*

then, $\frac{3}{5}$ *x* participates contributed 40*x*

Required average $=\frac{10x}{\frac{2}{5}x}=\frac{50}{2}$ = Rs 25

23. Let the original price = Rs 100

Price after 1st year = Rs 125

Price after 2nd year = Rs 100

Hence, after 180 year, there will be no change in the price

24. Salary = Rs 100

Savings = Rs 10, Expense = Rs 90

Next year, Expense = Rs 108,

90% of salary = Rs 108

Salary = $\frac{10}{90}\times 10$ Rs 120

Hence, 20% increase in salary

25. 15% failed in both the subjects ⇒ 85% passed in at least one of the subjects.

So, percentage of students passing both the subjects = 80 + 70 – 85 = 65%

So, 65% = 390 ⇒ 100% = 600

26. Salary of Dheeraj = Rs 100

Salary of Anil = Rs 80

Salary of Vinit = Rs 70

Required percent $=\frac{10}{70}\times 100 = 14.28\%$

27. Let adult males = x and adult females = *y*

then $0.55y = 0.4x$

$x=\frac{0.55y}{0.40}=\frac{11}{8}y$

Now, required percentage $=\frac{0.4x+0.55y}{x+y}=46.32\%$

28. Sona spends = $\frac{3}{25}$ S and Ellaraputtu spends

$=\frac{3}{25}$ S

Let Ellaraputtu spends = *x*% of her income

Then *x* % of $E=\frac{3}{25}x$

or $x=12\frac{S}{E}$

30. No. of voters on the voter's list = *x*

No. of voters participated = $x-\left(\frac{x}{10}+60\right)$

$=\frac{10x-x-600}{10}=\frac{9x-600}{10}$

then $\frac{47}{100}x+\left(\frac{47x}{100}+380\right)+\left(\frac{9x-600}{10}\right)=x$

We get *x* = 6200

31. Percentage of rainy day = 10%

Percentage of days having rainbows = 5%

Hence, required percentage = 95%

32. $x + y = 180$ (1)

$x - 3y = 0(2)$

Solving the equations, we get *x* = 135, *y* = 45

33.

Price	Quantity
$\uparrow\frac{1}{5}$	$\downarrow\frac{1}{6}$

Hence *x*/6 = 2, or, *x* = 12

34. 10×100=1000, 100 = no. of visitors

Now, 7.5 × No. of visitors = 1200

No. of visitors = 160

35. $6000 = 13350\left(1-\frac{125}{1000}\right)^n$

$\frac{600}{1335} = \left(\frac{7}{8}\right)^n$

36. $\frac{15}{100}\cdot\frac{9x}{10} = 47.25$

$1\,x = 350$

38. $x - 200 = \frac{x}{2} \rightarrow x = 400$

A got 400 votes and B got 200 votes

But is information 18 given regarding invalid votes.

So (d) is the answer.

39. Population after 2000 = 3244800

Population after 2001 = 2985216

Population at the end of 2003 = 3228810

40. $\frac{\frac{30}{100}x+15}{x+30}\times 100 = 45$

$x = 10$

41. Money spend in buying things = Rs 63000

Money kept in bank = Rs 27000

Money he got after selling the things = Rs 50400

Money he received from bank = Rs 35707.5

Loss = Rs 3892.5

Loss % = 4.32%

42. $\frac{378}{125} = 3 + \frac{3}{125} = 302.4\%$

Let original salary be Rs 100

And now going through option, we get (c) as answer

43. Sugar in original solution = $\frac{4}{100}\times 6 = 0.24$

Required percentage $= \frac{0.24}{5}\times 100 = 4.8\%$

44. Let B's Income = Rs x

A's Income = Rs $\frac{3}{5}x$

And B's expenditure = Rs y

A's expenditure = Rs $\frac{7}{10}y$

Also, $\frac{3}{5}x = \frac{3}{4}\cdot\frac{7}{10}y$

$$\frac{\text{A'savings}}{\text{B'savings}} = \frac{x-y}{\frac{3}{5}x-\frac{7}{10}y} = \frac{\frac{7}{8}y-y}{\frac{3}{5}\cdot\frac{7}{8}y-\frac{7}{10}y} = \frac{-y/8}{\frac{21y}{40}-\frac{7}{10}y}$$

$= \frac{5}{25} \approx 1:5$

45. No. of science book initially = $\frac{40}{100}\times 110000 = 44000$

No. of books later on = 130000

No. of science books = $\frac{45}{100}\times 130000 = 58500$

Required no. of books = (58500 – 44000)

= 14500

46. 2 trained labors = 5 untrained labourers

33 untrained labors= $\frac{2}{5}\cdot 3$ trained labours.

= 13.2 trained labours

No. of labour	Days	Hrs	Works
13.2	15	12	1
x	11	9	1.5

$\frac{13.2\times 15\times 12}{1} = \frac{x\times 11\times 9}{1.5}$

$x = 36$

47. y's income = Rs A

x's income Rs $\frac{3}{4}$A

y' expenditure = Rs B

x' expenditure = Rs $\frac{4}{5}$B

Also, $\frac{3}{4} = \frac{9}{10}\cdot\text{B}$

$\text{A} = \frac{6}{5}\text{B}.$

Required ratio $= \frac{\frac{3}{4}\text{A}-\frac{4}{5}\text{B}}{\text{A}-\text{B}} = 1:2$

48. Let she had Rs 100

70 mangoes – Rs 100 and 40 apples – Rs 100

35 mangoes = Rs 50

Rs 15 retained for auto fare

Remaining money = Rs 35

Number of apples = $\frac{40}{100}\times 35 = 14$

50. Let L = 100, B = 100

A = 10000

New length = 120, new breadth = 150

New Area = 18000

Moderate

1. Let MP = Rs 100

CP after 1st discount = Rs 98

CP after 2nd discount = Rs 94.08

But discount cannot be more than 100%

2. Proceeding the solution of 1, we get (b) as the answer.

3. Let B = Rs 100, service tax = Rs 10

Final Bill = Rs 100, Discount = Rs 10

S% = 10%

D% = 9.09

4. $\frac{2}{5}A = \frac{3}{5}B$

$2A = 3B$

5. & 6. Let no. of oranges with Alok = x

Shazmi bought = $\frac{1}{4}x$

Sandeep = $\frac{1}{3}\cdot\frac{3}{4}x = \frac{1}{4}x$

Siddharth bought = $\frac{1}{4}x$

$\frac{1}{4}x = 4$ and $x = 16$

7. Let the class has x students

No. of students receiving grade B = $\frac{1}{3}x$

No. of students receiving B + = $\frac{1}{4}x$

No. of students receiving B – = $\frac{1}{6}x$

No. of students failed = $\frac{1}{8}x$

No. of students receiving

$$A = x - \left(\frac{1}{3}x + \frac{1}{4}x + \frac{1}{6}x + \frac{1}{8}x\right)$$

$$= x - \left(\frac{21x}{24}\right) = 3x/24$$

For (3x/24) to be an integer, min. value of x = 24.

9. Let original price = Rs 100

No. of Units gold = 100

Turn-over = Rs 10000

Now, No. of Units sold = 80

Turn-over = Rs 12000

Price per unit = $\frac{1200}{80}$ = Rs 150

10. Let Rs x is kept in bank

After 3 years money will become

$$x\left(1+\frac{15}{100}\right)^3 = \text{Rs}\,1.521x$$

Price of AC after 3 years = Rs 1.52 x

Hence, Ravi will gain.

11. The best way to solve the question is through options.

12. N = 22.1875% of M

$$N = \frac{221875}{1000000}M$$

Values of M and N should be integer. So, Min. value of M+N = 391

13. No. of students that can sing = 30

No. of students, that cannot dance = 8

Maximum no. of students that can neither sing nor dance = 8

14. Using statement ii only we can find the winner. In case of statement i, there may be two contestants with 49% valid votes each and remaining two getting 1% valid votes each.

15. Let total no. of students appeared = x

No. of males students appeared = $\frac{9}{10}x$

No. of female students appeared = $\frac{x}{10}$

Now, $\frac{3}{5}\cdot\frac{9}{10}x + \frac{4}{5}\cdot\frac{x}{10} = 1240$

$\frac{31x}{50} = 1240$

$x = 2000$

16. Hydrogen = 300 gm

or, $\frac{5}{6}$ of water vapor = 300 gm

water vapour = 360 gm = 0.36 kg

17. No. of students taking only science = 4

or, 10% of total students = 4

or, total students = 40

Required percentage $= \frac{7}{40} \times 100 = 17.5\%$

18. Total no. of students = x

No. of boys = $\frac{3}{5}x$

No. of girls = $\frac{2}{5}x$

No. of girls scoring more than 40 marks

$= \frac{4}{5}\cdot\frac{2}{5}x\cdot\frac{8}{25}x;$

total number of students scoring more than 40 marks

$= \frac{3}{5}x$

Required fraction $= \frac{7}{25}\cdot\frac{5}{3} = \frac{7}{15}$

19. The best way of solving this question would be the method of elimmation, i.e., going through the options.

Obviously, option (d) cannot be the answer, because if 500 people are voting in against the resolution, then motion cannot be passed.

Checking option (a); If 400 people are voting against the resolution for the first time, then 500 are voting in favour. Now, in the second voting, persons voting against the resolution would be = 600, so persons voting in favour = 200

In the first voting, resolution was passed by 100 votes and in the second voting, resolution was defeated by 400 votes. And it is 300% more than 100.

20. No. of males $= \frac{11}{18}\times 7200 = 4400$

No. of males married $= \frac{40}{100}\times 4400 = 1760$

No. of females married = 1760

Required percentage $= \frac{1760}{2800}\times = 62\frac{6}{7}\%$

21. Original salary = Rs 100

Actual hiked salary = Rs 150

Salary for three months = Rs 450

Salary to be deducted = Rs 90

Required percentage $= \frac{60}{150}\times 100 = 40\%$

22. Let total no. of respondents = 100

People favoured Mondale = 60

People favoured blesh = 40

New No. of people favouring mandale = 58

New No. of people favouring bush = 42

Required per cent = 14%

23. Let the value of Index = 100

Value of Reliance share = 7

Value of HLL = 13

Value of Infosys = 15

Value of remaining = 65

New value of Reliance = 7.63

New value of HLL = 14.3

New value of Infosys = 15.60

New value of remaining = 68.47

Hence, Required percentage $= \frac{3.47}{65}\times 100 = 5.34\%$

24. Let total donation raised = Rs x

No. of people in group $= y$

then $\frac{4}{5}x = 400y$

$x = 500\,y$

Now, let z = Average donation per person from the new group

$(2y/3)z = x/5$

$z = 150$

25. Total employees $= x$

Female employees $= \frac{3}{5}x$, male employees $= \frac{2}{5}x$

No. of male employees who earn more than

Rs $25000 = \frac{3}{4}\times\frac{2}{5}x = \frac{3}{10}x$

No. of employees who earn more than Rs 25000 = $\frac{9}{20}x$

No. of female employees who earn more than

Rs $25000 = \frac{3}{20}x$

Required fraction $= \dfrac{\frac{3x}{5} - \frac{3x}{20}}{\frac{3}{5}x} = \frac{3}{4}$

26. The question is based upon the concept of successive discount.

Percentage pollution at the end of one day = 20% (1 – 0.8) = 4%

Percent6age pollution at the end of two days = 20% (1 – 0.8) (1 – 0.8) = 4%

Similarly, go ahead now.

27. $M = 1.5E$ ………… (1)

$E + M + S + H = 64$……………. (2)

$S = H + 48$ …………….. (3)

$\frac{3}{2}E + M + S + H = 70$…………………..(4)

Using all the equations, we get

$$M + \frac{2M}{3} + S + S - 48 = 64$$

5M + 6S = 336 ……………..(A)

Similarly, using (1), (2), and (4) we get another equation (B).

Solving both (A) and (B), we get (b) as the answer.

28. Amount = x,

rate = r

$$15=\frac{x\times r\times 1}{100}\text{ ,or, } rx=1500$$

$$\frac{(x+100)\times r\times 1}{100}=\left[420-(x+100)\right]$$

Solving,

we get x = 300, r = 5

29. $(x+2520)=x\left(1+\frac{10}{100}\right)^2$, x = Amul's Profit

x = 12000

y = Binit's Profit

$$4200=\frac{y\times 20\times 1}{100}$$

y = 21000

Chand's profit = Rs 9000

Ratio of their profit

= 12000 : 21000 : 9000

= 12 : 21 : 9 = 4 : 7 : 3

Chand's Share = $\frac{3}{14}\times 70000$ = Rs 15000

31. Go through the optoins.

32. $x\left(1+\frac{r}{100}\right)^2=4x\left(1-\frac{r}{100}\right)^2$

$$\left(1+\frac{r}{100}\right)=2\left(1-\frac{r}{100}\right)$$

$$\frac{3r}{100}=1$$

$$r=\frac{100}{3}=33\frac{1}{3}\%$$

33. For statement A

Share of ATF in 2002 – 03 = $\frac{2.3}{104}\times 100=2.32\%$

Hence, A is true

For statement B

Share of diesel in 2002 – 03 = $\frac{36}{104.2}\times 100$

= 34.55%

Share of diesel; in 2003–04 = $\frac{36.6}{107.4}\times 100=34.0\%$

39. Salary of Abhay = Rs 10000

Salary of Bhaskar = Rs 12500

Chitra's salary = Rs 12500

Dany's Salary = Rs 12000

40. Chitra's New Salary = Rs 13200

Advanced

1. $\frac{1}{5}\times 160.4=32.8$

Production in 99–00 = (160.4-32.8) MT = 128

2. Wheat = $\frac{2.7}{18.9}\times 100=14.28\%$

Cotton = $\frac{4.2}{14.7}\times 100=28\%$

Coffee = $\frac{2.5}{12.3}\times 100=20\%$

Tea = $\frac{2}{20.5}\times 100=10\%$

3. Rice= $\frac{162.7}{160.4}>1$, Coffee $\frac{12.3}{14.3}<1$

Wheat = $\frac{18.9}{16.2}>1$

Tea = $\frac{20.5}{22.5}<1$

Cotton = $\frac{14.7}{18.9}<1$

4. $6x^2+\frac{27x^2}{16}+\frac{3}{4}x^2=4860$, x = no. of people at the beginning

x = 24

5. Required percentage = $\frac{357.5-310.8}{310}\times 100$

$=\frac{46.7}{310.8}\times 100=15\%$

6. Required distance = $\frac{18.9\times 1000}{1.3\times 95}$

7. Auto drivers M3 shift = $\frac{18.9\times 1000}{7}$ = 2700

Taxi drivers – M2 shift = $\frac{240200}{8}$ = 30025

Taxi drivers M3 shift = $\frac{42300}{6}$ = 7050

Auto drivers M1 shift = $\frac{57300}{9}$ = 6366.66

11. Jan 2002 = Rs x

Feb 2002 = Rs $\frac{5}{4}x$

March 2002 = Rs x

April 2002 = Rs $1.2\,x$

May = Rs $\frac{54x}{50}$

12. Female population in Patna in the previous year = 40,000

For Lucknow

$x+\frac{1}{8}x=90000 \quad \frac{9x}{8}=90000$

$x = 80000$

Male = 40000, female = 40000

Next year

Male = 40000, female = 50000

13. Male population of Patna in the previous year = 44000

Male : Female = 11:10

In the next year, male : female = 11:10

$\frac{\text{Male}}{60000}=\frac{11}{10}$, Hence males = 66,000

Total population = 66000 + 60000 = 126000

15. For Arpit Pharma

Constituent A = $\frac{12}{100}\times 300=36$ gm

Constituent B = $\frac{15}{100}\times 300=45$ gm

Constitutent C = $\frac{20}{100}\times 300=60$ gm

Constituent D = $\frac{28}{100}\times 300=84$ gm

For Bittu Pharma:

Constituent A = $\frac{37}{100}\times 300=111$ gm

Constituent B = 45 gm

Constituent C = 39 gm

Constituent D = 45 gm

For Chintu Pharma

Constituent A = 78 gm

Constitutent B = 45 gm

Constituent C = 30 gm

Constituent D = 102 gm

The most affective constituent is B, B is present equally in all three capsules.

Now, it is turn of A.

16. For Arpit drug, side effect = $\frac{60}{100}=\frac{1}{2}=0.5$

For Bittu's drug, side effect = $\frac{.39}{105}=\frac{13}{35}=0.37$

For Chintu's drug, side effect = $\frac{30}{90}\times\frac{1}{3}=0.33$

17. Go through options

18. $\frac{3000}{1000}\times[60+39+30]=3\times 129=387\,\text{kg}$

19. Let population = 100

People read Dainik Jagran = 25

People read Prabhat Khabar = 20

People reading both = 8

People reading only Dainik Jagran = 17

People reading only Prabhat Khabar = 12

Required percentage = (5.1 + 4.8 + 4) = 13.9%

23. For last year

Honda = 100 cars

Toyota = 110 cars

This year

Honda = 120 cars

Toyota = 132 cars

Hyundai = 26.4 cars

Last year, Hyundai = 68.4 cars

Required percentage $=\frac{68.4}{278.4}\times 100=25\%$

24. Let the equations be

$x^2-2x+1=0$ (1)

and $x^2-x-2=0$ (2)

Required percentage $=\frac{1-(-2)}{1}\times 100=300\%$

25. $25000\left(1+\frac{10}{100}\right)^2-25000=\frac{25000\times 11\times x}{100}$

$25000\left[\left(\frac{11}{10}\right)^2-1\right]-\frac{25000\times 11\times x}{100}$

$\left(\frac{11}{10}\right)^2-1=\frac{11x}{100}\Rightarrow\left(\frac{121}{100}-1\right)=\frac{11x}{100}$

$\frac{21}{100}=\frac{11x}{100}$ and $x=\frac{21}{11}$

26. 2 pounds = Rs 132

Rs 54 = 1 euro

Rs $132=\frac{1}{54}\times 132$ Euro$==\left(\frac{66}{27}\right)$Euro$=\frac{22}{9}$Euro

= 2.44 Euro

27. 1 Canadian Dollar = Rs 35.2

1 USD = Rs 48.4

Required No. $=\frac{35.2\times 5}{48.4}=3.67$

28. Required no. of notes $=\frac{1000}{32}+\frac{2000}{32}=93.47=93$ notes

29. Data is in %age and question is about numbers, so it cannot be determined.

30. Through observation.

Questions 31 to 34

Total money with them can be either Rs 400, or Rs 300, or Rs 200, or Rs 200, or Rs 100.

From statement (3) Eeshwar = 200 – 35 = 165

From statement (5) Chandu = 200

From statement (4) Akshay = 400 – 360 = 40

(not least)

Now reading the question meticulously, we get to know that we don't need to calculate the sum of money with Bindu and Dev.

31. Five friends have Rs 400, Rs 300, Rs 200, Rs 200 and Rs 100

Eeshwar had Rs 200	Eeshwar left with Rs 165
Binod had Rs 300	Binod left with Rs 95
Chandu had Rs 200	Akshay left with Rs 40
Akshay had Rs 400	Chandu left with Rs 10
Dev had Rs 100	Dev left with Rs 70

35. Let population = 100

At least 50 people read a newspaper

At most 12.5 people read more than a newspaper. Hence, at least 37.5 people read only one newspaper.

36. Solution = 100 ml and Alcohol = 40 ml

For first vessel

$$\frac{40+x}{100+x}=\frac{1}{2},$$

so, x = 20ml

For second vessel

$$\frac{40+\frac{3}{5}y}{100+\frac{2}{5}y}=\frac{1}{2},$$

so, y = 25ml

Required percentage $=\frac{5}{25}\times 100=20\%$

Questions 37 to 40

This is basically a question that can be solved by forming equations.

Let DS = QA = EU = RC = x and GD = PI = y

So $5x + 2y = 250$, and similarly generate another equations to solve.

4

CHAPTER

Profit, Loss and Discount

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Basic terms involved
- Mechanism of profit generation
- Honest and dishonest ways
- Kinds of questions which are asked in the CAT
- Methods of solving questions

Introduction to the topic

As far as the CAT is concerned, this chapter is important because the application of the concept of profit, loss and discount ranges from simple mathematical calculations in data interpretation (DI) to complex logical evaluations in QA. This chapter can be considered as an extension of percentage, the only difference being the terminology used in both the concepts. The terms we use in percentage are initial value, final value, percentage increase etc. But when we use the same terms in profit, loss and discount (PLD), the names get converted to cost price, selling price, profit percentage etc. Let us understand the concepts and learn problem solving methods for this concept.

TERMINOLOGY

Cost Price

The price (amount) paid to purchase a product or the cost incurred in manufacturing a product is known as the cost price (CP) of that product.

Types of cost

1. **Fixed cost** As obvious from the name, it is that kind of cost which is fixed in all the cases.
2. **Variable cost** Variable costs are those costs which vary according to the number of units produced.
3. **Semi-variable cost** Semi variable costs are those costs which are fixed in one particular strata, but the costs varies among the different stratas.

One good example of fixed cost, variable cost and semi-variable cost is the bill we receive for the telephone connections at our home. A part of that bill, the rental, is fixed cost; and the rest of the part of the bill is calculated on the basis of the numbers of calls made.

Selling Price

The price at which a product is sold is called the selling price (SP) of the product.

Marked Price

The marked price or the mark-up price (MP) is the price which the shopkeeper/retailer fixes in anticipation of some discount they may be asked by a customer.

List Price

List price or the tag price, as the name suggests, is the price that is printed on the tag of the article.

For our calculations related to the concept of PLD, till the moment nothing is stated in the questions we would not see much difference between Marked Price and List price. This can be understood in the following example.

Suppose, the CP of an article is Rs 100 and the shopkeeper wants to earn a profit of 20%, i.e., he wants to have a profit of Rs 20. Now, he anticipates that a customer will ask for some discount and if he fixes the SP at Rs 120 he would not be able to give any discount to the customer. So, he now marks up his price over Rs 120 so that he can give some discount to his customer.

Remember that if nothing else is stated, then mark-up percentage is always calculated over CP.

$$\text{MP} \xrightarrow{\text{\% decrease}} \text{SP} \xrightarrow{\text{\% profit/\% loss}} \text{CP}$$

Example 1 After selling an article at a discount of 50%, profit percentage obtained is 20%. What is the mark-up over CP?

Solution Let us assume that CP = Rs 100.

So, SP = Rs 120.

Now, after giving a discount of 50% over MP, Rs 120 is the SP.

Using product stability ratio, 50%↓ = 100%↑

So, MP = 100%↑ over SP = Rs 240

Hence, percentage mark-up = 140%

Alternatively, 0.5 MP = 1.2 CP

MP/CP = 1.2/0.5 = 2.4

So, percentage mark-up $= \left(\frac{\text{MP}}{\text{CP}} - 1\right) \times 100$

= (2.4 – 1) × 100 = 140%

Further, as we know

If SP > CP, then there will be a profit and if SP < CP, then there will be a loss.

SP – CP = Profit

CP – SP = Loss

Profit % = Profit/CP × 100

Loss % = Loss/CP × 100

Profit% or Loss % is always calculated upon CP, unless something else is mentioned.

It can be seen that whenever, there is a profit, then MP ≥ SP > CP.

However, when there is a loss, then we can not have some definite relationship between the above written three quantities.

Margin

When we calculate profit percentage as a percentage of SP, then it is known as margin.

Calculating CP/SP Profit % or Loss %

If we say that there is a profit of 20%, then

CP × 1.2 = SP

So, if CP = Rs 120, then SP = Rs 120 × 1.2 = Rs 144

Or, if SP = Rs 144, then CP = 144/1.2 = Rs 120

(Needless to say that if there is a profit of 30%, then we will take 1.3 in the place of 1.2, and if there is a loss of 27%, then we will take 0.73 at the place of 1.2)

So, if there is a profit of R%, and CP = C, then,

$$\text{SP} = \frac{\text{C}}{100+\text{R}} \times 100$$

And in case of loss of R%, SP $= \frac{\text{C}}{100\text{-R}} \times 100$

Alternatively, we can use product stability ratio also to find out SP if CP is given.

Example 2 Two shopkeepers sell some article for Rs 4000 each. A shopkeeper calculates his profit per cent on his CP and another calculates his profit per cent wrongly on SP. What is the difference in their actual profit if both claim to have a profit of 20%?

Solution In 1st case,	In 2nd case
SP = Rs 4,000	SP = Rs 4,000
Profit = 20% of CP	Profit = 20% of SP
CP = Rs 3,333.33	CP = Rs 3,200
Profit = Rs 666.66	Profit = Rs 800

So, the difference in profit = Rs 133.33

PROFIT GENERATION

The purpose of studying Profit, Loss and Discount is to understand the mechanism of profit generation under different situations. Broadly it can be seen in two ways:

Honest Ways

Questions which you will find here will be based either in terms of Rs or in terms of goods.

Type 1: Questions in terms of goods

In these types of questions, CP of a fixed number of goods are compared with the SP of another field number of goods. Let us see through an example:

Example 3 The CP of 30 articles is equal to the SP of 40 articles. What is the profit or loss percentage?

Solution To obtain the same amount of money, which was needed to purchase 30 articles, we need to sell 40 articles, which is more than what we have got for the same sum. It means we need to arrange 10 more articles apart from the articles which we have purchased. So, there will be a loss.

Now, CP of 30 articles = SP of 40 articles
Or, CP/SP = 30/40 = ¾
Or, 1 – CP/SP = 1 – ¾ = ¼
So, Loss percentage
= (1 – CP/SP) × 100 = ¼ × 100 = 25%

Alternatively,
CP of 30 articles = SP of 40 articles
= Rs 120 (Assume)
So, CP of one article = Rs 4
SP of one article = Rs 3
Obviously, there is a loss of Re 1
Loss percentage = ¼ × 100 = 25%
Or, Otherwise we can use Profit/loss Percentage

$$= \frac{(\text{Goods left/added})}{(\text{Goods sold})} \times 100$$

Type 2: Questions in terms of money

Basically questions in terms of money relate to the CP or SP. However, sometimes the results given below also come handy in solving problems.

Some Important Results

When SPs of two articles are same

i. First one is sold at a profit of x% and second one is sold at a profit of y%.
Ratio of $CP_1:CP_2 = (100 + y):(100 + x)$

ii. First one is sold at a profit of x% and second one is sold at a loss of y%.
Ratio of $CP_1:CP_2 = (100 - y):(100 + x)$

iii. First one is sold at a loss of x% and second one is sold at a loss of y%.
Ratio of $CP_1:CP_2 = (100 - y):(100 - x)$

iv. First one is sold at a loss of x% and second one is sold at a profit of y%.
Ratio of $CP_1:CP_2 = (100 + y):(100 - x)$

Example 4 SPs of two articles are same. One is sold at a loss of 20% and another one at a profit of 20%. What is the net loss/profit in the whole transaction?

Solution Assume that SPs of each of the article = Rs 100

So, CP_1 (For the article which is sold at a loss of 20%) = 100/0.8 = Rs 125

CP_2 (For the article which is sold at a profit of 20%) = 100/1.2 = Rs 83.33

So, net CP = Rs 125 + Rs 83.33 = Rs 208.33

As we can observe now that SP<CP, so, there is a loss.

Loss % = (8.33/208.33) × 100 = (1/25) × 100 = 4%

Alternatively, if SPs of two articles are same, and one is sold at a profit of x% and another is sold at a loss of x%, then in that case there will be a loss always. And loss percentage

$$= \frac{x^2}{100}\%$$

Example 5 Two articles are sold at Rs 12,000 each. One is sold at a profit of 20% and another one at a loss of 20%. What is the net loss?

Solution SP_1 = Rs 12,000
CP_1 = Rs 12,000/ 1.2 = Rs 10,000
SP_2 = Rs 12,000
CP_2 = Rs 12,000/ 0.8 = Rs 15,000

So, total CP = Rs 25,000 and total SP = Rs 24,000

So, loss = Rs 1,000.

Alternatively, since there is a loss of 4%. So, for every Rs 100 invested, Rs 96 is coming back and Rs 4 is lost. In our case, 96% = Rs 24,000; so, 4% = Rs 1,000.

Example 6 Due North Inc. is the number 1 idea developer company worldwide. One day, they sold Idea 1 to CL and Idea 2 to AMS at a profit percentage of 20% and 30%, respectively. If the sum of developing Idea 1 and Idea 2 is Rs 25,000, what is the developing cost of Idea 1?

Solution The ratio of developing cost of Idea 1 and Idea 2 = 130 : 120

So, developing the cost of Idea 1 = $\frac{130}{250} \times 25{,}000$ = Rs 13,000

Dishonest Ways

Dishonest ways of earning profit includes adulterating or using faulty balance or both of these simultaneously.

Example 7 A shopkeeper sells his articles at his CP but uses a faulty balance which reads 1000 gm for 800 gm. What is his actual profit percentage?

Solution This question can be solved in several methods.

Method 1 Shopkeeper's net profit = 200 gm
CP of 1000 gm = SP of 800 gm
So, profit % = 200/800 × 100 = 25%

Method 2 Since while selling, 800 g = 1 kg
So, 200 gms = 1/4 kg
So, profit = 1/4 = 25%

Method 3 While selling, since the shopkeeper is branding 800 grams as 1 kg, so 1000 grams will be branded as 1000/800 = 1.25 kg and while purchasing, he has paid for just 1000 grams.
So, net profit = (1.25 – 1) = 0.25 kg
So, profit percentage = 0.25/1 × 100 = 25%

Example 8 A shopkeeper marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance also, which reads 1000 gm for 800 gm. What is his net profit percentage?

Solution Let us assume his CP/1000 gm = Rs 100
So, his SP/kg (800 gm) = Rs 126
So, his CP/800 gm = Rs 80
So, profit = Rs 46
So, profit percentage = 46/80 × 100 = 57.5%

Some more examples

Example 9 A cloth store is offering "Buy 3, get 1 free." What is the net percentage discount being offered by the store?

Solution Suppose price of one article is Re 1. So, price of 4 articles = Rs 4. Now the whole scene can be understood as – We are paying only Rs 3 at the place of Rs 4. So, discount = Re 1. So, discount percentage = ¼ (discount/total price) = 25%

Example 10 A shopkeeper sells his goods at its CP only. But still he manages to gain a profit of 40% because he has manipulated his weights. Find how many grams he is actually selling at the place of 1000 gm.

Solution To earn a profit of 40%, shopkeeper needs to make 1.4 kg out of 1 kg. So, he will be selling $\frac{1000}{1.4}$ grams = 728 grams at the place of 1 kg.

Example 11 When an article is sold at 20% discount, SP is Rs 24. What will be SP when the discount is 30%?

Solution 20% discount = 0.8 of MP = Rs 24
So, MP = Rs 30
So, SP when discount is 30% = Rs 21

PRACTICE EXERCISES

■■■

WARM UP

Q.1. An article is bought for Rs 600 and sold for Rs 750. What is the profit percentage?
(a) 20% (b) 25%
(c) 30% (d) None of these

Q.2. By selling a VCD player for Rs 1,950, I got a profit of 30%. At what price should I have sold it in order to get a profit of 40%?
(a) Rs 2,000 (b) Rs 2,100
(c) Rs 2,500 (d) None of these

Q.3. The profit obtained by selling a book for Rs 56 is the same as the loss obtained by selling this book for Rs 42. What is the cost price of the book?
(a) Rs 40 (b) Rs 50
(c) Rs 49 (d) None of these

Q.4. A man bought a shirt at 3/4th of its list price and sold it at 50% above its list price. What is his percentage gain in the transaction?
(a) 50% (b) 70% (c) 80% (d) 100%

Q.5. By selling a CD for Rs 150, a shop owner lost 1/16 of what it costs. What is the cost price of CD?
(a) Rs 120 (b) Rs 140
(c) Rs 150 (d) Rs 160

Q.6. Saurabh sold 250 cycles and had a gain equal to the selling price of 50 cycles. What is his profit percentage?
(a) 25% (b) 20%
(c) 10% (d) None of these

Q.7. If profit as a percentage of the selling price is 25%, then what is profit as a percentage of the cost price?
(a) 25% (b) $33\frac{1}{3}\%$
(c) 20% (d) 15%

Q.8. A man sold an article at a profit of 10%. If he had charged Rs 45 more his profit percentage would have been 25%. What is the cost price?
(a) Rs 200 (b) Rs 300
(c) Rs 250 (c) None of these

Q.9. Richa purchased a pen drive at 4/5th of its list price and sold it at 20% above the original list price. What is Richa's profit percentage?
(a) 25% (b) 40%
(c) 50% (d) None of these

Q.10. A man buys 200 oranges for Rs 10. How many oranges a rupee can he sell so that his profit percentage is 25%?
(a) 10 (b) 14
(c) 16 (d) 20

Q.11. The CP of 24 apples is the same as the SP of 18 apples. Find the percentage gain.
(a) $12\frac{1}{2}\%$ (b) $14\frac{2}{7}\%$
(c) $16\frac{2}{3}\%$ (d) $33\frac{1}{3}\%$

Q.12. A manufacturer earns Rs 20 on selling his product at 10% above his cost. What is the cost price?
(a) Rs 125 (b) Rs 150
(c) Rs 200 (d) None of these

Q.13. By selling an article for Rs 360, the loss incurred is 10%. At what minimum price should he sell that article to avoid loss?
(a) Rs 320 (b) Rs 324
(c) Rs 396 (d) Rs 400

Q.14. Nitika buys a kinetic for Rs 16,000 and sells it for Rs 18,500. If she wants to gain 40%, how much more should she charge for the kinetic?
(a) Rs 3,800 (b) Rs 4,000
(c) Rs 4,200 (d) None of these

Q.15. The owner of Book Point does not get either profit or loss by selling 15 books for Rs 225. How many books should he sell for the same amount to gain 25% profit?
(a) 10 (b) 11 (c) 12 (d) 13

Q.16. A reduction 20% in the price of the rice enables to get 6 kg more of rice for Rs 120. What is the initial price of rice (Rs/kg)?
(a) 4 (b) 5
(c) 6 (d) None of these

Q.17. CP of 12 apples is equal to the SP of 9 apples and the discount on 10 apples is equal to the profit on 5 apples. What is the percentage difference between the CP and SP of apples?
(a) 20% (b) 25%
(c) 16.66% (d) None of these

Q.18. A shopkeeper professes to sell his articles at CP but gives only 800 gm in the place of 1000 gm. What is his profit percentage?
(a) 20% (b) 25%
(c) 16.66% (d) 33.33%

Q.19. In winters, a metre scale shrinks and starts reading 900 cm for every 100 cm. If the shopkeeper is selling the cloth at his CP, what is his loss percentage?
(a) 11.11% (b) 10%
(c) 12.5% (d) None of these

Q.20. A shopkeeper professes to sell his articles at CP but gives only N grams in the place of 1000 gm. If his profit percentage is 100%, what is the value of N?
(a) 750 (b) 500
(c) 0 (d) Not possible

FOUNDATION

Q.1. A car is sold for Rs 2,400 at a profit of 20%. What is the CP?
(a) Rs 2,100 (b) Rs 2,000
(c) Rs 1,800 (d) Rs 2,200

Q.2. A car is sold for Rs 2,400 at a profit of 20% over SP. What is the CP?
(a) Rs 2,000 (b) Rs 1,920
(c) Rs 1,980 (d) Rs 1,800

Q.3. A car is sold for Rs 2,400 at a profit of 20% over SP. What is the actual profit percentage?

(a) 16.66%	(b) 25%
(c) 21.21%	(d) 14.28%

Q.4. Apples are purchased @ 10 apples/Re. How many apples should be sold for one Rupee to obtain a profit of 25%?

(a) 6 (b) 8 (c) 12 (d) 4

Q.5. Sharat sells some articles to Chandra at a profit of 20%. Chandra now sells this article to Mayank at a loss of 30% and Mayank sells this article at a profit of 20%. If CP of Chandra is Rs 150, then what is the SP of Mayank?

(a) Rs 105	(b) Rs 87.5
(c) Rs 125	(d) Rs 126

Q.6. Petrol is purchased @ Rs 5/ litre and sold @ 5 litres/ Re. What is the profit/loss %?

(a) Loss of 96%	(b) No profit, no loss
(c) Profit of 2400%	(d) None of these

Q.7. Despite giving a discount of 11.11% on the mark-up price, profit obtained is equal to 14.28%. What is the per cent mark up over CP?

(a) 14.28%	(b) 28.56%
(c) 25%	(d) 50%

Q.8. A shopkeeper wants to earn a profit of 20% and at the same time, the minimum discount which he wants to offer is 25%. What should be the minimum percentage mark-up over CP?

(a) 60%	(b) 42.5%
(c) 62.5%	(d) 35%

Q.9. In Q. No. 8, what should be the maximum percentage mark-up over CP?

(a) 50%	(b) 100%
(c) 71.4%	(d) Cannot be determined

Q.10. An article when sold at 10% discount on the marked price gave a profit of Rs 70. What is the CP?

(a) Rs 700	(b) Rs 350
(c) Rs 125	(d) Cannot be determined

Q.11. 100 kgs of gold are purchased for Rs 1,100. It is sold in such a way that after selling the whole quantity, the quantum of loss is equal to the amount obtained by selling 20 kgs of gold. What is the selling price?

(a) Rs 9.16	(b) Rs 18.32
(c) Rs 11.11	(d) Rs 25

Q.12. A milkman buys two cows for Rs 750. He sells first cow at a profit of 22% and the second cow at a loss of 8%. What is the SP of second cow if in the whole transaction there is no profit no loss?

(a) Rs 312	(b) Rs 506
(c) Rs 484	(d) Rs 532

Q.13. Two cars are sold for Rs 24,000 each. One is sold at a profit of 20% and another at a loss of 20%. What is the net profit percentage/loss percentage, and amount of profit/loss??

(a) 4% profit, Rs 2,000 profit
(b) 4% loss, Rs 2,000 loss
(c) 1% profit, Rs 500 profit
(d) 1% loss, Rs 500 loss

Q.14. A supplier sells 20 pencils at the marked price of 16 pens to a retailer. The retailer, in turn, sells them at the marked price. What is the percentage profit or percentage loss of the retailer?

(a) Loss 25%	(b) Profit 25%
(c) Loss 20%	(d) Profit 20%

Q.15. Sum of the CP's of two cars is Rs 1,00,000. 1st car is sold at a profit of 20% and 2nd car is sold at a loss of 20%. However, their SPs are same. What is the CP of 1st car?

(a) Rs 40,000	(b) Rs 60,000
(c) Rs 52,400	(d) Rs 47,600

Q.16. Sum of CP's of two cows is Rs 13,000. Both the cows are sold at a profit of 20% and 40% respectively with their SP's being the same. What is the difference of CPs of both the cows?

(a) Rs 1,000	(b) Rs 2,000
(c) Rs 1,500	(d) Rs 2,500

Q.17. Buy three, get one free. What is the percentage discount being offered here?

(a) 33.33%	(b) 25%
(c) 20%	(d) 28.56%

Q.18. CP of 40 articles is equal to the SP of 30 articles. What is the profit/loss percentage?

(a) 25% Profit	(b) 33.33% profit
(c) 25% loss	(d) 33.33% loss

Q.19. Due to a price hike of 20%, 4 kgs less tea is available for Rs 120. What is the original price of tea?

(a) Rs 4/kg	(b) Rs 5/kg
(c) Rs 6/kg	(d) Rs 4.5/kg

Q.20. A shopkeeper sells his goods at its CP only. But he uses 750 grams weight at the place of 1000 gms weight for a kg. What is his net profit percentage?

(a) 25%	(b) 20%
(c) 16.66%	(d) 33.33%

Q.21. A shopkeeper sells his goods at its CP only. But still he manages to gain a profit of 30% because he has manipulated his weights. Find out how many grams he is actually selling at the place of 1000 gm.

(a) 700 (b) 769 (c) 800 (d) 820

Q.22. Which of the following discount options is better for a customer?

i. Successive discounts of 20%, 30% and then pay a service tax of 10%.

ii. Successive discounts of 30%, 20% and then pay a service tax of 10%.
iii. Pay service tax of 10% first, then successive discounts of 20% and 30%.

(a) i or ii
(b) iii
(c) Either of these three
(d) i and iii or ii and iii

Q.23. A wholeseller supplies few chips to a retailer every year. Each chip costs Rs 10,000 to the wholeseller. 5% of the chips are defective and they are to be replaced without charging anything extra. If the wholeseller still makes a profit of 20%, at what price is he selling it to retailer?

(a) Rs 12,300 (b) Rs 12,600
(c) Rs 13,200 (d) None of these

Q.24. A seller calculated his intended selling price at 6% profit on the cost of a product. However owing to some mistake while selling, the units and tens digits of the selling price got interchanged. This reduced the profit by Rs 9 and profit percentage to 2.4%. What is the cost price of the product?

(a) Rs 240 (b) Rs 250
(c) Rs 400 (d) Rs 480

Q.25. A shopkeeper wrongly calculates his profit on SP and finds it to be 25%. What is the actual profit percentage?

(a) 20% (b) 33.33%
(c) 40% (d) 25%

Q.26. There are two shopkeepers selling the same article at the same price for same quantity. One day, first shopkeeper offers a price discount of 25% for the same quantity whereas second shopkeeper offers 25% more quantity for the same price. From a customer's point of view, which deal is better?

(a) 1st shopkeeper's deal
(b) 2nd shopkeeper's deal
(c) Both are equal
(d) Cannot be determined

Q.27. A shopkeeper sells two cows in such a way that CP of first cow is equal to the SP of second cow and SP of first cow is equal to the CP of second cow. Which of the following is final result after the whole transaction?

(a) There is a loss (b) There is a profit
(c) No profit, no loss (d) Cannot be determined

Q.28. When an article is sold for Rs 180, a profit of 20% is made. At what price should the article be sold so that profit percentage is double?

(a) Rs 200 (b) Rs 210
(c) Rs 192 (d) Rs 240

Q.29. Vinit sells a car to Amit at a profit of 20%. Now Amit sells this car to Vicky at a profit of 12%. And finally Vicky sells this car to Nishu at a loss of 21%. What is the sum of CPs of Amit and Nishu?

(a) Rs 250 (b) Rs 475
(c) Rs 540 (d) Cannot be determined

Q.30. Anoop sells a book to Mayank at a profit of 20% and Mayank sells this book to Siddharth at a profit of 25%. Now Siddharth sells this book at a loss of 10% to Shishir. At what percentage loss should Shishir sell this book now so that his SP becomes equal to Anoop's CP?

(a) 36.68% (b) 25.92%
(c) 48.66 (d) Cannot be determined

Q.31. When an article is sold for Rs X, loss percentage is equal to L%. However, when the same article is sold for Rs Y, profit percentage is equal to P%. What is the CP of that article?

(a) 100 (P+L)/(Y−X) (b) (P+L)/(Y−X)
(c) 100 (Y−X)/(P+L) (d) (Y−X)/(P+L)

Q.32. A milkman professes to sell milk at its CP only. But still he is making a profit of 20% since he has mixed some amount of water in the milk. What is the percentage of milk in the mixture?

(a) 80% (b) 83.33%
(c) 75% (d) 66.66%

Q.33. There is some profit when an article is sold for Rs 720. However when the same article is sold for Rs 420, there is some loss. If the quantum of loss is two times the quantum of profit, find the cost price of the article.

(a) Rs 620 (b) Rs 700
(c) Rs 520 (d) None of these

Q.34. While I was shopping in Kamla Nagar market for my baby girl Tatto, I came across two shopkeepers who were giving me a discount of same amount for a top. While first shopkeeper was offering a discount of 14.28%, second shopkeeper was offering a discount of 12.5% over their respective mark-ups. Whose cost price is higher?

(a) 1st shopkeeper
(b) 2nd shopkeeper
(c) Either of them depending upon their respective MP
(d) Cannot be determined

Q.35. A seller offers discounts on the basis of the number of articles purchased. He gives a discount of 10% when 5 articles are bought and a discount of 12.5% when 8 articles are bought. If the profit he makes in each case is the same, then the ratio of the marked price to the cost price of the article is

(a) 4:3 (a) 3:4
(c) 6:5 (d) None of these

Q.36. A shopkeeper purchases his goods at a discount of 10% on the list price. He normally gives a discount of 6% to his customers. If during a special scheme, he gets an additional discount of 2% on his purchase price, find his approximate profit percentage.

(a) 8.8% (b) 7.8%

(c) 6.6% (d) None of these

Q.37. A shopkeeper purchases a packet of 50 pencils at Rs 10 per pencil. He sells a part of the packet at a profit of 30%. On the remaining part, he incurs a loss of 10%. If his overall profit on the whole packet is 10%, the number of pencils he sold at profit is

(a) 25 (b) 30 (c) 20 (d) 15

Q.38. The costs of a pen and a book are Rs 3 and Rs 4 respectively. A student has to spend a sum of exactly Rs 100 to buy pens and books, buying at least one pen and one book. How many combinations of books and pens can he buy?

(a) 6 (b) 8 (c) 12 (d) 10

Q.39. Anam bought a jute bag at 30% discount on the list price. He then sold it at a price which is 160% of the list price thereby making a profit of Rs 81. What is the list price of the bag?

(a) Rs 90 (b) Rs 100

(c) Rs 180 (d) Rs 200

Q.40. What is the profit percentage of a dishonest cloth merchant who professes to sell his articles at CP but uses a scale which weighs less by 16.67%?

(a) 20 (b) 15

(c) 16 (d) None of these

Q.41. A shopkeeper calculated his profit on the marked price and finds it to be 30%. He forgets the fact that he gave a discount of 20%. What is his actual profit percentage?

(a) $14^{2}/_{7}\%$ (b) 20% (c) 25% (d) $28^{4}/_{7}\%$

Q.42. Santa Singh, the local fruit vendor, buys a certain number of oranges at Rs 7 a dozen and equal number at Rs 6 a dozen. He sells the oranges at Rs 7.50 a dozen and makes a profit of Rs 80. How many oranges does he buy?

(a) 20 dozens (b) 30 dozens

(c) 40 dozens (d) 80 dozens

Q.43. A milkman who wants to make a fast buck by using a false measure, measures only 900 ml instead of a litre. If his cost price and selling price per litre of milk are Rs 10 and Rs 12 respectively, what is his daily profit and profit percentage if he sells 20 litres in one day?

(a) Rs 60, 20% (b) Rs 60, $33^{1}/_{3}\%$

(c) Rs 40,25% (d) Rs 40, $33^{1}/_{3}\%$

Q.44. Two lots of oranges with equal quantity, one costing Rs 20 per dozen and the other costing Rs 30 per dozen, are mixed together and the whole lot is sold at Rs 24 per dozen. Then what is the profit or loss?

(a) 4% profit (b) 5.6% profit

(c) 4% loss (d) 5.6% loss

Q.45. A shopkeeper marks up the price of his product by 40%. If he increases the discount from 5% to 10%, the profit would decrease by Rs 14. How much profit would he earn if he gives a discount of 20% on the marked price?

(a) Rs 56 (b) Rs 28

(c) Rs 32 (d) Rs 24

Q.46. Navneet is a bananawallah who buys a gross of bananas at Rs 10 a dozen. During transport and storage two dozen bananas get spoilt. If Navneet spends 10% of his outlay on transport costs and wants an overall profit of 20%, at what price must he sell a dozen bananas?

(a) Rs 14.64 (b) Rs 15.84

(c) Rs 16.24 (d) Rs 17.54

Q.47. Krishna sells his watch at a loss of 5%. If he had sold it for Rs 56.25 more he would have gained 10%. What will be his gain or loss percentage if he sells it for Rs 450?

(a) 10% (b) 15% (c) 20% (d) 30%

Q.48. A person sold his watch for Rs 24 and got a percentage of profit equal to the cost price; then the cost price is

(a) Rs 145 (b) Rs 18

(c) Rs 22 (d) Rs 20

Q.49. A merchant buying goods abroad at a discount of 33.33% on the catalogue price, has to pay import duty of 20% on the net cost of goods, realize a profit of 25% on his outlay, what must he charge for an article priced Rs 228 in the catalogue?

(a) 228

(b) 322

(c) Both of the above

(d) Cannot be determined

Q.50. A group of students prepared stuffed toys as part of their group activity. They spent Rs 100 on velvet, Rs 5 on thread and needle and Rs 27 on miscellaneous items. They made 30 toys, 50% of which were purchased by force by some senior students causing them a loss of 50%. At what % profit should they sell the remaining toys so as to gain 50% on their total cost?

(a) 100% (b) 150%

(c) 75% (d) None of these

MODERATE

Direction for questions 1 to 5: Read the passage below and solve the questions based on it.

Patna-mart, sister concern of Wal-mart, is planning to establish a mechanism to give discount. It is actually a Khul Ja Simsim kind of game where if you win any round, you will get some discount specific to that round, and it you lose, you will be losing all the discounts so far won.

Now, round 1 is having discount coupons of 2%.

Round 2 is having discount coupons of 4%.

Round 3 is having discount coupons of 6%.

And so on…………

There are infinite rounds and you can participate in all the rounds but you can participate in any round only once and before participating in round N (any particular round), you should have participated in round 1 – round N - 1 (all the rounds before that particular round.)

All the discounts won will be successive, e.g., if you win Round 1 and then round 2 also, you will get a successive discount of 2% and then 4%. But again as the rule goes on, if you lose round 3, you will be losing all the discount coupons won so far. But despite losing round 3, you can participate in round 4 and hence.

Q.1. What is the minimum number of rounds to obtain maximum possible discount?

(a) 50 (b) 100

(c) Infinite (d) Cannot be determined

Q.2. What is the maximum possible discount?

(a) 1000% (b) 50%

(c) 100% (d) Infinite

Q.3. What is the minimum possible discount after 20 rounds provided that customer has won a minimum of 18 rounds?

(a) 100% (b) 50%

(c) 0% (d) None of these

Q.4. In the above question, what is the maximum possible discount?

(a) 100% (b) 50%

(c) 0% (d) None of these

Q.5. In a bid to attract more customers, Patna-mart is on a mass-compaigning move. With a new punch line “You can always check-out, but you can never leave”, they are organizing several contact programs across the city. As a result, there is an increase in sale of 10% and 20% respectively on weekdays and weekends (Saturday and Sunday). If each of the articles sold by Patna-mart is sold at a profit of 40%, what is the maximum possible percentage growth in the profit of Patna-mart?

(a) 32% (b) 72%

(c) Minimum 50% (d) Cannot be determined

Direction for questions 6 to 8: Read the passage below and solve the questions based on it.

Had the CP been 10% less and SP been 10% more, profit % would have been double than that of earlier case.

Q.6. What is the actual CP?

(a) Rs 70 (b) Rs 100

(c) Rs 140 (d) Cannot be determined

Q.7. What is the profit % in original case?

(a) 20% (b) 28.56%

(c) 14.28% (d) Cannot be determined

Q.8. How many values for actual SP is/are possible?

(a) 1 (b) 0

(c) 3 (d) Infinite

Direction for questions 9 and 10: Read the passage below and solve the questions based on it.

King Nandan is the king of Divyagarh. One day he decides to sell all the animals living in his big kingdom. Now, there are five states in his kingdom and the only animals which his kingdom is having are horses, cows and sheeps. King Nandan sells all the animals to eight neighboring states. Each of the neighboring states purchases same number of animals at the following rate.

Horse – Rs 1,700 each

Cow – Rs 200 each

Sheep – Rs 200 each

King Nandan receives Rs 28,500 in all?

Q.9. What can be the maximum number of animals?

(a) 80 (b) 130

(c) 160 (d) None of these

Q.10. What are the respective numbers of horses, cows and sheeps in any state?

(a) (3, 109, 8) (b) (1, 37, 3)

(c) (2, 74, 6) (d) Cannot be determined

Q.11. Sunny marks up his goods by 40% and gives a discount of 10%. Apart from this, he uses a faulty balance which reads 800 gms for 1000 gms. What is his net profit/loss percentage?

(a) 8% (b) 57.2%

(c) 37.6% (d) None of these

Q.12. A shopkeeper marks up his goods by 20% and then gives a discount of 20%. Besides he cheats both his supplier and customer by 100 grams i.e., he takes 1100 gram from his supplier and sells only 900 grams to his customer. What is his net profit percentage?

(a) 24.5% (b) 17.33%

(c) 25% (d) 32.5%

Q.13. In question number 12, shopkeeper takes a discount of 20% from his supplier which he disregards while preparing his mark-up. What is his net profit percentage?
(a) 37.5% (b) 46.66%
(c) 52.33% (d) 58.33%

Q.14. In addition to all the data of question number 13, now he adulterates his goods in the ratio of 1:5 (adulterant: original). Assuming that adulterant comes free of cost, what is the net profit percentage?
(a) 62.5% (b) 66.66%
(c) 72.08% (d) None of these

Q.15. In question number 14, if the cost of adulterant is 20% the cost of original substance, what is the net profit percentage?
(a) 77.77% (b) 81.08%
(c) 86.75% (d) None of these

Q.16. Some mangoes are purchased at the rate of 8 mangoes/Re and some more mangoes at the rate of 6 mangoes/Re, investment being equal in both the cases. Now, the whole quantity is sold at the rate of 7 mangoes/Re What is the net percentage profit/loss?
(a) 0.6% profit (b) 0.6% loss
(c) 1.2% loss (d) No profit/no loss

Q.17. Some olives are purchased at the rate of 8 olives /Re and same number of olives at the rate of 6 olives/ Re. Now, the whole quantity is sold at the rate of 7 olives/Re. What is the net percentage profit/ loss?
(a) 0.6% profit (b) 0.6% loss
(c) 1.2% loss (d) No profit/ no loss

Direction for questions 18 to 22: Read the passage below and solve the questions based on it.

A shop has the following discount layers
1st layer – Buy 1, get 2 free.
2nd layer – Buy 2, get 3 free.
3rd layer – Buy 3, get 4 free.
And so on there are infinite layers on the same pattern.

Q.18. For how many of these discount layers discount percentage offered is more than 50%?
(a) 4 (b) 27
(c) 99 (d) None of these

Q.19. The shopkeeper wishes to lessen the percentage of discount by clubbing any two layers. Which two layers should be clubbed so that discount is minimized by maximum percentage?
(a) 1st + 2nd (b) 2nd + 3rd
(c) 3rd + 4th (d) None of these

Q.20. What is the net percentage discount if we club 1st three layers?
(a) 43.33% (b) 46.66%
(c) 53.33% (d) None of these

Q.21. A new condition has been introduced by the shop that anybody can avail one layer discount only once (i.e., if you are planning to purchase 6 articles, then you can not purchase 2 articles in 1st layer and get rest four free. Since you have availed 1st layer discount, you can now ask for only other layer discounts.) Sona is purchasing 8 articles from the store. What is the maximum possible discount that she can avail?
(a) 72% (b) 26%
(c) 38% (d) None of these

Q.22. In Q.21, what is the minimum percentage discount that she can avail?
(a) 0% (b) 22%
(c) 44% (d) None of these

Direction for questions 23 and 24: Read the passage below and solve the questions based on it.

To produce a certain type of engine, two components are imported— Component P is imported from USA and component Q is imported from Japan. P and Q account for 30% and 40% of the production cost of the engine. A profit of 25% is to be made but discount of 10% is to be offered to customers for promotional reasons.

Q.23. If the price of US Dollar appreciates by 20% and the price of Japanese Yen appreciates by 30%, what percentage above the original CP should the product be marked?
(a) 33.3% (b) 48.5%
(c) 63.88% (d) 71.4%

Q.24. US Dollar appreciates by 4% and Japanese Yen by 32%. If marked price remains same, then what is the change in profit percentage?
(a) 4.38% (b) 3.33%
(c) 6.25% (d) 7.69%

Direction for questions 25 and 26: Read the passage below and solve the questions based on it.

A shopkeeper sells two types of custom confiscated goods, A and B, both at same price of Rs 120 whereas they cost him Rs 90 and Rs 160 respectively. Sale for A started from 1 for the first day and then went on increasing every day by 4 units, whereas sale of article B was 450 on the first day and went on decreasing every day by 6 units.

Q.25. Shopkeeper has planned the things in such a way that he starts earning profit exactly on diwali which is after N days. What is the value of N?
(a) 103 (b) 105 (c) 107 (d) 109

Q.26. On which day will he earn profit for the first time on day-to-day basis?
(a) 49 (b) 51 (c) 53 (d) 55

Q.27. In the year 2002, the cost price of an item was 90% of the selling price while in 2003, the cost price was 95% of the selling price. If the gross profit remains the same for both the years, What is the percentage increase in the cost price of the item from 2002 to 2003?
(a) 60% (a) 80%
(c) 100% (d) None of these

Direction for questions 28 to 33: Read the passage below and solve the questions based on it.

Mittal has recently acquired four companies viz., Bank of Bozoland (BOB), My Own Bank (MOB), Zany Obliterated Bank (ZOB), and Dogmatically Obscure Bank (DOB). He noticed that the sales of DOB are half that of BOB whereas the profits of DOB are double that of BOB. The expenses of ZOB are Rs 3 crores less than that of DOB whereas the profits of MOB is Rs 1 crore less than that of ZOB. The expenses of BOB are three times that of DOB. It is also known that the sales of ZOB are Rs 15 crore or one-fourth that of MOB. All figures are for 1992-93. An insider further informs Mittal that the sales of DOB are Rs 10 crores more than that of ZOB and the expenses of BOB are 90% of its own sales. Sales – Expenses = Profit

Q.28. The total sales of all the four companies is (Rs crores)
(a) 200 (b) 150 (c) 125 (d) 160

Q.29. The expenses of BOB exceed that of ZOB by (Rs crores)
(a) 31 (b) 43 (c) 33 (d) 62

Q.30. Which bank had the maximum profit?
(a) BOB (b) MOB (c) ZOB (d) DOB

Q.31. Total profits of the four companies (Rs crores) is
(a) 51 (b) 52
(c) 53 (d) None of these

Q.32. What is the total expense (Rs crores) of all the four banks together?
(a) 110 (b) 130 (c) 150 (d) 160

Q.33. The profits of MOB form what percentage of the expenses of ZOB?
(a) 16% (b) 20% (c) 15% (d) 23%

Q.34. By selling a watch at a profit of 10 per cent, a man got Rs 15 more than half its price. What is the price of the watch?
(a) 10 (b) 15 (c) 25 (d) 5

Q.35. A man would gain 25% by selling a chair for Rs 47.5 and would gain 15% by selling a table for Rs 57.5. He sells the chair for Rs 36; what is the least price for which he must sell the table to avoid any loss on the two together?
(a) Rs 51.2 (b) Rs 58.5
(c) Rs 62.5 (d) None of these

Q.36. A bookseller marks his books at an advance of 69% on the actual cost of production. He allows a discount of 15% and also given a copy free for every dozen sold at a time. What rate per cent profit does the bookseller make, if books are sold in lots of 12?
(a) 32.6 (b) 47.5
(c) 24.9 (d) None of these

Q.37. While returning from Dubai festival, Mr Sanjay Singh purchased a number of articles, all at a discount of $33\frac{1}{3}$% of the list price. However he is required to pay a duty of 20% on the his cost of goods. If he realizes a profit of 25% on his outlay, what must he charge for an article priced Rs 228 as the list price?
(a) 228 (b) 322
(c) Neither (a) nor (b) (d) Data insufficient

Q.38. A certain manufacturer sells a product to the distributor at 10% profit. Then the distributor sells it to the dealer and the dealer sells to the retailer at a mark up of 10% and 20% respectively. The retailer marks up his cost by 20% and then offers a 10% discount to the customer. If the customer had bought it from the distributor directly at the distributor's selling price, then how much reduction in price would he have got with respect to buying it from the retailer?
(a) 29.6% (b) 18% (c) 32% (d) 22.8%

Direction for questions 39 and 40: Read the passage below and solve the questions based on it.

Machine X produces articles at the rate of 50 units/h. SP of each article is Rs 100 and the cost of production is Rs 40 each. However, 20% of the articles produced are defective and hence cannot be sold. The rate of production of machine can be increased, but with every increase of N units/h production cost would increase by $2x$% and the number of defectives would become $(20 + 1.5x)$%.

Q.39. Find the maximum increase in production that can be undertaken without incurring losses.
(a) 14 unit (b) 15 unit
(c) 16 unit (d) 17 units

Q.40. What will be the profit in 1 hour if the machine produces 58 units per hour?
(a) Rs 1,839 (b) Rs 1,178
(c) Rs 1,253 (d) Rs 1,624

Direction for questions 41 and 42: Read the passage below and solve the questions based on it.

There is a growing demand of platinum in the Indian market. A firm dealing in platinum wants to earn a profit to the tune of Rs 5 lacs per annum. Platinum can be produced from a mine. It is required to employ 25 people whose average salary is Rs 2,000 per month. Cost of the raw material i.e., platinum ore is Rs 200 per ton of pure platinum produced. However, the fixed cost associated with the plant is Rs 1,00,000 annually.

Q.41. If the production rate is 20 ton/month, then what should be the price per ton of platinum to earn the required profit?

(a) Rs 4,300	(b) Rs 5,200
(c) Rs 6,300	(d) None of these

Q.42. If the price per ton is Rs 3,000, then how much ton of platinum is required annually to fulfill the profit target?

(a) 300	(b) 350
(c) 400	(d) None of these

Q.43. Two pieces of the same cloth together cost Rs 910. On selling from the first piece as many metres as the second piece originally contained and from the second piece half as many metres as the first piece contained originally. The remaining of the first piece turned out to exceed the remaining of the second piece by 10 m. Each metre of the cloth costs Rs 14. How many metre of cloth did each piece originally contain?

(a) 45 m; 20 m	(b) 45 m; 25 m
(c) 36 m; 29 m	(d) 40 m; 25 m

Q.44. Rupesh marks up an article by p%, gives a discount of $\frac{p}{4}\%$ and gets a profit of $\frac{p}{4}\%$. Had he marked up by $\frac{p}{2}\%$ and given a discount $\frac{p}{6}\%$, what would be his profit percentage?

(a) 25%	(b) $33\frac{1}{3}\%$
(c) 50%	(d) $66\frac{2}{3}\%$

Q.45. Amit brought two cars. He then sold the first car at 10% profit and the second one at 25% profit. The selling price of the second car is 25% more than the selling price of the first car. What is the approximate profit per cent in both the cars together?

(a) 17.85%	(b) 18.36%
(c) 16.19%	(d) Cannot be determined

ADVANCED

Q.1. A publisher printed 3,000 copies of *'Future Shock'* at a cost of Rs 2,400. He gave 500 copies free to different philanthropic institutions. He allowed a discount of 25% on the published price and gave one copy free for every 25 copies bought at a time. He was able to sell all the copies in this manner. If the published price is Rs 3.25, then what is his overall gain or loss percentage in the whole transaction?

(a) 113% (b) 130% (c) 162% (d) 144%

Q.2. A car manufacturing company XYZ can make 300 cars per month at a cost of Rs 2 lacs per car and pay an excise duty of 30% per car. It has 500 cars in its storage room in a month and is able to sell 300 cars per month at a rate of Rs 3 lacs per car. In the budget in the last week of February, the excise duty was reduced by 10% and it will take effect from 1st April. But, the customers cannot wait till 1st April. So the company decides to give this rebate to every customer who buys a car in March as well, but for this particular month the company has to pay 30% duty. The company can reduce or increase manufacturing of car per month by a maximum of 25%. What is the profit or loss in the revenue to the company in the month of March?

(a) 15 lac	(b) 60 lac
(c) 45 lac	(d) 30 lac

Q.3. A shopkeeper uses a "point of sales" software which prepares the bill for customers based on the number of units per unit. Both of the above are 2-digit numbers. While entering the data, he erroneously swaps the digits of both quantity of units and price. As a result, the closing stock balance is lowered by 72 units, and the sales value shown is Rs 1,368. How many units have actually been sold?

(a) 27
(b) 91
(c) 83
(d) None of these

Direction for questions 4 and 5: Read the passage below and solve the questions based on it.

Given below is the production of fertilizers bags by ABC fertilizers during different days of the week.

Monday	*Tuesday*	*Wednesday*	*Thursday*	*Friday*	*Saturday*
650	750	700	800	750	600

There are two modes of transporting these bags to warehouse. A truck has a capacity of 800 bags and cost Rs 1,100 per trip. A Matador has a capacity of 500 bags and cost Rs 700 per trip. The cost of storing the material in case it is not transported is Re 1 per bag per day. The cost of transportation includes the storing cost.

Q.4. In order to minimize transportation cost, how many trips should the Matador and the Truck respectively make to the warehouse during the week?
(a) 3, 4 (b) 2, 4 (c) 1, 5 (d) 4, 3

Q.5. What should be the minimum cost of storing 1 bag/ day, at which it is best to hire a truck everyday?
(a) Rs 0.4/day (b) Rs 1.4/day
(c) Rs 1.6/day (d) Rs 1.2/day

Q.6. Mohit goes to furniture shop to buy a sofa set and a center table. He bargains for a 10% discount on the center table and 25% discount on sofa set. However, the shopkeeper, by mistake, interchanged the discount percentage figures while making the bill and mohit paid accordingly. When compared to what he should pay for his purchases, what percentage did mohit pay extra given that the center table costs 40% as much as the sofa set.
(a) 12.3% (b) 7.2% (c) 8.1% (d) 6.3%

Q.7. In a laboratory experiment, a sample of air which is a mix of only oxygen and vapour is taken. Water contains hydrogen and oxygen gases. If air contains 70% oxygen (including that contained in the water vapour) by weight while water vapour contain 16.66% oxygen by weight, how many kilograms of water vapour is present in 1kilogram of water?
(a) 0.25 (b) 0.34 (c) 0.36 (d) 0.3

Q.8 A shopkeeper purchases a packet of 50 pens at Rs 10 per pen. He sells a part of the packet at a profit of 30%. On the remaining part, he incurs a loss of 10%. If his overall profit on the whole packet is 10%, the number of pens he sold at profit is
(a) 25 (b) 30 (c) 20 (d) 15

Q.9. Amar sold his moped to Bharat at 20% profit and Bharat sold it to Sridhar at 10% profit. Sridhar sold the same to a mechanic and received Rs 2,316. If Amar had sold the same moped to the mechanic and receive the same amount the mechanic paid to Sridhar, what profit percentage would Amar have made?
(a) 52% (b) 48%
(c) 33.3% (d) Cannot be determined

Direction for questions 10 to 13: Read the passage below and solve the questions based on it.

Newspaper major *The Asian Times* is trying to enter the city of Paperabad, where already two newspapers *The Deccan City* and *Good Morning Paperabad* are in circulation. The population of the city is 35 lakhs of which, according to the recent survey, 60% read newspaper. The two evening newspapers sell at Rs 1.75 per copy on weekdays and Rs 3.00 per copy on Sundays. *The Asian Times* promised its readers newspapers at Rs 1.50 per copy per day. Since people were aware of the quality of the paper, the management thought that it can capture a major chunk of the newspaper reading population. On weekdays, the operational cost of the *The Asian Times* were Rs 62,500 per day and the cost for the material and printing charges was Rs 2.75 per newspaper. For the Sunday edition the cost of the newspaper per copy was Rs 3.00 and the operational cost were Rs 75,000. The number of the copies of the newspaper *The Asian Times* sold on Sunday was 16.66% less than the number of copies sold on any of the week days. The number of copies sold on any weekday being the same. The amount collected through the advertisement on any weekday was the same and equaled Rs 2,50,000 per day while for Sunday it was 4,35,000.

Q.10. If *The Asian Times* wanted to break even, how many copies of the newspaper are required to be sold every week?
(a) 21,25,800 (b) 16,97,143
(c) 47,50,500 (d) 50,50,500

Q.11. What percentage of the newspaper readers should read *The Asian Times* so that the newspaper makes a profit of Rs 12,500 every weekday?
(a) 7.61% (b) 12.4 %
(c) 12.8 % (d) None of these

Q.12. If 50,000 of the readers of *The Deccan City* now read *The Asian Times* on all weekdays, how much profit would *The Asian Times* expect to make every week?
(a) 2,55,000 (b) 3,85,000
(c) 4,57,000 (d) Cannot be determined

Q.13. How many copies of the newspaper should *The Asian Times* sell on Sunday so that there is a profit of Rs 60,000 every Sunday?
(a) 1,88,000 (b) 2,40,000
(c) 3,20,000 (d) 3,76,000

Direction for questions 14 to 16: Read the passage below and solve the questions based on it.

In the "Get-One" supermarket, a new software was installed in the computerized billing section which made the calculations of each of the item sold very easy on the first day of the operations. To verify the workability of the software only one of the item was billed on the computer.

Due to some technical problem in the software, it reversed both the numbers of the item sold and price per piece as a result of which the closing of the stock items showed 63 items less and the value on the items still left was shown Rs 936. It was known that both the stocks left and the price per piece are two digit numbers.

Q.14. How many items were actually left at the end of the day?
(a) 29 (b) 81
(c) 92 (d) None of these

Q.15. What is the correct collection for the day if 44 pieces were sold?
(a) Rs 2,288 (b) Rs 1,744
(c) Rs 1,444 (d) None of these

Q.16. By how much would the incorrectly calculated value of the items sold be more/less than the correctly calculated value?
(a) Rs 887 more (b) Rs 887 less
(c) None of these (d) Cannot be determined

Q.17. Mahesh and Umesh purchased a radio each for the same price and both marked-up their respective radios by the same amount. Mahesh gave a discount of Rs 20 followed by another discount of 20% on the reduced price, while Umesh gave a discount of 20% followed by a discount of Rs 20. If Mahesh's profit percentage is equal to thrice of Umesh's loss percentage, what is the profit (in Rs) of Mahesh on his radio?
(a) Rs 2 (b) Rs 3 (c) Rs 4 (d) Rs 5

Q.18. Vinod is a very shrewd shopowner. He adjusted his electronic weighing scale in a typical way. Error percentage of his sale is directly proportional to the displayed weight of the commodity. But he gives a discount of 10% in every transaction. His weighing scale shows 5 kg for 4 kg. What is the actual profit % when the displayed weight is 10 kg?
(a) 35% (b) 30% (c) −5% (d) 50%

Direction for questions 19 and 20: Read the passage below and solve the questions based on it.

A shopkeeper believes in running his business on the sentiments of market i.e., quick response to the demand-supply relationship. He starts every morning by selling product X at Rs 500 for the first hour. If the number of customers for any hour is more than 10, he will increase the price by Rs 10 over the price of the previous hour. If less than 10, he will decrease the price by Rs 10 over the previous hour. If it is equal to 10, then he will maintain the same price level of the previous hour. CP of product X is Rs 440.

Q.19. If the customer for the first four hours are 15, 10, 6 and 12, then find the profit made by the shopkeeper.
(a) Rs 2,050 (b) Rs 2,270
(c) Rs 2,740 (d) None of these

Q.20. The shopkeeper never sells for a loss. If he shuts his shop after 10 hours due to this principle and there is exactly one hour when the number of customers was 10, then the minimum number of hour when the number of customers is more than 10 is
(a) 2 (b) 1 (c) 4 (d) 3

Q.21. In a joint venture, Ramesh invested Rs 6 lakhs as fixed cost. While Suresh had to pay the variable cost for every three months in advance (i.e., for the first three months at the start of first month and for the next three months at the start of the fourth month and so on).
Table for costs is given below

Quantity produced/month	*Variable cost per unit (Rs)*	*Fixed cost (Rs) for 6 months*
0-1,00,000	1.5	6,00,000
1,00,001-2,00,000	1.25	6,00,000
2,00,001-3,00,000	1	9,00,000

For the first three months they produced 1,00,000 units per month and for the next three months they produced 1,50,000 units per month.

All units produced were sold at Rs 4 each. After six months they sold their company for Rs 2,00,000 and divided total profit according to their investments. Find out the total profit.
(a) 15.875 lakhs (b) 15.125 lakhs
(c) 10.125 lakhs (d) None of these

Q.22. In the above question, what is the share of Suresh?
(a) 8.7 lakhs (b) 6.2 lakhs
(c) 4.3 lakhs (d) 5.6 lakhs

Q.23. In an orchard, there are only mango and apple trees. Fruits available on each tree are equal to the total number of trees of the same kind. Also, mango trees are twice in number as compared to the apple trees. Only 40% of total mangoes and 70% of total apples are in good condition. These have to be stored before selling. This year the farmer could sell only 75% of the stored mangoes and $\frac{6}{7}$ of the stored apples. The rest was wasted. Selling price per mango is Rs 10 and per apple is Rs 5. Investment to store (consider only this as the cost price) is Rs 20 per mango and Rs 10 per apple. In this transaction, the farmer lost Rs 15,000. What is the total number of trees in the garden?
(a) 75 (b) 80
(c) 25 (d) 60

Q.24. A spring balance reads 1 kg when actually 800 g is weighed on it. The shrewd grocer has already marked up the price of his goods by 20%. One day, a police inspector detects this and orders him to repair his spring balance and revert to the original list price of the goods. Moreover, he asked him to give a discount of 10% to all the customers for a month from then onwards as a punishment. A day after, Ramu paid Rs 16 for 1 kg rice. Before the raid, how much he would have paid for the same quantity?

(a) Rs 26.66 (b) Rs 27.5
(c) Rs 24 (d) Rs 28

Q.25. The evergreen shrubs at Ravi's nursery are planted in rows on a square plot of land measuring 2,401 ft^2. The shrubs are planted in such a manner that the centers of the adjacent shrub are 7 ft apart and the outer shrubs are planted along the edges of the plot, with a shrub at each corner. Ravi spent $ 896 to cover all the costs necessary for raising this crop of the evergreen shrubs. If Ravi succeeds in selling each shrub for $ 35, his profit will be what percentage of his total cost?

(a) 50% (b) 90% (c) 150% (d) 200%

Q.26. A retailer keeps Reebok shoes, which are listed at a mark-up of 10% above their factory price. However, on Monday he realized the shortage of demand and hence, decreased the list price by 10%. On the very next day, i.e., Tuesday, he realized he is making a loss so, he increased the list price by 10% again. He continues this trend indefinitely. On which day he will suffer a loss of more than 10% per shoes for the first time?

(a) Monday (b) Tuesday
(c) Thursday (d) Sunday

Direction for questions 27 and 28: Read the passage below and solve the questions based on it.

A person purchased 50,000 cups of glasses at a rate of Rs 10 per cup. For transporting these cups to his shop, he has four options as follows:

Mode of transportation	*Rent for one trip*	*Capacity of number of glass in one trip*	*Breakages during transportation*
Truck	Rs 2,000	20,000	8%
Small truck	Rs 1,000	12,000	10%
Jeep	Rs 750	10,000	15%
Rickshaw	Rs 500	5,000	10%

Even if a transport vehicle is not full to its capacity, rent for the trip will remain the same.

Q.27. Which mode of transportation is most profitable?

(a) Truck (b) Small truck
(c) Jeep (d) Rickshaw

Q.28. If the most profitable mode of transportation is chosen, then what should be his selling price per cup so as to get the profit of 10%?

(a) Rs 12.45 (b) Rs 14.67
(c) Rs 13.80 (d) Rs 12.10

Direction for questions 29 to 31: Read the passage below and solve the questions based on it.

The air conditioning market is one of the few segments that have been stable since the last three-four years. The total air-conditioning industry is estimated to be around Rs 3,000 crores with central air-conditioning including packaged and ducted air-conditioning systems estimated to be around Rs 1,200 crore. Windows and mini-split air-conditioners are estimated at Rs 1,800 crores, with window ACs cornering 80% of the market.

Prior to 1997, corporations, institutions and the government dominated the consumer segment. They did around 85–90 per cent of the buying. It was only in the year 1997 that the home segment picked up. The institutional segment accounted for 65 per cent while the rest went to the home segment. The average growth for the last five years in the residential segment has been around 24 per cent.

Q.29. LG has priced their split-ACs at Rs 35,000. If they have estimated a market share of 70% in the split-AC market, then how many split-ACs would they be selling approximately?

(a) 72,000 (b) 68,000
(c) 71,000 (d) 70,000

Q.30. If prior to 1997, the institutional buyers comprised of Rs 2,700 crore worth of market, what is the approximate share of Samsung ACs in 1997 if they sold only to home buying segment? Their share prior to 1997 was 20% in the home AC segment, while it had increased by 15% in 1997 from its earlier levels. The overall market size for total AC market had remained the same.

(a) 300 crores (b) 69 crores
(c) 76 crores (d) 241.5 crores

Q.31. With respect to the previous question, if Samsung entered the institutional market in 1998 and had a share of 10% by 1999. What was the ratio of its share in the institutional market in 1998-1999 with respect to its share in the home market prior to 1997? The market size of institutional segment showed a 10% increase over its size in 1997. (Use the data from previous questions if required). (Ratio is required with respect to revenue).

(a) 3 : 1 (b) 2.75 : 1
(c) 3.57 : 1 (d) 2.8 : 1

Q.32. The cost price of a dozen mangoes is equal to the selling price of 10 mangoes. The selling price of apples per piece is the cost price of a mango (per piece). If the profit % made on selling an apple is half that of selling a mango, then what is the cost of an apple if the selling price of one mango is Rs X?

(a) Rs $\frac{X}{6}$ (b) Rs $\frac{5X}{6}$

(c) Rs $\frac{25X}{33}$ (d) Rs $\frac{33X}{25}$

Direction for questions 33 to 37: Read the passage below and solve the questions based on it.

Various costs involved in printing news papers

All the given cost are calculated in Rs

Now, the total cost of printing is calculated by the following formula:

Total cost = P × (Columns × Cost per column + Rows × Cost per row + Labour cost per print) + Plate cost + Setup cost

Where, P = The number of prints

Fixed cost = Plate Cost + Set up Cost

Total cost = Fixed cost + Variable Cost

Newspapers	*The Times of India*	*The Hindustan Times*	*The Pioneer*	*The Statesman*	*Dainik Jagran*
Columns	3	4	2	4	2
Cost/column	20	15	35	20	10
Rows	400	350	400	600	200
Cost/row	0.5	0.8	0.6	0.3	0.5
Plate	1000	800	750	800	600
Set-up cost	200	100	150	300	150
Labour cost per print	1.5	2	1.2	0.9	1

Q.33. If the number of prints of each newspaper (in thousands) is equal, which newspaper is more costly?
(a) *The Times of India*
(b) *Dainik Jagran*
(c) *The Hindustan Times*
(d) *The Pioneer*

Q.34. If one has to print hundred prints of *The Pioneer*, then what is the ratio of the variable cost to the fixed cost (approx)?
(a) 20 (b) 30
(c) 40 (d) 35

Q.35. Which is the costliest newspaper in terms of cost per ten prints?
(a) *The Times of India*
(b) *The Hindustan Times*
(c) *The Pioneer*
(d) None of these

Q.36. How many copies of the *Dainik Jagran* should be printed so that the total cost is equal to the cost of 100 prints in *The Times of India* (approx)?
(a) 150 (b) 175
(c) 225 (d) 200

Q.37. Which newspaper has the highest fixed cost?
(a) *The Pioneer*
(b) *The Statesman*
(c) *The Hindustan Times*
(d) None of these

Direction for questions 38 to 40: Read the passage below and solve the questions based on it.

The following table shows the relationship between production volumes, unit price and unit cost of a particular product. Refer to the following table to answer the questions that follow:

Contribution = volume x (price – cost)

Production Volumes (No. of Units)	*Unit Price (Rs)*	*Unit Cost (Rs)*
1000	140	100
2000	125	95
3000	110	90
4000	100	80
5000	90	75
6000	75	70
7000	65	60

Q.38. At what volume will the contribution be maximum?
(a) 2,000 units (b) 3,000 units
(c) 4,000 units (d) 5,000 units

Q.39. If the unit cost goes down by Rs 10 at all levels of production, what will be the maximum possible contribution (Rs)?
(a) Rs 75,000 (b) Rs 1,25,000
(c) Rs 1,20,000 (d) Rs 80,000

Q.40. A breakeven point is the volume of production at which for the first time the contribution equals fixed costs. If the fixed costs are Rs 60,000, then the production level at which the break even is reached, is
(a) 2,000 units (b) 4,000 units
(c) 5,000 units (d) 6,000 units

ANSWERS

■■■

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(c)	(d)	(d)	(a)	(b)	(b)	(c)	(c)	(d)	(c)	(d)	(d)	(c)	(b)	(d)	(b)	(b)	(b)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(b)	(d)	(a)	(b)	(a)	(d)	(d)	(a)	(b)	(b)	(b)	(a)	(a)	(b)	(b)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(c)	(b)	(b)	(b)	(a)	(c)	(b)	(d)	(b)	(c)	(b)	(c)	(d)	(c)	(d)	(a)	(a)	(a)	(a)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(a)	(d)	(b)	(c)	(d)	(b)	(c)	(d)	(a)	(b)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(c)	(d)	(d)	(b)	(b)	(d)	(d)	(d)	(a)	(b)	(b)	(c)	(d)	(d)	(b)	(d)	(a)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(a)	(c)	(a)	(c)	(b)	(d)	(b)	(c)	(d)	(d)	(b)	(a)	(c)	(d)	(a)	(a)	(d)	(d)	(c)
Q. No.	41	42	43	44	45															
Answer	(b)	(d)	(d)	(c)	(a)															

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(c)	(b)	(c)	(c)	(c)	(a)	(a)	(d)	(b)	(d)	(d)	(b)	(b)	(d)	(b)	(b)	(a)	(c)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(a)	(a)	(a)	(c)	(d)	(a)	(d)	(a)	(d)	(c)	(c)	(b)	(d)	(b)	(b)	(d)	(c)	(b)	(a)

HINTS AND SOLUTIONS

Warm up

1. Profit percentage $=\frac{150}{600}\times 100=25\%$

2. CP = $\frac{\text{Rs }1950}{1.3}$ = Rs 1500

   Hence required Selling Price = Rs 1500 × 1.4 = Rs 2100

3. Let us assume that the cost price = Rs X

   $56-x=x-42$

   $2x=98$

   Hence, $x=49$

4. MP = x

   CP = $3\,x\,/4$

   SP = $\frac{3}{2}x$

   Profit Percentage $=\frac{\frac{3}{2}x-\frac{3}{4}x}{\frac{3}{4}x}\times 100=100\%$

5. Assume cost price = Rs X

   $x-150=\frac{x}{16} \qquad \Rightarrow x=160$

6. Profit Percentage $=\frac{50}{200}\times 100=25\%$

7. SP = x

   Profit Percentage $=\frac{\text{Profit}}{x}\times 100$

   $25=\frac{\text{Profit}}{x}\times 100$

   Profit $=\frac{1}{4}x$

   CP = $x-\frac{1}{4}x=\frac{3}{4}x$

   Profit Percentage $=\frac{\frac{1}{4}x}{\frac{3}{4}x}\times 100=33\frac{1}{3}\%$

8. CP = Rs x

   Profit $=\frac{x}{10}$

   Now, $\frac{\frac{x}{10}+45}{x}\times 100=25$

   $X=300$

9. List Price = x

   CP $=\frac{4}{5}x$, SP $=\frac{6}{5}x$

   Profit Percentage $=\frac{\frac{6x}{5}-\frac{4x}{5}}{\frac{4}{5}x}\times 100=50\%$

10. Cost price of 1 orange = Rs $\frac{10}{200}$ = Rs $\frac{1}{20}$

    SP of 1 orange = Rs $\left(\frac{1}{20}+\frac{1}{4}\cdot\frac{1}{20}\right)$ = Rs $\frac{1}{16}$

    So, he should sell 16 oranges in a rupee to make a profit of 25%.

11. Profit Percentage $=\frac{6}{18}\times 100=33\frac{1}{3}\%$

12. $\frac{1}{10}x=20$

    $x=200$

13. CP $=360+360\cdot\frac{1}{9}$ = Rs 400

14. Profit in order to gain 40% = Rs 6400

    More money charged = (Rs 6400–2500) = Rs 3900

15. CP of 1 book = Rs 15

    SP of 1 book in order to gain 25% profit = Rs 18.75

    Required No. of Books = $\frac{22.5}{18.75}=12$

16. Suppose initially x kg of rice was bought.

    Now, $\frac{1}{4}x=6$

    $x=24$

    Initial price = Rs $\frac{120}{24}$ = Rs 5

17. CP of 1 apple = Re 1

    CP or 12 apples = Rs 12, SP of 9 apples = Rs 12

    Required Difference = $\frac{3}{9}\times 100=33\frac{1}{3}\%$

18. Let CP of 1000 gm = Rs 1000

    CP of 800 gm = Rs 800

    SP of 800 gm = Rs 1000

    Profit Percentage $=\frac{200}{800}\times 100=25\%$

19. CP of 100 cm = Rs 100

CP of 110 cm = Rs 110

Loss = Rs 110

Loss Percentage $= \frac{10}{100} \times 100 = 10\%$

20. Go through options.

N = 500Fdfs

Foundation

1. CP = $2400 - 2400 \times \frac{1}{6} - 2400 - 400 - 2000$

2. Profit Percentage $= \frac{2400 \times 20}{100} = 480$

CP = Rs (2400–480) = Rs 1920

3. Actual Profit Percentage = $\frac{480}{1920} \times 100 = 25\%$

4. CP of 10 Apples = Rs 1

CP of 1 Apple = Rs $\frac{1}{10}$

SP of 1 Apple = Rs $\left(\frac{1}{10} + \frac{1}{4} \cdot \frac{1}{10}\right) = \frac{1}{8}$

5. CP of Mayank = CP of Chandra × 0.7 = Rs 150 × 0.7 = Rs 105

SP of Mayank = Rs 126

6. CP of 1 liter petrol = Rs 5

SP of 1 liter Petrol = Rs $\frac{1}{5}$

Loss % = $\frac{5 - \frac{1}{5}}{5} \times 100 = \frac{24}{25} \times 100 = 96\%$

7. MP = x

S.P. = $x - \frac{1}{9}x = \frac{8}{9}x$

$\text{CP} = \frac{8}{9}x - \frac{8}{9}x \cdot \frac{1}{8} = \frac{8x}{9} - \frac{x}{9} = \frac{7x}{9}$

Required Percentage $= \frac{x - \frac{7x}{9}}{\frac{7}{9}x} \times 100 = \frac{\frac{2x}{9}}{\frac{7x}{9}} \times 100$

=28.56%

8. Let CP = Rs 100

SP = Rs 120

MP = x

$\frac{3}{4}x = 120$

x = Rs 160

Hence, 60%

9. Maximum can be anything beyond 60%.

10. Without knowing the profit percentage, we cannot determine the CP.

11. Assume x is the SP per kg.

$1100 - 100x = 20x$

$120x = 1100$

$x = \frac{110}{12} = 9.16$

12. $\frac{22}{100}x = (750 - x) \times \frac{8}{100}$

$x = 200$

SP of 2nd Cow = (550-44) = Rs 506

13. Net Loss Percentage $= \left(\frac{20}{10}\right)^2 = 4\%$

For every Rs 100 invested, Rs 4 is the loss, or, for every Rs 96 realized through selling, Rs 4 is the loss.

Loss amount = Rs $\left(48000 \times \frac{1}{24}\right) = \text{Rs}\,2000$

14. MP of 1 Pencil = Re 1

For supplier, SP of 20 pencils = Rs 16

For retailer, SP of 20 pencils = Rs 20

Profit Percentage $= \frac{4}{16} \times 100 = 25\%$

15. $x + \frac{1}{5}x = (100000 - x) - \frac{1}{5}(100000 - x)$

$x + \frac{1}{5}x = 100000 - x - 20000 + \frac{x}{5}$

$x = 40000$

16. $x + \frac{1}{5}x = (13000 - x) + \frac{2}{5}(13000 - x)$

$x + \frac{1}{5}x = 13000 - x + 5200 - \frac{2}{5}x$

$2x + \frac{3}{5}x = 18200$

Or, $\frac{13}{5}x = 18200$

$x = 7000$

17. Required Difference = 7000 – 6000 = 1000

18. CP of 1 article = Re 1

CP of 40 articles = Rs 40

SP of 30 articles = Rs 40

Profit Percentage $= \frac{10}{30} \times 100 = 33.33\%$

19. Due to price hike of 20%, quantity of less tea available

$= \frac{1}{6} x = 4 \quad x = 24$ (where x = initial quantity)

Original price = $\text{Rs}\,\frac{120}{24} = \text{Rs}\,5$

20. Profit Percentage $= \frac{250}{750} \times 100 = 33.33\%$

21. Weight he is selling instead of 1000 grams = 1000/1.3 = 769 grams.

22. Let MP = Rs 100

(i) Customer has to pay = Rs 61.6

(ii) Customer has to pay = Rs 61.6

(iii) Customer has to pay = Rs 61.6

23. Let No. of chips supplied = 100

For whole seller, Net CP = (10000 × 100 + 5 × 10000) = Rs 1050000

Profit $= \text{Rs}\left(1050000 \times \frac{1}{5}\right) = \text{Rs}\,210000$

SP for whole seller = Rs 1260000

SP of 1 chip = Rs 12600

24. Go through options.

25. Let SP = Rs 100

Profit = Rs 25

CP = Rs 75

Actual Profit Percentage $= \frac{25}{75} \times 100 = 33.33\%$

26. Let SP of 1 kg article = Rs 100

For 1st Shopkeeper

SP of 1 kg Article = Rs 75

For 2nd Shopkeeper

SP of 1.25 kg Article = Rs 100

SP of 1 kg Article = Rs 80

27. Let CP of 1st Cow = Rs x

CP of 2nd Cow = Rs Y,

Hence net CP = Rs $(x + y)$

SP of 1st Cow = Rs y

SP of 2nd Cow = Rs X

Net SP = Rs $(x + y)$

Hence, No Profit No Loss

28. CP = $\text{Rs}\left(180 - 180 \cdot \frac{1}{6}\right) = \text{Rs}\,150$

29. In the whole question, there is nowhere the mention of any Rs figures. Since amount is not given, we cannot calculate any value (in Rs).

30. Let CP of book for Anoop = Rs 100

CP of book for Mayank = Rs 120

CP of book of Siddharth = Rs 150

CP of book of Shishir = Rs 135

Required loss percentage $= \frac{35}{135} \times 100 = 25.92\%$

31. Let CP = Rs 700

x = Rs 90

L = 10%

Y = Rs 110

P = 10%

Putting these values, we get (c) as the answer.

32. CP of 100% Pure Milk = Rs 100

CP of the Mixture = $\text{Rs}\left(100 - 100 \times \frac{1}{6}\right) = \text{Rs}\,83.33$

Hence, 83.33% milk is present 1 liter of mixture.

Alternatively, if he has initially 10 L of milk, he must have made it 12 L of mixture to get a profit of 20% (since SP per litre = CP per litre). Hence out of 12 litres of mixture, 10 litres is milk and 2 litres is water.

33. Let us assume that CP = Rs x

$720 - x = 2\,(x - 420)$

$720 - x = 2x - 840$

$3x = 1560$

$x = 520$

34. According to the question, 14.28% of Mark up of 1st shopkeeper = 12.5% of mark up of 2nd shopkeeper. With this we can compare the markup prices.

However nothing can be said about the cost prices of the shopkeepers.

35. Let MP = Rs 100 and CP = Rs X

then, $450 - 5x = 700 - 8x$

$x = \frac{250}{3}$

Required ratio $= \frac{100}{250/3} = 6 : 5$

36. Since, cost price and selling price are not known therefore, we cannot find profit percentage.

37. $13x + 9\,(50 - x) = 550$

$4x = 100$

$x = 25$

38. Go through options.

39. Assume that MP = Rs 100

then CP of Anam = Rs 70

and his SP = Rs 160

So, his Profit = Rs 90.

Now using unitary method, when profit of Anam = Rs 90, then his MP = 100. So when his MP = Rs 81, his MP = 90.

40. CP of 100 m = Rs 100

$$\text{CP of } \frac{500}{6}\text{m} = \text{Rs } \frac{500}{6}$$

$$\text{Profit Percentage } = \frac{100-\frac{500}{6}}{\frac{500}{6}}\times 100 = 20\%$$

Alternatively, profit is 16.66% by selling 83.33% quantity.

$$\text{Hence profit percentage} = \frac{16.66\times 100}{83.33}\% = 20\%$$

41. Let MP = Rs 100

CP = Rs 70 and SP = Rs 80

Hence, profit percentage = 14.28%

42. $(7.50 \times 2x) - (7x + 6x) = 80$

$2x = 80$

$x = 40$

43. CP of 1000 ml milk = Rs 100

CP of 900 ml milk = Rs 9

Profit = Rs (3 × 20) = Rs 60

$$\text{Profit Percentage } = \frac{60}{9\times 20}\times 100 = 33.33\%$$

44. Net CP = Rs $50x$

Net SP = Rs $48x$

$$\text{Loss Percentage } = \frac{2x}{50x}\times 100 = 4\%$$

45. Let CP = Rs 100

MP = Rs 140

Profit, when discount is 5% = Rs 33

Profit, when discount is 10% = Rs 26

$$\text{Hence, CP} = \frac{100}{7}\times 14 = \text{Rs } 200$$

MP = Rs 280

SP = Rs 224

46. 1 Gross = 12 dozen

Net CP = Rs (120 + 12) = Rs 132

Net SP = Rs 158.4

$$\text{Required price } = \text{Rs } \frac{158.4}{10} = \text{Rs } 15.84$$

47. 15% of CP = Rs 56.25, Hence CP = Rs 375

New SP = Rs 450, Hence profit = Rs 75

Profit percentage = 20%

48. $x = \frac{24-x}{x}\times 100$, So. $X = 20$

49. $$\text{Net CP} = \text{Rs}\left(228-\frac{1}{3}\times 228+\frac{1}{5}\times 152\right)$$

= Rs (152 + 30.4) = Rs 182.4

$$\text{SP} = \text{Rs}\left(182.4+\frac{1}{4}\times 182.4\right) = \text{Rs } 228$$

50. Net CP = Rs 132

Money paid by seniors = Rs 33

$$\text{Required Percentage } = \frac{99}{66}\times 100 = 150\%$$

Moderate

1. Maximum possible discount in any case can be only 100% and any discount more than 100% is actually 100% only. Now, whatever is the discount, till the moment we get a discount of 100%, value of the article cannot be zero (even infinite discounts of 99.99% won't make it equal to a discount of 100%) So, number of rounds needed to obtain 100% discount = 50.

2. As discussed above, maximum possible discount can be only 100%.

3. Despite the fact that a customer has won 18 rounds out of the first 20 rounds, if he looses in the 20th round, net discount = 0%

6.

Actual CP	C
New CP	0.9C
Actual SP	S
New SP	1.1 S
Actual Profit %	100(S – C)/C
New Profit %	100 (1.1 S – 0.9C)/0.9C

So, [S/C] – 1 = 2/7

Profit percentage = 28.56%

And obviously CP cannot be calculated.

6. As done above, CP cannot be calculated.
Alternatively, since in the question, nowhere the term Rs has been used, the value of CP cannot be calculated.

7. Profit percentage as calculated above = 28.56%.

8. Since there can be any value of CP, the number of values of SP can also be anything, provided profit percentage = 28.56%.

11. Assume CP/1000 gm = Rs 1000
So, MP = Rs 1400
SP = Rs 1260
Now, when he sells 1000 gms, he actually obtains the money for only 800 grams (Go through the statement carefully).
So, when he sells articles worth Rs 1000, money obtained after selling will be equal to = Rs 1260 × 0.8 = Rs 1008
So, profit percentage = 8%

12. Assume his CP = Rs 1000/1100 gm
MP = Rs 1200 and SP = Rs 960/900 gm
So, SP/1100 gm = Rs 1173.33
So, profit = Rs 173.33
Profit percentage = 17.33%

18. Since in all these cases, more articles are given free than the purchased article, so the net discount is more than 50%. Hence in all these layers, the discount offered is more than 50%.
(If the articles given free is equal to the articles purchased, discount percentage will be equal to 50%).

20. If we club the first three layers, it is like: buy 6, get 9 free. So, net discount = 60%

[illegible]

	Sales	*Profit*	*Expenses*
BOB	$2x = 50$	$y = 5$	$3z = 45$
MOB	60	$t - 1$	$60 - 2 = 58$
ZOB	15	$t = 3$	$15 - 3 = 12$
DOB	$x = 25$	$2y = 10$	$25 - 10 = 25$

34. Price = Rs X
SP = Rs 1.1 $x = 0.5x + 15$, So, $0.6x = 15$
So, $x = 25$
Alternatively, we can do this question very easily by using options.

36. CP = Rs X/dozen = 0.833/copy
So, MP = 1.69 x/dozen
SP = Rs 1.4365x /13 copies = 0.1105 x/copy
So, profit = 32.6%

37. List price = 228
CP (before duty) = Rs 152
CP (after duty) = Rs 152 × 1.2 =
SP = Rs 152 × 1.2 × 1.25 = Rs 228

38. This whole sequence goes like this:
100 →10% UP → 110 →10% UP → 121→20% UP→145.2→ UP by 20% and down by 10%→ 156.816
Distributors' SP = 121
So, percentage reduction = 22.8%

43. Go through the options.

44. This question can be done by forming equation by assuming the CP to be Rs X.
Alternatively, doing a bit of hit and trial gives the value of p = 200.
Using this, we get the answer as option (d) = 66.66%

Advanced

1. Cost Rs 2400
Published Price Rs 3.25
SP = 75/100 × 3.25 = Rs 2.4375
No. of free copy = (3000/25) = 120 + 500 = 620
So, total SP = 2380 × Rs 2.4375 = Rs 5801.25
Hence percentage gain = 5801.25 – 2400/2400 – 100 = 144%

3. $9(x - y) = 72$, $x - y = 8$
Therefore, only possibility is 19 or 91.

4. Matador's capacity is just 500, so it will be used minimum, i.e., 1 day.
So truck will be used for the rest 5 days.

6. Centre table – 40 sofa set – 100
According to Mohit, cost of centre table-
90/100 × 40 = 36
Cost of sofa set – 75/100 × 100
According to shopkeeper,
CT – 75/100 × 40 = 30
SS – 90/100 × 100 = $\frac{90}{120}$
Therefore, extra = 8.1%

8. Through the method of alligation
+30 −10
+10
20 20
1 : 1

9. Amar–100, Bharat–120, Sridhar–132
No profit or loss is mentioned about the deal between Sridhar and the mechanic. So the answer cannot be determined.

[illegible]

10.

	Weekdays	*Weekend*
Fixed Cost	Rs 3,75,000	Rs 75,000
Variable Cost	$2.75 \times 6x$	$(5x/6) \times 3$
Total Cost	Rs 3,75,000 + $2.75\,x$	Rs 75,000 + $(5x/6) \times 3$
Reveneue through advertisement	Rs 15 lacs	Rs 4,35,000

Where x is the number of copies sold any week day.
For breaking even,
[Rs 375000 + $16.5x$] + [Rs 75,000 + $(5x/6) \times 3$] = Rs 19, 35,000 + $10.25x$
Rs 450000 + $19x$ = Rs 1935000 + $10.25x$
So, $x = 1697142.28$
Hence, to obtain break-even point, minimum 1697143 copies are to be sold.

12. Since we are not having any information regarding the total number of copies sold (the information is talking about the reader of "The Deccan City"), we cannot determine the profit.

13. Rs 75, 000 + $3x$ = Rs 4,35000 + $1.5x$, where x is the number of copies sold on Sunday.

[illegible]

14. Number of items – Shown number of items = 63
$9(x - y) = 63$, $(x - y) = 7$ where x and y are its Ten's and Unit's digits
So, the number may be 81 and 18 or 92 and 29.
However value = Rs 936, which is divisible by 18 only.
So, actually left items = 81.

15. Rate = 936/18= 52
It is given that price/piece was also reversed.
So, price/piece = Rs 25
Collection = Rs 25 × 44 = Rs 1100

27.

Mode of Transportation	*Rent for one trip*	*Breakages During Transportation*	*Total Cost*
Truck	Rs 6,000	Rs 40,000	Rs 46,000
Small truck	Rs 5,000	Rs 50,000	Rs 55,000
Jeep	Rs, 3750	Rs 75,000	Rs 78,750
Rickshaw	Rs 5,000	Rs 50,000	Rs 55,000

Obviously, truck is the most profitable mode.

28. For 10% profit, he should obtain 1.1 × (Rs 5,00,000 + Rs 6,000) = Rs 5,56,600
Number of cups left = 46,000
So, SP per cup = Rs 12.1

[illegible]

38. Using the options,
(a) 2,000 units, C = 2,000 × 30 = Rs 60,000
(b) 3,000 units, C = 3,000 × 20 = Rs 60,000
(c) 4,000 units, C = 4,000 × 20 = Rs 80,000
(d) 5,000 units, C = 5,000 × 15 = Rs 75,000

39. At 5,000 units, C = 5,000 × 25 = Rs 1,25,000

40. From question number 38 At 2,000 units,
C = Rs 60,000.

5

CHAPTER

Average and Its Applications

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Average and its various interpretations
- Definitions and properties
- Applications of average in different cases such as alligation
- Different cases of mixtures and alligation
- Methods of solving questions

Introduction to the topic

Averages and their applications was once an important topic for CAT, but the number of questions asked from this topic has been reduced in the last couple of years. However, this concept is used vigorously in other exams. As a concept, average should be seen more as a logical concept of group formation than a pure mathematical concept. And this is true about its applications also.

AVERAGE

Traditionally, average is calculated by dividing the sum of all the numbers by the number of numbers.

$$\text{Average} = \frac{\text{Sum of numbers}}{\text{Number of numbers}}$$

For example, the average of the four numbers 214, 215, 219, 224 will be

$$\text{Average} = \frac{214+215+219+224}{4} = 218$$

Central Value Meaning of Average

Average can also be seen as the central value of all the given values.

Applying this definition for the above example let us assume the central value of all the given numbers = 214

Now, find the deviations of all the numbers from 214

214 215 219 224

When assumed central value is (214), the sum of the deviations

= 0 + 1 + 5 + 10

Now, finding the average of deviations gives us

$$\frac{0+1+5+10}{4} = \frac{16}{4} = 4$$

So, average = assumed central value + average of deviations = 214 + 4 = 218

Thus, we can assume any value to be the assumed average and then find the average of all the deviations; and when we add all the numbers and divide it by number of numbers, 0 is assumed to be the central value.

Example 1 Average age of A, B and C is 84 years. When D joins them the average age of A, B, C and D becomes 80 years. A new person E, whose age is 4 years more than D, replaces A and the average age of B, C, D and E becomes 78 years. What is the age of A?

Solution Since the average age of A, B and C is 84 years so, we can assume that age of A, B and C is 84 years.

A = 84 years

B = 84 years

C = 84 years

After D has joined them,

	Initially	*Finally*
A	84 years	80 years
B	84 years	80 years
C	84 years	80 years
D	…..	80 years

Decrease in the age of A, B and C can be attributed to the increase in the age of D. So, after getting 12 years in total (4 years each from A, B and C) D is at 80 years. The original age of D = 80 − 12 = 68 years.

So, age of E = 72 years

Now, the average age of A, B, C and D = 80 years; A + B + C + D = 320

And average of B, C, D and E = 78 years; B + C + D + E = 312

(Since the average difference between the age of A and E is 2 years,)

Difference (A – E) = 2 × 4 = 8 years

Since E = 72 years, so A = 80 years

By using central value method of averages every question of average can be done by mental calculation only.

Example 2 Average of ten two digit numbers is S. However when we reverse one of the numbers AB as BA from the given 10 numbers, then the average becomes S + 1.8. What is the value of B – A?

Solution Average of 10 numbers is increasing by 1.8, so it can be assumed that 1.8 has been added to all the numbers.

So, BA is 1.8 × 10 = 18 more than AB.

There are so many two-digit numbers which satisfy above condition. Using hit and trial method, the number can be 13, 24, 35, 46, 57, 68, 79. In every case, difference between the digits = 2

Otherwise, we can use the formula

$(BA - AB) = 9 \times (B - A)$

Where BA and AB are two-digit numbers.

Example 3 The average score of Rahul Dravid after 25 innings is 46 runs per innings. If after the 26th innings, his average runs increased by 2 runs, then what is his score in the 26th inning?

Solution Runs in 26th inning = Total runs after 26th innings – Total runs after 25th innings

$$= 26 \times 48 - 25 \times 46 = 98$$

Alternatively, this question can be done by the above given central value meaning of average. Since the average increases by 2 runs per innings, we can assume that 2 runs have been added to his score in each of the first 25 innings. Now, the total runs added in these innings have been contributed by the runs scored in the 26th inning, which must be equal to 25 × 2 = 50 runs.

And after contributing 50 runs, his score in the 26th inning is 48 runs.

Hence, runs scored in the 26th inning = new average + old innings × change in average

$$= 48 + 25 \times 2 = 98.$$

To have a mental mapping, we can see the whole situation as:

Number of innings	*Average in the 1st 25 innings*	*Averate in the 1st 26 innings*	*Addition*
1	46	48	2
2	46	48	2
3	46	48	2
...	...	...	...
...	...	...	...
...	...	...	...
25	46	48	2
26		48	

Properties of Average

1. Average always lies in between the maximum and the minimum value. It can be equal to the maximum or minimum value if all the numbers are equal.
   For example—A1, A2, A3 and A4 are four numbers given where A1>A2>A3>A4.
   Average of these four numbers will always lie in between A1 and A4.
   However, if all the four numbers are equal (A1 = A2 = A3 = A4) then the average will be equal to each of these numbers.
   Average = A1 = A2 = A3 = A4.
2. Average is the resultant of net surplus and net deficit, as used in the central tendency method.
3. When weights of different quantities are same, then simple method is used to find the average. However, when different weights of different quantities are taken, then it is known as weighted average. Here the method of weighted average is used to find the average.
   For example, assume per capita income of India is USD 500 and per capita income of US is USD 200. Now if we merge India and US into one country then it is observed that per capita income of this new country will not be equal to $\frac{500+200}{2}$ = USD 350
4. If the value of each quantity is increased or decreased by the same value S, then the average will also increase or decrease respectively by S.
5. If the value of each quantity is multiplied by the same value S, then the average will also be multiplied by S.
6. If the value of each quantity is divided by the same value S ($S \neq 0$) then the average will also be divided by S.

Example 4 The average of 4 positive numbers is A and the average of all the possible triples formed out of these four positive numbers is B. Which of the following is true regarding A and B?

(a) A = B (b) A > B
(c) A < B (d) Cannot be determined

Solution Let us assume that the numbers are 1, 2, 3 and 4

Average of 1, 2, 3 and 4 : 1 + 2 + 3 + 4 = 10/4 = 2.5

The triplets are 1, 2 and 3; and the average = $\frac{6}{3}$

1, 2 and 4; and the average = $\frac{7}{3}$

1, 3 and 4; and the average = $\frac{8}{3}$

2, 3 and 4; and the average = $\frac{9}{3}$

Average of these four averages = $\frac{\frac{6}{3}+\frac{7}{3}+\frac{8}{3}+\frac{9}{3}}{4} = 2.5$

So, option a (A = B) is the answer.

Central value method

It should be observed here that when we find the average of all the possible triplets, all the numbers (1, 2, 3, 4) are added thrice. So effectively we are adding 12 numbers. Hence, the average should be equal to:

$$\frac{3(1+2+3+4)}{12} = 2.5$$

Thus, the average will be equal in all the cases.

Extension to this problem The average of four positive numbers is A and the average of all the possible pairs formed out of these four positive numbers is B. Which of the following is true?

(a) A = B (b) A > B
(c) A < B (d) Cannot be determined

Some Special Cases

1. Average involving time, speed and distance

$$\text{Average speed} = \frac{\text{Total distance}}{\text{Total time}}$$

However, while solving the questions involving time, speed and distance, we should assume some distance, preferably the LCM of all the given speeds.

Example 5 Lovely goes to Patna from New Delhi at a speed of 40 km/h and returns with a speed of 60 km/h. What is her average speed during the whole journey?

Solution Assuming that the total distance between Patna and New Delhi is 120 km (LCM of 40 and 60) the total time taken (Patna – New Delhi and New Delhi – Patna) = 3 + 2 = 5 h

So, average Speed = $\frac{240}{5}$ = 48 km/h

2. Average involving age

Average of a group of n persons given at any point of time can be calculated in the following way

5 years ago	*10 years ago*	*Now*	*10 years later*	*5 years later*
N – 5	N – 10	N	N + 10	N + 5

Example 6 The average age of the 5 members of a family is 20 years. The youngest member of the family is 4 years old. At the time of his birth the average age of the rest of the family was N years. What is the average age of the family (in terms of N) excluding the youngest member?

Solution Sum of ages of all the members of the family = 100

Sum of ages of all the members of the family excluding the youngest number = 100 – 4 = 96

So, average age of all the members of the family excluding the youngest number = 96/4 = 24 = N

What is the average age of the family(in terms of N) excluding the youngest member = N + 4

3. Average involving number system

1. Average of 1st n consecutive natural numbers $= \frac{n+1}{2}$
   For example, the average of 1st five natural numbers = 3
2. The average of 1st n consecutive even natural numbers $= n + 1$
   Sum of 1st n consecutive even natural numbers $= n(n + 1)$
   For example, the average of 1st five even natural numbers = 6
3. The average of 1st n consecutive odd natural numbers $= n$
   Sum of 1st n consecutive odd natural numbers $= n^2$
   For example, the average of 1st five odd natural numbers = 5

Weighted Average

It is observed that the average can be calculated only if the weights of all the factors, is same. So, the weighted average is a more generalized form of average. This can be further understood with the following illustration

	Class A	*Class B*
No. of students	10	10
Average age	12 years	16 years

Now, if we combine both these classes, then the average age of all the students $= \frac{12+16}{2} = \frac{28}{2} = 14$ years. This is one standard example of Average.

Let us see another example:

	Class A	*Class B*
No. of students	12	16
Average age	10 years	14 years

Now, if we combine these two classes, then the average can not be calculated by the above mentioned method, since the weights attached to different averages are different.

Some more cases of weighted average

1. As we have observed above in the case of average, if per capita income of India is USD 500 and per capita income of US is USD 200, and if we merge India and US into one country, then the per capita income of this new country (India + US) cannot be found by just adding the per capita income of both the countries and dividing it by 2.

   The weights, i.e., the population attached to the different averages, i.e., the per capita income would also have to the considered.
2. Average speed cannot be calculated by just adding the different speeds and then dividing it by 2. This can be understood by the following example:

   A person goes to A from B at a speed of 40 km/h and returns with a speed of 60 km/h, then the average speed for the whole journey can not be equal to 50 km/h.

   We know that average speed $= \frac{\text{Total distance}}{\text{Total time}}$

Finding expression for weighted average

Let us assume there are N groups with the following structure:

Group no.	*No. of members*	*Avg. age of the group*
G_1	N_1	A_1
G_2	N_2	A_2
G_3	N_3	A_3
...	...	...
...	...	...
G_N	N_N	A_N

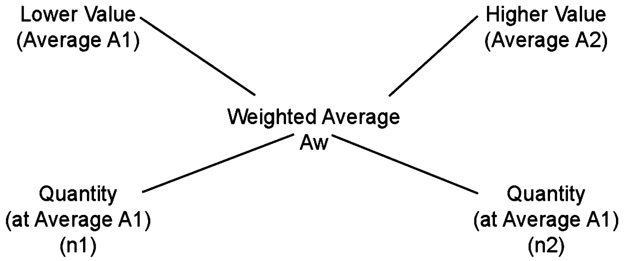
If we combine all these groups, then the average age of all the members = $(N_1 \times A_1 + N_2 \times A_2 \ldots + N_N \times A_N)/(N_1 + N_2 + N_3 + \ldots + N_N)$

$$= \frac{\sum AN}{\sum N}$$

Group no.	*No. of members*	*Avg. age of the group*
G_1	N_1	A_1
G_2	N_2	A_2

Considering that there are only two groups and both the groups are combined then the average age of all the members $= (N_1 \times A_1 + N_2 \times A_2)/(N_1 + N_2) = A_w$

Simplifying the above written expression, we get the conventional criss-cross method as given below



And we write this as: $\frac{n_1}{n_2} = \frac{A_2 - A_w}{A_w - A_1}$

i.e.,

$$\frac{\text{Quantity (Lower Priced)}}{\text{Quantity (Higher Priced)}} = \frac{\text{Higher Price} - \text{Averge Price}}{\text{Average Price} - \text{Lower Price}}$$

It is quite obvious that the ratio of the number of persons/items in different groups is proportionate to the deviations of their average from the average of all the people combined.

This average of all the members combined is known as weighted average, and is denoted by A_w. This process of mixing the two groups is also referred as alligation.

Elements of weighted average

As we can see from the above derivation there are five quantities

i. Number of members in 1st group (n_1)
ii. Number of members in 2nd group (n_1)
iii. Average of 1st group (A_1)
iv. Average of 2nd group (A_2)
v. Weighted average (A_w)

Normally, in the case of weighted average, we get questions in which one of these five elements is missing, and with the help of the remaining four quantities, the value of that missing quantity is found. Different possibility (situations) are given below (Y represents – data given, N represents – data not given):

Situations	n_1	n_2	A_1	A_2	A_w
First	Y	Y	Y	Y	N
Second	Y	Y	Y	N	Y
Third	Y	Y	N	Y	Y
Fourth	Y	N	Y	Y	Y
Fifth	N	Y	Y	Y	Y
Sixth	N	N	Y	Y	Y

First Situation

Example 7 10 kg of rice priced at Rs 12 per kg is mixed with 6 kg of rice priced at Rs 16 per kg. What is the average price of the whole mixture?

Solution Lower priced value = Rs 12 per kg and its quantity = 10 kg

Higher priced value = Rs 16 per kg and its quantity = 6 kg

Using Alligation,

$\frac{10}{6} = \frac{16 - A_w}{A_w - 12}$, or, A_w = Rs 13.5/kg

However, in my opinion, in this situation it is better to use the normal method rather than using the weighted average method of finding A_w.

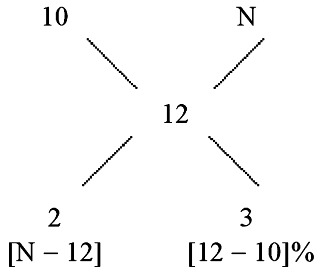
Normal method – Total value = 12 ×10 + 16 × 6 = 216

So, average price = $\frac{216}{16}$ = Rs 13.5/kg

Second/Third Situation

Example 8 Two varieties of rice are mixed in the ratio 2 : 3. The price of the mixture is Rs 12 per kg and the price of the variety having lower weight is Rs 10 per kg. Find the price of the other variety.

Solution



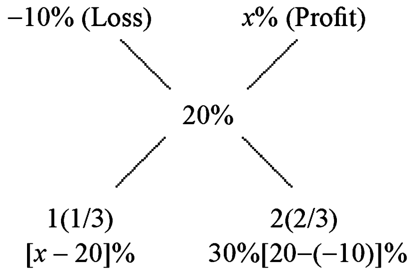
Now, $\frac{2}{3} = \frac{(N-12)}{(12-10)} = \frac{N-12}{2}$

So, N = Rs 13.33 per kg

Fourth/Fifth Situation

Example 9 Some articles are purchased for Rs 450. 1/3rd of the articles are sold at a loss of 10%. At what percentage profit should the remaining articles be sold to obtain a net profit of 20% on the whole transaction?

Solution



Now, = [20−(−10)]/[x − 20] $= \frac{2}{1}$

So, $x = 35\%$

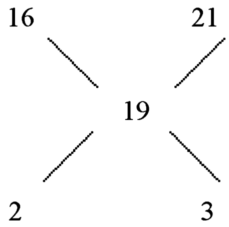
It is seen that the quantities are in the ratio of 1 : 2, so the deviation from mean percentage profit in the loss percentage and profit percentage will also be same.

2 corresponds to 30%, so 1 will correspond to 15%. So, $x = 35\%$

Sixth Situation

Example 10 Two different qualities of sugar are mixed in some ratio. The price of one quality of sugar is Rs 16/kg and that of another quality is Rs 21/kg. In what ratio have the sugar of two qualities been mixed if the price of the mixture is Rs 19/kg?

Solution



So, the ratio of quantity of sugar of different qualities = 2 : 3

MIXTURES

When two or more than two pure substances/mixtures are mixed in a certain ratio, they create a mixture. Here we shall confine ourselves to mostly homogenous mixtures in view of the questions commonly asked in CAT.

Mixing without Replacement

In this particular type of mixing, two or more than two substances are mixed without any part of any mixture being replaced.

Example 11 In a mixture of 420 litres the ratio of milk and water is 6 : 1. Now, 120 litres of the water is added to the mixture. What is the ratio of milk and water in the final mixture?

Solution Volume of milk = 360 litres and volume of water = 60 litres.

When 120 litres of water is added, volume of water = 180 litres

So, the ratio of milk water = 2 : 1

Example 12 A milkman mixes 20 litres of water with 80 litres of milk. After selling one-fourth of this mixture, he adds water to replenish the quantity that he had sold. What is the current proportion of water to milk?

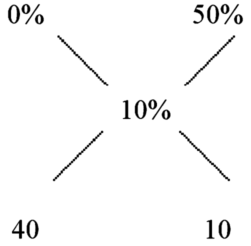
Solution Ratio of milk and water = 20 : 80

When 1/4th of this mixture is sold, total volume of mixture will be reduced by 25%, so 25% of milk and water both will reduce. So, volume of milk and water after selling out 1/4th of mixture = 60 litres and 15 litres respectively. Addition of 25 litres of water will finally give us the following: volume of milk = 60 litres and volume of water = 40 litres. Hence, the ratio of water and milk = 40 : 60 = 2:3.

Example 13 How many litres of fresh water should be mixed with 30 litres of 50% milk solution so that resultant solution is a 10% milk solution?

Solution

Method 1 *Using Alligation*



So, the ratio of fresh water added: milk solution = 4 : 1

Hence, 120 litres of fresh water should be added.

Method 2 *Principle of constant volume of one component*

Since we add fresh water, the volume of milk will be constant.

Now volume of milk = 15 litres = 10% of the new mixture.

So, 100% of the new mixture = 150 litres

So, volume of fresh water added = 150 − 30 = 120 litres.

Method 3 *Principle of inverse proportion*

We know that concentration is inversely proportional to the volume of solute added.

So, in this case 30 × 50% = 10% × (30 + x), where x is the volume of water added.

So, $x = 120$ litres

Method 4 *Using equation*

In the final mixture, $\frac{\text{Milk}}{\text{Total}} = 10\% = \frac{15}{30+x}$

So, $x = 120$ litres

Mixing with replacement In this particular type of mixing, two or more than two substances are mixed by replacing some part of a mixture. In these types of questions, total volume may or may not be the same and information regarding the same can be obtained from the question.

Case 1 When the quantity withdrawn and the quantity replaced are of the same volume.

Initially there are 40 litres of milk, and 4 litres of milk is replaced with 4 litres of water

Obviously, there will be 36 litres of milk and 4 litres of water.

Now, 4 litres of mixture is replaced with 4 litres of water

The quantity of milk and water being withdrawn here will be in the ratio of 9 : 1 (36:4). So, quantity of milk withdrawn = 9/10 × 4 = 3.6 l.

So, the volume of milk = 32.4

And the volume of water = 7.6

Now, again 4 litres of mixture is replaced with 4 litres of water

The quantity of milk and water being withdrawn here will be in the ratio of 81 : 19 (32.4:7.6). So, the quantity of milk withdrawn = (81/100) × 4 = 3.24 l

So, the volume of milk = 29.16

And the volume of water = 10.84

If we summarize the above values, then it looks like

	1st operation		*2nd operation*		*3rd operation*	
	Taken out	*Left*	*Taken out*	*Left*	*Taken out*	*Left*
Milk	4	36	3.6	32.4	3.24	29.16
Water	0	4	0.4	7.6	0.76	10.84

It can be seen that the quantity of water or milk withdrawn is 10% of the existing volume of milk or water because only 10% of the total volume of 40 litres taken out.

With this we can deduce a standard formula for these kinds of calculations.

If V is the initial volume of milk (or any liquid), and x litres of milk is always replaced by water, then quantity of milk left after n such operations = $V\left(1-\frac{x}{v}\right)^n$

This formula is very similar to the standard formula we have seen in the case of Compound Interest $\left[P(1+\frac{R}{100})n\right]$.

The only difference between the two formulae is that while the interest is being added every year (or for the given time-period), volume of milk gets reduced after every operation.

Using the values of the above example, quantity of milk left after 3 operations $= 40\times\frac{36}{40}\times\frac{36}{40}\times\frac{36}{40} = 19.16\,\text{litres}$

The same problem can be solved with straight-line approach of percentage also Since 10% of existing volume is taken out every time, the percentage of milk in the final mixture after the third operation = 72.9%

$$\left(100\% \xrightarrow{10\%\downarrow} 90\% \xrightarrow{10\%\downarrow} 81\% \xrightarrow{10\%\downarrow} 72.9\%\right)$$

Since 100% = 40, so 72.9% = 29.16 litres

Case 2 When the quantity withdrawn and the quantity replaced are of the same volume, but the total volume before replacement does not remain the same.

Initially, there are 40 litres of milk, and 4 litres of milk is taken out and 4 litres of water is poured in

So, there will be 36 litres of milk and 4 litres of water.

Now, 5 litres of mixture is taken out and 5 litres of water is poured in.

The quantity of milk and water being withdrawn here will be in the ratio of 36 : 4. So, the quantity of milk withdrawn $= \frac{36}{40}\times 5$

Milk left = $40\times\frac{36}{40}\times\frac{35}{40}$

Again, if now 6 litres of mixture is taken out and 6 litres of water is poured in

Milk left = $40\times\frac{36}{40}\times\frac{35}{40}\times\frac{34}{40}$

Case 3 When the quantity withdrawn and the quantity replaced are not of the same volume.

Initially there are 40 litres of milk, and 4 litres of milk is taken out and 5 litres of water is poured in

Obviously, there will be 36 litres of milk and 5 litres of water.

Now, 5 litres of mixture is taken out and 6 litres of water is poured in then the quantity of milk and water being withdrawn will be in the ratio of 36 : 5. So, the quantity of milk withdrawn = $\frac{36}{41}\times 5$

Milk left = $40\times\frac{36}{40}\times\frac{36}{41}$

Again 6 litres of mixture is taken out and 7 litres of water is poured in.

Thus, the volume of milk in the final mixture

$= 40\times\frac{36}{40}\times\frac{36}{41}\times\frac{36}{42}$

Example 14 Two vessels A and B of equal capacities contain mixtures of milk and water in the ratio 4 : 1 and 3 : 1, respectively. 25% of the mixture from A is taken out and added to B. After mixing it thoroughly, an equal amount is taken out from B and added back to A. The ratio of milk to water in vessel A after the second operation is

(a) 79:21 (b) 83:17
(c) 77:23 (d) 81:19

Solution Assume there is 20 litres of the mixture in both the vessels.

In vessel A, milk = 16 litres and water = 4 litres

25% from A to B = milk in B = 15 + 4 = 19 litres

= water in B = 5 + 1 = 6 litres

ratio = 19:6

Equal amount from vessel B to vessel A

= milk in $A = 12 + \frac{19}{5} = \frac{79}{5}$

= water in $A = 3 + \frac{6}{5} = \frac{21}{5}$

Hence, the ratio is 79:21

PRACTICE EXERCISES

■■■

WARM UP

Q.1. Find the average age of a family of five members, whose ages are 42, 49, 56, 63 and 35 years respectively.
(a) 50 years (b) 49 years
(c) 45 years (d) 48 years

Q.2. Rahul Ghosh walks from A to B at 8 km/h and comes back from B to A at 12 km/h. What is his average speed for the entire journey?
(a) 8.8 km/h (b) 9.6 km/h
(c) 10.2 km/h (d) 11.4 km/h

Q.3. The average of five numbers is 39.20 and the average of three of these numbers is 41. Find the average of the remaining two numbers.
(a) 35.5 (b) 36.5 (c) 37.5 (d) 38.5

Q.4. In a class of 60 students, the average height of 30 students is x cm and that of the remaining students is y cm. Find the average height of the whole class.
(a) $(x + y)$ cm (b) 30 cm
(c) $\frac{x+y}{2}$ cm (d) None of these

Q.5. What is the average of the first 15 multiples of 3?
(a) 21 (b) 27 (c) 24 (d) 15

Q.6. The average age of 7 members of a family is 25. If one of the members whose age is 28 years, is excluded, then others are in the ratio 1:2:3:4:5:6. Find the age of the eldest member of the family.
(a) 28 (b) 35 (c) 14 (d) 42

Q.7. The average of seven consecutive even numbers is 10. If the next three even numbers are included, then what is the new average?
(a) 10 (b) 12 (c) 13 (d) 14

Q.8. Average weight of three students is 40 kg. If the first student is twice the second and the second student is thrice the weight of the third student, then what is the weight of the student with lightest weight?
(a) 18 (b) 20
(c) 14 (d) None of these

Q.9. In a test, the average marks of five students was found to be 50. However during cross checking of the papers, it was discovered that the mark of one student was misread as 84 instead of 48. Find the correct average.
(a) 57.2 (b) 42.8 (c) 46.5 (d) 43.2

Q.10. The average weight of seven members of a family is 18 kg. If the head of the family is not considered, the average weight of the others would be 5 kg less. The weight of the head of the family is
(a) 42 kg (b) 48 kg
(c) 52 kg (d) 36 kg

Q.11. In a prep school, the average weight of 30 girls in a class of 50 students is 16 kg and that of the remaining students is 15.5 kg. What is the average weight of the all the students in the class?
(a) 15.2 kg (b) 15.8 kg
(c) 15.4 kg (d) None of these

Q.12. The average of the amount of money that 10 girls have is Rs 45. When two more girls join, the average increases by Rs 2. Find the average amount of the two new girls.
(a) Rs 57 (b) Rs 54
(c) Rs 48 (d) None of these

Q.13. In what ratio must 35% spirit solution be mixed with pure spirit to get a resultant solution of 56% spirit?
(a) 35:44 (b) 44:21 (c) 8:3 (d) 56:9

Q.14. A beggar has 50 coins with an amount of Rs 15 in the denominations of 25 paise and 50 paise. How many 25 paise coins are there with the beggar?
(a) 36 (b) 40 (c) 30 (d) 25

Q.15. Two litres of pure spirit is added to 6 litres of a spirit solution containing $16\frac{2}{3}\%$ spirit. What is the concentration of the spirit in the resultant solution?
(a) 12.5% (b) 25%
(c) 37.5% (d) None of these

Q.16. Three litres of pure milk is added to 10 litres of a milk solution containing 9% milk. Find the concentration of the resultant solution.
(a) $33\frac{1}{3}\%$ (b) 40%
(c) 50% (d) 30%

Q.17. In what ratio must a quality of sugar @ Rs 15.50/ kg be mixed with another quality of sugar @ Rs 21.80/ kg so that selling the mixture at Rs 20.02/ kg results in 10% profit?
(a) 5:4 (b) 3:2 (c) 2:1 (d) 4:3

Q.18. Parul Jaiswal has Rs 11,200 in Rs 100 and Rs 500 denomination. What is the ratio of the number of notes, of the two types, if she has a total of 40 notes with her?
(a) 3:2 (b) 7:3 (c) 8:7 (d) 11:9

Q.19. Ordinary sugar contains 32% impurities, while processed sugar has 18% impurities. How many kilograms of ordinary sugar must be processed to yield 102 kg of processed sugar?
(a) 185 (b) 246
(c) 123 (d) None of these

Q.20. A shrewd milkman mixes water and milk in the ratio of 2:3. What part of this mixture should be removed and replaced with water so that the solution contains water and milk in the ratio 1:1?
(a) 1/6 (b) 1/4 (c) 1/3 (d) 1/2

Q.21. A milkman sells his mixture of milk and water at the cost price of milk itself and thus he gains a profit of 20%. What is the ratio of milk and water in the mixture?
(a) 4:1 (b) 5:1
(c) 1:4 (d) None of these

FOUNDATION

Q.1. Neo is going to NDLS from PNBE by his car at a speed of 40 km/h. While coming back, he returns with a speed of *x* km/h. What should be the value of *x* so that his average speed during the intre journey is 80 km/h?
(a) 160 km/h (b) 40 km/h
(c) 120 km/h (d) It is not possible

Q.2. There are 125 middle level employees in Due North Inc. The average monthly salary of the middle level employees is Rs 5,500 and that of the senior level employees is Rs 14,000. If the average monthly salary of all these employees is Rs 8,687.5, find the total number of employees in the company, if middle and senior level employees of Due North Inc. form 80% of their total employees.
(a) 175 (b) 200 (c) 220 (d) 250

Q.3. I went to a hotel along with 12 friends. I paid Rs 145 and all the others paid an equal amount. In the end, we did some calculations and found that the average sum paid by all of us was Rs 5 more than what was originally paid by each of my friends. How much money did each of my friends pay?
(a) Rs 120 (b) Rs 100
(c) Rs 80 (d) Rs 70

Q.4. Saketprakash married ten years ago at the age of 27 years. His wife was 23 years old then. Six years after their marriage, the average age of Saketprakash, his wife and their son was 22 years. After how many years of Saketprakash's marriage was his son born?
(a) 6 years (b) 3 years
(c) 2 years (d) 4 years

Q.5. Mr Anant Roy, the renowned author, recently got his new novel released. To his utter dismay he found that for the 1,007 pages on an average there were 2 mistakes every page. While, in the first 612 pages there were only 434 mistakes, they seemed to increase for the latter pages. Find the average number of mistakes per page for the remaining pages.
(a) 6 (b) 4
(c) 2 (d) None of these

Q.6. The class X of Nav Yug Vidhyalaya has four sections— A, B, C and D. The average weight of the students of A, B, C together and A, C, D together are 45 kg and 55 kg respectively, while the average weight of the students of A, B, D together and B, C, D together are 50 kg and 60 kg respectively. Which of the following could be the average weight of the students of all the four sections together?

(a) 47.6 kg (b) 52.5 kg
(c) 53.7 kg (d) 56.5 kg

Q.7. The average score of a cricketer in a certain number of innings is 44. He then played another eight innings in which he got out at 97, 3, 23, 0, 68, 40, 50 and 71 runs respectively. Then, which of the following statements is true regarding his new average score in all the innings together?

(a) The new average is more than the old average
(b) The new average is less than the old average
(c) The new average is the same as the old average
(d) Cannot conclude any of the above unless we know the number of innings played

Q.8. Average age of A, B and C is 84 years however when D joins them, then the average comes down to 80. Now, a new person E whose age is 3 years more than D replaces A and the new average is 79 years. What is the age of A?

(a) 75 (b) 79 (c) 81 (d) 77

Direction for questions 9 and 10: Read the passage below and solve the questions based on it.

The average age of the students in a class is 35 years.

Q.9. If a student whose age was 25 is absent from the class, the average of those present goes up by 1. How many students were there in the class originally?

(a) 9 (b) 10 (c) 11 (d) 12

Q.10. If 2 students whose ages were 30 years and 20 years left, by what value would the average of the remaining increase? (use the result from question 9.)

(a) $\frac{20}{9}$ (b) 2
(c) $\frac{20}{11}$ (d) None of these

Q.11. If the average of m numbers is a, and on adding x to the m numbers, the average of the $1 + m$ numbers is b. Then find the value of x.

(a) $m(b - a) + b$ (b) $m(b + a) + a$
(c) $m(a - b) + a$ (d) None of these

Q.12. Brian Lara, the famous batsman, scored 6,000 runs in certain number of innings. In the next five innings he was out of form and hence, could make only a total of 90 runs, as a result of which his average fell by 2 runs. How many innings did he play in all, if he gets out in all the innings?

(a) 105 (b) 95 (c) 115 (d) 104

Q.13. The average weight of A, B and C is x kg. A and C lose y kg each after dieting and B gains $\frac{y}{2}$ kg. After this their average weight decreases by 1 kg. Find y.

(a) 1 kg (b) 2 kg
(c) 3 kg (d) Cannot be determined

Q.14. One out of the five weights A, B, C, D and E is of a different weight. A person makes a measurement and finds the following: $A + B = C + D + E$
Which of the following should be known to find out which one is of a different weight?

(a) $A + C > B + D + E$
(b) $A + D + E < B + C$
(c) $A + B + C > D + E$
(d) $A + C < B + D + E$

Q.15. In a class, with a certain number of students, if another student weighing 50 kg joins, then the average weight of the class increases by 1. If one more student weighing 50 kg joins then the average weight of the class increases by 1.5 over the original average. What is the original average weight of the class?

(a) 46 (b) 4 (c) 2 (d) 47

Q.16. The average weight of the books carried by Kuku to school is 1.5 kg and 2.5 kg is the average weight of the exercise books. If Kuku is carrying only text books and exercise books in the ratio of 3 : 4, what is the total weight of his school bag? (Ignore any other weights such as that of school bag, stationary etc.)

(a) 10.5 kg (b) 1 kg
(c) 6 kg (d) Data insufficient

Q.17. The average of n numbers is 32. If 3/4 of the numbers are increased by 4 each and the remaining are decreased by 6 each, then what is the new average?

(a) 32 (b) 32.5 (c) 33.5 (d) 34.5

Q.18. The average of eight numbers is 25, that of the first two is 20 and of the next three is 26. The sixth number is less than the seventh by 4, and less than the eighth by 6. The last number is

(a) 30 (b) 32 (c) 40 (d) 36

Q.19. The average amount of sales per day in a shop for 5 days starting from Monday is Rs 1,800. The average amount of sales per day for 5 days from Tuesday is Rs 1,890. By how much does the sale on Saturday exceed that on Monday?

(a) Rs 450 (b) Rs 500
(c) Rs 600 (d) Rs 650

Q.20. Three years ago, the average age of a family of 5 members was 17 years. Inspite of the birth of a child in the family, the present average age of the family remains the same. The present age of the child is

(a) 3 years (b) 1 year
(c) 2 years (d) 1.5 years

Q.21. A batsman has a certain average of runs for 16 innings. In the 17th inning, he makes a score of 85 runs, thereby increasing his average by 3 runs. What is the average after the 17th inning?

(a) 58 (b) 37
(c) 35 (d) None of these

Q.22. If the average marks of 'a' students in a class is 'c' and that of the remaining 'b' students is 'd', what is the average marks of the class?

(a) $\frac{ac+bd}{b+d}$ (b) $\frac{ab+cd}{a+d}$
(c) $\frac{ac+bd}{a+b}$ (d) $\frac{ad+cd}{b+d}$

Q.23. The average marks of a class of 48 students is 35. Of them, two score zero, of the rest, the first 30 scored an average of 40, the next fourteen scored an average of 20. If the remaining two scored equal marks, what are their individual marks?

(a) 80 (b) 75 (c) 90 (d) 100

Q.24. The total salary of A, B and C is Rs 444. If they spend 80%, 85%, 75% of their salaries respectively, their savings are as 7:6:9. Then salary of B is:

(a) Rs 140 (b) Rs 160
(c) Rs 144 (d) None of these

Q.25. The average age of 2/3rd of the class is 17. What should be the average the age of the remaining 1/3rd students so that the average age of the entire class is 20?

(a) 21 (b) 22 (c) 24 (d) 26

Q.26. Sidharth Tripathi travels from Delhi to Lucknow at a speed of 75 km/h and returns to Delhi at a speed of 50 km/h. He again leaves for Lucknow immediately at a speed of 30 km/h and returns back to Delhi at a speed of 60 km/h. What is his average speed for the entire journey?

(a) 54 km/h (b) 48 km/h
(c) 56 km/h (d) Cannot be determined

Q.27. Average marks of a class of n students is 64. When eight new students with an average marks of 73 join the class, the new average of the entire class is a whole number. Find the number of students in the class given that n lies between 25 to 60.

(a) 44 (b) 32 (c) 36 (d) 72

Q.28. Average of 5 consecutive integers is N. What will be the new average when next two integers are also included?

(a) N + 2
(b) N + 2/7
(c) N + 1
(d) N + K / 7, where K depends upon the values taken.

Q.29. The average of 10 two-digit positive integers is Z. However, one number AB is taken as BA, then the average increases to Z + 2.7. What is the value of $|B - A|$?

(a) 1 (b) 2 (c) 3 (d) 4

Q.30. In the above question, how many set of values for A and B are possible?

(a) 5 (b) 6 (c) 7 (d) 8

Q.31. In a country, the average salary per month is calculated on the basis of 14 months and average expenses is calculated on the basis of 10 months. What will happen to the average savings per month with respect to actual savings (calculated on the basis of 12 months)?

(a) Decrease (b) Increase
(c) No change (d) Cannot be determined

Q.32. In the above question, what will be the net percentage change in the savings with respect to the actual savings?

(a) 11.11% (b) 22.22%
(c) 33.33% (d) Cannot be determined

Q.33. The average rainfall for the first 3 days out of five days was recorded to be 0.45 inches. The rainfall on the last two days was in the ratio 2:3. The average of five days was 0.40 inches. What was the rainfall on the last day?

(a) 0.45 inches (b) 0.39 inches
(c) 0.35 inches (d) 0.30 inches

Q.34. The average height of the girls of a class is 155 cm and the average height of the boys of the class is 4 cm more than the average of the class. If the number of girls is 205 less than the number of boys, what is the average height of the boys?

(a) 140 cm (b) 162 cm
(c) 152 cm (d) None of these

Q.35. The average price of a share is the average of 5 readings taken at regular intervals in a day. The index price is taken by a weighted arithmetic average price of a class A and class B stock. The respective weights are 1.1 and 0.9 for the two kinds of stocks. If the five readings of a class A stock were 19, 26, 31, 35, 39 and for a class B stock the readings were 7, 8, 17, 20, 23 then what was the index price that day?

(a) 46.5 (b) 25 (c) 23.25 (d) 45

Q.36. Ankit travels from New Delhi to Patna. He stops at equidistant places, i.e., Lucknow and Mirzapur during his journey. He covers the first 1/3rd distance at a speed of 50 km/h, the 2nd 1/3rd distance at a speed of 40 km/h and the last 1/3rd distance at a speed of 60 km/h. However, while returning, his uniform speed is 75 km/h and he stops only at Jaunpur, which is exactly at the mid-point between New Delhi and Patna. What is the average speed (in km/h) of Ankit during the whole journey, not considering his halt time?

(a) 59.8 (b) 59.01
(c) 60.8 (d) None of these

Q.37. The average age of 24 students and the class teacher is 16 years. If the age of the class teacher is excluded the average reduces by 1 year. What is the age of the class teacher?

(a) 38 (b) 39
(c) 40 (d) None of these

Q.38. First ten multiples of 1, 2,... 10 are taken. What is the average of all these 100 numbers?

(a) 25.5 (b) 50.5
(c) 75.5 (d) None of these

Q.39. There are 20 students in Mr Rahul Ghosh's class. He conducts an examination out of 100 and then arranged the marks in an ascending order. He found Chandan, the topper of the class, had slipped to the tenth position. When he was adding the scores of the last 11 students the average was 64 and that of the top 10 was 67. If the average marks obtained by all the students of his class was 65, how many marks did Chandan score?

(a) 64 (b) 67
(c) 74 (d) Cannot be determined

Q.40. The average of 38, 84, 63, 45, 32 and *n* lies between 50 and 60. If n is an integer greater than the average of the above numbers, then *n* lies between

(a) $53 < n < 97$ (b) $50 < n < 100$
(c) $52 < n < 98$ (d) None of these

Q.41. Milawati Sharma, the local grocer mixed three qualities of tea T_1, T_2 and T_3 priced at Rs 74 per kg, Rs 68 per kg and Rs 63 per kg in the ratio of 1:2:4. He added some more quantity of T_1 to 4 kg of this mixture. He sold this new mixture for Rs 84 per kg, thereby making a profit of 20%. How much of T_1 did he mix with the mixture?

(a) 4 kg (b) 6 kg
(c) 8 kg (d) None of these

Q.42. 60 litres of a 75% solution of wine is taken into a laboratory. 4 litres of the solution is flushed and 4 litres of water is added to compensate the loss. Then 5 litres of the solution is flushed again and 5 litres of water is added. Again 6 litres of the solution is flushed and 6 litres of water is added. The final concentration of wine in the solution is

(a) 255% (b) 57.75%
(c) 60.25% (d) 59%

Q.43. A and B are two alloys of gold and copper prepared by mixing the respective metals in the proportions of 7:2 and 7:11 respectively. If the alloys are melted and mixed to form a third alloy C, which has an equal proportion of gold and copper. What is the ratio of alloys A and B in the mixture?

(a) 2:5 (b) 5:2 (c) 11:2 (d) 1:2

Q.44. A vessel contains a mixture of two liquids A and B in the ratio of 4:1. When 10 litres of the mixture is replaced with liquid B, the ratio becomes 2:3. How many litres of liquid A was present in the jar initially?

(a) 12 litres (b) 14 litres
(c) 16 litres (d) 10 litres

Q.45. From a vessel containing pure milk, 9 litres is taken out. It is replaced with an equal quantity of water. This operation is repeated twice. The ratio of milk and water now in the vessel is 16:9. What was the volume of milk in the vessel initially?

(a) 54 litres (b) 36 litres
(c) 42 litres (d) 45 litres

Q.46. There are only deers and ducks in a zoo. When heads are counted, it is 204 and when the legs are counted, it is 512. How many deers are there in the zoo?

(a) 52 (b) 152 (c) 176 (d) 26

Q.47. Sharat invests a total of Rs 10,000 in two parts, some money @ 6% per annum and rest of the money @ 8% per annum of SI. After two years, he got a total of Rs 1,440 as interest. What is the amount put at 6% per annum interest?

(a) Rs 4,000 (b) Rs 6,000
(c) Rs 3,000 (d) Rs 7,000

Q.48. A vessel contains a mixture of two liquids A and B in the ratio of 7:5. When 9 litres of the mixture is taken out and the vessel is filled with B, the ratio of A and B is now 7:9. How many litres of liquid A was there in the vessel initially?

(a) 24 litres (b) 21 litres
(c) 18 litres (d) 27 litres

Q.49. A milkman dilutes 36 litres of pure milk with water. The percentage of milk in the solution is now 75%. How many litres of water did he add?

(a) 6 litres (b) 8 litres
(c) 12 litres (d) 16 litres

Q.50. Doodhimal, the local milkman was notorious for the quality of milk that he used to sell. Initially he had 60 liters of milk in his can. He removed one-third of it and replaced that with water. He then removed one-third of the mixture and again added water to it. How many times should this process be repeated if Doodhimal wants the concentration of milk to be just below 20%?

(a) 3 (b) 4 (c) 5 (d) 7

MODERATE

Q.1. The average marks of the girls in a class is equal to the number of boys and the average marks of boys is equal to the number of girls. If the class average is 4 less than the average of both the boys' and the girls' average marks, what will be the number of students in the class?

(a) 24 (b) 48
(c) 50 (d) 64

Q.2. In the above question, what can be the minimum number of students in the class?

(a) 0 (b) 2
(c) 8 (d) None of these

Q.3. A, B, C, D and E are five consecutive integers and the average of these five numbers is less than $\frac{1}{4}$th of A. Then A is

(a) Positive (b) Negative
(c) Even (d) Data Sufficient

Q.4. Find the difference between the average of the first *n* even numbers and the average of all the even numbers up to *n* (*n* is an even number).

(a) $\frac{n-1}{2}$ (b) $\frac{n+1}{2}$
(c) $\frac{n}{2}$ (d) None of these

Q.5. I was born 30 years after my father was born. My sister was born 25 years after my mother was born. The average age of my family is 26.25 years right now. My sister will get married 4 years from now and will leave the family. Then the average age of the family will be $\frac{107}{3}$ years. What is the age of my father?

(a) 30 year (b) 35 year
(c) 40 year (d) 45 years

Direction for questions 6 to 9: Read the passage below and solve the questions based on it.

India is supposed to send its boxing team for Olympics in each of the following 10 weight group divisions.

A	(48 kg - 52 kg)	B	(52 kg - 56 kg)
C	(56 kg - 60 kg)	D	(60 kg - 64 kg)
E	(64 kg - 68 kg)	F	(68 kg - 72 kg)
G	(72 kg - 76 kg)	H	(76 kg - 80 kg)
I	(80 kg - 84 kg)	J	(84 kg - 88 kg)

After selecting one player from each group, their average weight comes out to be 68 kg. If one of the players named *X* leaves the team, their average weight comes down to 66.5 kg.

Q.6. Player *X* is from group

(a) A (b) E
(c) I (d) None of these

Q.7. If X leaves the team, and two new players join the group, then their average weight increases to 68 kg. These players can be from group ....

(a) A, C (b) D, J
(c) Both from G (d) Either 2 or 3

Q.8. What is the average weight (in kg) of all the players taken together?

(a) 56.7 (b) 58.8
(c) 61.4 (d) Cannot be determined

Q.9. In the average of all the groups together, which group contributes most in overall average?

(a) F (b) G
(c) H (d) Cannot be determined

Q.10. A shipping clerk has to weigh 6 distinct packets. He weighs them four at a time, weighing all the possible combinations of the packets from the six. The average weight of all the weighing combinations is found to be 500 gm. What is the combined weight of all the six packets?

(a) 375 gm (b) 1250 gm
(c) 750 gm (d) 1000/3 gm

Direction for questions 11 to 21: Read the passage below and solve the questions based on it.

There are three sections in class *X* at A. R. Academy. Below is the table indicating the number of students, and the range of marks obtained by the students of that particular section.

	Section A	*Section B*	*Section C*
No. of students	25	30	20
Minimum marks	22	24	20
Maximum marks	30	28	26

The minimum and the maximum marks provides us with only the range of marks which can be obtained by the students of that particular section.

Q.11. What is the average marks obtained by all the students of class X together?

(a) 24.25 (b) 26.33
(c) 26.75 (d) Cannot be determined

Q.12. If we shift 5 students from section A to section B and then '*x*' students from section B to section C, then what

should be the value of 'x' so that the minimum possible average of section B and the maximum possible average of section C are the same?
(a) 6 (b) 7
(c) 9 (d) Not possible

Q.13. If we combine section A and section C, then what will be the ratio of the maximum possible average of this combined section and the average of section B?
(a) 2:1 (b) 3:2
(c) 1:2 (d) Cannot be determined

Q.14. What is the maximum number of students that can be transferred from section A to section C, so that the maximum possible average of section A is still more than the maximum possible average of section C?
(a) 20 (b) 22
(c) 24 (d) None of these

Q.15. What can be the maximum possible average of class X when all the three sections are combined?
(a) 26.33 (b) 28.13 (c) 27.28 (d) 28.5

Q.16. We take a sample size of the following numbers from different sections

	Section A	*Section B*	*Section C*
Total no. of students selected	10	10	10
No. of students of minimum marks	5	6	4
No. of students of maximum marks	5	4	6

What is the average of this sample size?
(a) 25.06 (b) 26.16 (c) 25.83 (d) 26.71

Q.17. In the above question, which section has contributed the maximum in the average of the whole sample size?
(a) A
(b) B
(c) C
(d) Two sections contributed equal

Q.18. If we shift 5 students from section A to section B and then 10 students from section B to section C, what will be the maximum average of section C?
(a) 27 (b) 26.66 (c) 26 (d) 27.33

Q.19. In the above question, what can be the minimum average of section C?
(a) 20 (b) 21 (c) 21.33 (d) 22

Q.20. 10 students are shifted from each of the sections A and C to section B and then 10 students each are shifted from section B to each of the section A and section C. Now what is the ratio of maximum possible average of section A and section C?
(a) 1:1 (b) 15:14
(c) 14:15 (d) Cannot be determined

Q.21. In the above question, what is the ratio of the minimum possible average of section A and section C?
(a) 11:12 (b) 12:11
(c) 1:1 (d) Cannot be determined

Q.22. The average of 5 positive numbers is A. The average of the average of all the possible distinct triplets of these 5 numbers is B. Which of the following is true?
(a) A>B (b) A<B
(c) A=B (d) Cannot be determined

Q.23. The average of 5 positive numbers is A. The average of the average of all the possible distinct pairs of these 5 numbers is B. Which of the following is true?
(a) A>B (b) A<B
(c) A=B (d) Cannot be determined

Direction for questions 24 to 26: Read the passage below and solve the questions based on it.

The following operators have been defined for two numbers A and B.

@(A, B) = Average of A and B
*(A, B) = Product of A and B
/(A, B) = A divided by B

Q.24. If A = 2 and B = 4, then the value of @ (/(*(A, B), B), A) would be
(a) 2 (b) 4 (c) 6 (d) 16

Q.25. The sum of A and B is given by
(a) *(@ (A, B), 2) (b) /(@ (A, B), 2)
(c) @(*(A, B), 2) (d) @ (/(A, B) 2)

Q.26. The sum of A, B and C is given by
(a) * (@ (* (@ (B, A), 2), C), 3)
(b) * (@ (* (@ (B, A), 2), C), 2)
(c) /(* (@ (* (B, A), 2), C), 3)
(d) None of these

Q.27. The average age of Mr and Mrs Sinha at the time of their marriage in 1972 was 23 years. On the occasion of their anniversary in 1976, they observed that the average age of their family had come down by 4 years compared to their average age at the time of their marriage. This was due to the fact that their son Vicky was born during that period. What was the age of Vicky in 1980?
(a) 6 (b) 7 (c) 8 (d) 5

Q.28. A man makes 60 articles in the 1st hour. His efficiency decreases by 25% in the 2nd hour, increases by 40% in the 3rd hour, decreases by 33% in the 4th hour and increases by 50% in the 5th hour. If he has to work for more than 1 hour, then in which hour the average number of articles produced per hour then would be minimum?

(a) 2nd hour
(b) After 5th hour
(c) 3rd hour
(d) None of these

Q.29. Sachin Tendulkar has a certain batting average *N* (a whole number) in his career of 86 innings. In the 87th inning, he gets out after scoring 270 runs which increases his batting average by a whole number. The batting average is calculated by dividing the total number of runs scored by the total number of innings played by the player. How many values of his new average is/are possible?

(a) 0
(b) 1
(c) 2
(d) None of these

Q.30. In a school there are two sections for class X – section A and section B. The number of students in section A and section B are 60 and 70 respectively and their respective average weight is 35 kg and 38 kg. If one of the students from section A is shifted to section B then the average weight of both the sections decreases. What can be said about the weight of that student?

(a) His weight is less than 35 kg.
(b) His weight is more than 38 kg.
(c) His weight is more than 35 kg and less than 38 kg.
(d) Cannot be determined

Direction for questions 31 and 32: Read the passage below and solve the questions based on it.

The average score of a batsman for a certain number of innings was 21.75 per inning. He played 3 innings more and scored 28, 34 and 37 runs respectively, thus increasing his average by 1.25.

Q.31. How many innings did he play in all?

(a) 18
(b) 21
(c) 27
(d) None of these

Q.32. What was his total score?

(a) 520
(b) 601
(c) 621
(d) None of these

Q.33. A curious student of Statistics calculated the average height of all the students of his class as A. He also calculated the average of the average heights of all the possible pairs of students (two students taken at a time) as B. Further, he calculated the average of the average heights of all the possible triplets of students (three students taken at a time) as C. Which of the following is true of the relationship among A, B and C?

(a) A + 2B = C
(b) A + B = 2C
(c) A = B = 3C
(d) None of these

Q.34. A succession of numbers is said to be in the Arithmetic Progression (AP) if the difference between any given term and the previous term is constant throughout. In a AP $n_1, n_2, n_3, \ldots, n_{46}$, the sum of odd numbered terms $n_1, n_3, n_5, \ldots, n_{45}$, is 1272. What is the sum of all the 46 terms of this AP?

(a) 2,491
(b) 2,500
(c) 2,400
(d) Cannot be determined

Q.35. There are two vessels of milk of different prices with the volume of 220 litres and 180 litres respectively. Equal amounts of milk were poured off simultaneously from the two vessels and the milk poured off from the first vessel was poured into the second vessel and the milk poured off from the second vessel was poured into the first vessel. Then the price of milk in both the vessels becomes the same. How much milk was poured from one vessel into the other?

(a) 90
(b) 80
(c) 160
(d) 99

Q.36. There are two solutions of Sulphuric acid (acid + water) with concentration of 50% and 80% respectively. They are mixed in a certain ratio to get a 62% sulphuric acid solution. This solution is mixed with 6 litres of water to get back 50% solution. How much of the 80% solution has been used in the entire process?

(a) 15 litres
(b) 12 litres
(c) 10 litres
(d) None of these

Q.37. Two-fifths of the volume of the mixture of the milk and water is of the ratio of 4:3 is converted into a mixture of the ratio of 5:3 by the method of replacement with milk. The 4:3 mixture was prepared from a 4:1 mixture by the method of 'addition of the substances of the mixture'. If the replacement volume is 14 litres, what is the volume of the substance added?

(a) 12 litres
(b) 60 litres
(c) 80 litres
(d) 24 litres

Q.38. Rakesh has a stock of 1000 litres of pure milk. He decides to decrease the concentration to half, using the method of successive dilution, replacing 200 litres of contents by water, in each step of dilution. After the fourth dilution, he realizes that the concentration is less than the required value. Each of the contents removed during the dilution process is stored separately. He also has ample supply of water. Will he be able to make the mixture of desired concentration?

(a) Not possible
(b) Possible
(c) Cannot be determined
(d) None of these

Q.39. An industrial solvent of 90% strength is prepared and stored in a 150 litres capacity container. The container is filled to 80% of its capacity. It is required to decrease the concentration of the solvent in the container to less than 60%. For doing this, either pure water can be added to it and/or solution can be removed. But removal can only be done if there is no place for adding water. At a given time, only 10 litres of the solution can be added or removed. How many times should water be added to the solution?

(a) 4 (b) 5 (c) 9 (d) 6

Q.40. Four gallons are drawn from a case full of wine. It is then filled with water. Four gallons of mixture are again drawn and the cask is re-filled with water. The ratio of the quantity of wine now left in the cask to that of the mixture in it is 36 : 49. How much does the cask hold?

(a) 30 gallons (b) 25 gallons
(c) 35 gallons (d) 28 gallons

Q.41. There is a vessel holding 40 litres of milk. 4 litres of milk is initially taken out from the vessel and 4 litres of water is then poured in. After this, 5 litres of mixture is replaced with 5 litres of water. And finally 6 litres of the mixture is replaced with 6 litres of water. How much milk (in litres) is there in the vessel?

(a) 26.775 (b) 29.16
(c) 24.72 (d) 27.42

Q.42. There is a vessel holding 40 litres of milk. 4 litres of milk is initially taken out from the vessel and 5 litres of water is poured in. After this, 5 litres of the mixture from this vessel is replaced with 6 litres of water. And finally 6 litres of mixture from this vessel is replaced with 7 litres of water. How much of the milk (in litres) is there in the vessel now?

(a) 22.42 (b) 27.09
(c) 24.72 (d) 29.42

Q.43. There is a vessel holding 40 litres of milk. 4 litres of milk initially is taken out from the vessel and 5 litres of water is poured in. After this, 6 litres of mixture from this vessel is replaced with 7 litres of water. And finally 8 litres of mixture from this vessel is replaced with 9 litres of water. How much of the milk (in litres) is there in the vessel now?

(a) 22.82 (b) 20.92
(c) 26.78 (d) 24.87

Q.44. There are three vessels A, B and C having a mixture of milk and water. Concentration of milk in the vessels is C_1, C_2 and C_3 respectively. When a part of vessel A is replaced with the mixture in vessel B, then the concentration of milk in the final mixture is less than C_1. However, when vessels B and C are mixed in the ratio of 1 : 1, then the quantity of milk in the final mixture is double than the quantity of milk in the vessel B. Which of the vessels is having the mixture of maximum concentration of milk?

(a) A (b) B
(c) C (d) Cannot be determined

Q.45. In the above question, which of the vessels is having the mixture of minimum concentration of milk?

(a) A (b) B
(c) C (d) Cannot be determined

Q.46. In question number 44, if we arrange the concentration of milk in different vessels in ascending order, then which of the following is true?

(a) ABC (b) BAC
(c) CAB (d) Cannot be determined

Q.47. In two alloys, the ratios of copper to zinc is 5:2 and 3:4. How many kilograms of the first alloy and the second alloy respectively should be melted together to obtain 28 kg of a new alloy with equal copper and zinc?

(a) 8 kg and 20 kg (b) 4 kg and 24 kg
(c) 3 kg and 25 kg (d) 7 kg and 21 kg

Q.48. From a 3 : 5 solution of milk and water, 20% is taken out and replaced by milk. How many times should this process be done to make the ratio of milk to water as 17:8?

(a) Once (b) Twice
(c) Thrice (d) Four times

ADVANCED

Direction for questions 1 and 2: Read the passage below and solve the questions based on it.

In the National Baseball League (NBL), the batting average (AB) of a hitter is computed from the home runs scored, innings played, completed innings and incomplete innings.

R1 = Number of home runs scored in complete innings.

N1 = Number of complete innings.

R2 = Number of home runs scored in incomplete innings.

N2 = Number of incomplete innings.

$$AB = \frac{R_1 + R_2}{N_1}$$

To accurately assess a hitter's accomplishment, the NBL has devised two other statistical computations called the Quick Average (AQ) and the Comprehensive Average (AC), which are defined as follows:

$$AQ = \frac{R_1}{N_1} + \frac{N_2}{N_1}\max\left[0, \left(\frac{R_2}{N_2} - \frac{R_1}{N_1}\right)\right]$$

$$AC = \frac{R_1 + R_2}{N_1 + N_2}$$

Q.1. Based on the information, which of the following is true?

(a) $AQ \le AB \le AC$ (b) $AB \le AC \le AQ$
(c) $AC \le AB \le AQ$ (d) None of these

Q.2. An experienced hitter with no incomplete innings has AB of 50. The next time he bats, the inning is incomplete and he scores 45 runs. It can be inferred that

(a) AB and AQ will increase, while AC will decrease
(b) AB will increase, while AC will decrease
(c) AB will increase and we cannot comment on the changes in AQ and AC
(d) None of these

Q.3. If the average marks of 17 students in a class is A. The marks of the students when arranged in either an ascending or a descending order was found to be in arithmetic progression. The class teacher found that the students who were ranked 3rd, 7th, 9th, 11th, 15th had copied in the exam and hence got all of them rusticated. The average of the remainder of the class was B. Then

(a) A = B (b) A > B
(c) A < B (d) Data Insufficient

Q.4. The average weight of the students in four sections A, B, C and D is 60 kg. The average weights of the students of A, B, C and D individually are 45 kg, 50 kg, 72 kg and 80 kg respectively. If the average weight of the students of section A and B together is 48 kg and that of the students of B and C together is 60 kg, what is the ratio of the number of students in sections A and D?

(a) 4:3 (b) 3:2 (c) 8:5 (d) 12:7

Q.5. Illara, Puttu and Munchun had three bottles containing equal amounts of orange juice, mango juice and water respectively. Since Illara and Puttu wanted to dilute their drinks, Munchun gave one-fourth of the water that she had, to each of them. After mixing their bottles well, each of them gave to Munchun the same quantity of the solution that they had borrowed from her. Which of the following statements is true?

i. The concentration of water in Munchun's drink is more than the concentration of orange juice in Illara's drink.
ii. The concentration of orange juice in Illara's drink is more than the concentration of mango juice in Puttu's drink.
iii. The concentration of orange is more than the concentration of mango in Munchun's drink.

(a) i and ii (b) i, ii and iii
(c) i, ii and iii (d) None of these

Q.6. We write down all the digits from 1-9 side by side. Now we put '+' between as many digits as we wish to, so that the sum of numbers become 666. It is explained below

1 2 3 4 5 6 7 8 9 = 666

Now suppose we put plus signs at following places:

12 + 345 + 67 + 89 = 513

Since there are four numbers, so the average can be calculated by dividing the sum by 4. What is the average if the sum is 666?

(a) 166.5 (b) 111
(c) 133.2 (d) Cannot be determined

Q.7. In the above question, what is the average if the sum is 261?

(a) 54 (b) 52.2
(c) 55 (d) None of these

Direction for questions 8 to 11: Read the passage below and solve the questions based on it.

Go through the following definitions

A + B – All A are B.
A * B – Some A are B.
A @ B – No A are B.
A # B – Only A are B.

Following table gives the average age break-up of all the students at IIM-A (2005 batch)

Group of students	*Average age (years)*	*No. of persons in the group*
Males M	23	140
Females F	22	100
Engineers E	25	160
Fresher R	20	80
Working Professionals W	27	160

Assume that all the students mentioned above are either males or females. The minimum entry age in the IIM is 20 years (at the time of taking the entrance exam to qualify for IIM-A). No fresher can be a working professional and males/females/engineers can be either a fresher or a working professional. Everybody is either a fresher or a working professional.

Q.8. Following statement is definitely true: M + E. What is the average age (in years) of all the engineers who are not male?
(a) 39 (b) 40
(c) 38 (d) Cannot be determined

Q.9. Total number of working professionals is divided in the ratio of 3 : 2 among males and females. If A * B, then what is the range of the average age (x) of all the males who are engineers?
(a) $32 \le x \le 38$ (b) $39 \le x \le 43$
(c) $44 < x \le 49$ (d) Cannot be determined

Q.10. R + M and F + W are definitely true. What is the average age (in years) of all the males who are working professionals?
(a) 35 (b) 35.33
(c) 36 (d) None of these

Q.11. Consider the following statements – F @ E. What is the average age (in years) of all the working females?
(a) 23 (b) 24
(c) 24.5 (d) Data inconsistent

Direction for questions 12 to 14: Read the passage below and solve the questions based on it.

Ten different samples of milk solution— A, B, C, D, E, F, G, H, I and J are taken and the concentration of each sample (total quantity of milk as a percentage of total quantity of solution) was found out and the results were tabulated as follows

Sample	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*I*	*J*
Concentration (%)	78	57	82	84	98	66	34	87	79	71

Q.12. If two samples of milk are taken to form a sample whose concentration is more than 80%, then a maximum of how many distinct pairs of samples are there, which will never give the desired result for any ratio of their volumes?
(a) 30 (b) 48 (c) 12 (d) 15

Q.13. A triplet is a set of three distinct samples taken in some ratio of their volumes. How many such distinct triplets are there, which when mixed can give a sample with more than 80% concentration?
(a) 50 (b) 100 (c) 150 (d) 200

Q.14. There are how many distinct samples such that if two or mote than two such samples are mixed in any ratio, the resulting solution is always a sample of more than 80% concentration?
(a) 4 (b) 6 (c) 8 (d) 10

Q.15. There are three containers C_1, C_2 and C_3 having milk and water solutions in the ratio of a : b, where a ε (1, 2, 3) and b ε (4, 5, 6). Containers C_1, C_2 and C_3 can take any value of a and b. For how many different set of values of concentration of containers C_1, C_2 and C_3, the resultant mixture will have the ratio of milk and water as 1:1?
(a) 12 (b) 0
(c) Infinite (d) None of these

Q.16. In the above question, a ε (1, 2, 6) and b ε (4, 5, 3). Containers C_1, C_2 and C_3 can take any value of a and b. For how many different set of values of concentration of containers C_1, C_2 and C_3, the resultant mixture will have the ratio of milk and water as 1 : 1?
(a) 12 (b) 0
(c) Infinite (d) None of these

Q.17. The average weight of a class of 25 students is 30 kg. The average weight of the girls is 5 kg more than that of the boys. If the class teacher's weight, which is between 64 kg and 106 kg, is included, the average weight of the male members of the class equals that of the female members. If the average weight of the boys (in kg) is an integer, then what is the number of girls in the class?
(a) 20 (b) 15
(c) 10 (d) Cannot be determined

Q.18. A succession of numbers is said to be in Arithmetic Progression (AP) if the difference between any given term and the previous term is constant throughout. In an AP $n_1, n_2, n_3, \dots, n_{47}$, the sum of odd numbered terms $n_1, n_3, n_5, \dots, n_{47}$ is 1272. What is the sum of all the 47 terms of this AP?
(a) 2491 (b) 2500 (c) 2400 (d) 2591

Direction for questions 19 to 21: Read the passage below and solve the questions based on it.

In the entrance examination of IIMs, there were 200 questions, each of which carried the same marks. A correct answer gets 2 marks and there is 100% negative marking. A total of 70 candidates took the exam and it was later found that the average marks obtained by these 70 candidates was 240. The candidates were not required to attempt all the questions. None of the candidates got more number of incorrect answers than correct answers.

Q.19. Which of the following is always true?
(a) None of the applicants had a net score less than 240
(b) The net scores of the individuals would always be an even integer
(c) The net scores of any candidate would always be a positive integer
(d) The net scores of the topper could be exactly 24% more than the average of the entire group

Q.20. If the topper had a net score of 380 and the sixth ranker had a net score of 370 and the net scores of the top six rankers were distinct, then what would be the average net score of the 64 students who got ranks from the seventh to the last?

(a) 225 (b) 245
(c) 275 (d) None of these

Q.21. When the scores of the top four students are deleted, the average score of the remaining 66 students falls by 6 marks. Assume that it is possible for two or more students to have the same net score. What is the minimum score possible for the fourth ranking student if no student got a net score of more then 352?

(a) 280 (b) 300 (c) 308 (d) 320

Q.22. Two alloys composed of gold and silver together weigh 20 kg. One lump contains 75% gold and 31.25 gm kg silver. Another alloy contains 85% gold and 30 gm per kg silver. The total quantity of silver in two lumps is 617.5 gm. If the two lumps are melted and formed into one, what percentage of gold will it contain?

(a) 50% (b) 89% (c) 78% (d) 67%

Q.23. Let N be a set of real numbers such that p is any real number in the set. There exists two numbers in N whose average is p, then

(a) N is a finite set
(b) N is a set containing all real numbers
(c) N is a set of all numbers in the interval (2, 3)
(d) None of these

Direction for questions 24 and 25: Read the passage below and solve the questions based on it.

There are two brands of TV, Onida and Videocon, in the market. Onida is an old brand launched in 1990 and Videocon is a new brand launched in 1997. For both these brands, 30% of the TV's purchased every year are disposed off as out-dated exactly two years later. It is known that 10 Onida TV's were disposed off in 1997. The following table gives us the number of TV's in operation at the end of the year from 1995 to 2000.

	1995	*1996*	*1997*	*1998*	*1999*	*2000*
Onida	120	162	182	222	236	236
Videocon	0	0	30	80	124	134

Q.24. What is the average number of Videocon TVs disposed every year by 2000?

(a) 4 (b) 12
(c) 8 (d) None of these

Q.25. What is the average number of Onida TVs purchased every year till 1995?

(a) 20 (b) 24
(c) 28 (d) Cannot be determined

TRUE/FALSE

1. For any given series of 100 terms, if we know the last term and the first term, we can always calculate the average of all the 100 terms.

State whether True or False

2. A, B, C, D, E and F are six numbers, not necessarily natural numbers. They are divided into two groups—each group having three numbers each and we do not know which number belongs to which group. Final average of all the numbers can be calculated by calculating the average of individual average of the two groups.

State whether True or False

3. In the above question, final average will depend upon the set of numbers forming the group.

State whether True or False

4. 10 natural numbers are taken. Average of all the possible pairs of numbers formed out of these ten natural numbers are calculated. It is possible to determine the average of all the ten natural numbers using this information.

State whether True or False

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(c)	(c)	(d)	(c)	(d)	(b)	(b)	(b)	(a)	(b)	(b)	(c)	(d)	(d)	(d)	(c)	(d)
Q. No.	21																			
Answer	(b)																			

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(c)	(c)	(b)	(b)	(c)	(a)	(c)	(a)	(a)	(a)	(b)	(d)	(d)	(d)	(c)	(a)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(c)	(d)	(b)	(d)	(b)	(c)	(c)	(c)	(b)	(a)	(d)	(b)	(d)	(c)	(b)	(c)	(b)	(c)	(a)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(d)	(b)	(a)	(c)	(d)	(a)	(a)	(b)	(c)	(b)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(a)	(d)	(d)	(c)	(d)	(d)	(d)	(c)	(d)	(d)	(d)	(c)	(b)	(a)	(a)	(a)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(c)	(c)	(a)	(a)	(b)	(b)	(d)	(d)	(c)	(c)	(c)	(d)	(d)	(d)	(c)	(c)	(b)	(d)	(d)
Q. No.	41	42	43	44	45	46	47	48												
Answer	(a)	(b)	(d)	(a)	(d)	(d)	(d)	(c)												

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(a)	(a)	(d)	(d)	(b)	(a)	(d)	(b)	(d)	(d)	(b)	(b)	(b)	(c)	(b)	(a)	(b)	(d)
Q. No.	21	22	23	24	25															
Answer	(b)	(c)	(b)	(a)	(d)															

■■■ True/False

1. False

   Formula to find out the average = (1st term + Last term)/2 can be used only if the series is in AP (i.e., the difference between any two consecutive terms is same).

2. True

   Average is only the central value of the numbers, and it does not matter what ways the numbers has been taken.

3. False

4. True

   Average is only the central value of the numbers, and it does not matter what ways the numbers has been taken – numbers are taken independently or they are first paired up and then taken together to find out the average.

HINTS AND SOLUTIONS

Warm up

1. $\frac{42+49+56+63+35}{5}=\frac{245}{5}=49$

2. Average speed = $\frac{2.8.12}{8+12}=\frac{192}{20}=9.6\,\text{km/h}$

3. Required Average = $\frac{39.2\times 5-41\times 3}{2}=\frac{196-123}{2}$

   $=\frac{73}{2}=36.5$

4. $\frac{30x+30y}{60}=\frac{x+y}{2}$

5. $\frac{15}{2}[2.3+14.3]=\frac{1}{2}[6+42]=24$

6. $25\times 7-28=21x$

   $21x=147$

   $x=7$

   Age of eldest member = 6 × 7 = 42 years

7. $7x+42=70$

   $7x=28$

   $x=4$

   Required average = $\frac{10x+90}{10}=x+9=13$

8. $6x+3x+x=120$

   $10x=120$

   $x=12$

9. Correct Average = $\frac{250-84+48}{5}=\frac{214}{5}=42.8$

10. Total weight of the whole family = 7× 18 = 126

    Weight of family excluding head of the family =13 × 6 = 78 kg

    Weight of head of the family = 48 kg

11. $\frac{30.16+20\times 15.5}{50}=\frac{480+310}{50}=\frac{790}{50}=15.8\,\text{kg}$

12. $\frac{47.12-45.10}{2}=\frac{564-450}{2}=\frac{114}{2}=57$

13. $\frac{x}{y}=\frac{100-56}{56-35}=\frac{44}{21}$

14. $x+y=50$

    $\frac{1}{4}x+\frac{1}{2}y=15$

    Solving both the equations, we get $x = 40$ and $y = 10$

15. $\frac{1+2}{6+2}=\frac{3}{8}=37.5\%$

16. $\frac{3+\frac{9}{100}\times 10}{10+3}\times 100=\frac{3.9}{13}\times 100=30\%$

17. $\text{CP}=\text{Rs}\left(20.02-20.02\times\frac{1}{11}\right)=\text{Rs}\left(20.02-1.82\right)$

    Rs 18.20

18. $\frac{x}{y}=\frac{20000-11200}{11200-4000}=\frac{88000}{7200}=\frac{22}{18}=\frac{11}{9}$

20. Let x part of mixture is removed

    then, $\frac{2-\frac{2}{5}x+x}{3-\frac{3}{5}x}=1$

    $x=\frac{5}{6}$

21. Let CP of milk be Rs 100
then CP of mixture = Rs

$$\left(100-\frac{1}{6}\times 100\right) = \text{Rs } \frac{250}{3}$$

Using alligation, we get

$$\frac{\text{Milk}}{\text{Water}} = \frac{\frac{250}{3}-0}{100-\frac{40}{3}} = \frac{5}{1}$$

Foundation

1. $$80 = \frac{2\cdot x\cdot 40}{40+x}$$

$40 + x = x$

Hence, not possible

Alternatively, the average speed for the whole journey has to be less than double the speed of one-side. In this case, speed in one direction = 40 kmph, hence average speed for the whole journey has to be less than 80 kmph.

2. Let x = no. of senior lever employees

then $\frac{125\times 5500 + x\times 14000}{125+x} = 8687.5$

$x = 75$

$\frac{4}{5}$ of total employees = (125 + 75) = 200

Hence no. of total employees = $\frac{5}{4}\cdot 200 = 250$

3. $$\frac{140+12x}{13} = x+5$$

$145 + 12x = 13x + 65$

$x = 80$

4. After 6 years from Marriage

Age of Satya Prakash = 33 years

Age of his wife = 29 years

Let the age of their son be x years.

Then, $\frac{33+29+x}{3} = 22$

$62 + x = 66$

$x = 4$

Hence, option (c) is the answer.

5. Total mistakes = 1007 × 2 = 2014

Let x be average mistake per page for the remaining pages

$434 + 395x = 2014$

$395x = 1580$

$x = 4$

6. Let a, b, c, d the number of students in section A, B, C, D respectively

then,

$$= \frac{45(a+b+c)+55(a+c+d)+50(a+b+d)+60(b+c+d)}{3(a+b+c+d)}$$

$$= 50 + \frac{5b+10c+15d}{3(a+b+c+d)}$$

Clearly, a, b, c, d are natural no. put a = b = c = d = 1

Then, required average = $50 + \frac{30}{12} = 50 + 2.5$

= 52.5

7. Old average = 44

Average of 8 new innings

$$= \frac{97+3+23+0+68+40+50+71}{8}$$

$$= \frac{352}{8} = 44$$

8. Age of D = 80 × 4 – 84 × 3 = 320 – 252 = 68

Age of E = 71 years

Age of A = 80 × 4 – 79 × 4 + 71

= 75

9. Let x students were in the class

$$\frac{35x-25}{x-1} = 36$$

Or, $35x - 25 = 36x - 36$

$x = 11$

10. $$\frac{11\times 35-30-20}{9} = \frac{385-50}{9} = \frac{335}{9} = 37\frac{2}{9}$$

Required No. $= 37\frac{2}{9} - 35 = \frac{335-315}{9} = \frac{20}{9}$

11. $\frac{ma+x}{1+m} = b$ $\qquad ma + x = b + bm$

$x = b + m\,(b - a)$

12. Go through options

For (a)

He played 100 matches previously,

His old average = $\frac{6000}{100} = 60$

His new average = 6090/105 = 58

Option (a) is satisfied. Hence option (a) is the answer.

13. $$\frac{3x-y-y+\frac{y}{2}}{3} = x-1$$

$$3x - \frac{3y}{2} = 3x - 3$$

$$\frac{3y}{2} = 3,\quad y = 2$$

14. Assign some numerical values, then proceed we will get (d) as answer.

16. No data regarding the numbers of books is given, hence we cannot determine the weight of school bag.
Hence, (d)

17. $$\frac{32\times n+\frac{3}{4}n.4-\frac{1}{4}n.6}{n}=\frac{70n-3n}{2n}=33.5$$

18. $40 + 78 + x - 6 + x - 2 + x = 200$
$3x = 90$
$x = 30$

19. Total sales from Monday to Friday = Rs 9000
Sales from Tuesday to Saturday = Rs 9450
Thus, a sale of Rs 450 more was on Saturday than on Monday.

20. Total age of family 3 years ago = 17 × 5 = 85 years
Total age of family now = 17 × 6 = 102 years
Total age of family excluding the child now = (85 + 15) = 100 years
Age of child = 2 years

21. Let a be the average till 16th inning.
$$\frac{16a+85}{17}=a+3$$
$16a + 85 = 17a + 51$
$a = 34$
Required average = 37

22. $$\frac{\text{Total Marks}}{\text{Total no. of Students}}=\frac{ac+bd}{a+b}$$

23. $$\frac{2\times 0+30\times 40+14.20+2x}{48}=35$$
$1200 + 280 + 2x = 1680$
$2x = 200$
$x = 100$

25. Let n = total no. of students
$$\frac{\frac{2}{3}n\cdot 14+\frac{1}{3}n\cdot x}{n}=20$$
$$\frac{34n+nx}{3n}=20$$
$34n + nx = 60\,n$
$nx = 26x$
$x = 26$

26. Average speed for 1st Journey
$$=\frac{2.75.50}{125}=60\,\text{km/h}$$
Average speed for 2nd Journey
$$=\frac{2.30.60}{90}=40\,\text{km/h}$$
Average speed for culture Journey
$$=\frac{2.60.40}{100}=48\,\text{km/h}$$

27. $$\frac{64n+8.73}{n+8}=\frac{64n+584}{n+8}$$
Put the values of n from the options to get the answer

28. Let the integers be 1,2,3,4,5
$$\text{N}=\frac{1+2+3+4+5}{5}=\frac{15}{5}=3$$
When next two integers are also included then new average $=\frac{28}{7}=4=\text{N}+1$

29. $10\text{Z} - (10.\text{A} + \text{B}) + (10\text{B} + \text{A}) = 10\,(\text{Z} + 2.7)$
$10\text{Z} - 10\,\text{A} - \text{B} + 10\,\text{B} + \text{A} = 10\text{Z} + 27$
$9\,\text{B} - 9\,\text{A} = 27$
$\text{B} - \text{A} = 3$
$|\,\text{B} - \text{A}\,| = 3$

30. Clearly B > A, and none of them is zero

A	1	2	3	4	5	6
B	4	5	6	7	8	9

33. $$\frac{3\times 0.45+2x+3x}{5}=0.40$$
$1.35 + 5x = 2$
$5x = 0.65$
$x = 0.13$
Rainfall on last day = $3x = 3 \times 0.13 = 0.39$

35. Average price of $\text{A}=\frac{19+26+31+35+39}{5}$
$$=\frac{150}{5}=30$$
Average price of $\text{B}=\frac{7+8+17+20+23}{5}=\frac{150}{5}=30$

Index price $=\frac{30\times 1.1+15\times 0.9}{2}=23.25$

36. Average speed when Ankit travels from New Delhi to Patna $=\dfrac{d}{\frac{-d/3}{50}+\frac{d/3}{40}+\frac{d/3}{60}}$
$$=\frac{d}{\frac{d}{150}+\frac{d}{120}+\frac{d}{180}}=\frac{1800d}{12d+15d+10d}$$

$$= \frac{1800d}{37d} = \frac{1800}{37}\text{km/h}$$

Required Average speed $= \dfrac{2\cdot\frac{1800}{37}\cdot 75}{\frac{1800}{37}+75}$

$$= \frac{270000}{4575} = 59.01$$

37. $\frac{25.16-x}{24} = 15,\ \ x$ = age of teacher

$x = 25.16 - 24.15\ = 400 - 360 = 40$

38. Average of 1st 10 multiple of

$$1 = \frac{1+2+3+......+10}{10} = 5.5$$

Average of 1st 10 multiple of

$$2 = \frac{2+4+........+20}{10} = 11$$

Average of 1st 10 multiple of

$$3 = \frac{2+6+........+30}{10} = 16.5$$

Hence, required Average

$$= \frac{5.5+11+16.5+.....to\ 10\ \text{term}}{10}$$

$$= \frac{\frac{10}{2}[2\times 5.5+9\times 5.5]}{10}$$

$$= \frac{1}{2}\times 60.5 = 30.25$$

39. Score of Chandan = 64 × 11 + 10 × 67 – 20 × 65
= 704 + 670 – 1300
= 1374 –1300 = 74

40. $\frac{38+84+63+45+32+n}{6} = \frac{262+n}{6} = 43.66+\frac{n}{6}$

only (c) satisfies the condition

41. Type of T_1 Tea in 4 kg mixture = $\frac{1}{7}\times 4\text{kg} = \frac{4}{7}$ kg

Type of T_2 tea in 4 kg mixture= $\frac{2}{7}.4$ kg = $\frac{8}{7}$kg

Type of T_3 tea in 4 kg mixture = $\frac{4}{7}\times 4 = \frac{16}{7}$kg

CP of 4 kg Tea mixture = Rs $\left(\frac{296+504+1008}{7}\right)$

= Rs $\frac{1808}{7}$

SP of New mixture = Rs 84

CP of New mixture = Rs $\left(84-84.\frac{1}{4}\right)$

= Rs 63 per kg

Suppose x kg of T_1 Tea is added to 4 kg mixture

CP of $(4 + x)$ kg tea mixture = $(4 + x)63$ = Rs $(252 + 63x)$

Also, CP of $(4 + x)$ kg Tea mixture

= Rs $\left(\frac{1808}{7}+\frac{x}{7}.74\right)$

= Rs $\left[\frac{1808+74x}{7}\right]$

Thus, $252 + 63x = \frac{1808+74x}{7}$

$1764 + 441x = 1808 + 74x$

$367x = 47$

$x = \frac{47}{367}$

42. Initial Concentration of wine = 75%
Concentration of wine after 1st flushing

$$= \frac{45-75\%\text{ of }4\times 100}{60} = 70\%$$

Concentration of wine after 2nd flushing

$$= \frac{42-70\%\text{ of }5\times 100}{60}$$

= 63.66%
Concentration of wine after 3rd flushing

$$= \frac{38.5-63.66\%\text{ of }38.5\times 100}{60}$$

= 57.75%

43. x= amount of alloy B
y = Amount of Alloy A

In the Alloy

Take the quantity of gold in both the Alloy under consideration and then take alligation

$$\frac{x}{y} = \frac{7/9\ \ 1/2}{\frac{1}{2}-\frac{7}{18}} = \frac{\frac{14-9}{18}}{\frac{9-7}{18}} = \frac{5/18}{\frac{2}{18}} = 5:2$$

Hence, required ratio = 2 : 5

44. Suppose x litre was the initial volume of mixture. Then volume of liquid A $= \frac{4}{5}x$, and volume of liquid B = $x/5$

Now, $\dfrac{\frac{4}{5}x - \frac{4}{5}\cdot 10}{\frac{1}{5}x - \frac{1}{5}\cdot 10 + 10} = \dfrac{2}{3}$

$\dfrac{\frac{4}{5}x - 8}{\frac{1}{5}x - 2 + 10} = \dfrac{2}{3} \Rightarrow \dfrac{\frac{4}{5}n - 8}{\frac{1}{5}n + 8} = \dfrac{2}{3}$

$12x - 120 = 2x + 80$

$10x = 200$

$x = 20$

Volume of liquid = $\frac{4}{5}\cdot 20 = 16$ I

45. Let $25x$ be the volume of pure milk present in vessel. Applying the formula,

Amount left = Initial amount $\left(1 - \dfrac{\text{amount take out}}{\text{Initial Amount}}\right)^{n}$

n = no. of times operation carried out

$16x = 25x\left(1 - \dfrac{9}{25x}\right)^{2}$

$x = \dfrac{9.5}{25} = \dfrac{9}{5}$

Hence, volume of milk present initially = $25\cdot\frac{9}{5}$ 45I

46. Let x = no. of deer

y = no. of ducks

Total no. of heads are 204 i.e.,

$x + y = 204$ …… (1)

total no. of legs = 512

i.e., $4x + 2y = 512$ …… (2)

Solving 1 and 2

$x = 52$

$y = 152$

47. Net interest earned by him $= \frac{1}{2}\times\frac{1440}{1000}\times 100$ =7.2%

Let Rs x is invested at 6% p.a. and Rest invested at 8% p.a.

$\dfrac{x}{y} = \dfrac{8-7.2}{7.2-6} = \dfrac{.8}{1.2} = \dfrac{2}{3}$

Required Amount = $\frac{2}{5}\times 10000 = \text{Rs } 4000$

48. Let $7x$ and $5x$ volume of liquids A and Liquid B are present in vessel then,

$\dfrac{7x - \frac{7}{12}\cdot 9}{5 - \frac{5}{12}\cdot + 9} = \dfrac{7}{9}$

$x = 3$

Hence, 7.3 = 21 l liquid A was present.

49. Let x = volume of water he added

$\dfrac{36}{36 + x} = \dfrac{3}{4}$

$144 = 108 + 3x$

$3x = 36$

$x = 12$

50. After 1st replacement concentration of milk = $\frac{40}{60}\times 100 = 66.66\%$

After 2nd replacement concentration of milk = $\frac{80/3}{60}\times 100 = 44.44\%$

A sequence is found - 66.66%, 44.44%, 22.22% us, this process is done for 4 times.

Moderate

1.

Number of Boys	B	Average of Girls
Number of Girls	G	Average of Boys

And class average = [BG + BG]/2

Average of Boys' average and Girls' average

= [B + G]/2

Now, [BG + BG]/2 = [B + G]/2 – 4

Or, $[B - G]^2 = 8\,[B+G]$

Since LHS is a perfect square, RHS should also be a perfect square. Now see the options that satisfy this.

2. Now different values of [B + G] possible are – $0(2\times 0^2)$, $2(2\times 1^2)$, $8(2\times 2^2)$, $18(2\times 3^2)$ etc.

We cannot take 0, since number of boys + number of girls cannot be equal to zero.

We cannot take 2, since in this case, B – G will exceed B + G.

We cannot take 8, since in this case, B – G will become equal to B + G, making G = 0. And this will result into making all the equations wrong.

So, the minimum possible value = 18

3. Take five consecutive integers such that it satisfies the condition.

4. Sum of 1st n even no. = $n(n + 1)$

   Average of 1st n even number = $\frac{n(n+1)}{n} \equiv n+1$

   Let $n = 2m$

   Average of all the even no. upto n i.e., $2m$

   $= \frac{2m(2m+1)}{2m} = (2m+1)$

   $= (n + 1)$

   Hence, required difference = 0.

5. Let present age of father = x year

   and present age of mother = y year

   therefore present age of son = $(x – 30)$

   and present age of daughter = $(y – 25)$

   Sum of their ages = 4 × 26.25 = 105 years

   i.e., $x + y + x – 30 + y – 25 = 105$

   $2x + 2y = 160$

   $x + y = 80$ ...(1)

   After 4 years, their total ages will be (excluding the daughter)

   $x + 4 + y + 4 + x – 30 + 4 = 107$

   $2x + y – 18 = 107$

   $2x + y = 125$ ...(2)

   Solving (1) and (2)

   $x = 45$

6. Total weight of all players initially = 68 × 10 = 680 kg

   Total weight of players when 1 player left the team = 598.5

   Different in weight = (680 – 598.5) kg = 81.5

7. Total weight of 11 players (68 × 11) kg = 748 kg

   Increase in weight = (748 – 598.5) kg = 149.5

8. Since the exact weight is not known, we cannot find out the average weight of all the players taken together.

9. Exact weight of players are not known hence (d)

10. If he is taking 4 packages at a time, then he is taking each of packet 15 times (${}^6C_2$). So, total weight taken this way = 15 × 500. So, actual weight = [7500 × 3/2]/15

11. What is given here is the range, and not the exact marks.

13. Since we do not have the exact average, we cannot calculate this.

14. Try with option. Also, max student that can shifted cannot be more than 24

    when (c) then average of section A = 30

    and average of section c

    $= \frac{24 \times 22 + 1 \times 20 + 19 \times 26}{44} = 23.6$

15. Add in the existing solution

    Max. possible average of all three sections

    $= \frac{1 \times 22 + 24 + 30 + 1 \times 24 + 29 \times 28 + 1 \times 20 + 19 \times 26}{77}$

    = 28.13

16. Required average

    $= \frac{5 \times 22 + 5 \times 30 + 6 \times 24 + 4 \times 28 + 20 \times 4 + 6 \times 26}{30}$

    25.06

17. Add in the existing solution

    Marks contributed by Section

    A = 5 × 22 + 5 × 30 = 260

    Marks contributed by Section

    B = 6 × 24 + 4 × 28 = 256

    Marks contributed by Section

    C = 20 × 4 + 6 × 26 = 236

18. First shift 5 students from A to B having the max. marks then, shift 5 students from B having max. marks and 5 those came from A.

    Required Average

    $= \frac{5 \times 30 + 5 \times 28 + 1 \times 20 + 19 \times 26}{30} = 27$

19. Now, shift 5 students from A having minimum marks and shift 5 from B having minimum marks and 5 those came from A

    Required Average

    $= \frac{5 \times 22 + 5 \times 24 + 19 \times 20 + 1 \times 26}{30} = \frac{636}{30}$

    $= 21.2 \approx 21$

22. Take the values as 1, 2, 3 and 4 and check what happens.

    Otherwise this question is very similar to the No. 10 of Moderate.

23. Let the numbers be 1,2,3,4 and 5 them solve

24. To calculate @ (/ (* (A, B), B), A); we will start with the innermost part.

    * (A, B) = * (2, 4) = 8

    / (* (A, B), B = / 8, 4 = 2

    @ (/ (* (A, B), B), A) = @ (2, A) = @ (2, 2) = 2

25. Check the options to find the answer as option (a).

27. Sum of ages of Mr and Mrs Sinha in 1972 = 46 years

    Sum of age of their family in 1976 = 19 × 3 = 57 years

Sum of ages of Mr & Mrs Sinha in 1976 = (46 + 8) years = 54 years

their son's age in 1976 = (57 – 54) years = 3 years

28. Number of articles made in 1st hour = 60
Number of articles made in 2nd hour = 45
Number of articles made in 3rd hour = 63
Number of articles made in 4th hour = 42
Number of articles made in 5th hour = 63
So, obviously articles made in 4th hour is minimum.

29. Total number of runs scored till 86th inning = 86 N
Now, 86 N + 270 = 87 (N + S), where S is the increase in batting average.
Different values of S possible now S = 0, 1, 2 and 3.

30. Since, a boy in increased in Section B and the average is decreased ⇒ Hence, his weight is less than 38 kg. Again, the average weight is also decreased in both the sections. Hence, his weight is more than 35 kg. Hence, (c)

31. Let the no. of innings played be N

$$\text{then, } \frac{21.75\text{N}+28+34+37}{\text{N}+3} = \text{N}+1.25$$

Solving, we get N = 27

32. Total score = 21.75 × 24 + 28 + 34 + 37 = 621

33. Let the height of four students be 150, 160, 170, 180 cm then

$$\text{A} = \frac{150+160+170+180}{4} = \frac{660}{4} = 165\,\text{cm}$$

$$\text{B} = \frac{\frac{150+160}{2}+\frac{160+170}{2}+\frac{170+180}{2}+\frac{150+180}{2}+\frac{150+170}{2}+\frac{160+180}{2}}{6}$$

$$= \frac{155+165+175+165+160+170}{6} = \frac{990}{6} = 175\,\text{cm}$$

$$= \frac{670}{4} = 167.5$$

Similarly, C = 167.5

Now going through the options,

34. Given $n_1 + n_3 ...... + n_{45} = 1273$
From above we cannot find the value of d.

35. Go through options.

36. Let x litres of 50% solution and y litres of 80% solutions are used

$$\frac{x}{y} = \frac{80-62}{62-50} = \frac{18}{12} = \frac{x}{y} = \frac{3}{2}$$

Solution get mixed in the ratio 3 : 2.

Now, suppose the value of acid is Z litres⇒

$$\frac{0.62z}{z+6} = \frac{1}{2}$$

⇒ 1.24 $Z = Z + 6$ ⇒ 0.24 Z= 6

⇒ Z = 25

Hence, required rate = $\frac{2}{5}\times 25 = 10$ litres

37. Solve with the help of options.

38. It is possible because each contents removed during the dilution process is stored separately.

39. Value of solvent in container = $\left(\frac{4}{5}\times 150\right)l = 120l$

Concentration of solvent = $\left(\frac{9}{10}\times 120\right)l = 108\,l$

When first time water is added the concentration of solvent

$$= \frac{108}{130}\times 100 = 83\%$$

When 2nd time water is added the concentration of solvent

$$= \frac{108}{140}\times 100 = 77\%$$

When 3rd the water is added the concentration of

Solvent $= \frac{108}{150}\times 100 = 72\%$

Now, 10 l solution should be removed
Solvent present in container = (108 – 7.2) l = 100.8 l
Again 10 l water is added and 10 l solvent is removed. To get the required solution, water is added for 6 times.

40. Let the initial volume be 85x

$$36x = 49x\left(1-\frac{4}{49x}\right)^2$$

$$\left(\frac{6}{7}\right)^2 = \left(1-\frac{4}{49x}\right)^2$$

$$1-\frac{4}{49x} = \frac{6}{7}$$

⇒ x = 4/7

Required volume = 49 × 4/7 = 28 g

41. After 1st operation milk left =

$$40\left(1-\frac{4}{40}\right) = 40\cdot\left(1-\frac{1}{10}\right) = 40\cdot\frac{9}{10} = 36\,L$$

After 2nd operation milk left = 36 [1– (5/40)] = 31.5

After 3rd operation milk left $\left(36.5 - 6\times\frac{31.5}{40}\right)$

= (26.775L)

42. When 4 litres of milk is taken out, volume of milk left in the vessel = 36 litres [40{1–(4/40)}]
When 5 litres of mixture is taken out, volume of milk left in the vessel = [40{1–(4/40)}{1–(5/41)}
When 6 litres of mixture is taken out, volume of milk left in the vessel = [40{1–(4/40)}{1–(5/41) {1–(6/42)}]

43. Do as given in 42.

44. When a part of vessel A is replaced with the mixture in vessel B, then the concentration of milk in the final mixture is less than C_1 – It implies that $C_1 > C_2$
When vessels B and C are mixed in the ratio 1:1, then the quantity of milk in the final mixture is double than the quantity of milk in the vessel B – It implies that $C_2 = C_3$
Hence C_1 is greatest.

45. Obviously, $C_2 = C_3$ is the right answer.

46. When a part of vessel A is replaced with the mixture in vessel B, then the concentration of milk in the final mixture is less than C_1 – It implies that $C_1 > C_2$
When vessels B and C are mixed in the ratio 1 : 1, then the concentration of milk in the final mixture is double than the concentration of milk in the vessel B. It implies that $C_3 > C_2$. Now we don't know which of C_1 or C_3 is greater. So answer is cannot be determined.

47. Let x and y kg of 1st Alloy and 2nd Alloy are taken

$$\frac{x}{y} = \frac{\frac{5}{7} - \frac{1}{2}}{\frac{1}{2} - \frac{3}{7}} = \frac{\frac{10-7}{14}}{\frac{7-6}{14}} = \frac{3}{1}, \ \frac{x}{y} = \frac{3}{1}$$

Hence, (d)

48. Let the value of milk = $3x$ litres and volume of water = $5x$ litres
When 20% of mixture is taken out

Volume of milk left = $\left(3x - \frac{1}{5} \times 3x\right) = \frac{12x}{5}$ litres = $2.4\,x$ litres

Volume of water left = $\left(5x - \frac{1}{5} \times 5x\right) = 4x$ litres

And $1.6\,x$ litres of milk is added, New ratio = 1 : 1
Similarly, proceeding we get
(c) as answer

Advanced

1. Obviously, in this case, AC ≤ AB. Now we are required to compare AB and AQ.

$$AB = \frac{R_1}{N_1} + \frac{R_2}{N_1} \text{ and } AQ =$$

$$\frac{R_1}{N_1} + \frac{R_2}{N_1} \max\left[0, \left(\frac{R_2}{N_2} - \frac{R_1}{N_1}\right)\right]$$

So, basically we have to compare $\frac{R_2}{N_1}$ and

$$\frac{R_2}{N_1} \max\left[0, \left(\frac{R_2}{N_2} - \frac{R_1}{N_1}\right)\right]$$

Since we don't have any information regarding values of R_2, R_1, N_2 and N_1, we cannot determine anything. So, Answer is option (d) None of these.

2. Since total number of runs is increasing without increase in the number of completed innings, AB will increase. However, since runs scored is less than 50 in the last inning, so AC will decrease.

3. This question is one standard example of the definition of average in terms of surplus and deficit of the values. (Refer to the definition given)
Now, what we were expected to mark here was the fact that whatever is the average (in this case, it is A), surplus generated by the marks of 3rd student will be same as deficit incurred due to 15th student. So, rusticating both of them is not going to create any difference on average marks of the class (remember marks are in AP). And similar will be the impact of rusticating 7th student and 11th student and then finally 9th student.
So, A = B

5. Basically in these types of situations, we should start with an attempt of refuting the statements given. The statement that cannot be refuted will be true, otherwise false.
Checking statement i, since Munchun is donating ½ of water from her bottle and Illara is donating only ¼ of mixture from her bottle, concentration of water in Munchun's drink will be less than the concentration of orange juice in Illara's drink. So, statement i. is not true.
Checking statement ii, since Illara and Puttu are receiving and then donating the same amount of liquid, so concentration of orange juice in Illara's drink will be equal to the concentration of mango juice in Puttu's drink. So, statement ii. is also not true.
Checking statement iii, concentration of orange will be equal to the concentration of mango in Munchun's drink. So, statement iii is also wrong.
Hence option (d). None of these is the answer.

6. We can get the sum 666 in two ways:
1 + 2 + 3 + 4 + 567 + 89 = 666 or
123 + 456 + 78 + 9 = 666
So, average cannot be uniquely determined.

7. 261 is possible only if we take 123 + 45 + 6 + 78 + 9
So, the average will be 261/5 = 52.2

8. It is equal to (Sum of ages of all the Engineers – Sum of ages of all the males)/(Number of engineers – Number of males) = 780/20 = 39
9. Since the relationship here is: Some A are B, so it cannot be determined.
10. Required average = (Sum of ages of all working professional – sum of ages of all females) / (No. of working professional – No. of females)

$$=\frac{27.160-22.100}{160-100}=\frac{4320-2200}{40}=35.33$$

11. If no females (F) are engineers (E), then it is not possible to have 160 engineers from 140 males. So, F @ E is not possible.
12. The required result will be produce when we choose sample of A, B, F, G, I, J

No. of pairs = ${}^6C_2 = \frac{6.5}{1.2} = 15$

14. The required result well be produced by taking the samples C, D, E and H.
Hence, required = ${}^4C_2 = 6$
15. All the ratios formed from a and b will be resulting into a ratio less than 1 (like 1 : 4, 2 : 5 etc). To obtain a resultant ratio of 1:1, we should have atleast one of the ratios greater than 1. Hence we cannot get the desired ratio of 1 : 1.
16. When three solutions are taken together we can have infinite values.
18. Average of the sum of n_1, n_3, n_5,........ n_{47} terms $=\frac{1272}{24}=53$

Required sum = average of odd terms × no. of terms = 53 × 47 = 2491
22. Eliminating the option, we get (c) as answer because average always lies between greatest and lowest.
23. A set of all real number can give the above conditions. Hence, (b)

[illegible]

19. All the additions in the net score would be in the multiple of 2 and similarly all the subtractions will also be in the multiple of 2. Hence, net scores will also be a multiple of 2.
21. The sum of scores of top four students = 1356

Maximum possible score possible for top three students = 352 × 3 = 1056

Hence, the minimum possible score for the 4th student = 300
24. Total number of TVs disposed off in 1999 = 20% of 30 = 6

Total number of TVs disposed off in 2000 = 20% of 50 = 10

So, Total number of TVs disposed off by 2000 = 16

So, average number of TVs disposed off = $\frac{16}{4}=4$
25. Since we do not have the figure of disposed off TVs every year, we cannot determine the average number of TVs purchased every year till 1995.

6

CHAPTER

Ratio, Proportion and Variation

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Ratio, proportion and variation
- Definitions and properties
- Different applications of ratio, proportion and variation
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

Ratio and proportion is important not only for the QA paper but also for DI, especially for calculations. Questions from this topic are based on conceptual clarity and their different applications are often tested in the CAT. However, this is a favourite topic for all the non-CAT and non-XAT level examinations. While going through this chapter, students should try to internalize the mathematical as well as the logical method of solving the problems.

RATIO, PROPORTION AND VARIATION

Ratio is a quantity which expresses the relationship between two similar quantities. It expresses a magnitude by which one quantity is a part of or a multiple of another quantity.

If the value of A and B are 8 and 6 respectively, then they are in the ratio 8:6 (read as 8 is to 6). Ratio can be understood also as the relationship which one quantity bears with the other of the same kind. Due to this reason, we cannot compare salary of one person with the percentage expenditure of another person.

The ratio of two quantities A and B is written as A:B. Here, A is known as an antecedent and B is known as a consequent. It can also be said that A:B = kA:kB, where k is any constant known as constant of proportionality, $k \neq 0$.

If the antecedent is more than the consequent (or, the numerator is more than the denominator), then the ratio is known as an improper ratio.

For example, 5/3, 55/29 etc.

If the antecedent is less than the consequent (or, the numerator is less than the denominator), then the ratio is known as a proper ratio.

For example, 3/7, 7/18, etc.

Since Ratio compares two similar quantities, it cannot have any units.

Example 1 Consider any ratio $\frac{a}{b}$. Now x is added to the numerator and the denominator of this fraction. Which of the following is greater: $\frac{a+x}{b+x}$ or $\frac{a}{b}$?

Solution It depends upon two factors:

i. If the ratio is proper or improper
ii. x is positive or negative.

If $\frac{a}{b} > 1$ and $x > 0$, or, $\frac{a}{b} < 1$ and $x < 0$

$$\frac{a}{b} > \frac{a+x}{b+x}$$

and if $\frac{a}{b} > 1$ and $x < 0$, or, $\frac{a}{b} < 1$ and $x > 0$

$$\frac{a}{b} < \frac{a+x}{b+x}$$

RATIO

Ratio can be understood in two ways:

1. Ratio as a bridging element
2. Ratio as a multiplier

1. Ratio as a Bridging Element

Ratio as a bridging element helps us in establishing the relationship between more than two quantities. This can be further understood with the following example

Suppose conversion rate of our currency Rupee is given with respect to US dollar and also with respect to Pound sterling. If we have to find the conversion ratio of US dollar with respect to pound sterling, we can do it by making rupee as the bridge between US dollar and pound sterling.

Example 2 The ratio of the age of A and B is 2:5 and ratio of the age of B and C is 3:4. What is the ratio of the age of A, B and C?

Solution Since B is the common platform which associates A and C, so we will try to make B equal in both the cases.

Age of A : Age of B = 2:5] × 3
Age of B : Age of C = 3:4] × 5
Or, Age of A : Age of B = 6:15 ...(1)
Age of B : Age of C = 15:20 ...(2)

Since ratio of B is same in both the cases, hence age of A:Age of B:Age of C = 6:15:20.

Example 3 Given that

Salary of A: Salary of B = 1:2
Salary of B: Salary of C = 3:4
Salary of C: Salary of D = 5:6
Salary of D: Salary of E = 7:8
Salary of E: Salary of F = 9:10

What is the ratio of the salaries of A, B, C, D, E, and F?

Solution Salary of A : Salary of B:Salary of C:Salary of D:Salary of E:Salary of F = (1 × 3 × 5 × 7 × 9):(2 ×3 × 5 × 7 × 9):(2 × 4 × 5 × 7 × 9):(2 × 4 × 6 × 7 × 9):(2 × 4 × 6 × 8 × 9):(2 × 4 × 6 × 8 × 10).

(Understand the above mechanism with the help of the method given in Example 2. In these cases, this method can be used as a shortcut to find the ratios in the following way: For A, take all the leftmost digits, and now keep shifting towards the right digits by quitting one by one all the leftmost digits. So, B = Right digit of 1st ratio and so on for C, D, E and F.)

Example 4 If A : B = 3:4,

B : C = 5:7
C:D = 10:11

What is the ratio of A:D?

Solution A = 3 × 5 × 10 and D = 4 × 7 × 11.

So, the ratio = 150 : 308

Alternatively, (A/B) × (B/C) × (C/D) = (3/4) × (5/7) × (10/11) = (3 × 5 × 10)/(4 × 7 × 11) = 150:308

Example 5 A, B, C and D purchase a gift worth Rs 60. A pays 1/2 of what others are paying, B pays 1/3rd of what others are paying and C pays 1/4th of what others are paying. What is the amount paid by D?

Solution Since A is paying 1/2 of what others are paying, so A is paying 1/3rd of the total amount.

(To understand this, let us assume that B, C and D are paying Rs $2x$. So A is paying Rs x. The total amount being paid by A, B, C and D = $3x$ = Rs 60, hence, the amount paid by A = $x/3x$ = 1/3rd of the total.)

So, the amount paid by A = 60/3 = Rs 20

Similarly, B is paying 1/4th of the total and C is paying 1/5th of the total.

Hence, the amount paid by B and C are Rs 15 and Rs 12 respectively.

So, the amount paid by D = Rs 13

2. Ratio as a Multiplier

The moment we say that the ratio of two numbers A and B is 5 : 1, what we mean to say that A is 5 times of B.

It can also be seen that A : B : C in A/2:B/3:C/4 = K is not same as A:B:C = 1/2:1/3:1/4 since multiplier of A, B and C are not the same in both the cases.

Ratio of A : B : C in A/2:B/3:C/4 = K can be calculated in the following way

Since A/2 = B/3 = C/4 = K, so A = 2K, B = 3K and C = 4 K

Hence, the ratio of A:B:C = 2:3:4.

While calculating the ratio of A, B and C in A : B : C = 1/2:1/3:1/4, we will multiply each of A, B and C by the LCM of the denominator of all the ratios i.e., 12.

So, A : B : C = 6:4:3

Example 6 10 persons can cut 8 trees in 12 days. How many days will 8 persons take to cut 6 trees?

Solution Let us see this question in a changed perspective

Suppose if the question is—10 persons can cut 8 trees in 12 days. How many days will 10 persons take to cut 4 trees?

Answer to this question is:Since the amount of work is getting halved, so the number of days will also get halved.

There are three factors, namely, the number of men, the number of days and the number of trees, which are responsible for the final answer.

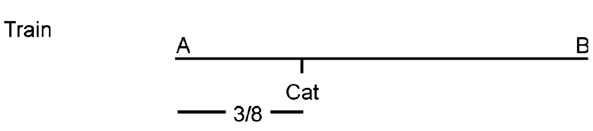
Since the number of men are less in the final situation, so more number of days will be required. Hence, multiplier = 10/8 (had there been 12 persons, multiplier would have been 10/12.)

The number of trees are less in the final situation, so less number days will be required. So, multiplier = 6/8

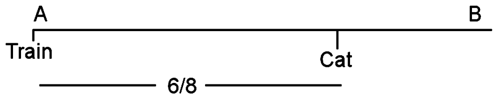
Hence, the total number of days = 12 × 10/8 × 6/8 = 90/8 = 11.25 days

Example 7 A train approaches a tunnel AB. Inside the tunnel, a cat is sitting at a point that is 3/8th of the distance of AB measured from the entrance A. When the train whistles, the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat exactly at the exit. What is the ratio of the speed of the train and the speed of the cat?

Solution



Initially, this was the position of the train and the cat. Now, let us assume that the cat is moving towards exit B. The moment the cat covers 3/8th of AB distance in the direction of exit B, the train will be at the entrance A.



Now, if the cat moves in the direction of exit B, the train is catching up with the cat at the exit B. So, in the time cat covers 2/8th distance, the train is covering the whole distance from A to B.

So, the ratio of the distance covered by train and the distance covered by the cat = 4 : 1

So, the ratio of speed = 4:1

Example 8 Pranesh can do a work in 15 days. In how many days will the work be completed by his brother Saket if efficiency of Saket is 60% more than that of Pranesh?

Solution Since the ratio of efficiency of Pranesh and Saket = 100 : 160 = 5:8 the number of days taken by Pranesh and Saket will be in the ratio of 8 : 5

Since Pranesh takes 15 days to do this work, Saket will take 15 × 5/8 = 9.37 days

Comparison of Ratio

This is one of the most important calculations and is extensively used in DI. On an average, if somebody does 100 calculations in DI at least 8–10 calculations will be from comparing the ratios. Normally, there are two methods to compare two or more than two ratios:

1. Cross multiplication method

E.g., Let us compare 11/15 and 13/18.

$$\frac{11}{15} \times \frac{13}{18}$$

Cross multiplying numerator of 1st fraction with the denominator of 2nd fraction and denominator of 1st fraction with the numerator of 2nd fraction,

11 × 18 13 × 15

198 195

Since, 198 is greater than 195 the 1st fraction (11/15) is greater than the 2nd fraction (13/18).

2. Decimal calculation

$$\frac{11}{15} = 0.733 \qquad \frac{13}{18} = 0.722$$

Obviously, here the 1st fraction $\left(\frac{11}{15}\right)$ is greater than the 2nd fraction $\left(\frac{13}{18}\right)$.

However, if we have to compare 3156/5438 and 3423/5822, then using any of the above two methods becomes cumbersome and time-consuming.

Here, we will compare ratios with the help of percentage.

3. Percentage comparison

Let us first understand this with the help of the following ratios

1st Case $\frac{10}{15}\xrightarrow[100\%]{}\xrightarrow{100\%}\frac{20}{30}$

2nd Case $\frac{10}{15}\xrightarrow[100\%]{}\xrightarrow{200\%}\frac{30}{30}$

3rd Case $\frac{10}{15}\xrightarrow[300\%]{}\xrightarrow{200\%}\frac{30}{60}$

In the 1st case, percentage change in numerator (100%↑) = percentage change in denominator (100%↑), so ratios are equal.

In the 2nd case, percentage change in the numerator (200%↑)>percentage change in the denominator (100%↑), so the 2nd ratio is greater than the 1st ratio.

In the 3rd case, percentage change in the numerator (200%↑)<percentage change in the denominator (300%↑), so the 1st ratio is greater than the 2nd ratio.

This particular example can also be seen as a general rule for determining the order of ratios.

PROPORTION

In case of an analogy, two quantities share same kind of relationship. For example, what Macbeth is to William Shakespeare, Dr Zivago is to Boris Pasternak.

In QA, the same is true for proportion. It is basically the equality of the two ratios.

$$\frac{A}{B}=\frac{C}{D}$$

When A, B, C and D are in proportion, then A and D are known as 'extremes' and B and C are known as 'means'.

Thus, we can say,

Product of Extremes = Product of Means

Example 9 What is the value of x in the following expression?

$$\frac{5}{8}=\frac{x}{12}$$

Solution $\frac{5}{8}=\frac{x}{12}$

$\Rightarrow x = \frac{60}{8}=7.5$

It can be calculated with the help of percentages also. In this question, the percentage increase in the denominator is 50%, so the numerator will also increase by 50%.

Standard Results/Definitions on Ratio/ Proportion

1. Continued proportion

   a, b and c are said to be in continued proportion if $\frac{a}{b}=\frac{b}{c}$

   So, $b^2 = ac$. Here b is known as the mean proportion.

   Similarly, if a, b, c and d are in continued proportion, then we get:

   $$\frac{a}{b}=\frac{b}{c}=\frac{c}{d}$$

2. Componendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{b}=\frac{c+d}{d}$

3. Dividendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a-b}{b}=\frac{c-d}{d}$

4. Componendo and Dividendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a+b}{a-b}=\frac{c+d}{c-d}$

5. Invertendo

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{b}{a}=\frac{d}{c}$

6. Alterando

   If $\frac{a}{b}=\frac{c}{d}$, then $\frac{a}{c}=\frac{b}{d}$

7. $\frac{a}{b}=\frac{c}{d}=\frac{a+c}{b+d}$

   In general, if $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots=\text{K}$

   Then, $\frac{a}{b}=\frac{c}{d}=\frac{e}{f}=\ldots\ldots\ldots=K=\frac{a+c+e+\ldots\ldots}{b+d+f+\ldots\ldots}$

   = (any combination of the numerator/any combination of the corresponding denominator)

   For example, 1/2 = 3/6 = 4/8 = … = (1 + 3 + 4)/ (2 + 6 + 8) = (3 + 4)/(6 + 8)

8. If we multiply the numerator and the denominator of a ratio by any number N (N ≠ 0), then the ratio remains same.

   A/B = NA/NB

9. If we divide the numerator and the denominator of a ratio by any number N (N ≠ 0), then the ratio remains same.

   A/B = (A/N)/(B/N)

10. If a/b, c/d, e/f … etc. are all unequal ratios, then the value of $(a + c + e + \ldots)/(b + d + f + \ldots)$ lies in between the minimum and the maximum of all these ratios.

VARIATION

Two quantities A and B are said to be varying with each other if there exists some relationship between A and B such that the change in A and B is uniform and guided by some rule.

Some typical examples of variation

- Area (A) of a circle = π R^2, where R is the radius of the circle.

Area of a circle depends upon the value of the radius of a circle, or, in other words we can say that the area of a circle varies as the square of the radius of a circle.

- At a constant temperature, pressure is inversely proportional to the volume.
- If the speed of any vehicle is constant, then the distance traversed is proportional to the time taken to cover the distance.

Direct Proportion

If A varies directly to B, then A is said to be in direct proportion to B.

It is written as – $A\ \alpha\ B$.

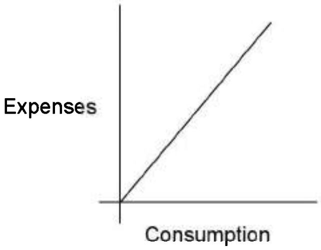
It can be understood with the typical example of percentage relating to expenses, consumption and price of the article.

If the price of a article is constant, then Consumption α Expenses.

$\Rightarrow$ Consumption = K. Expenses, where K is proportionality constant.

If we increase consumption by 20%, then the expenses will also increase by 20%.

At a constant price, if a graph is drawn between consumption and expenses by taking them at X-axis and Y-axis respectively, then this graph will be a straight line.



This is what we mean to say with direct proportion.

Inverse Proportion

If A varies inversely to B, then A is said to be in inverse proportion to B.

It is written as – $A\ \alpha\ 1/B$

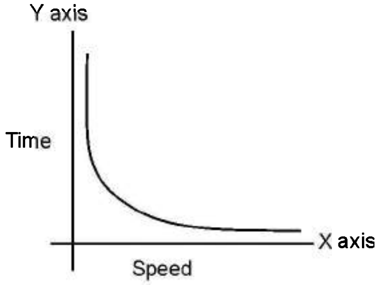
It can be understood with a time-speed-distance example, where if the distance is constant, then speed α 1/time.

Assuming the distance between New Delhi to Patna is 1000 km then consider the following table:

Speed (km/h)	*Time (in h)*
500	2
2 × 500	1/2 × 2
3 × 500	1/3 × 2
4/3 × 500	3/4 × 2

It can be seen here that the multiplier of time is reciprocal of the multiplier of speed.

For any fixed distance, if we draw a graph between speed and time by taking them at X-axis and Y-axis respectively, then this graph will be a curve.



Thus, when the speed is minimum, the time is maximum and when the speed is maximum, the time is minimum.

Example 10 The height of a tree varies as the square root of its age (between 5 and 17 years). When the age of a tree is 9 years, its height is 4 feet. What will be the height of the tree at the age of 16?

Solution Let us assume the height of the tree is H and its age is A years.

So, $H\ \alpha \sqrt{A}$, or, $H = K \times \sqrt{A}$

Now, $4 = K \times \sqrt{9}$

$\Rightarrow K = 4/3$

So, height at the age of 16 years = $H = K \times \sqrt{A} = 4/3 \times 4 = 16/3$ = 5 feet 4 inches.

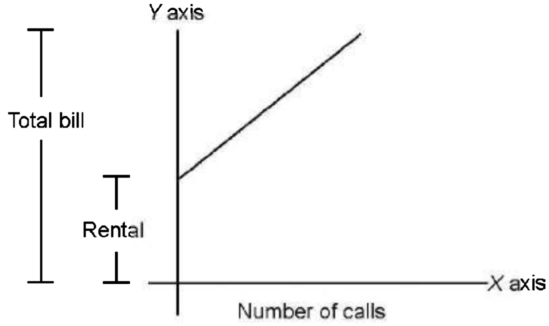
Direct Relation

A is directly related to B and as B changes, A also changes but not proportionally.

It is written as – A = C + K. B, where C and K are constants.

One classical example of direct relation can be seen as the telephone connection in a house. In a telephone connection we pay some money as the rent along with the phone charges according to the rate and number of calls made. So, the total bill = rental + K. (number of calls).

If we draw a graph between the number of calls and the total bill by taking them at X-axis and Y-axis respectively, this graph will be a straight line in the following way:



Example 11 Total expenses at a hostel is partly fixed and partly variable. When the number of students is 20, total expense is Rs 15,000 and when the number of students is 30, total expense is Rs 20,000. What will be the expense when the number of students is 40?

Solution Expenses = F + K. V; where F is the fixed cost and V is the number of students.

Rs 15,000 = F + K.20 ....(1)
Rs 20,000 = F + K.30 ....(2)
Solving (1) and (2),
Rs 5,000 = 10. K ⇒ K = Rs 500
So, F = Rs 5000
So, F + 40 K = Rs 5,000 + 40 × 500 = Rs 25,000

APPLICATION OF RATIO, PROPORTION AND VARIATION

1. Income–expense ratio

When the ratio of incomes and expenses of two persons are given and their savings is being asked.

Example 12 The ratio of the incomes of Mr Vinay Singh and Mr Arun Sharma is 3 : 5 and the ratio of their expenses is 1:3. Who is saving more?

Solution Let us assume the values of income and expenses of A and B,

	Income	*Expense*	*Savings*
Vinay Singh	Rs 3	Rs 1.5	Rs 1.5
Arun Sharma	Rs 5	Rs 4.5	Rs 0.5

So, Vinay saves more than Arun.
In the other case,

	Income	*Expense*	*Savings*
Vinay Singh	Rs 3	Rs 1	Rs 2
Arun Sharma	Rs 5	Rs 3	Rs 2

So, savings of both of them is equal.

	Income	*Expense*	*Savings*
Vinay Singh	Rs 3,000	Rs 1,600	Rs 1,400
Arun Sharma	Rs 5,000	Rs 4,800	Rs 200

So, in this case Mr Singh is saving less than Mr Sharma.
Thus, it is difficult to determine who is saving more.

The concept tells us: If the value of the ratio of income is more than the value of the ratio of expenses, then we cannot determine who is saving more. If the value of the ratio of expenses is more than the value of the ratio of income, then we can determine who is saving more. (Ratio should be taken in such a way that the value of ratio is less than 1, i.e., the numerator should be less than the denominator.)

In the above case, the value of the ratio of income = 3/5 = 0.6 and value of ratio of expenses = 1/3 = 0.33

Since the value of ratio of expenses < value of the ratio of income, we cannot determine who is saving more.

However, in the above question, if we take the ratio of income of Vinay and Arun as 3 : 5 and the ratio of their expenses as 3:1, then Arun is saving more.

2. Partnership

If two or more than two persons are investing their money in a joint venture, then the profit or loss incurred is distributed among everybody in the ratio of the time period for which the money has been invested by each of them.

It can be observed here that the partnership is a typical case of direct variation.

Example 13 A, B, C, … , K, L invested money in the ratio of 12:11:10:… : 2:1. The duration for which they invested the money is in the ratio of 1:2:3 :… : 11:12. Who will obtain the maximum profit at the end of year?

Solution Profit will be distributed in the ratio of product of the time and money of every individual.

So, the ratio of profit = 12 × 1:11 × 2:10 × 3:9 × 4:8 × 5:7 × 6:6 × 7:5 × 8:4 × 9:3 × 10:2 × 11:1 × 12.

So, the maximum profit will be obtained by F and G both.

PRACTICE EXERCISES

■■■

WARM UP

Q.1. The speed of three buses are in the ratio 2 : 3 : 4. The ratio between the time taken by these buses to travel the same distance is
(a) 2 : 3 : 4 (b) 4 : 3 : 2
(c) 4 : 3 : 6 (d) 6 : 4 : 3

Q.2. The difference between the two positive numbers is 10 and the ratio between them is 5 : 3. Find the product of the two numbers.
(a) 375 (b) 325 (c) 275 (d) 125

Q.3. If 30 oxen can plough 1/7th of a field in 4 hours, in how many hours will 18 oxen take to do the remaining work?
(a) 30 hours (b) 20 hours
(c) 15 hours (d) 40 hours

Q.4. Four numbers in the ratio of 1 : 3 : 4 : 7 add up to give a sum of 75. Find the value of the biggest number.
(a) 42 (b) 35 (c) 49 (d) 63

Q.5. In the above question, what is the difference between the biggest and the smallest number?
(a) 42 (b) 30 (c) 49 (d) 63

Q.6. When we increase both the numerator and the denominator by 7, a fraction changes to 3/4. Find the original fraction.
(a) 5/12 (b) 7/9 (c) 2/5 (d) 3/8

Q.7. According to the Boyle's law, at a constant temperature, pressure of a definite mass of gas is inversely proportional to the volume. If the pressure is reduced by 20%, find the respective change in volume.
(a) −33.33% (b) +25%
(c) −25% (d) +33.33%

Q.8. What number must be subtracted from both the numerator and the denominator 27 : 35, so that it becomes equal to 2 : 3?
(a) 8 (b) 12
(c) 6 (d) None of these

Q.9. P is directly proportional to Q, and Q = 7 when P = 15, find P when Q = 14.
(a) 36 (b) 54 (c) 30 (d) 60

Q.10. The concentration of three milk solutions A, B and C are 10%, 20% and 30% respectively. They are mixed in the ratio 2 : 3 : N resulting in a 23% concentration solution. Find N.
(a) 7 (b) 6 (c) 5 (d) 4

Q.11. Which of the following will have the maximum change in their values if 10 is added to both the numerator and the denominator of all the fractions?
(a) 3 : 4 (b) 2 : 3 (c) 4 : 7 (d) 5 : 7

Q.12. If $p/q = 3/4$, then find the value of the expression $(5p - 3q)/(7p + 2q)$.
(a) 3/29 (b) 5/27 (c) 7/31 (d) 5/58

Q.13. In a mixture of milk and water of volume 30 litres, the ratio of milk and water is 7:3. Find the quantity of water to be added to the mixture to make the ratio of milk and water 1:2.
(a) 30 (b) 32 (c) 33 (d) 35

Q.14. A and B invested Rs 12,000 and Rs 18,000 respectively in a business for the whole year. At the year-end, there was a total profit of Rs 2,000. What is the share of A in the profit?
(a) Rs 800 (b) Rs 1,200
(c) Rs 1,600 (d) None of these

Q.15. A and B invested the same capital in a business. At the year-end they share the profit in the ratio of 3 : 2. If A has invested his capital for the whole year, for how many months B has invested his capital?
(a) 6 months (b) 8 months
(c) 9 months (d) None of these

Q.16. A and B invest Rs 12,000 and Rs 16,000 respectively in a business. At the year-end, they share the profit in the ratio of 3 : 1. If A has invested his capital for the whole year, for how many months B has invested his capital?
(a) 4 months (b) 3 months
(c) 6 months (d) 8 months

FOUNDATION

Q.1. King Dashrath decided to distribute gold coins to his three queens in the following way: 2nd queen would get 5/7th of what the 1st queen would get and the 3rd queen would get 3/5th of what the 2nd queen would get. 1st queen got 60 gold coins more than the 3rd queen. How many gold coins were distributed to the three queens?
(a) 120 (b) 175 (c) 225 (d) 250

Q.2. The sum of the ages of the 4 members of Sinha family is 140 years. 5 years ago the ages of the 4 members Nishu, Vicky, Mrs Sinha and Mr Sinha were in the ratio of 2:3:7:8. After how many years would Nishu be as old as the present age of his mother?
(a) 10 yrs (b) 17 yrs
(c) 30 yrs (d) 32 yrs

Q.3. In the prime minister office, one-third of the workers are women, half of the women are married and one-third of the married women have children. If three-fourth of the men are married and two-third of the married men have children, what part of the persons working at the PMO are without children?
(a) 5/18 (b) 4/9 (c) 11/18 (d) 17/36

Q.4. Two-fifth of Anil's salary is equal of Bhuvan's salary and seven-ninth of Bhuvan's salary is equal to Chandra's salary. The sum of the salary of all of them is Rs 770. Which of the following is the salary of each?
(a) 300, 225, 250 (b) 500, 425, 375
(c) 450, 180, 140 (d) 520, 610, 475

Q.5. Inspector Vijay Shankar spends one-fourth of his salary on the house rent, one-third on food and one-sixth on travel. After spending one-tenth of the remaining amount on clothes, he is left with Rs 13,500. What is the difference between the amounts spent on the house rent and food?
(a) Rs 7,500 (b) Rs 10,000
(c) Rs 4,000 (d) Rs 5,000

Q.6. Bidhan is planning to buy a bike worth Rs 35,000, provided his brother agrees to lend him 3/2 times the money. Bidhan contributes and the financer provides 3 times the brother's contribution. How much does Bidhan's brother contribute?
(a) 5,000 (b) 7,500
(c) 10,000 (d) 7,000

Q.7. A group of students decided to buy a book jointly which costs between Rs 170 to 195. However at the last moment two students decided not to contribute and so, each of the remaining students had to pay one rupee each more. What was the price of the book if the students paid equal shares?
(a) Rs 182 (b) Rs 188.12
(c) Rs 192.4 (d) Rs 180

Q.8. An alloy of manganese, tin and bronze contains 90% bronze, 7% manganese and 3% tin. A second alloy of bronze and tin is melted with the first and the mixture contains 85% of bronze, 5% of manganese and 10% of tin. What is the percentage of bronze in the second alloy?
(a) 67.5% (b) 72.5%
(c) 77.5% (d) 82.5%

Q.9. Vinit is carrying some money with him. He gives 25% of the money to his friend, Sachin. He spends $\frac{1}{3}$ rd of the remaining, on a movie ticket. With 40% of the remaining money he does shopping. After all this he meets his friend Gita who gives him 66.66% of the money which he was having at that time. Finally, Vinit gives $\frac{4}{5}$ th of the total money to his mother. What is left is the square root of the initial amount he was carrying. How much money does Gita give him?
(a) 40 (b) 25 (c) 10 (d) 20

Q.10. Two cogged wheels of which one has 32 cogs and other 54 cogs, work into each other. If the latter turns 80 times in three quarter of a minute, how often does the other turn in 8 seconds? (Assume equal size cogs and equi-spaced).
(a) 24 (b) 16 (c) 32 (d) 8

Q.11. Kunal, Saurav and Gopi assemble for a dutch party. Kunal brings 3 guavas while Saurav brings 5. Since Gopi did not have any guavas, he contributed Rs 8. How many rupees should Kunal and Saurav respectively get, assuming each of the three consumes an equal portion of the guavas?
(a) 1, 6 (b) 2, 5
(c) 5, 7 (d) None of these

Q.12. A bag contains Rs 600 in the form of one rupee, 50 paise and 25 paise coins in the ratio of 3:4:12. Find the total number of 25 paise coins in the bag.
(a) 75 (b) 200 (c) 300 (d) 900

Q.13. A certain sum of money was divided among Anand, Bidhan and Chandra in the following way: Chandra got half as much as Anand and Bidhan together got. Anand got one-third of what Bidhan and Chandra

together got. What is the ratio of Anand's share to that of Chandra's share?
(a) 1:2 (b) 3:5
(c) 4:7 (d) None of these

Q.14. Anil started a manufacturing unit with a certain amount of money. After a few months, Dheeraj became his partner, contributing three times of what Anil had contributed. At the end of the year, each was entitled to half the total profit. If Anil started the unit in January, then when did Dheeraj join as a partner?
(a) August (b) September
(c) July (d) October

Q.15. Pawan, Qureshi and Ravi entered into partnership, and provided capital of Rs 22,000, Rs 26,000 and Rs 34,000 respectively. Some months later Rs 10,000 extra capital was supplied by Qureshi. At the end of 12 months, the total profit was Rs 50,274, and Pawan's share was Rs 12,474. When did Qureshi supply the extra capital?
(a) After 4 months (b) After 5 months
(c) After 6 months (d) None of these

Q.16. A's salary is $\frac{5}{4}$ of B's salary, and B's salary is $\frac{4}{3}$ of C's salary, while M's salary is equal to the sum of salaries of A, B and C. If salary of C is Rs 1,500, What is the salary of M?
(a) 5,500 (b) 6,500
(c) 7,000 (d) 6,000

Q.17. Four years ago, the ratio of the ages of A and B was 13:9 and eight years hence it would be 4:3. The difference of their present ages is
(a) 56 years (b) 40 years
(c) 16 years (d) 24 years

Q.18. The work done by a man, a woman and a child is in the ratio of 3:2:1. There are 20 men, 30 women and 36 children in a factory. Their weekly wages amount to Rs 780, which is divided in the ratio of work done by the men, women and children. What will be the wages of 15 men, 21 women and 30 children for 2 weeks?
(a) Rs 585 (b) Rs 292.5
(c) Rs 1,170 (d) Rs 900

Q.19. On the eve of the foundation day party of Due North Inc., the ratio of males to females was 5:3. However, when 10 males left the party, this ratio got changed to 1:1. How many people were there originally at the party?
(a) 48 (b) 32 (c) 64 (d) 40

Q.20. What is the angle in degrees made by a sector, the ratio of whose area with the area of the semicircle is equal to 1:10?
(a) 36 (b) 18 (c) 24 (d) 9

Q.21. Two candles of the same length are lighted at 12 noon. The first is consumed in 6 hours and the second in 4 hours. Assuming that each candle burns at a constant rate, in how many hours after being lighted, was the first candle twice the length of the second?
(a) 3 p.m. (b) 2 p.m.
(c) 1.30 p.m. (d) 2.30 p.m.

Q.22. The value of a diamond varies directly as the square of its weight. If a diamond worth Rs 10,000 is divided into 2 pieces in the ratio of 4:6, what is the loss in value?
(a) 52% (b) 48%
(c) 36% (d) None of these

Q.23. The monthly income of A and B are in the ratio of 3:2, and their expenditures are in the ratio of 5:3. If each of them saves Rs 1,000 a month, then what is the income of A?
(a) Rs 4,000 (b) Rs 6,000
(c) Rs 9,000 (d) None of these

Q.24. If ab, bc, x and c^2 are in proportion, then find x, if none of these are equal to zero.
(a) a^2c (b) c^2a
(c) ac (d) None of these

Q.25. The first, second and third class fares between two railway stations, Patna and Lucknow were 10:8:3 and the number of first, second and third class passengers between the two stations was is 3:4:10. If total sales of the ticket is Rs 16,100, find the money obtained by the sales of second class tickets.
(a) Rs 5,250 (b) Rs 5,600
(c) Rs 6,400 (d) Rs 6,650

Q.26. If A:B = 3:4. What will be the value of $(3A^2 + 4B)/(3A - 4B^2)$?
(a) 43:55
(b) −43:55
(c) Either of (1) or (2)
(d) Cannot be determined

Q.27. The sum total of the salary of Atul, Binod and Charu is Rs 350. If they spend 60%, 80% and 75% of their respective salaries, then their savings are in the ratio of 8:8:5. Find the salary of Atul.
(a) Rs 50 (b) Rs 100
(c) Rs 200 (d) Rs 250

Q.28. Two horses cost as much as 5 dogs, 6 dogs as much as 8 oxen, 10 oxen as much as 50 sheep, 14 sheep as much as 9 goats. If the price of one goat is Rs 70, how much will one horse cost?
(a) Rs 670 (b) Rs 730
(c) Rs 830 (d) Rs 750

Q.29. A pond has a rare breed of lotus that triples in number every minute. If $\frac{1}{3}$rd of the pond is full of lotus in

30 min, what is the total time taken for the whole pond to be full?

(a) 1 min (b) 31 min
(c) 90 min (d) 60 min

Q.30. Satish used 6 litres of oil paint to paint a map of India, 6 m high. How many litres of paint would Santosh need for painting a proportionally scaled map of the height of 18 m?

(a) 54 (b) 18
(c) 30 (d) Cannot be determined

Q.31. A is proportional to B. B is inversely proportional to C. C is proportional to the square of D. D is directly proportional to the cube root of E. Assuming positive integers, if A increases then E

(a) Increases
(b) Decreases
(c) Cannot say
(d) Could increase or decrease

Q.32. Measurement of the temperature is carried out using thermometers in which the mercury expands linearly with the change in temperature. If the temperature range from the melting point of water to the boiling point of water is divided into 0 to 100 in Celsius scale and 32 to 212 in Fahrenheit scale, what is the temperature at which both the scales indicate the same value?

(a) 0 (b) 40 (c) 32 (d) – 40

Q.33. The sum of four numbers is 255. The ratio of the first number to the second number is 2:3. The ratio of the second number to the third number is 5:6. The ratio of the third number to the fourth number is 8:9. What is the average of the second number and the third number?

(a) 72 (b) 132
(c) 60 (d) None of these

Q.34. The population of the Balearic Island situated in Mediterranean Sea is 18,000. Balearic Island has three ports A, B and C. Every year, the entire population of each port moves to the other two ports, half going to one and the remaining half going to the other. The current populations of A, B and C are 2,000, 6,000 and 10,000 respectively. Then four years from now, the population of port A will be

(a) 5,000 (b) 6,500
(c) 6,000 (d) 5,750

Q.35. Arun has a certain amount of money in the denomination of 1 rupee and 10 rupee notes. The number of 1 rupee notes multiplied by the number of 10 rupee notes is equal to the total money (in rupees) that he has. What is the total number of ten rupee notes that he can have?

(a) 11 (b) 13
(c) 15 (d) None of these

Q.36. Divya and Nandan go to a bakery and buy some black currant pastries. Divya buys 9 black currants more than Nandan. Later in the day, they go to the bakery again and buy some butter scotch pastries. In the entire day, Divya and Nandan buy a total of 12 and 21 pastries respectively but spend the same amount of money. Find the ratio of the price of a black current to that of a butter scotch.

(a) 3:2 (b) 2:1 (c) 5:2 (d) 3:1

Q.37. Rohan purchases some pastries of two varieties— one variety costing Rs 3 per piece and the other Rs 7 per piece. He wanted to spend exactly 50 rupees for the same. What is the maximum number of pastries he can purchase of the costlier variety?

(a) 6 (b) 5 (c) 4 (d) 2

Q.38. Three friends A, B and C started a venture with capitals in the ratio of 4:1:15. At the end of every quarter A halves his capital, while B doubles his capital and C leaves his capital untouched. This process is repeated till the end of the year. If at the end of the year B's share of the profit is Rs 22,000, what is the total profit?

(a) Rs 88,000 (b) Rs 1,10,000
(c) Rs 1,21,000 (d) None of these

Q.39. Rakesh, Shyam, and Prashant have some money with them. If Rakesh gives Rs 15 to Shyam then he would have the same amount. If Prashant takes half of Shyam's money, he would have 5 times as much money left with Shyam. After this, the ratio of money with Rakesh, Shyam and Prashant is 5:2:4. What is the total amount with them?

(a) Rs 55 (b) Rs 110
(c) Rs 165 (d) Rs 231

Q.40. King Fahd of Saudi Arabia distributed his property equally among his 6 sons as follows— to his eldest son he gave 1 lakh gold coins and 1/7th of the remaining coins. To the second son, he gave 2 lakh gold coins and 1/7th of the remaining gold coins and so on and to his youngest son he gave 6 lakh gold coins. The entire number of gold coins was distributed without any remainder. What was the total number of gold coins with the king?

(a) 22 lakh (b) 50 lakh
(c) 71 lakh (d) 36 lakh

Q.41. Abhishek bought a total of 'N' dozen fruits consisting of apples and bananas in the ratio of 3:7. The number of dozen of apples he bought is equal to the cost per dozen of bananas and vice versa. Total amount spent by him is equal to Rs 1,050. Find the number of dozen of apples and the number of dozen of bananas he bought respectively.

(a) 12 dozen and 38 dozen
(b) 14 dozen and 36 dozen

(c) 15 dozen and 35 dozen
(d) 18 dozen and 32 dozen

Q.42. If $xy = r, yz = r^3, xz = r^2$, also $x + y + z = 13$ and $x^2 + y^2 + z^2 = 91$, then what is the value of $z:y$?
(a) 3 (b) $\frac{7}{3}$ (c) 13 (d) $\frac{13}{3}$

Q.43. Gandhiji owns cows, some black and some white. He finds that 4 black cows and 3 white cows gave the same amount of milk in 5 days as 3 black cows and 5 white cows gave in 4 days. What is the ratio of milk given by a black cow in a day to that given by a white cow in a day?
(a) 8 : 5 (b) 5 : 8 (c) 3 : 5 (d) 5 : 3

Q.44. Two software developers Vinit and Harpal share a mainframe and the final cost is Rs 74,750. Vinit uses 5 type I terminals for 7 weeks and 22 type II terminals for 4 weeks. Harpal uses 10 type I terminals for 6 week and 8 type II terminals for 6 weeks. If cost of operating each of type I and type II terminal is in a ratio of 2:3, find the share of Vinit and Harpal in the final cost.
(a) Rs 41,750, Rs 33,000
(b) Rs 40,000, Rs 34,000
(c) Rs 41,000, Rs 3,80,000
(d) Rs 50,000, Rs 24,750

Q.45. A metal trader buys 2 kinds of silver foils, the ratio of their prices being 1:4. He sells the alloy at Rs 90 per kg so that he can make a profit of 20%. If the ratio of their quantities present in a alloy is 6:1 respectively, find the purchase price of the foil present in lesser quantity.
(a) Rs 52.5 (b) Rs 55
(c) Rs 47.5 (d) Rs 45

Q.46. A heap of topaz, rubies and emeralds is required to be formed. A topaz is worth Rs 5 crores, a ruby Rs 2 crores and an emerald Rs 3 crores. The weight of a topaz is 0.7 kg, weight of a ruby is 0.2 kg and that of an emerald is 0.3 kg. If the total weight allowed is 8 kgs, then which combination maximizes the worth of the entire heap?
(a) 10 topaz, 2 rubies 2 emeralds
(b) 6 topaz, 1 ruby, 12 emeralds
(c) 11 topaz, 1 emerald
(d) 26 emeralds, 1 ruby

Direction for questions 47 and 48: Read the passage below and solve the questions based on it.

A book having pages between 4,000 and 5,000 is divided into four parts, each part being divided into chapters. The total number of pages in each of the four parts is the same. The ratio of the chapters across all the parts is 6:5:10:14. The number of chapters in the fourth part is 70.

Q.47. What is the total number of pages in the book?
(a) 4,000 (b) 4,800 (c) 4,200 (d) 4,600

Q.48. What is the total number of chapters in the book?
(a) 113 (b) 226
(c) 127 (d) None of these

Q.49. A rat takes 5 leaps for every 4 leaps of a cat, but 3 leaps of the cat are equal to 4 leaps of the rat. What is the ratio of the speed of the rat to that of the cat?
(a) 11 : 15 (b) 15 : 11 (c) 16 : 15 (d) 15 : 16

Q.50. The ratio of the age of Sita and her mother is 2:3. N years from now, ratio of their ages will become 3:5. What is the value of N?
(a) Infinite (b) 1
(c) More than 1 but finite (d) Not possible

MODERATE

Direction for questions 1 to 3: Read the passage below and solve the questions based on it.

Two of my friends Nishant and Rajesh, have the same number of children. Each son of Nishant has 5 times as many sisters as the number of brothers, while each son of Rajesh has as many sisters as the number of brothers. Each daughter of Nishant has twice the number of the brothers but each daughter of Rajesh has twice the number of brothers as the number of sisters.

Q.1. How many children did Rajesh have?
(a) 3 (b) 4 (c) 6 (d) 7

Q.2. How many sons does Nishant have?
(a) 1 (b) 2
(c) 4 (d) Cannot be determined

Q.3. What is the ratio of the number of sons to the number of daughters in the 2 families?
(a) 1:3 (b) 3:1
(c) 7:2 (d) None of these

Direction for questions 4 to 6: Read the passage below and solve the questions based on it.

Three persons, Anny, Berry and Cherry, started playing a game. They had money in the ratio of 3:2:1. It was decided that if Anny lost the game he would distribute 1/4th of the money that he had prior to the round, equally between Berry and Cherry. If Berry lost the game he would distribute half of his money equally between Anny and Cherry and if Cherry lost the game he would distribute 2/3rd of his money prior to that round equally between Anny and Berry. Berry lost the game first, and then Cherry lost the next game followed by Anny and then followed by Berry again. In the process Anny gained Rs 10.

Q.4. What was the total sum of money they had initially?
(a) Rs 60 (b) Rs 120
(c) Rs 150 (d) Rs 240

Q.5. After which round the game would have been stopped for Anny to gain the maximum profit?
(a) Round 1 (b) Round 2
(c) Round 3 (d) Round 4

Q.6. Which of these did not have any chance of making profit in any of the rounds?
(a) Both Berry and Cherry
(b) Both Anny and Cherry
(c) Only Cherry
(d) Only Berry

Direction for questions 7 to 9: Read the passage below and solve the questions based on it.

Anu, Bunty, Christy and Dorthy are 4 salesmen. In the 1st year they received a commission of Rs 4,200 from their company and divided it in the ratio of 2 : 3 : 4 : 5. In the 2nd year the commission doubled, the amount was divided in the ratio of 3 : 4 : 5 : 2. In the 3rd year the commission was tripled when compared to the first year and they shared in the ratio of 4 : 5 : 3 : 2 and in the 4th year the commission was half compared to the previous year and they shared in the ratio of 4 : 3 : 5 : 2.

Q.7. What was the average yearly income of Christie over the period?
(a) Rs 2,287.5 (b) Rs 2,252.5
(c) Rs 1,680.25 (d) None of these

Q.8. How much should Bunty have earned so that his average yearly income would have been Rs 3,500?
(a) 2,450 (b) 3,250
(c) 4,850 (d) 5,750

Q.9. The ratio of the rate of tomatoes to the rate of brinjals is
(a) 1:2 (b) 1:3
(c) 1:5 (d) None of these

Direction for questions 10 and 11: Read the passage below and solve the questions based on it.

Narendrajee is the office assistant at LC's North Campus centre. One day he realized that (i) among all the visitors the ratio of the number of men and women was the same as that between women and children, (ii) maximum number of the visitors to the centre were children. At the end of the day, he finds that 7 women visited the centre.

Q.10. How many children visited the centre that day?
(a) 44 (b) 49 (c) 52 (d) 57

Q.11. How many men visited the centre that day?
(a) 25 (b) 68
(c) 49 (d) None of these

Q.12. P works twice as fast as Q, whereas P and Q together can work three times as fast as R. If P, Q and R together work on a job, in what ratio should they share the earnings?
(a) 2:1:1 (b) 4:2:1
(c) 4:3:2 (d) 4:2:3

Direction for questions 13 to 15: Read the passage below and solve the questions based on it.

Sachin has 3 children Ten, Dul and Kar. One day he brought 117 sweets for his children and planned to divide the sweets in such a manner that two times the number of sweets that Ten would have got should be equal to 2/3 times that Dul got and 4 times that Kar got. Since Sachin was poor at counting, he divided the sweets in the ratio of 2:3:4.

Q.13. How many sweets did Kar get as a result of wrong sharing?
(a) 16 (b) 20
(c) 22 (d) None of these

Q.14. If Kar were to get the same number of sweets that Dul got now after the wrong distribution had the distribution being done correctly, how much should the total number of sweets be?
(a) 150 (b) 169
(c) 216 (d) None of these

Q.15. If X bulbs consume X^2 units of electricity in X^3 days, find the time in which Y bulbs will consume Y units of electricity.
(a) X^1 days (b) X^2 days
(c) X^3 days (d) X^0 days

Direction for questions 16 and 17: Read the passage below and solve the questions based on it.

There are certain number of apples, guavas and oranges in a basket. The number of each variety is more than one. The ratio of the number of apples to the number of guavas is equal to the ratio of the number of guavas to the number of oranges.

Q.16. If the total number of fruits is 61, then find the number of guavas.

(a) 16 (b) 20
(c) 25 (d) Cannot be determined

Q.17. If the number of guavas is 21, then which of the following can be the total number of fruits in the basket?
(a) 63 (b) 89 (c) 101 (d) 117

Q.18. The time taken by a jet, to reach a certain place while flying at a speed, slower by 300 m/h than the normal, is double the time taken by it while flying at a speed faster by 300 m/h than normal. What is the ratio of the 'delay time' while flying slower to the 'early time' while flying faster?
(a) 3:2 (b) 1:2
(c) 2:1 (d) Cannot be determined

Q.19. Anand and Bhavana have a few one rupee, 50 paise and 25 paise coins each. The total amount with each of them is Rs 5. The number of coins of two of the three types of coins that Anand has are equal and same is the case for the coins that Bhavana has. If Anand has the maximum possible total number of coins, then find the maximum possible total number of coins that Bhavana can have given that she has fewer coins than Anand.
(a) 16 (b) 12 (c) 10 (d) 9

Q.20. In the above question, what is the number of 50 paise coins with Anand?
(a) 1 (b) 2 (c) 3 (d) 4

Q.21. In a flood-affected village called "Waterland", there are 500 adult residents. They planned to construct a dam to overcome the situation. Each of the men contributed a sum equal to the number of women in the village and each woman a sum equal to the number of men. The village-chief came to know about this and contributed an amount equal to the contribution of the villagers. The total contribution then became Rs 2.4 lakhs. Find the number of men in "Waterland".
(a) 250 (b) 300
(c) 350 (d) Cannot be determined

Q.22. If the man initially had Rs 82, instead of Rs 100, when he went to the market, then the ratio of the rate of tomatoes to the rate of cucumber was
(a) 19:4 (b) 2:3
(c) 4:19 (d) None of these

Q.23. The sides of a right angled triangle are a, $a + x$ and $a + 2x$ with a and x both positive. The ratio of a to x is
(a) 3:1 (b) 5:2 (c) 2:1 (d) 3:4

Q.24. If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}$, then each fraction is equal to
i. −1 ii. 1/2 iii. 1
(a) i and ii only (b) ii and iii only
(c) i, ii and iii only (d) None of these

Direction for questions 25 and 26: Read the passage below and solve the questions based on it.

Ujala Jewellers is a well known diamond jewellery shop. Owner of the shop has priced his diamond necklaces such that the cost varied as the square of the number of diamonds, for example the necklace placed at the first right top corner has 8 diamonds and costs Rs 1,60,000. There are lots of necklaces and the price of all necklaces have been found by the same method.

Q.25. What would be the price of the bracelet with 10 diamonds?
(a) 1,75,000 (b) 2,25,000
(c) 2,50,000 (d) 4,00,000

Q.26. Vijay Shanker asked for diamonds in a necklace to be divided in the ratio of 5:3:2. Owner of Ujala Jewellers knew that he would suffer some loss. What is the % decrease in the price that he would incur, if he has to oblige Vijay Shanker?
(a) 22% (b) 38% (c) 62% (d) 78%

Q.27. Between two stations Kanpur and Mumbai the first, second and third class fares were fixed in the ratio of 8:6:3. However, when Mr L Prasad, the railway minister, presented the railway budget, he reduced the first class fares by 1/6 and the second class by 1/12 in order to rationalize the fares vis-à-vis the second-class fares. In a year, ratio of the number of first, second and third class passengers were respectively 9:12:26 and the money at the booking offices was Rs 1,08,800. How much was paid by the first class passengers?
(a) Rs 32,080 (b) Rs 16,800
(c) Rs 24,400 (d) None of these

Direction for questions 28 and 29: Read the passage below and solve the questions based on it.

In the famous Hawthorne studies by Elton Mayo, employees of the organization were given the options of going to several places for entertainment. One-seventh of the employees preferred to go to the polo ground, a number equal to the square root of the remaining employees went to the museum. One-fifth of the remaining employees went to the fair. One-third of the remaining to the science exhibition and the rest to watch movie ASHOKA, which was running house full in the nearby theatre. The total number of employees who went to the polo ground and the museum is twice that of the employees who went to the fair.

Q.28. What is the strength of the organization?
(a) 14 (b) 35 (c) 42 (d) 49

Q.29. The entry tickets for museum, fair, exhibition and the movie were Rs 5, Rs 4, Rs 2 and Rs 25 respectively. How much money did the employees pay for all these on that Sunday?

(a) Rs 275 (b) Rs 320
(c) Rs 388 (d) Rs 470

Direction for questions 30 and 31: Read the passage given and solve the questions based on it.

In the last year's "Tycoon Contest", nine teams from different colleges participated. Navneet Anand, the quiz master, asked the nine teams— "I have 9 coins, the ratio of the weight of the coins are same, except one. I give the different numbers to these 9 coins starting from 11 to 19. The team who can find the coin with different weights in the least number of weighings can take that coin and will be the winner."

Q.30. Team number 5 said that they knew the answer and if they had won the coin, what was their answer?
(a) One weighing (b) Two weighings
(c) Three weighings (d) Four weighings

Q.31. Navneet Anand now said that the coin whose weight was different from the rest was the heaviest of the all. What is the minimum number of weighings required to find that coin? Contestant 8 gives the correct answer and his answer is
(a) One weighing (b) Two weighings
(c) Three weighings (d) Four weighings

Q.32. The cost of building a wall is Rs 1,347. Wages of workmen is increased by 1/8 of the former wages and working hours per day have been increased by 1/20 of the former duration. What is the new cost (approximately) of building a wall, the length of which is two times the length of this wall and the other dimensions of this wall are same?
(a) Rs 2,692 (b) Rs 2,724
(c) Rs 2,886 (d) Rs 2,484

Q.33. If $a^2 = by + cz$, $b^2 = cz + ax$, $c^2 = ax + by$, then $\frac{x}{a+x} + \frac{y}{b+y} + \frac{z}{c+z} = ?$
(a) abc (b) 1 (c) $\frac{abc}{xyz}$ (d) $\frac{xyz}{abc}$

Q.34. If 29 goats can feed on a field of uniformly grown grass in 7 days or 25 goats can feed on the same field in 9 days, how many goats will feed in 6 days?
(a) 36 (b) 44
(c) 42 (d) 32

Q.35. Given, $x:y = a:b = 1:5$. Then, the value of

$\frac{x^2 + a^2}{y^2 + b^2}$ is

(a) $\frac{1}{5}$ (b) $\frac{1}{50}$

(c) $\frac{1}{250}$ (d) None of these

Q.36. a, b and c are three positive numbers and $s = \frac{(a+b+c)}{2}$.
If $(s-a):(s-b):(s-c) = 1:7:4$, the ratio of $a:b:c =$
(a) 8 : 10 : 6 (b) 9 : 4 : 12
(c) 7 : 8 : 9 (d) 11 : 5 : 8

Q.37. In an election, each voter may vote for two candidates. Half of the voters vote for A, but divide their votes between B, C and D in the ratio of 3:2:1. Of the remaining, half vote for B and divides their votes between C and D in the ratio of 2:1. Of the remaining, half vote for C and D and the remaining 840 do not vote for anyone. How many votes were received by A, B, C and D respectively?
(a) 3,360, 3,360, 3,080, 1,960
(b) 3,360, 3,360, 1,960, 3,080
(c) 3,450, 3,210, 4,120, 1,210
(d) 5,000, 5,000, 2,500, 1,250.

Direction for questions 38 and 39: Read the passage below and solve the questions based on it.

Two friends Pankaj and Pranav decide to play a game where the person losing that round gives half of his money to the one who wins. They start playing with Rs 100 each.

Q.38. At the end of 3 rounds which of the following cannot be the amount that Pankaj has?
(a) Rs 137.5 (b) Rs 162.5
(c) Rs 112.5 (d) Rs 125

Q.39. What is the maximum amount that changes hands in the game in 3 rounds?
(a) Rs 100 (b) Rs 75
(c) Rs 11.5 (d) Rs 87.5

Q.40. The square of the time taken by a planet to complete one revolution around the sun varies as the cube of its mean distance from the sun. If the ratio of the mean distances of Mars and Saturn from the sun is 16:3. Find the number of days on Saturn that are equal to a year on Mars.
(a) 4,496 days (b) 4,294 days
(c) 5,598 days (d) 4,696 days

Q.41. In a forest reserve, the ratio of the number of deers, bear and fox is 3:7:5. If the difference between the number of deers and bears is a multiple of 3 as well as 7, what is the minimum number of animals in the park?
(a) 315 (b) 310
(c) 45 (d) Cannot be determined

Q.42. A watermelon is cut into two pieces in the ratio of 3:5 by weight. The bigger of the two is further cut in the ratio of 5:7 by weight. Find the ratio of each of the three pieces.

(a) 3:5:7 (b) 36:25:35
(c) 15:25:56 (d) None of these

Q.43. Shalaj and Associates, a shoe-manufacturing company, has three machines producing shoes. If machine X produces 1/4th as many as machine Y produces in the same time, and machine Y produces twice as many as machine Z produces in the same time, then during a fixed time, what fraction of the total number of shoes is produced by machine Z?

(a) $\frac{1}{14}$ (b) $\frac{1}{3}$
(c) $\frac{2}{7}$ (d) $\frac{1}{2}$

Direction for questions 44 and 45: Read the passage below and solve the questions based on it.

A man went to the market with Rs 100 in his pocket. He bought 3 kgs of brinjals, *y* kgs of tomatoes and 4 kgs of cucumber. The amount spent on tomatoes was double than that spent on cucumbers and also is same as one third of that spent on brinjals. After the shopping the man was left with Rs 10 in his pocket.

Q.44. Which of the following statements is true?
(a) The ratio of the rate of tomatoes to the rate of brinjals is 1:4
(b) The ratio of the rate of tomatoes to the rate of cucumber is 1:8
(c) The ratio of the rate of cucumber to the rate of brinjals is 8:1
(d) None of these

Q.45. If the man was left with Rs 19 instead of Rs 10 and the total weight of vegetables bought is 10 kgs then, which of the following statement is true?
(a) The value of *y* is 4 kg.
(b) The value of *y* is one more than the rate of tomatoes.
(c) The value of *y* is one-sixth of the amount spent on tomatoes.
(d) None of these

ADVANCED

Q.1. The Road Transport Corporation has hired a consultancy firm to determine the standard time that should be taken for a journey between two local destinations. The consultant after a detailed study and discussion submitted in his report that the time of journey is proportional to the product of the square root of number of passengers and the distance to be covered. For two distinct trips, the times taken are in the ratio of 2 : 1 and the distances covered are 80 km and 30 km respectively. How many passengers are there in the first bus, if the second bus has 64 passengers?
(a) 9 (b) 16 (c) 36 (d) 24

Q.2. The inhabitants of Hawaii Island are taxed in a peculiar manner. They have to pay a fixed sum irrespective of their income strata. In addition to this they have to pay an amount which is proportional to the excess of their salary over Rs 50,000. Mr Kalyan pays a total tax of Rs 6,200 when his annual salary is Rs 6,0000 and his wife Mrs. Kalyan pays a total tax of Rs 7,700 when her annual salary is Rs 75,000. What is the yearly salary of Mr Ajay who pays an annual tax of Rs 8,200?
(a) Rs 80,000 (b) Rs 82,000
(c) Rs 90,000 (d) None of these

Q.3. Rohit finds that the distance he covers is equal to the sum of two quantities, one of which varies with time 'T' and the second varies with the square of the time 't' that has elapsed from the start. After 5 seconds Rohit has covered a distance of 100 metres and after 6 seconds, he has covered a distance of 138 metres. How far is he from the starting point after a time of 20 seconds?
(a) 450 m (b) 880 m
(c) 1,220 m (d) None of these

Q.4. While developing Tejas, the Light Combat Aircraft developed by DRDO to replace MIG series aircraft, it was found that the distance a man falls under gravity is $5t^2$ m in t s and the distance a man falls with an open parachute under gravity is $2t^2 - 3t$ m in t s. Squadron leader Ajay Rathod jumps out of a Tejas at a height of 2,300 m with a closed parachute. Find the height at which he can open the parachute so as to touch the ground in exactly 40 s.
(a) 1,050 m (b) 1,775 m
(c) 1,175 m (d) 1,250 m

Q.5. A manufacturing unit is to be set up at Geneva. The cost of setting up a unit is proportional to the square of the raw material it handles. If for all the processes four separate units are deployed, then the raw material that the individual unit is going to handle is in the ratio of 6:3:3:1. But if a single unit is deployed, then the total raw material that is to be handled is summation of raw material that is to be handled by an individual unit. By installing separate units, the company saved Rs 57 lakh. What is the cost of installing a single unit?

(a) Rs 84.5 lakh (b) Rs 285 lakh
(c) Rs 338 lakh (d) None of these

Direction for questions 6 and 7: Read the passage given and solve the questions based on it.

At 70° F, 100 litres of milk can dissolve a maximum of 1.5 kg of sugar. For every increase in temperature by 1° F, the solubility of milk increases by 0.1 litre. For example, at 71° F, 100 litre of milk can dissolve 1.6 kg of sugar. The boiling point of milk is 100° F i.e., when the temperature of milk reaches 100° F, it stops increasing further and the milk starts evaporating @ 200 gm/min.

Q.6. If 20 litres of milk at 80° F with 180 gm sugar is heated till it starts to evaporate, then what is the maximum amount of sugar that can be added so that it does not sediment?

(a) 900 g (b) 520 g (c) 720 g (d) 770 g

Q.7. At 75° F, 75 litres of milk with 1 kg of sugar is heated till it starts to evaporate, and appropriate quantity of sugar is added so that it becomes fully saturated. Thereafter, it is heated for 10 min. How much sugar would have deposited at the bottom after 10 min?

(a) 120 g (b) 90 g (c) 100 g (d) 60 g

Q.8. Pranav, Raj and Santosh entered into a partnership with a sum of Rs 6,200, Rs 9,300 and Rs 12,400 respectively. It was decided that Pranav will manage the business for the whole year. But after some months due to some personal problems Pranav handed over his responsibility of managing the business to Raj for the rest of the year. The person who manages the business gets 10% of the profit as commission. If the total income of Pranav at the end of the year is Rs 290 and the total profit is Rs 1,200, for how many months did Pranav manage the business?

(a) 2 (b) 4
(c) 6 (d) None of these

Q.9. The total profit made by running a train between New Delhi to Lucknow consists of two parts : (i) a constant amount of Rs 24 lakh per trip and (ii) it varies as the square of the number of coaches attached to the engine that amounts to Rs $7P^2$, where P is the number of coaches attached to the engine in the trip. If the average profit per coach per trip should not fall below Rs 169 lakh, then what is the minimum number of coaches that have to be attached to the engine?

(a) 23 (b) 24
(c) 25 (d) None of these

Direction for questions 10 and 11: Answer the questions based on the following information.

A simple weighing balance having two pans is in perfect balance when no weights are kept in the two pans even though both the pans are not of the same weight. Further even when 10 kg is kept in the first pan and 5 kg is kept in the second pan, it remains in balance.

Q.10. What is the ratio of the weight of the first pan to the second pan?

(a) 1:2 (b) 2:1
(c) 3:1 (d) Cannot be determined

Q.11. What weight should be kept in the second pan if a weight of 5 kg is kept in the first pan so that the weighing scale is in balance?

(a) 2.5 kg (b) 1.25 kg
(c) 7.5 kg (d) None of these

Q.12. There are two alloys of gold and copper. In the first alloy there is twice as much gold as copper and in the second alloy there is 5 times less gold than copper. How many times of the second alloy should be taken than the first, in order to obtain a new alloy in which there would be twice as much copper as gold?

(a) Two times (b) Three Times
(c) Four Times (d) None of these

Q.13. The average age of the students in a class of 50 is 13. The weight of each student is directly proportional to the height. A 165 cm tall student has a weight of 33 kg. The average weight of the class is:

(a) 33 kg (b) 39 kg
(c) 36 kg (d) Data insufficient

Q.14. Nishit Transport Corporation (NTC) operates buses between Kolkata and Patna. To increase the profit, NTC has sought the services of Dipanjan Das and Sons consultants. After a thorough study of the mechanism of the whole business, Dipanjan Das and Sons suggested that— (i) profit in running a bus is directly proportional to the distance when the number of passengers is constant, and (ii) directly proportional to the number of passengers in excess of a certain fixed number when the distance is constant. The profit is Rs 80 when 30 passengers are carried over a distance of 40 km, and Rs 180 when 35 passengers are carried over 60 km. What should be the advice, furnished by Dipanjan Das and Sons consultants, regarding the minimum number of passengers to be carried, so that there is no loss?

(a) 19 (b) 18 (c) 20 (d) 25

Q.15. During the acceleration and breaking test by two men for a new bike launched by Kajaj, the 1st man starts the scooter and accelerates upto 40 km/h for a distance of 30.2 m and drives the remaining distance at 40 km/h. Second man starts and accelerates upto 60 km/h for a distance of 80.9 m and travels with a speed of 60 km/h and then applies brakes to 0 km/h at the distance of 24.2 m. The total distance travelled is 200 m. If both of them started at the same time, then who will reach the 200 m mark first?

(a) Ist man (b) 2nd
(c) Same time (d) Data insufficient

Q.16. Two alloys of chromium have different percentage of chromium in them. The first one weighs 6 kg and second one weighs 12 kg. One piece each of equal weight was cut off from both the alloys and the first piece was alloyed with the second alloy and the second piece was alloyed with the first one. As a result, the percentage of chromium became the same in the resulting two new alloys. What was the weight of each cut-off piece?

(a) 4 kg (b) 2 kg (c) 3 kg (d) 1 kg

Q.17. The cost C of manufacturing a brick can be estimated by the formula $C = 0.03\ ABt^2$, where A and B are the amounts in Rs of the two major ingredients sand and clay and t is the production time in hours. If A is increased by 50%, B is increased by 20% and t is decreased by 30% by what percentage will the estimated cost of manufacturing the brick change?

(a) 12.4% (b) 11.8%
(c) 14.6% (d) 18.2%

Q.18. During the bank wiring experiment to understand the impact of pattern of informal relationship among employees in any organization, it is found that the quantity of work done by a man in an hour is directly proportional to his pay per hour and inversely as the square root of the number of hours he works per day. He can finish a piece of work in six days when he works 9 hours a day at Re 1 per hour. How many days will he take to finish the same work when working 16 hours a day at the rate of Rs 1.50 per hour?

(a) 10 days (b) 3 days
(c) 8 days (d) 4 days

Q.19. According to the personnel administration prevalent in India, people are awarded pensions in proportion to the square root of the number of years they have served. One has served 9 years longer than the other and receives a pension greater by Rs 250. If the length of service of the first had exceeded that of the second by $4\frac{1}{4}$ years their pensions would have been in the ratio of 9:8. How long have they served respectively?

(a) 16 years and 25 years
(b) 9 years and 16 years
(c) 9 years and 25 years
(d) None of these

Q.20. The speed of man while walking, running and driving are in the ratio of 1:3:9. If he travels equal distance while walking, running and driving, what is the ratio of $\frac{\text{time taken}}{\text{velocity}}$ in the above three cases?

(a) 9 : 3 : 1 (b) 1 : 1 : 1
(c) 81 : 9 : 1 (d) 27 : 9 : 1

Q.21. Given that $2^x = 4^y = 8^z$ and $xyz = 288$. What is the value of $\frac{1}{2x}+\frac{1}{4y}+\frac{1}{8z}$?

(a) $\frac{18}{74}$ (b) $\frac{18}{96}$ (c) $\frac{11}{74}$ (d) $\frac{11}{96}$

Q.22. According to census 2001 of India, population of 5 states: state 1, state 2, state 3, state 4, state 5, showed that their population figures are in the ratio of 1 : 3 : 2 : 3 : 2. Further, the ratio of the percentage of women in the population in these states is 1 : 3 : 2 : 5 : 4. If the total population of women in the five states is 3.7 million, then what is the excess number of women in state 2 over that of state 5?

(a) 10 lakh (b) 1 lakh
(c) 2.2 lakh (d) Cannot be determined

Q.23. The time period of oscillation T for a pendulum is given by $T = k\sqrt{\frac{\ell}{g}}$, where ℓ is the length of the pendulum and g is the acceleration due to gravity, 'k' is any const. If 'k' and 'g' remain unchanged under any condition, what should be the percentage change in ℓ such that the time period T increases by 10%?

(a) 10% increase (b) 10% decrease
(c) 20% increase (d) 21% increase

Q.24. A tap A can fill an acid into a vessel in 30 min. Another tap B can fill alkali into a vessel in 20 min and a leak C at the bottom of the vessel can empty the vessel in 12 min. Initially, both the taps A and B are opened for 6 min. Then tap B is closed and leak C is opened. For how many more minutes should both these taps A and C be kept opened so that the proportion of acid and alkali is same? (Assume tap C removes acid and alkali in exactly the same ratio as they were present in the solution when tap C was just opened and no reaction takes place).

(a) 1 min (b) 2 min
(c) 3 min (d) 4 min

Q.25. If after the operation of question 24 is performed (till the ratio is 1 : 1), tap C is closed and B is opened again and then taps A and B are kept open till the vessel is full, what will the ratio of acid and alkali be in the full vessel?

(a) 19:11 (b) 11:19 (c) 14:11 (d) 11:14

Direction for questions 26 to 29: Read the passage below and solve the questions based on it.

On 13th of November, I kept some money in a magical purse in which on every alternate day starting from the 13th, money increases with respect to the closing amount on the evening of the previous day, that the ratio of money becomes 2:1 but on every alternate day starting from the 14th, the money decreases by Rs 10, when compared to the closing amount on the evening of the previous day. I had Rs 500 on the 19th and the magical purse kept on multiplying money for me till the 25th of that November.

Q.26. What is the initial amount of money that I kept into the purse?
(a) 50 (b) 80
(c) 140 (d) Cannot be determined

Q.27. How much money was there on the 22nd of November?
(a) 440 (b) 360
(c) 240 (d) None of these

Q.28. Had I stopped on the 21st, how much less money would I have received with respect to the money which I would have obtained on the 25th?
(a) 2,880 (b) 3,260
(c) 4,270 (d) None of these

Q.29. Had I started with Rs 100, how much more/less money would I have received than I am receiving now?
(a) 400 (b) 760 (c) 1,280 (d) 1,640

Q.30. The square of the ratio of the number of sweets with Aldrin's sons and daughters is equal to the cube of the ratio of the number of sweets with Bladimir's sons and daughters. The total number of chocolates with all the children of Aldrin and Bladimir is a prime number The number of sweets with Bladimir's sons equals the number of sweets with Aldrin's daughters. The number of sweets with Bladimir's daughters is 1/32 of the number of sweets with Aldrin's sons. What is the total number of sweets with all the children?
(a) 31 (b) 41 (c) 61 (d) 101

Direction for questions 31 and 33: Read the passage below and solve the questions based on it.

Surya Pratap Singh has 4 children (two daughters who are older to the two sons). He decides to divide his property among his children such that both the daughters together get the same share as the two sons together. Elder son gets more than the younger son and the elder daughter gets more than the younger daughter. His entire property consists of cash, gold coin and copper plate. Each gold coin is worth Rs 5,000 and copper plate in worth Rs 1,000. The eldest daughter got 1/3rd of the gold and 1/5th of the copper while elder son got Rs 1,50,000 in cash, 50% of his wealth as gold and 250 copper plates. The younger daughter got 18% of the copper and 15% of the cash. The younger son got cash equal to 90% of the cash the elder daughter got and 1/6th of the gold and 12% of the copper. Surya Pratap Singh has gold worth 3 times that of copper.

Q.31. Who got the maximum wealth?
(a) Elder son (b) Elder daughter
(c) Younger son (d) Both (1) and (2)

Q.32. How much cash did the younger daughter get?
(a) Rs 50,000
(b) Rs 60,000
(c) Rs 75,000
(d) Cannot be determined

Q.33. If the younger son wants to sell all the copper he has, who is capable of buying it by paying cash?
(a) Elder son (b) Elder daughter
(c) Younger son (d) All the three

Q.34. Two items having the same cost price and marked price are given two different discounts, which differ by 20 percentage points. If their profits differ by 32 percentage points, what is the ratio of their cost price to the marked price?
(a) 5:8 (b) 4:7
(c) 2:3 (d) Cannot be determined

Q.35. A, B and C run around a circular track of the length 300 m. A and B run with speeds of 10 m/s and 12 m/s in the same direction respectively and C runs in the opposite direction with a speed of 15 m/s. If all the three start at the same time, which of the following is true?
(a) For a given period of time, the ratio of the number of meeting points of A and C to the number of meeting points of B and C is more than 1.
(b) For a given period of time, the ratio of the number of meeting points of B and C to the number of meeting points of A and C is more than 1.
(c) For a given period of time, the ratio of the number of meeting points of A and C to the number of meeting points of B and C is equal to 1.
(d) None of the above can be concluded.

Direction for questions 36 to 38: Read the passage below and solve the questions based on it.

In a class, every student plays exactly one of the following games—squash, table tennis, hockey, handball, cricket. Exactly 1/12th of the girls and 1/8th of the boys play squash, 1/15th of the girls and 1/6th of the boys play hockey, 1/4th of the girls and 1/12th of the boys play handball, 1/5th of the girls and 3/8th of the boys plays table tennis and the remaining boys and girls play cricket. The total strength of the class is less than 120.

Q.36. How many girls are there in the class?
(a) 60 (b) 48
(c) 90 (d) Cannot be determined

Q.37. Which of the following cannot be the ratio of boys and girls in the class?
(a) 2:5 (b) 4:5
(c) 3:4 (d) Cannot be determined

Q.38. What is the total strength of the class?
(a) 84 (b) 108
(c) 132 (d) Either (1) or (2)

Direction for questions 39 and 40: Read the passage below and solve the questions based on it.

There are four friends Sharat, Chandra, Mayank and Sid living in the same village of Bhagalpur. Once Sharat had nothing to eat, so, Chandra, Mayank and Sid collected their rotis and gave a part of their rotis to Sharat. It was later found that the number of rotis given by Chandra is one more than the rotis given by Mayank, and the number of rotis given by Mayank is one more than the rotis given by Sid. Now Sharat was required to pay the cost of rotis in the ratio of the rotis contributed by all three of them. It was found that Chandra should get twice as much money as Mayank. Price of one roti = Rs 5.

(All the values, number of rotis and money are in integers.)

Q.39. How much money did Chandra get?
(a) Rs 8 (b) Rs 12
(c) Rs 16 (d) None of these

Q.40. How many rotis were consumed by all of them?
(a) 6 (b) 9
(c) 12 (d) 15

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Answer	(d)	(a)	(d)	(b)	(b)	(c)	(b)	(d)	(c)	(c)	(b)	(a)	(c)	(a)	(b)	(b)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(c)	(c)	(c)	(d)	(b)	(d)	(b)	(d)	(a)	(d)	(d)	(d)	(b)	(d)	(d)	(c)	(c)	(d)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(b)	(b)	(c)	(b)	(d)	(a)	(d)	(b)	(a)	(b)	(d)	(d)	(d)	(d)	(b)	(b)	(d)	(b)	(d)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(c)	(a)	(b)	(a)	(a)	(d)	(c)	(d)	(d)	(d)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(d)	(b)	(b)	(d)	(d)	(c)	(d)	(b)	(d)	(a)	(d)	(d)	(c)	(b)	(a)	(c)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(a)	(a)	(a)	(c)	(b)	(d)	(c)	(d)	(c)	(b)	(c)	(b)	(d)	(d)	(d)	(a)	(d)	(d)	(a)
Q. No.	41	42	43	44	45															
Answer	(a)	(b)	(c)	(d)	(c)															

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(a)	(d)	(c)	(d)	(c)	(b)	(d)	(b)	(b)	(a)	(a)	(b)	(c)	(b)	(a)	(b)	(b)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(b)	(d)	(b)	(d)	(b)	(d)	(a)	(c)	(b)	(a)	(b)	(d)	(a)	(b)	(a)	(d)	(d)	(d)	(c)

HINTS AND SOLUTIONS

Warm up

1. Required ratio = 1/2 : 1/3 : 1/4
   To find the simple ratio, we will multiply each of these by the LCM of 2, 3 and 4.
   = 12 (1/2 : 1/3 : 1/4) = 6 : 4 : 3
2. Let the nos. be $5x$ and $3x$.
   then, $5x - 3x = 10$
   $2x = 10, x = 5$
   Required product = $5x \times 3x = 5 \times 5 \times 3 \times 5 = 375$
3. No. of oxen work time
   30 1/7 4
   18 $\frac{6}{7}$ x
   Applying the variation, we get
   $30 \times 4 \times 7 = \frac{18 \times x \times 7}{6}$ $x = 40$ hrs.
4. Let the number be x, $3x$, $4x$ and $7x$.
   then, $x + 3x + 4x + 7x = 75$
   $15x = 75 \quad \Rightarrow x = 5$
   Required value = $7 \times 5 = 35$
5. Required difference = 35 – 5 = 30
6. Go through options. (c) is the answer.
7. p = pressure, v = volume
   $p = \frac{k}{v}$ k = constant term
   k = pv
   p is reduced by 20% i.e., by $\frac{1}{5}$
   then V is increased $\frac{1}{4}$ i.e., 25%
8. $\frac{27-x}{35-x} = \frac{2}{3}$
   $81 - 3x = 70 - 2x, x = 11$
9. $p \propto q \Rightarrow p = k\, q$
   $15 = k \times 7,\ k = \frac{15}{7}$
   P = kq = 14 × 15/7 = 30
10. Go through options
11. Go through options
12. $\frac{5p-3q}{7p+2q} = \frac{5.\frac{3}{4}q - 3q}{7.\frac{3}{4}q + 2q} = \frac{15q-12q}{21q+8q}$ = 3/29
13. Volume of milk in mixture = $\frac{7}{10} \times 30 = 21\ l$
    Volume of water = 9 L
    Let x litres of water is added
    $\frac{21}{9+x} = \frac{1}{2}$
    $42 = 9 + x$
    $x = 33$
14. Ratio of their investment = 12000 : 18000 = 2: 3
    A's share in profit = (2/5) × 2000 = Rs 800
15. Suppose B invested for n months
    Hence, $\frac{12 \times x}{n \times x} = \frac{3}{2}$, x = their investment
    $\frac{12}{n} = \frac{3}{2},\quad n = 8$
16. Let B invested for n months
    then, $\frac{12000 \times 12}{16000 \times n} = \frac{3}{1}$
    $n = 3$

Foundation

1. Let x gold coins were given to 1st queen. Then no. of gold coins given to 2nd queen = $\frac{5}{7}x$
   And no. of gold coins given to 3rd queen $= \frac{3}{7}x$
   then, $x - \frac{3}{7}x = 60 \Rightarrow \frac{4x}{7} = 60$
   $x = 105$
   Required no. = $105 + \frac{5}{7} \times 105 + \frac{3}{7} \times 105$ = 225

2. Let their ages 5 years ago be $2x$, $3x$, $7x$ and $8x$.
Their ages now $2x + 5$, $3x + 5$, $7x + 5$, $8x + 5$.
Or, $20x + 20 = 140$
$\Rightarrow 20x = 120 \Rightarrow x = 6$
Present age of Nishu = $2 \times 6 + 5 = 17$ years
Present age of mother = $7 \times 6 + 5 = 47$ years
Hence, required years (47 –17) years = 30 years

3. Let x = total no. of workers

No. of women workers = $\frac{1}{3}x$

No. of men workers = $\frac{2}{3}x$

No. of married women = $\frac{1}{6}x$

No. of women having children = $\frac{1}{3}.\frac{1}{6}x = \frac{1}{18}x$

No. of women without children = $5x/18$

No. of married men = $\frac{3}{4}.\frac{2}{3}x = \frac{1}{2}x$

No. of men having children = $x/3$

No. of men without children = $\left(\frac{2}{3}x - \frac{1}{3}x\right) = \frac{x}{3}$

Required fraction = $\dfrac{\frac{5x}{18} + x/3}{x} = \dfrac{\frac{5x+6x}{18}}{4} = \frac{11}{18}$

4. Let salary of Anil = Rs x

Bhuvan's salary $= \frac{2}{5}x$

Chandra's Salary = $\frac{7}{9}.\frac{2}{5}x = \frac{14x}{45}$

$x + \frac{2}{5}x + \frac{14x}{45} = 770$

$\Rightarrow \frac{77x}{45} = 770$

$x = 450$

5. Let his salary be Rs x

Money spent on house rent = $\frac{1}{4}x$

Money spent on food = $\frac{1}{3}x$

Money spent on travel = $\frac{1}{6}x$

Money spent = $\frac{1}{4}x + \frac{1}{3}x + \frac{1}{6}x = \frac{6x + 8x + 4x}{24}$

$= \frac{18x}{24} = \frac{3}{4}x$

Money left = $\frac{x}{4}$

Money spent on clothes = $\frac{1}{10}.\frac{x}{4} = \frac{x}{40}$

Money left = $\frac{x}{4} - \frac{x}{40} = \frac{10x - x}{40} = \frac{9x}{40}$

i.e., $\frac{9x}{40} = 13,500$

$x = 1500 \times 40 = 60000$

Required amount = 5000

6. Let Rs x be Bidhan' s contribution. Then his brother's contribution = $\frac{3}{2}x$

Financers contribution = $3.\frac{3}{2}x = \frac{9x}{2}$

then, $x + \frac{3}{2}x + \frac{9x}{2} = 35000$

then, $\frac{14x}{2} = 35000 \Rightarrow x = \frac{35000 \times 2}{14} = 5000$

His brother's contribution = Rs 7500

7. Go through options.

8. Let x and y be the mass of 1st alloy and 2nd alloy respectively

Bronze in 1st alloy = $\frac{9}{10}x$

Manganese in 1st alloy = $\frac{7}{100}x$

Tin in 1st alloy = $\frac{3}{100}x$

For manganese, we have

$\dfrac{\frac{7}{100}x}{x+y} \times 100 = 5 \Rightarrow 7x = 5x + 5y \Rightarrow 2x = 5y.$

Hence, $x/y = 5/2$

Applying alligation,

t = % of bronze in 2nd Alloy

$\frac{y}{x} = \frac{5}{85-t}$

$\frac{2}{5} = \frac{S}{85-t}$, $t = 72.5$

9. Let Rs x was initially with Vineet

Money given to Sachin = $\frac{1}{4}x$

Money spent on movie ticket = $\frac{1}{3}.\frac{3}{4}x=\frac{1}{4}x$

Money left = $\frac{1}{2}x$

Money spent on shopping = $\frac{2}{5}.\frac{1}{2}x=\frac{1}{5}x$

Money given by Gita = $\frac{2}{3}.\frac{3x}{10}=\frac{x}{5}$

Total money he had = $\frac{3x}{10}+\frac{x}{5}=\frac{5x}{10}=\frac{1}{2}x$

Money given to mother = $\frac{4}{5}.\frac{1}{2}x=\frac{2x}{5}$

Money left = $\frac{1}{2}x-\frac{2}{5}x=\frac{x}{10}$

Given that,

$\sqrt{x}=\frac{x}{10} \qquad x=\frac{x^2}{100}$

$100x - x^2 = 0 \Rightarrow x\,(100 - x) = 0 \Rightarrow x = 100$

Money given by Gita $=\frac{100}{5}=$ Rs 20

10. No. of cogs remains inversely proportional to the number of turns.

No. of turns $=\frac{80}{45}\times\frac{54}{32}\times 8$

$= 24$

12. Let the no. of coins denomination Rs 1, 50 paise and 25 paise be $3x$, $4x$ and $12x$ respectively.

Then, $3x.1+4x.\frac{1}{2}+12x.\frac{1}{4}=600$

$3x + 2x + 3x = 600 \Rightarrow 8x = 600 \Rightarrow x = 75$

No. of 25 paise coins = 12 × 75 = 900

13. Let Anand, Bidhan and Chandra got Rs x, y and z respectively then, Chandra's Share = $\frac{1}{2}$ $(x + y)$ and

Anand's Share = $\frac{1}{3}(y+z)$

We donot know the values of x, y, z, therefore (d)

14. Let Anil invested Rs X, then Dheeraj's investment = Rs $3x$

Let Dheeraj join him for n month

then, $\frac{x\times 12}{3x\times n}=1\Rightarrow x\times 12=3x\times n\Rightarrow n=4$

Hence, Dheeraj joined him is September.

15. Since, we don't know the share of either of them, we cannot find the required time.

16. Let salary of C = Rs x = Rs 1500

Salary of B = $\frac{4}{3}x=\frac{4}{3}\times 1500=\text{Rs}\,2000$

Salary of A = $\frac{5}{4}.\frac{4}{3}x=\frac{5}{3}x=\text{Rs}\,\frac{5}{3}\times 1500=Rs\ 2500$

Salary of M = (Rs (1500 + 2000 + 2500) = Rs 6000

17. Four years ago, let ages of A and B be $13x$ and $9x$ years respectively.

then, $\frac{13x+12}{9x+12}=\frac{4}{3} \Rightarrow 39x + 36 = 36x + 48$

Or, $3x = 12 \Rightarrow x = 4$

Hence, present age of A = $13x + 4 = 13 \times 4 + 4 = 56$

And present age of B = $9 \times 4 + 4 = 40$

18. Let the wages of a man, a woman and a child he $3x$, $2x$ and x respectively,

then, $20 \times 3x + 30 \times 2x + x \times 36 = 780$

$60x + 60x + 36x = 780$

$\Rightarrow 156x = 780 \Rightarrow x = 5$

Required Amount = 2(15 × 3 × 5 + 21 × 2 × 5 + 30 × 5)

= 2 (225 + 210 + 150), = Rs 1170

19. Let $5x$ and $3x$ be the no. of males and females originally at the party.

Now, $\frac{5x-10}{3x}=1$, $5x - 10 = 3x$

$\Rightarrow 2x = 10 \Rightarrow x = 5$

No. of people originally at the party = $8x = 40$

20. Area of semicircle = $\frac{1}{2}\pi r^2$

Area of sector = $\frac{\theta}{360}\times\pi r^2$

$$\frac{\frac{\theta}{360}\times\pi r^2}{\frac{1}{2}\pi r^2}=\frac{1}{10},\ \frac{\theta}{180}=\frac{1}{10}=\theta=18^\circ$$

22. $V \alpha W^2$

$V = KW^2$, V = Value of diamond and W = Weight of diamond

K = constant

$10000 = K\,.\,(10)^2 \Rightarrow K = 100$

Hence, $V = 100 \times 4^2 = 1600$

$V = 100.6^2 = 3600$

Net loss = (Rs (10000 – 5200) = Rs 4800 i.e., 48%.

23. Monthly income of A and B be $3x$ and $2x$ respectively.

Monthly Expenditure of A and B be $5y$ and $3y$ respectively.

$3x - 5y = 1000,\ 2x - 3y - 1000$

Solving, we get, $y = 1000$, $x = 2000$

Income of A = Rs 6000

24. $\frac{ab}{bc}=\frac{x}{c^2}$ and $x=\frac{ab.c^2}{bc}=ac$

25. Let the fare of 1st, 2nd and 3rd class be $10x$, $8x$, and $3x$ respectively and let the no. of passengers of 1st, 2nd and 3rd class be $3y$, $4y$ and $10y$ respectively.
then $30xy+32xy+30xy=16100 \Rightarrow 92xy=16100$
Hence $xy=175$
Required amount $=32xy=32\times175=5600$

26. $$\frac{3A^2+4B}{3A-4B^2}=\frac{3.\left(\frac{3}{4}B\right)^2+4B}{4.\frac{3}{4}B-4B^2}=\frac{\frac{27B^2}{16}+4B}{3B-4B^2}$$

$$=\frac{27B^2+64B}{16(3B-4B^2)}=\frac{27B+64}{16(3-4B)}$$

Further it cannot be solved. Hence (d) is the correct answer

Note: Had there been no square terms, we could have calculated the final value by just putting A = 3 and B = 4.

28. Price of one goat = Rs 70 ⇒ Price of 9 goats = Rs 9 × 70 = Rs 630
Price of 14 sheeps = Rs 630 ⇒ Price of 1 sheep = Rs $\frac{630}{14}=$ Rs 45
Price of 50 sheeps = Rs 45 × 50 = Rs 2250
Price of 10 oxen = Rs 2250
Price of 1 oxen = Rs 225 ⇒ Price of 8 oxen = Rs 1800
Price of 6 dogs = Rs 1800 ⇒ Price of 1 dog = 300
Price of 5 dogs = Rs 1500 ⇒ Price of two horses = Rs 1500
Price of one horse = Rs 750

29. Since in 30 minutes pond is $\frac{1}{3}$ full, in next 1 minute it will triple i.e., the pond well be full. Hence (b) is the correct answer

31. A α B
B α (1/C)
C α D^2
D α $E^{1/3}$
When A increase
⇒ B also increase ⇒ C decreases.
Decrease in C results in decrease in D and decrease in D results decrease in E.

32. Using the formula $\frac{C}{5}=\frac{F-32}{9}$
We well get [– 40] as answer

33. Let the 1st no. be $2x$ and 2nd no $=3x$.

3rd no. $=\frac{18}{5}x$

4th no. $=81x/20$

So, $x+3x+\frac{18}{5}x+\frac{81x}{20}=233$

$$\frac{20x+60x+72x+81x}{20}=233$$

$x=\frac{233\times20}{233} \Rightarrow x=20$

2nd no. $=3x=3\times20=60$

3rd no. $=\frac{18.20}{2}=72$

Average $=\frac{60+72}{2}=\frac{132}{2}=66$

34. After one year
Population of port A = 8000
Population of port B = 6000
Population of Port C = 4000
After 2nd year
Population of A = 5000
Population of B = 6000
Population of C = 7000
After 3rd year
Population of A = 6500
Population of B = 6000
Population of C = 5500
After 4th year
Population of A = 5750

35. Let x = no. of Re 1 notes and y = no. of Rs 10 notes
So, $xy=x+10y \quad \Rightarrow xy=x+10y$
Now, go through options

36. At first let Nandan buys x black currents, then no. of pastries bought by Divya $=x+9$
Now, let Divya bought y butter scotches, then no. of butter scotches bought by Nandan $=y+18$

$\frac{x+y+9}{x+y+18}=\frac{12}{21} \quad \Rightarrow x+y=3$

x and y cannot be zeroes.
When $x=1, y=2$ ……… A
When $x=2, y=1$ ……… B
Let the price of black current be Rs A/unit and price of butter scotch be Rs B/unit.
For Case (A)
$10a+2b=a+20b \Rightarrow 9a=18b \Rightarrow a:b=2:1$

37. Go through options.

38. Ratio of their investments for 1 year
$=4x\times3+2x\times3+x\times3+\frac{x}{2}\times3:x\times3+2x\times3+4x\times3+8x\times3:15x\times12$

$=\frac{45x}{2}:45x:180x=45x:90x:360x=1:2:4$

B's share in profit $=\frac{2}{7}$ of profit

Let y be profit, then $\frac{2}{7}y=22000$, $y=77000$

42. $y=\frac{r}{x}$, $\qquad z=\frac{r^2}{x^2}$

Then, $x + y + z = 13$

$$x + \frac{r}{x} + \frac{r^2}{x^2} = 13$$

$$x^2 + xr + r^2 = 13x^2 \qquad \ldots(1)$$

And, $x^2 + \frac{r^2}{x^2} + \frac{r^4}{x^4} = 91 \qquad \ldots(2)$

With the help of (1) and (2), we get (a) as answer.

43. Let 1 black cow gives x litres milk in 1 day and 1 white cow gives y litres milk in 1 day. Then
$5(4.x + 3y) = 4(3x + 5y) \Rightarrow 20x + 15y = 12x + 20y$
$\Rightarrow 8x = 5y$. Hence, $\frac{x}{y} = \frac{5}{8}$

44. Let the cost of type I terminal be Rs x per week and cost of type II terminal be Rs y per week.
$5 \times x \times 7 + 22 \times 4 \times y + 10 \times 6x + 8 \times 6x = 74750$
...(1)
Also, given $\frac{x}{y} = \frac{2}{3} \qquad$ ...(2)
Using (2) and (1), we get option (a) as answer.

45. S.P. of alloy = Rs 90
Profit = 20%= $\frac{1}{5}$
C.P. = Rs $\left(90 - \frac{1}{6}.90\right)$ = Rs 75
Prices of foils are in ratio 1 : 4.
Hence, price of cheaper quality = $\frac{1}{5} \times 75 = \text{Rs}\,15$
and price of dearer quality = $\frac{4}{5} \times 75 = \text{Rs}\ 60$

46. Go through options

47. Let the no. of chapters be $6x$, $5x$, $10x$ and $14x$.
Also, $14x = 70$, $x = 5$
Thus, chapters are 30, 25, 50 and 70.
We can easily eliminate (a) as no. of pages has to be in between 4000 and 5000.
For (b) Each parts well have 1200 pages which is not possible for part having 70 chapters. Only option (c) satisfies the conditions.

48. Total no. of chapters = $(6x + 5x + 10x + 14x) = 35x = 35 \times 5 = 175$

49. Required ratio = 15/16

50. Let their present ages be $2x$ and $3x$.
Given that
$$\frac{2x + \text{N}}{3x + \text{N}} = \frac{3}{5}$$
$10x + 5\text{N} = 9x + 3\text{N}$
$X = -2\text{N}$ which 18 not possible.
Hence, (d)

Moderate

[illegible]

	Brother	***Sister***
Mr Nishant's son	X	5X
Mr Rajesh' son	Y	Y
Mr Nishant's daughter	2Z	Z
Mr Rajesh' daughter	2A	A

Each son of Mr Rajesh has as many sisters as the number of brothers and each daughter of Mr Rajesh has twice the number of brothers as the number of sisters. It is possible only if the number of sons = 4 and number of daughters = 3. However, nothing concrete can be said about the children of Mr Nishant.

[illegible]

Number of men : number of women = Number of women : number of children

10. Only the second option is satisfying the values.

11. Since this ratio is less than 1, the number of men should be less than the number of females visited. So, number of males should be less than 7. So, the answer is option d. None of these.

12. If P is taking 3 days to do some work, then Q takes 6 days to do the same work. Now, both of them will take 2 days to complete the work. So, R takes 6 days to complete the same work.

Hence, earning should be distributed in the ratio of their efficiency, i.e., 2 : 1 : 1.

Questions 13 to 15

Assume that Ten, Dul and Kar are getting T, D and K sweets respectively.
2 T = 2/3 D = 4 K = 4P (assume)
So, T = 2 P, D = 6 P and K = P
So, T : D : K = 2:6:1

13. Kar would get – 117(4/9). So, the answer is option (d). None of these.

15. x bulbs consume $1/x$ units everyday.
So, 1 bulb will consume $1/x^2$ units of electricity everyday
Y bulbs consume y/x^2 units everyday
Now, y/x^2 units is consume in 1 day.
Hence, y units will be consumed in x^2 days.

[illegible]

16. Number of apples/Number of guavas = Number of guavas/Number of oranges
(Number of guavas)^2 = Number of apples × Number of oranges
This gives us a hint that Number of apples × Number of oranges should be a prefect square.
Now start using the options.
Option (b) satisfies the relationship (16/20 = 20/25)

17. We have to check all the possibilities in this question. If we take the ratio to be equal Number of apples/Number of guavas = Number of guavas/Number of oranges = 1, then Number of apples = Number of guavas = Number of oranges. So, one possible number of total fruits = 63
In 16 and 17, if we start using the concept of GP, then it will be difficult to find the number of fruits by GP as this questions contains a lot many variables.

19. Village-chief contribution = Rs 1.2 lakhs, so villagers' contribution = Rs 1.2 lakhs
Now, Go through the options.

[illegible]

28. Go through the options.

29.

No. of people	6	6	8	16
Event	Museum	Fair	Exhibition	Movie
Rate	5	4	2	25

36. Using options is the best way of solving this question.

37. Best way of solving this question is—Go through the options.

38. Its all about taking different values and checking which option cannot satisfy the money with Pankaj.

41. No. of deers, bears and foxes = $3x:7x:5x$
Difference between the no. of deers and bears = $4x$
Now, $4x$ has to be a multiple of 21. So, x has to be a multiple of 21.
So, Total number of animals = $15x = 15 \times 21 = 315$

42. Weight of two pieces of Watermelon
= $3 \times 12y : 5 \times 12y$
(We are assuming the weight to be the multiple of 12 because the bigger of two pieces has to be cut in the ratio 5 : 7.)
Now $5 \times 12y$ has to be cut in the ratio 5 : 7. So, the two parts will be $25y$ and $35y$.
So, finally the ratio is 36:25:35.

43. Assume that Y produces 4P units in unit time. Then X and Z produce P and 2P units in the same time. So, Total shoes produced = 7P, Fraction of the total number of shoes produced by machine Z = 2/7

48. Use options.

49. $$\frac{a}{a+b} = \frac{b}{c+a} = \frac{c}{a+b} = \frac{a+b+c}{2(a+b+c)}$$

Now if $a + b + c = 0$, then $a + b = -c$

So, $\frac{a+b+c}{2(a+b+c)} = -1$

And if $a + b + c \neq 0$, then $\frac{a+b+c}{2(a+b+c)} = \frac{1}{2}$

Advanced

1. Time a (√Number of passengers) × Distance
Expressing this in symbols, $T \alpha (\sqrt{N}) \times D$
Or, $T1 / T2 = (\sqrt{N1/N2}) \times D1/D2$
Or, $2 / 1 = (\sqrt{N1/64}) \times 80/30$

2. Tax = Fixed + Variable
Since Mr Kalyan pays Rs 6200 and Mrs Kalyan pays Rs 7700, so it can be inferred that for Rs 10,000 (Difference between their salaries), one has to pay Rs 1500 as the variable tax. So, Fixed component of tax = Rs 5200. The point worth to be noted here is the fact that tax is to be paid over an above Rs 50,000. So, the variable tax percentage above Rs 50,000 = 10%. Now the answer can be calculated using options.

Questions 6 and 7

Generate the equations first and then use options for faster calculations.

9. Total profit = Rs 24 lakh per trip + Rs $7P^2$, where P is the number of coaches.
Now go through the options.

10. When 10 kg is kept in first pan and 5 kg is kept in second pan, it remains in balance – It means that 1st pan is heavier than the second pan (or, more difficult to pull down), so the ratio of weight of pans = 2 : 1.

11. Do as given in the solution of 10.

12.

	Gold	Copper
First Alloy	2	1
Second Alloy	2	7
Finally	1	2

It is possible if we mix first alloy and second alloy in the ratio 1 : 3. So, the second alloy will be mixed two times more than the first alloy.

13. Weight α Height and Height α Age.

19. Go through the options.

20. Assume the distance = 9 units. So, the ratio of the time taken = 9 : 3 : 1
So, the answer = 9/1 : 3/3 : 1/9

22. Population state 1, state 2, state 3, state 4, state 5 = x : $3x$: $2x$: $3x$: $2x$. The ratio of the percentage of women in the population = y : $3y$: $2y$: $5y$: $4y$.
Now use the above values to establish the relationship between xy and the total population of women = 3.7 million.

23. It is possible if new length of the pendulum become 1.21 times of the old length of the pendulum.

Questions 26 to 29

Using the options given in question 26, we can see that it is possible to have Rs 500 on 19th only if he starts with Rs 80 on 13th of November.

Date	*Money*
13th	Rs 80
14th	Rs 70
15th	Rs 140
16th	Rs 130
17th	Rs 260
18th	Rs 250
19th	Rs 500
20th	Rs 490
21st	Rs 980
22nd	Rs 970
23rd	Rs 1940
24th	Rs 1930
25th	Rs 3860

27. It is Rs 970 on 22nd.

28. It is obvious from the table given above.

29. The new table would have been like this:

Date	*Money*
13th	Rs 100
14th	Rs 90
15th	Rs 180
16th	Rs 170
17th	Rs 340
18th	Rs 330
19th	Rs 660
20th	Rs 650
21st	Rs 1300
22nd	Rs 1290
23rd	Rs 2580
24th	Rs 2570
25th	Rs 5140

Now it can be calculated.

35. Total number of distinct meeting points for B and C = 9 and Total number of distinct meeting points for A and C = 5. Now it can be checked using the options.

Questions 36 to 38

Whatever is the total number of boys, it should be divisible by 8, 6, 12 and total number of girls should be divisible by 12, 15 and 4. Now use options.

7

CHAPTER

Time and Work

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Time and work equivalence
- Relationship between time and efficiency
- Different types of work
- Kinds of questions asked at the CAT
- Methods of solving questions

Introduction to the topic

A look at the CAT papers of the last 15 years shows that time and work is an important topic for this exam. There have been questions from this topic almost every year. The number of questions, however, have varied between 1 and 3. A basic understanding of the concept of time and work will enable you to solve problems from this topic with considerable ease.

TIME AND WORK

There is a definite relationship between the amount of time taken, the number of persons doing the work and the unit of work done. Under this topic we study the phenomenon of accomplishment of a work with relation to the time. There are certain formulae that demonstrate the above-mentioned relationship and exhibit how each of these factors influence the other two.

TIME–WORK EQUIVALENCE

The essence of time-work equivalence lies in the fact that it exhibits the most fundamental relationship between the three factors as mentioned above viz., work, time and the agent which is completing the work. That is,

Work done = Number of days × Number of men

$W = M \times D$

This gives us an important concept of man-days.

Suppose there are 20 persons working for 10 days to complete a job, then the total work done is equal to 200 man-days. Now, if we change the number of days in which the work is to be completed, then the other factor i.e., the number of persons will change accordingly, so that, the product of the factors becomes equal to 200 man-days. Product-stability-ratio (Chapter 3) is a very effective tool to calculate this.

Example 1 Seven persons can clean 7 floors by 7 mops in 7 days. In how many days can 5 persons clean 5 floors by 5 mops?

Solution This problem can be solved through several methods.

Method 1 To clean 7 floors, we need to have 7 × 7 = 49 man-days.

So, to clean 5 floors, we need to have 35 man-days.

So, $35 = D \times 5$. So, $D = 7$ days

Method 2 Using ratio proportion, less work and less men are involved here.

So, the number of days = $7\times\frac{5}{7}\times\frac{7}{5} = 7$ days

Method 3 Let us try to have a mental image of this situation: There is a building with seven floors namely $F_1, F_2, \dots, F_7$ and seven persons $P_1, P_2, \dots P_7$ are cleaning this building in such a way that one floor is being cleaned by each one of them. Since it takes 7 days to complete the whole work, it can be inferred that everybody is taking 7 days to clean his respective floor. So, if there are just floors and five persons are cleaning these five floors, then it will take them seven days (assuming that the top two floors have been demolished.)

Now, depending upon different situations, three conditions are possible, in the relationship W = MD (where W = quantity of work, M = number of persons, D = number of days)

Condition 1: W is constant

$M \times D = \text{Constant}$

$M \propto 1/D$

It can be observed that if the work done is constant, then the number of persons is inversely proportional to the number of days, which means that the multipliers of M and D will be reciprocal. Extending this situation, if 10 persons can do a work in 20 days, then 5 persons can do the same work in 40 days, or, 20 persons will do the same work in 10 days. Further it can be summarized as

W = M × D

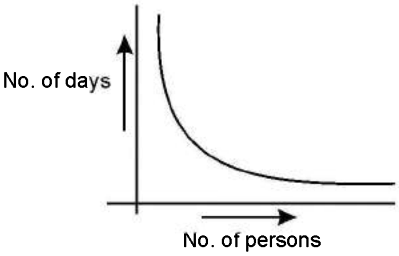
200 = 10 × 20

200 = 5(10 × 1/2) × 40(20 × 2)

200 = 20(10 × 2) × 10(20 × 1/2)

Thus, it can be said that the multiplier of M and multiplier of D are reciprocal to each other.

It can be seen with the help of the graph given below



Example 2 Yadavjee contractor undertakes to get a work done in 50 days by 50 labourers. After 40 days, he realizes that only 50% of the work is done. How many more men should be employed so that the work is complete on time?

Solution

W =	M ×	D	
50%	50	40	...(i)
Rest 50%	(50 + M)	10	...(ii)

Since work is constant in both the cases, so, the number of men and the number of days will be reciprocal to each other. As the number of days left in (ii) is 1/4th of initial period (i), so the number of persons will become 4 times of the initial number of persons.

Hence, the number of persons = 50 × 4 = 200. So, M = 150 men

Example 3 In the above question, if the schedule can go behind by 10 days, then how many extra men are required to complete the work?

Solution So, now we will have to complete the work in 60 days.

W =	M ×	D	
50%	50	40....................	(i)
Rest 50%	(50 + M) 20	(ii)	

Since work is constant in both the cases, so the number of men and the number of days will be reciprocal to each other. As, the number of days left in (ii) is 1/2 of initial period (i), so the number of persons will become 2 times of the initial number of persons.

Hence, the number of persons = 50 × 2 = 100. So, M = 50 men

Example 4 In question 2, he realizes after 40 days that work is only 20% complete. How many extra men should be employed now so that the work is completed on time?

Solution

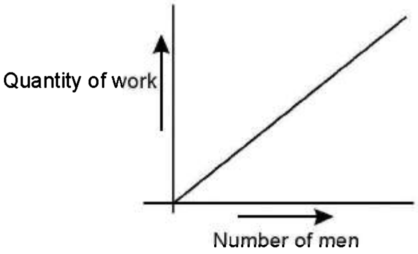
W =	M ×	D
20%	50	40 .... (i)
20%	200	10 .... (ii)
So, 80%	800	10 .... (iii)

Hence, 750 more persons are needed to complete the job on time.

Condition 2: D is constant

$W \alpha M$

More work will be done if we employ more men and vice-versa. It means that multiplier of W and M will be same. It can be seen with the help of the graph given below



Condition 3: M is constant

$W \alpha D$

In general, we can summarize that

$$\frac{M_1D_1}{W_1} = \frac{M_2D_2}{W_2}$$

Example 5 12 persons can cut 10 trees in 16 days. In how many days can 8 persons cut 12 trees?

Solution Here $W_1 = 10\, W_2 = 12$
$M_1 = 12\, M_2 = 8$
$D_1 = 16\ D_2 = ?$

Putting the values in the equation $W_1/W_2 = (M_1/M_2) \times (D_1/D_2)$

We get $\frac{10}{12} = \frac{12}{8} \times \frac{16}{D_2} \Rightarrow D_2 = 28.8\,\text{days}$

INDIVIDUAL WORK AND INDIVIDUAL EFFICIENCY

Individual Work

If Amit can do a certain work in 10 days, then he will finish 1/10th of the work in one day.

Example 6 Amit can do some work in 12 days and Vinit can do the same work in 15 days. In how many days will both of them do the work when working together?

Solution Assume total work = 1 unit

Work done by Amit in one day = 1/12 unit

Work done by Vinit in one day = 1/15 unit

Work done by both of them in one day when working together = (1/12) + (1/15) = 9/60 unit

Hence, they will be doing the whole work in 60/9 days $= 6\frac{2}{3}$ days

LCM method of solving time and work questions

This can be understood in terms of the above written example in the following way:

Let us assume total work to be equal to the LCM of the days taken by Amit and Vinit (i.e., of 10 and 15)

Assume work = 60 units

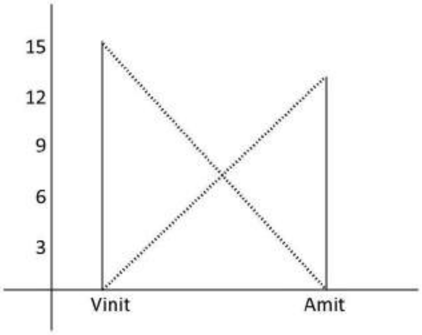
Work done by Amit in one day = 5 units

Work done by Vinit in one day = 4 units

Work done by both of them in one day when working together = 9 units

So, the number of days taken by both of them when working together = 60/9 = $6\frac{2}{3}$ days

Alternatively, we can do this problem with the help of a graph also.



We have created the pillars of the number of days of Amit and Vinit. Now the point of intersection of the top of the first pillar to the bottom of the second pillar and the top of second pillar and the bottom of the first pillar is the number of days taken by both of them when working together. In the above drawn graph, it can be clearly seen that the point of intersection of both the straight lines is a bit more than 6.

Despite graphical method appearing easier than the earlier two methods, the usage of this method should be avoided due to the complexity of denoting the points on the graph paper.

Example 7 A, B and C can do a piece of work individually in 8, 12 and 15 days respectively. A and B start working, but A quits after working for 2 days. After this, C joins B till the completion of work. In how many days will the work be complete?

Solution Let us assume that the Work = LCM (8, 12, 15) = 120 units

So, the work done by A in one day = 15 units

Work done by B in one day = 10 units

Work done by C in one day = 8 units

Work done by A and B in two days = 2 × 25 = 50 units
Remaining work = 70 units
Work done by C and B in one day = 18 units

Time taken to complete the remaining work by C and B = $\frac{70}{18} = 3\frac{16}{18}$ days

So, the total number of days = $5\frac{16}{18}$ days

Example 8 A and B together can do a work in 12 days, B and C together can do the same work in 10 days and A and C together can do the same work in 8 days. In how many days will the work be complete if A, B and C are working together?

Solution Let us assume work = LCM of (12, 10, 8) = 120 units

So, A and B are doing 10 units in one day, B and C are doing 12 units in a day and A and C are doing 15 units in a day.

Adding all these, 2 (A + B + C) are doing 37 units in a day.

$\Rightarrow$ (A + B + C) are doing $\frac{37}{2}$ = 18.5 units in a day

So, time taken to complete the work = $\frac{120}{18.5}$ days = 6.48 days

Individual Efficiency

Efficiency is also known as work-rate.

If A is taking less number of days with respect to B to complete the same work, we can say that the efficiency of A is more than the efficiency of B.

So, more the efficiency, less will be the number of days and less the efficiency, more will be the number of days to do a certain work. We have observed individual efficiency in case of percentage also (Product Stability ratio).

Now, assume A takes 20 days to complete a work and B takes 25 days to complete the same work. It means A is doing 5% (100%/20) work in one day and B is doing 4% (100%/25) work in a day. So, efficiency of A is 25% more than efficiency of B.

General expression correlating time taken and efficiency

If efficiency of A is x per cent more than the efficiency of B and B takes 'B' days to complete the work, then A will take $\left(\frac{B}{100+x}\times 100\right)$ days to complete the same work.

If efficiency of A is x% less than the efficiency of B and B takes 'B' days to complete the work, then A will take $\left(\frac{B}{100-x}\times 100\right)$ days to complete the same work.

So, if A is 20% more efficient than B and B takes 'B' days to complete the work, then A will take $\frac{B}{1.2}$ days to do the same work.

With this, it can also be observed that if work is constant then time taken is inversely proportional to efficiency.

Example 9 John is thrice as efficient as Abraham and hence completes a work in 60 days less than the number of days taken by Abraham. What will be the number of days taken by both of them when working together?

Solution Since John is thrice as efficient as Abraham, so the number of days taken by him will be 1/3rd the number of days taken by Abraham. If John is taking x days, then Abraham will take $3x$ days to complete the same work.

Now, $3x - x = 2x = 60$ days

So, $x = 30$ days and $3x = 90$ days

Let us assume that the total work = 90 units (LCM of 30 and 90)

So, the total work done by both of them in one day = 3 + 1 = 4 units of work.

So, the total number of days = $\frac{90}{4}$ days = 22.5 days

Example 10 A can do a work in 4 days. Efficiency of B is half the efficiency of A, efficiency of C is half the efficiency of B and efficiency of D is half the efficiency of C. After they have been grouped in two pairs it is found that the total number of days taken by one group is 2/3rd the time taken by the other group. Which of the following is a possible group? (CAT 2001)

(a) AB (b) BC (c) CD (d) AC

Solution Number of days taken by A = 4 days
Number of days taken by B = 8 days
Number of days taken by C = 16 days
Number of days taken by D = 32 days
Assume that the total work = 32 units
So, the work done by A = 8 units
Work done by B = 4 units
Work done by C = 2 units
Work done by D = 1 units

It can be observed that the work done by B and C together in one day is 2/3rd the work done by A and D. So the groups are—AD and BC.

Example 11 In a nuts and bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 min after production of every 1,000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 min after production of every 1,500 bolts. If both the machines start their production at the same time, what is the minimum duration required for producing 9,000 pairs of nuts and bolts?

(a) 130 min (b) 135 min (c) 170 min (d) 180 min

Solution

Machine I

Number of nuts produced in one min = 100
Time required to produce 1,000 nuts = 10 min
Cleaning time for nuts = 5 min
Overall time to produce 1,000 nuts = 15 min.

Over all time to produce 9,000 = 138 min – 5 min = 133 min … (1)

Machine II

Time required to produce 75 bolts = 1 min
Time required to produce 1,500 bolts = 20 min
Cleaning time for bolts = 10 in
Effective time to produce 1,500 bolts = 30 min
Effective time to produce 9,000 bolts = 30 × 6 – 10 = 170 min … (2)
From (1) and (2)
Minimum time = 170 min

Example 12 A, B and C are assigned a piece of work which they can complete by working together in 15 days. Their efficiencies (measured in terms of rate of doing work) are in the ratio of 1:2:3. After $\frac{1}{3}$rd of the work is completed, one of them has to be withdrawn due to budget constraint. Their wages per day are in the ratio of 3:5:6. The number of days in which the remaining two persons can complete the work (at optimal cost) is

(a) 18 (b) 20 (c) 15 (d) 12

Solution A, B and C together in 15 days

= A alone in 90 days, B alone in 45 days, C alone in 30 days

Wages per day per unit work for A, B and C are $\frac{3}{1}:\frac{5}{2}:\frac{6}{3}$

Hence, A is the least efficient and hence, must be done away with.

For B and C, the whole work can be finished in 18 days and hence, remaining 2/3rd of the work can be finished in 12 days only.

COLLECTIVE WORK AND COLLECTIVE EFFICIENCY

When people of different efficiencies start working together, the method of time-work equivalence to find the time or the amount of work done cannot be used. In those cases, we will be required to relate time with the efficiency of a group.

For example, if 5 men and 8 women can do a piece of work in 10 days, from this information we can not find out that in how many days a man or a woman can do the work individually, since we are not aware of their individual efficiencies. However, if we get a similar equation like x men and y women can do the same work in 'p' days, then we can correlate these two equations to find the number of days taken by one man and one woman to do the work.

Example 13 10 men and 9 women can do a piece of work in 20 days. The same work can be done by 6 men and 12 women in 30 days. In how many days can the same work be done by 1 man and 1 woman?

Solution 10 men and 9 women can do a work in 20 days ⇒ 20 × (10 men and 9 women) can do the same work in 1 day.

Similarly, 6 men and 9 women can do the same work in 30 days ⇒ 30 × (6 men and 12 women) can do the same work in 1 day.

So, efficiency of 20 × (10 men and 9 women) = efficiency of 30 × (6 men and 12 women)
Or, 200 men + 180 women = 180 men + 360 women
Or, 20 men = 180 women ⇒ 1 man = 9 women
So, the total work = 20 × (10 men and 9 women) = 20 × (90 + 9) women = 99 × 20 = 1980 units
Total number of persons employed =
1 man + 1 women = 10 women

So, the time taken = $\frac{1980}{10}$ = 198 days

EXTENSION OF THE CONCEPT OF TIME AND WORK

1. Pipes and cisterns

Pipes and cisterns is just another application of the concept of time and work. While we see only +ve work being done in normal cases of time and work, in case of pipes and cisterns, –ve work is also possible.

Given that pipes A and B can fill a tank in 20 min and 25 min working individually ⇒ this statement is similar to "A can do a work in 20 min and B can do the same work in 25 min."

Again, given that pipe C can empty a tank in 40 min we can say this statement is similar to "C can demolish a wall in 40 min (assuming that the work is building or demolishing the wall)

Let us understand this with the help of an example.

Example 14 A and B are two taps which can fill a tank individually in 10 min and 20 min respectively. However, there is a leakage at the bottom, which can empty a filled tank in 40 min. If the tank is empty initially, how much time will both the taps take to fill the tank (leakage is still there)?

Solution Let us assume the units of work = LCM of (10, 20, 40) = 40 units

Work done by Tap A/min = 4 units/min (Positive work)
Work done by Tap B/min = 2 units/min (Positive work)
Work done by leakage/min = 1 unit/min (Negative work)
Net work done/min = 5 units/min
Hence time taken = 8 min

Example 15 Pipe A can fill a tank in 3 hour. But there is a leakage also, due to which it takes 3.5 hour for the tank to be filled. How much time will the leakage take in emptying the tank if the tank is filled initially?

Solution Assume the total units of work = 10.5 units

Work done by Tap A/h = 3.5 units/h (Positive work)
Work done by leakage/h = 3 units/h (Negative work)
Net work done/h = 0.5 units/h

So, the time taken = $\frac{10.5}{0.5}$ = 21 hour

Alternatively, due to the leakage, the pipe is required to work for an extra half an hour. So, the quantity filled by pipe in half an hour is being emptied by the leakage in 3.5 hour. Hence, the quantity filled by pipe in 3 hour will be emptied by the leakage in 21 hour.

2. Variable work

The concept of variable work comes from the possibility

⇒ that the rate of working can be different or

⇒ can be dependent upon some external agent.

In these cases, the rate of work will be proportional to some external factor.

Understand this with the help of a simple statement: The rate of the flow of water from a pipe is directly proportional to the area of the cross section of the pipe.

Example 16 There are three inlet taps whose diameters are 1 cm, 2 cm and 3 cm respectively. The rate of flow of the water is directly proportional to the square of the diameter. It takes 9 min for the smallest pipe to fill an empty tank. Find the time taken to fill an empty tank when all the three taps are opened.

Solution The rate of flow of a diameter^2, or, rate of flow = K × diameter^2 (where K is a constant)

For 1st tap, rate of flow = $K \times 1$

For 2nd tap, rate of flow = $K \times 4$

For 3rd tap, rate of flow = $K \times 9$

We know, the quantity filled will be equal to the product of the rate of flow and time.

So, the quantity filled by the smallest pipe = $K \times 1 \times 9 = 9K$ = Capacity of tank

Quantity of water filled by all the taps together in 1 min $= 9K + 4K + 1K = 14K$

Assume that all the taps working together take ‘t’ min.

So, $14\,K \times t = 9\,K$

So, the time taken $t = \frac{9K}{14K} = \frac{9}{14}$ min

3. Alternate work

The concept of alternate work is analogous to the concept of man-days. As we have seen in the concept of man-hour that if 20 men can do a work in 10 days, then this work is equivalent to 200 man-days. However, in the case of alternate work, two or more than two people of different efficiencies work alternately or in some particular pattern.

Example 17 Navneet can build a wall in 30 days and Rakesh can demolish the same wall in 40 days. If they work on alternate days with Navneet starting the job on the 1st days, then in how many days will the wall be built for the first time?

Solution Let us assume the total units of work = 120 units

So, the wall built by Navneet in one day = 4 units

And wall demolished by Rakesh in one day = 3 units

So, effectively in two days, total wall built = 1 unit

Now, they work on alternate days, so days taken to built 116 units = 116 days

On 117th day Navneet will add another 4 units and so completing the construction of wall in 117 days.

(This problem can be understood well with another very traditional problem—A frog climbs up a pole 4 inches in 1 hour and slips 3 inches next hour. If height of the pole is 120 inches, then what is the time taken by the frog to reach the top of the pole?)

PRACTICE EXERCISES

WARM UP

Q.1. Anil can do a piece of work in 18 days and Nishit can do the same work in 12 days (Nishit is more efficient than Anil). In how many days both of them will complete the work when working together?

(a) 7.2 days (b) 6 days

(c) 5.2 days (d) None of these

Q.2. A can do a piece of work in ‘a’ days and B can do the same piece of work in ‘b’ days, then the number of days in which both of them will complete the work working together is N. Which of the following can never be true?

(a) $2N$ is the Arithmetic Mean of a and b

(b) $2N$ is the Geometric Mean of a and b

(c) $2N$ is the Harmonic Mean of a and b

(d) None of these

Q.3. In the context of the above question, if $a \neq b$, then which of the following is true?

(a) $2N$ is the Arithmetic Mean of a and b

(b) $2N$ is the Geometric Mean of a and b

(c) $2N$ is the Harmonic Mean of a and b

(d) None of these

Q.4. Aftab can do a piece of work in 12 days and Ravi can do the same work in 18 days. If Aftab works for three days and then quits, in how many days will the remaining work be completed by B alone?
(a) 13½ (b) 9 (c) 16½ (d) 12

Q.5. Efficiency of Ravi and Mayank in doing the same work is 3:2. If together they can complete a work in 18 days, then in how many days Ravi can complete the work alone?
(a) 45 (b) 30 (c) 24 (d) 40

Q.6. A tap can fill an empty tank in 12 hours and a leakage can empty half the tank in 10 hours. If the tap and the leakage are working simultaneously, how long would it take for the empty tank to be filled to half its capacity?
(a) 60 hour (b) 30 hour
(c) 15 hour (d) 12 hour

Q.7. Pipe A can fill an empty tank in eight hours and pipe B can empty the full tank in four more hours. If both the pipes are opened simultaneously, then how long would it take to fill an empty tank?
(a) 12 hours (b) 24 hours
(c) 18 hours (d) 30 hours

Q.8. Sanjay can wash a truck in 45 min and he works for six hours a day. How many trucks can he wash in three days?
(a) 24 (b) 12 (c) 21 (d) 15

Q.9. Sumit Niyogi can build a wall measuring 20 ft × 3 ft × 15 ft in 6 days. Vartika Dikshit can build a wall measuring 30 ft × 4 ft × 10 ft in 5 days. What is the ratio of the work done by Sumit Niyogi and Vartika Dikshit in one day?
(a) 5:8 (b) 6:5 (c) 6:7 (d) 7:5

Q.10. Nandan is twice as good a workman as Divya and together they complete the work in 15 days. In how many days can Divya alone complete the work?
(a) 45 days (b) 40 days
(c) 35 days (d) None of these

Q.11. N men can do a piece of work in 8 days. The same work can be done by (N + 4) men in 6 days. Find the value of N.
(a) 10 (b) 12
(c) 14 (d) None of these

Q.12. Harveen can do a piece of work in 18 days. He worked for 12 days and left. Deepak finished the remaining work in 8 days. In how many days can Deepak alone complete the work?
(a) 21 (b) 22 (c) 23 (d) 24

Q.13. Munchun can do a work in 20 days and Chunmun can do it in 10 days. Both worked together and were paid Rs 1,800. The share of Chunmun is:
(a) Rs 1,200 (b) Rs 1,500
(c) Rs 900 (d) None of these

Q.14. Shahid Bhai can do 2/3rd of the work in 18 days. In how many days can he finish 1/4th of the work?
(a) 6½ (b) 6.75 (c) 6 (d) 13.5

Q.15. A can do half as much work as B in one day. B alone can do a certain work in 12 days. In how many days can A and B together finish that work?
(a) 6 days (b) 7 days
(c) 8 days (d) None of these

Q.16. If 35 persons can do a piece of work in 6 days, in how many days can 15 persons do it?
(a) 12 (b) 14
(c) 16 (d) 18

Q.17. A and B can complete a work in 15 days. A is 50% more efficient than B. How long would A take to complete the work alone?
(a) 20 days (b) 22 days
(c) 24 days (d) None of these

Q.18. Forty persons can finish a work in 60 days. If 40 persons start the work and at the end of every 10 days 40 more persons of the same efficiency join them, then how long will it take to complete the work?
(a) 30 days (b) 20 days
(c) 40 days (d) None of these

Q.19. A group of 18 men and 12 women can do a work in 18 days. A woman takes twice as much time as a man to do the work. How many days will 8 men take to finish the same work?
(a) 45 (b) 48
(c) 54 (d) None of these

Q.20. A team has a food stock for N number of days. After 20 days, 1/4th of the team quits and the food lasted for another N number of days for the remaining persons in the team. What is the value of N?
(a) 50 (b) 60 (c) 70 (d) 80

FOUNDATION

Q.1. To complete a work, A and B take 30 days, B and C take 24 days, and C and A take 20 days. They all start together and work for 5 days after which B and C leave. How many more days will A take to finish the remaining work?

(a) 11 (b) 28 (c) 33 (d) 18

Q.2. A water tank normally takes 7 hours to be filled by a tap but because of the leak, it takes another 2 hours. In how many hours will the leak empty a full water tank?

(a) 20.5 hours (b) 24.5 hours
(c) 30 hours (d) 31.5 hours

Q.3 The wages of 10 men and 8 boys amount to Rs 37. If 4 men together receive Re 1 more than 6 boys, what are the wages of each man and boy respectively?

(a) 2.5 and 1.5 (b) 11 and 9
(c) 3 and 4 (d) 6 and 5

Q.4. Fourteen machine can produce 500 units in 12 hours. After 4 hours of production, two more machines were started. How many more hours will it take to complete the production of 500 units?

(a) 4 (b) 5 (c) 6 (d) 7

Q.5 Pramod can do 2/5th of the work in 8 days and Binod can finish the remainder in 18 days. How long would they take to finish it together?

(a) 8 (b) 10 (c) 12 (d) 14

Q.6 A cistern can be filled separately by two pipes P and Q in 45 min and 35 min respectively. A tap R at the bottom can empty the full cistern in 30 min. If the tap R is opened 7 min after the two pipes P and Q are opened then after what time from the opening of pipe P and Q the cistern becomes full?

(a) 40th min (b) 61st min
(c) 37th min (d) None of these

Q.7. A can do a job in 20 days, B in 30 days and C in 60 days. If A is helped by B and C every 3rd day. How long will it take for them to complete the job?

(a) 12 days (b) 4 days
(c) 15 days (d) 18 days

Q.8. Saket and Prakash take 7.5 days working together. However, if Saket and Prakash work individually to complete the equal amount of work, it takes them 20 days to complete the job. How long will Prakash alone take to do the job if Saket is more efficient than Prakash?

(a) 18 days (b) 24 days
(c) 30 days (d) 27 days

Q.9. Four machines A, B, C and D are to be used for producing 10,000 units. Machine A alone can produce 50 units in a day, B is half as efficient as A, C is as efficient as A and D is twice as efficient as A. In how many days would the work be complete if all the four machines start working together?

(a) 19 days (b) 29 days
(c) 46 days (d) None of these

Q.10. In an organization, regular working day is of 8 hours and regular week is of 5 working days. A man is paid Rs 3.20 per hours overtime. If he has earned Rs 432 in 4 weeks, what is the total number of hours he worked in those 4 weeks?

(a) 180 (b) 175
(c) 160 (d) None of these

Q.11. Tap A fills a tank in 20 min while C empties it at 1/3rd the rate at which A fills it. At 12.00 noon, A and C are simultaneously started and when the tank is 50% full, tap A is turned off. At what time will the tank be empty?

(a) 12.35 P.M. (b) 12.45 P.M.
(c) 12.30 P.M. (d) 12.55 P.M.

Q.12. Two pipes X and Y can fill a tank in 6 hours and 9 hours respectively. If they are turned on alternately for one hour each starting with X, then what is the time taken to fill the tank?

(a) 6.25 hours (b) 5.66 hours
(c) 7 hours (d) 8 hours

Q.13. A 10 hectare field is reaped by 2 men, 3 women and 4 boys in 10 days. If a man, a woman and a boy work in the ratio of 5 : 4 : 2, then in how much time 6 men, 4 women and 7 boys reap a 16 hectare field?

(a) 5 days (b) 6 days
(c) 8 days (d) 7 days

Q.14. Four plumbers had to repair 364 taps. At the end of 3 days working 7 hours per day they had repaired 168 taps. In order to speed up the work, 3 more plumbers were added who also worked 7 hours daily. If efficiency of each of the plumbers is same, how many additional days would it take to complete the task?

(a) 2 days (b) 3 days
(c) 4 days (d) 1½ day

Q.15. Three factories are capable of manufacturing a particular part of the bike. Each of the two factories can produce 1,00,000 parts in 15 days. The third factory can produce them 30% faster. How many days (approx) would it take to produce a million parts with all the three working simultaneously?

(a) 43 (b) 45 (c) 48 (d) 52

Q.16. Five persons working 8 hours daily can complete a wall in 10 days. When they have worked for 5 and 1/2 days, 5 more persons are brought to work. In how many more days will the wall be complete?
(a) One more day (b) More than 2 days
(c) More than 4 days (d) More than 3 days

Q.17. Two filling pipes can fill a tank in 15 hours and 12 hours respectively while a third pipe C can empty it in 20 hours when filled. If the tank is empty and all the three pipes are opened, in how much time will the tank be full?
(a) 14 hours (b) 10 hours
(c) 6 hours (d) 8 horus

Q.18. A work can be finished by Arnold, Bill and Kennedy in 20 days, 24 days and 30 days respectively provided the working hours are 10 hours per day. If all of them are engaged to complete this work and they finished the work in 10 days, how many hours per day did they work?
(a) 7 hours (b) 10 hours
(c) 8 hours (d) 9 hours

Q.19. If A completes a work in t days, B in 1/r days and C in s days then in how many days will they complete the work together?
(a) st/(s+t+rst) (b) rt/(r+t+rst)
(c) rst/(rs+st+tr) (d) rs/(r+s+rst)

Q.20. A certain job was assigned to a group of people to complete it in 20 days. But 12 of them did not turn up for the job and the remaining persons did the job in 32 days. What is the original number of men in the group?
(a) 32 (b) 34
(c) 36 (d) None of these

Q.21. A machine P can print one lakh books in 8 hours, machine Q can print the same number of books in 10 hours while machine R can print them in 12 hours. If all of them work together for 6 hours, how many books will be printed?
(a) 60,000 (b) 1,80,000
(c) 1,85,000 (d) 2,85,000

Q.22. A pipe can fill a cistern in 6 hours. Due to a leak in its bottom, it is filled in 7 hours. When cistern is full, in how much time will it be emptied by the leakage?
(a) 42 hours (b) 40 hours
(c) 43 hours (d) 45 hours

Q.23. Tap A can fill a tank in 16 min and tap B can empty it in 8 min. If the tank is already 1/2 full and both the taps are opened together, the tank will be either filled or emptied? How long will it take before the tank is either filled or emptied?
(a) Emptied; 16 min (b) Filled; 8 min
(c) Emptied; 8 min (d) Filled; 12 min

Q.24. Three-fourth of a tank is full of water. If 5 liters are added to it. Then four-fifth of the tank becomes full. What is capacity of the tank?
(a) 75 litres (b) 80 litres
(c) 100 litres (c) 120 litres

Q.25. A 6-cm long cigarette will burn itself up in 15 min if no puff is taken. For every puff, it burns three times as fast as during the duration of the puff. If the cigarette burns itself in 13 min, then how many puffs has the smoker taken if his average puff lasted 3 s?
(a) 30 (b) 20
(c) 15 (d) None of these

Q.26. Two taps A and B can fill a vessel in 12 min and 15 min respectively, but a third tap C can empty the full tank in 6 min. A and B is kept open for 5 min in the beginning and then C is also opened. In what time will the vessel be emptied?
(a) 30 min (b) 33 min
(c) 45 min (d) None of these

Q.27. Two pipes A and B can separately fill a cistern in 15 and 10 min respectively and a waste pipe C can carry off 7 litres per min. If all the pipes are opened when the cistern is full, it is emptied in 2 hours. How many litres does the cistern hold?
(a) 30 litres (b) 40 litres
(c) 35 litres (d) 45 litres

Q.28. One pipe can fill a tank three times as fast as another pipe. If together the two pipes can fill the tank in 36 min, then how much time will the slower pipe take to fill the tank working alone?
(a) 81 min (b) 108 min
(c) 144 min (d) 192 min

Q.29. Two pipes A and B can separately fill a cistern in 10 min and 5 min respectively and a waste pipe C can carry off 5 litre per min. If all the pipes are opened when the cistern is full, it is emptied in 1 hour. What is the capacity of the cistern?
(a) 40 litres (b) 35 litres
(c) 95 litres (d) None of these

Q.30. Sanjay can do as much work in 16 hours as Balram can do in 24 hours. Pranesh can do the same work in 32 hours. If all of them work together, they can complete the same work in 24 hours. In how many hours can Pranesh alone complete the work?
(a) 80 (b) 88 (c) 104 (d) 112

Q.31. One fill-pipe A is 10 times faster than the second fill-pipe B. If B can fill a cistern in 55 min, then find the time when the cistern will be full if both the fill-pipes are opened together?
(a) 5 min (b) 6 min
(c) 7 min (d) 3 min

Q.32. A tank can be filled by one tap in 20 min and by another in 25 min. Both the taps are kept open for 5 min and then the second tap is turned off. In how much more time will the tank be filled?

(a) 11 min (b) 13 min
(c) 15.5 min (d) 18 min

Q.33. Fifteen men take 21 days working 8 hours each to do a piece of work. How many days of 6 hours each would 21 women take, if 3 women can do as much work as 2 men can do?

(a) 20 (b) 30
(c) 21 (d) 28

Q.34. Three hundred workers are set to build a dam. In 21 weeks they have done 60% of the work, but subsequent rains lasting 4 weeks washed away 25% of what they had done. The job is resumed by 200 workers. What is the additional time it will take to complete the work?

(a) 12 (b) 24
(c) 36 (d) None of these

Q.35. Tap A can fill a tank completely in 4 hours, while tap B can empty it in 8 hours. However the operator forgot to close the tap B, and as a result both the taps remained open. After 4 hours, the person realized his mistake and immediately closed the tap B. How much more time would the tank take to get full now?

(a) 1 hour (b) 2 hours
(c) 3 hours (d) 4 hours

Q.36. If work done by 2 men are equivalent to work done by 5 women and work done by 2 women are equivalent to work done by 5 children, then in how many days can the work be completed by one man alone if the same work can be completed by 100 children in 10 days?

(a) 120 days (b) 80 days
(c) 100 days (d) 160 days

Q.37. A certain number of people completed a job in 24 days. If there were 2 persons less, the work would have taken 6 more days. How many persons were there initially?

(a) 12 (b) 16
(c) 18 (d) 10

Q.38. Twelve men complete a work in 9 days. After they have worked for 6 days, 6 more men join them. How many days will they take to complete the remaining work?

(a) 2 days (b) 3 days
(c) 4 days (d) 5 days

Q.39. A certain number of men can finish a piece of work in 100 days. If however there were 10 men less, it would take another 10 days to finish the work. How many men were there originally?

(a) 50 (b) 75
(c) 82 (d) 110

Q.40. A Zamindar hired a certain number of men to plough his 480 acres of fields. Some of the men did not turn up at the job, and consequently each of those who reported had to plough 20 acres more of the field each than originally planned by the zamindar, assuming equal distribution of labour. How many men were initially appointed and how many abstained from the job?

(a) 15, 5 (b) 12, 8
(c) 10, 3 (d) 16,10

Q.41. Anoop takes 5 hours more than Mayank to build a wall. However, Anoop takes 9 hours more than Manoj. Anoop and Mayank together can do the same work in the same time as Manoj. How many hours will Anoop take to do it?

(a) 12 hours (b) 15 hours
(c) 18 hours (d) 24 hours

Q.42. A tank can be filled in by two taps A and B in 3 min. However, due to a leakage at the bottom of the tank, it takes 3.5 min to fill the tank. If tank is full, then how much time will the leakage take to empty the tank?

(a) 21 min (b) 12 min
(c) 24 min (d) 18 min

Q.43. A road construction agency takes up the work of making a road 189 km long in 15 months. He engages 256 men, but after 1/5th of the time, agency finds that only 1/6th of the work has been completed. How many more men should be now employed to complete the work in time?

(a) 57 (b) 59
(c) 64 (d) 67

Q.44. Twenty-five men are employed to do a piece of work in 24 days. After 15 days, 10 more men are engaged and the work was finished a day earlier than actually planned. In what time could they finish the work if extra men were not employed?

(a) 29.5 days (b) 24.6 days
(c) 26.2 days (d) 21.7 days

Q.45. Twelve men or 18 women can do some work in 7 days. In what time can 4 men and 8 women do the same work?

(a) 4 days (b) 8 days
(c) 7 days (d) 9 days

Q.46. A man works for 10 hours per day to finish a work in 3 days. A boy finishes the same in 5 days working 18 hours per day. How many boys are required to do the job in the time taken by one man?

(a) 1 (b) 2
(c) 3 (d) 4

Q.47. A tap can fill a tank in 60 min. Tap is turned on to fill the tank. When the tank is half full, it springs a leak at the bottom and the water level falls to 1/4 of the tank in 10 min. The tap is now closed. What is the time taken by the leakage to empty the tank now?

(a) 7 min (b) 4 min
(c) 8 min (d) None of these

Q.48. A piece of work can be done by A and B in 10 days, by B and C in 15 days and by C and A in 20 days. All three worked for 6 days and then A withdrew. B and C worked for another 4 days and then B stopped working. How many more days will C take to complete the remaining work?

(a) 12 (b) 12.5
(c) 10.33 (d) 10

Direction for questions 49 and 50: Read the passage given below and solve the questions based on it.

A group of 30 people can complete a job by working for 10 hours a day in 15 days. The group starts the work. But at the end of every day, starting from the first day, one person leaves the group and the remaining people work for 20 min less on the next day.

Q.49. On which day will the work be completed?

(a) 28th (b) 29th
(c) 30th (d) None of these

Q.50. What part of the work, approximately remains to be done at the end of 15 days?

(a) 0.51 (b) 0.61
(c) 0.49 (d) 0.39

MODERATE

Q.1. Mayank can do 50% more work than Shishu in the same time. Shishu alone can do a piece of work in 30 hours. Shishu starts working and he had already worked for 12 hours when Mayank joins him. How many hours should Shishu and Mayank work together to complete the remaining work?

(a) 6 (b) 12 (c) 4.8 (d) 7.2

Q.2. A man works twice as fast as a woman. A woman works twice as fast as a child. If 16 men can complete a job in 12 days, how many days would be required for 32 women, and 64 boys together to complete the same job?

(a) 2 day (b) 3 days
(c) 4 days (d) 6 days

Q.3. A tank with capacity T litres is empty. If water flows into the tank from pipe X at the rate of X litre per minute and water is pumped out by pipe Y at the rate of Y litres per minute and X>Y, then in how many minutes will the tank be filled?

(a) T/(Y-X) (b) T/(X-Y)
(c) (T-X) Y (d) (X-Y)/60T

Q.4. A can do a piece of work in 90 days, B in 40 days and C in 12 days. They work for a day each in turn, i.e., first day A does it alone, second day B does it alone and 3rd day C does it alone. After that the cycle is repeated till the work is finished. They get Rs 240 for this job. If the wages are divided in proportion to the work each had done. Find the amount A will get?

(a) 14 (b) 24 (c) 34 (d) 36

Q.5. There are 12 pipes connected to a tank. Some of them are fill pipes and the others are drain pipes. Each of the fill pipes can fill the tank in 8 h and each of the drain pipes can drain completely in 6 h. If all the pipes are kept open, an empty tank gets filled in 24 h. How many of the 12 pipes are fills pipes?

(a) 5 (b) 6 (c) 7 (d) 8

Direction for questions 6 and 7: Read the passage below and solve the questions based on it.

The tank at a water supply station is filled with water by several pumps. At first, three pumps of the same capacity are turned on; 2.5 hours later, two more pumps (both the same) of a different capacity are set into operation. After 1 hour, the additional pumps were set into operation; the tank was almost filled to its capacity (15 m^3 were still lacking); in another hour the tank was full. One of the two additional pumps could have filled the tank in 40 hours.

Q.6. What is the volume of the tank?

(a) 60 m^3 (b) 80 m^3 (c) 75 m^3 (d) 90 m^3

Q.7. How much water does one of the first three pumps fill in an hour?

(a) 5 m^3 (b) 4 m^3 (c) 3 m^3 (d) 2 m^3

Direction for questions 8 and 9: Read the passage below and solve the questions based on it.

A person can dig a trench 40 metres in depth in 4 days working 8 hours a day. However, after every day he finds that 1/5th of the depth got filled up with mud again.

Q.8. In how many days can he actually dig a trench if it is of 30 metres in depth?
(a) Exactly 4 days (b) On the 5th day
(c) Exactly 5 days (d) On the 6th day

Q.9. What was the depth of the trench at the beginning of the fourth day?
(a) 14.4 metres (b) 19.52 metres
(c) 23.616 metres (d) None of these

Direction for questions 10 to 12: Read the passage below and solve the questions based on it.

There are three taps A,B, C and an outlet pipe D. A, B and C can fill the tank in the Panikam locality in 10, 20 and 25 h respectively. The outlet pipe can empty the same tank in 100 h. There are 2,000 houses in the locality. The tank has a capacity of 50,000 litres.

Q.10. If all the taps and the outlet pipe are opened simultaneously, how much water is thrown into the tank every hour?
(a) 8000 litres (b) 9000 litres
(c) 1000 litres (d) Cannot be determined

Q.11. All the three taps were open and the emptying pipe is closed. At the time when the tank was supposed to be full, it was found that only 2/5th of the tank was full. It was discovered that all the residents had kept their water taps open during this period. At what rate were the residents of each house getting water? (Consider that each house has only one tap.)
(a) 1.1 litres/h (b) 2.22 litres/h
(c) 2.85 litres/h (d) 4.46 litres/h

Q.12. If the average requirement of every house in the area per day is 40 litres, for how many h, for the second time should the taps be opened? (Consider that all the three tapes were opened on both occasions).
(a) $3\frac{3}{19}$ h (b) $4\frac{1}{17}$ h
(c) 5 h (d) None of these

Direction for questions 13 and 14: Refer to the data below and answer the questions that follow

Fifty cows can graze a field for 40 days and 40 Cows can graze the same field for 80 days. Some grass is already there and the grass grows at a constant rate.

Q.13. For how many days can 60 cows graze the same field? (growth of grass is also there.)
(a) 20.66 (b) 50.66 (c) 60.66 (d) 26.66

Q.14. What is the maximum number of cows that can graze the same field for an infinite time?
(a) 30 (b) 25 (c) 8 (d) 35

Q.15. Anand got an order from a boutique for 480 shirts. He brought 12 sewing machines and appointed some expert tailors to do the job. However, many did not report to duty. As a result, each of those who reported had to stitch 32 more shirts than was originally planned by Anand, with equal distribution of work. How many tailors had been appointed earlier and how many had not reported to work?
(a) 12, 4 (b) 10, 3
(c) 10, 4 (d) None of these

Direction for questions 16 to 18: Refer to the data below and answer the questions that follow

Anoop and Sandeep can dig a well each in 10 and 5 days respectively. But on a particularly difficult terrain the work is such that due to fatigue every subsequent day the efficiency of a worker falls by 10%.

Q.16. If Anoop is given a task of digging one such well in the difficult terrain, then in how many days will he finish the work?
(a) 12th day (b) 15th day
(c) 11th day (d) never

Q.17. If Sandeep is given the task of digging one such well in the difficult terrain, then in how many days will he finish the work?
(a) 9th day (b) 7th day
(c) 8th day (d) Never

Q.18. If both Anoop and Sandeep work together to finish two such wells then, in how many days will the work finish?
(a) 11th day (b) 12th day
(c) 8th day (d) Never

Direction for questions 19 and 20: Refer to the data below and answer the questions that follow.

Anoop was writing the reading comprehension sections in the DOG entrance examinations. There were four passages of exactly equal length in terms of number of words and the four passages had 5, 8, 8 and 6 questions following each of them respectively. It is known that Anoop can answer exactly 12 questions in the time he takes to read any one of the four passages. Assume that his rate of reading and answering questions remains the same throughout the section.

Q.19. Anoop took 13 min more to finish the first three passages than the time he took to finish the last passage. Assuming that Anoop answered all the questions in each passage, What percentage of the total time did he spend on the first passage?
(a) 24.5% (b) 25.4%
(c) 22.6% (d) 26.2%

Q.20. By what per cent should Anoop increase his reading speed if he has to cut down on his total time spent on the section by 20%? Assume that the time spent on answering the questions is constant and as given in the directions.

(a) 36.36% (b) 54.54%
(c) 50.50% (d) 45.45%

Q.21. The kingdom of Charles had a circular wall around it and four-city gates pointing north, south, east and west respectively, Now the Shire of Nottingham and the Shire of Birmingham decided to race on the shortest route touching all the four gates. They start at the west gate and one of the shires rides five times as fast as the second. They will meet for the 19th time after they start, at the

(a) The north gate
(b) The south gate
(c) Exactly midway between the north and the west gate
(d) Exactly midway between the south and the east gate

Q.22. The packing process in a garment factory involves processing by 3 machines. The first machine, A, inserts the garment in a packet, after which the second machine, B, seals the packet and finally the third machine, C, attaches a price tag to the packet. Machine A takes at least 10 per unit. Machine B takes at least 15 per unit. Machine C takes at least 12 per unit. The factory starts at 8 a.m. and runs non-stop till 6 p.m., What is the maximum number of packets that can be processed in a one-hour period?

(a) 360 (b) 240 (c) 300 (d) 900

Q.23. P, Q and R can each complete a certain work in 16, 20 and 30 days respectively. The three of them start the work together. P leaves after 4 days; Q leave 4 days before the work is finished. How long did the work last?

(a) 12.6 days (b) 9.2 days
(c) 16.4 days (d) 11.4 days

Q.24. A certain work was started by 4 men and 10 women who completed 50% of the work in 6 days. Then another 2 men and 2 women joined them and they could complete 2/3rd of the remaining work in three days. How many men along with 6 women are required to complete the remaining work in two more days?

(a) 3 (b) 5 (c) 6 4.9

Q.25. A can work half as fast as B and C together. A and B together can work three times as fast as C. If all three of them complete a job in 15 days, how long would C take to complete the same work?

(a) 45 days (b) 54 days
(c) 60 days (d) 65 days

Q.26. A cistern can be filled by two taps A and B coming from the same main pipeline in 20 and 30 min respectively. Both the taps are opened at the same time but due to partial closing of a valve in the main pipeline, tap A was supplying only 4/5th of its capacity and tap B, 5/6th of its capacity. After some time, the valve in the main pipeline was opened fully thereby enabling the two taps to supply at full capacity. It took another 5 min to fill the cistern completely. How long (approx.) was it before the valve on the main pipeline opened?

(a) 7 minutes (b) 9 minutes
(c) 11 minutes (d) 13 minutes

Q.27. In what time would a cistern be filled by three pipes of diameter of 1 cm, 2 cm and 3 cm if the largest pipe alone can fill the cistern in 49 minutes, the amount of water flowing through each pipe being proportional to the square of its diameter?

(a) 31.5 minutes (b) 63 minutes
(c) 126 minutes (d) None of these

Q.28. Three workers— A, B and C— working simultaneously, produced 144 dyes in four hours. In one hour, A can produce as many dyes more than B as B can produce more than C. During a period of five hours, A can produce as many dyes as C can in seven hours. What is the difference between the number of dyes produced by B and C per hour?

(a) 1 (b) 2 (c) 3 (d) 4

Q.29. A can do a piece of work in 36 days, B in 54 days and C in 72 days. All three began the work together but A left after 8 days and B 12 days before the completion of the work. In all for how many days did C work?

(a) 24 days (b) 28 days
(c) 32 days (d) 36 days

Q.30. Three men with 5 boys can do a piece of work in 2 days and 4 men and 16 boys can complete the job in one day. How much time will it take for 1 boy together with a woman who can work twice as fast as the boy to complete a job that is 3 times as time consuming?

(a) 24 days (b) 28 days
(c) 32 days (d) 36 days

Q.31. Tatto, Bubby and Tappo are doing a work in 10 days. However, Tappo only worked for the first 3 days till 37/100th of total work is done. The remaining work was done by Tatto and Bubby alone. Efficiency of Tatto is 25% more than the efficiency of Bubby. What is the time taken by the fastest worker to finish the work alone?

(a) 16.66 days (b) 30 days
(c) 25 days (d) 20 days

Q.32. P works twice as fast as Q, whereas P and Q together can work three times as fast as R. If P, Q and R work together on a job, in what ratio should they share the earnings?

(a) 2:1:1 (b) 4:2:1
(c) 4:3:2 (d) 4:2:3

Q.33. Rakesh and Sid can complete a piece of work in 12 and 24 days respectively. Rakesh started the work and after a few days Sid joined him and the work was completed in 10 days. After how many days, did Sid join Rakesh?

(a) 6 (b) 7 (c) 8 (d) 10

Q.34. Sagar is 20% more efficient than Pranesh. If Pranesh can complete a piece of work in 30 days, then in how many days can both Sagar and Pranesh complete the work?

(a) (11+ 5/11) days (b) (13+ 7/11) days
(c) (15+ 5/11) days (d) (17+ 7/11) days

Q.35. Manoj and Harvinder can complete a piece of work in 16 days, which Harvinder can complete in 32 days. If Manoj and Harvinder work on alternate days, starting with Harvinder, in how much time would the work be completed?

(a) 17.25 days (b) 17.66 days
(c) 20 days (d) 32 days

Q.36. A factory manufactures dyes. It has 12 men and two machines which can be operated by all of its men. It takes 4 hours to manufacture one dye on the machine with the operator. The machines can work continuously without a break. Without the machine each of the men can manufacture a dye in 8 hours. The policy is such that the production is maximized and the men are ready to work in three shifts of 8 hours per day. What will be the average cost incurred per dye if 1 man hour costs Rs 20 and 1 machine hour costs Rs 15?

(a) Rs 140 (b) Rs 160
(c) Rs 147 (d) Cannot be determined

Q.37. Ramesh and Suresh could finish the garden cleaning in ‘p’ days. They worked together for ‘q’ days; then Ramesh fell sick and the remaining cleaning was completed by Suresh in ‘r’ days. In how many days could each of them clean the garden?

(a) Ramesh = $\dfrac{pr}{r-p+q}$ days; Suresh = $\dfrac{pr}{p-q}$ days

(b) Ramesh = $\dfrac{p}{r-p+q}$ days; Suresh $\dfrac{p}{p-q}$ days

(c) Ramesh = $\dfrac{pr}{r+p-q}$ days; Suresh $\dfrac{pr}{q-p}$ days

(d) Ramesh = $\dfrac{pr^2}{r-p+q}$ days; Suresh $\dfrac{pr^2}{q-p}$ days

Q.38. A, B, C, D and E are five taps. The capacity of B is 2 times that of A, the capacity of C is 3 times that of A. Capacities of D and E are 4 and 5 times that of A respectively. In the first case A, C and E act as input pipes and B and D act as output pipes.

In the second case, C, D, E act as input pipes and A and B act as output pipes.

If A and B working together as input pipes can fill the tank in 4 hours, then what is the difference in the time required to fill the tank in the first and second case stated above?

(a) 4.44 hours (b) 3.33 hours
(c) 2.22 hours (d) 1.11 hours

Q.39. A tank has 5 input pipes, A, B, C, D and E. Among them A takes the maximum time to fill the tank. Their capacities to fill the tank are in “Geometric Progression (GP)” in the given order with common ratio 2. The tank also has 5 output pipes F, G, H, I and J. Among them F takes the maximum time to empty the tank. G’s capacity to empty the tank is 2 times that of F. H’s capacity? is times that of F and so on. Working together A and F can fill the tank in 100 h. and all the taps working together can fill the tank in $\left(\dfrac{100}{63}\right)$ hr. How much time will E and J take to fill the tank when working together?

(a) $\dfrac{100}{62}$ (b) $\dfrac{100}{38}$

(c) $\dfrac{100}{48}$ (d) Cannot be determined

Q.40. A man makes 60 articles in the 1st hour. His efficiency decreases by 25% in the 2nd hour, increases by 40% in the 3rd hour, decreases by 33% in the 4th hour and increases by 50% in the 5th hour. If he has to work for more than 1 hour, then in which hour the average number of articles produced per hour till then would be minimum?

(a) The 2nd hour
(b) After the 5th hour
(c) The 2nd and the 4th hour
(d) None of these

Q.41. A worm climbs a vertical bamboo stick 32 cm tall. The worm climbs 3 cm in the day and slips 2 cm during the night each time. However, there are cavities in the stick at 2 cm, 4 cm,16 cm from the bottom that would not let the worm slip further. If the worm climbs only during the daytime and slips only during the night, on which day from the start will it reach the top?

(a) 29 days (b) 28 days
(c) 27 days (d) 26 days

Q.42. Anil, Bhuvan and Chandan take 10, 20 and 25 days to complete a job. What is the minimum time required to finish the job if not more than 2 of them work together on a single day and no two consecutive days have the same pair of people working?

(a) 6.5 days (b) $6\dfrac{13}{15}$ days

(c) 7.5 days (d) $7\dfrac{21}{28}$ days

Q.43. A swimming pool is fitted with three pipes. The first two pipes working simultaneously fill the pool in the same time as the third pipe alone. The second pipe alone fills the pool 5 hours faster than the first pipe and 4 hours slower than the third pipe. In what time will the second and the third pipe together fill the pool?

(a) 3.25 h (b) 6.5 h

(c) 5 h (d) None of these

Q.44. A contractor needs to complete a given work in 24 days. For every day that he finishes ahead of time, he gets an incentive 'I' and for every day that he finishes behind time, he needs to pay a penalty 'P'. He has a choice of 2 workers who work as follows: A completes the work in 24 days for 75% of the time and in 30 days for 25% of the time. B completes the work in 20 days for 60% of the time and in 30 days for 40% of the time. At what ratio of P to I will the contractor be indifferent of using A or B?

(a) $\frac{8}{3}$ (b) 2

(c) $\frac{7}{3}$ (d) $\frac{7}{4}$

Q.45. Anuj can do a piece of work in a certain number of days. To do the same piece of work, Bhanu takes thrice the number of days as Anuj takes whereas Chandu takes thrice as many days as Bhanu does and Dodo takes thrice as many days as Chandu does. Now, they are paired and two groups are formed. The first pair takes one-third the time taken by the second pair to complete the work. which is the first pair?

(a) Anuj and Dodo

(b) Anuj and Chandu

(c) Chandu and Bhanu

(d) Bhanu and Dodo

ADVANCED

Direction for questions 1 to 5: Read the passage below and solve the questions based on it.

There are infinite pipes attached to a very big tank. Pipes are numbered like P_1, P_2, P_3... and so on. Its also known that efficiency of every subsequent pipe is half the efficiency of earlier pipe i.e., the efficiency of P_2 pipe is half the efficiency of P_1 pipe and so on. However, in case of P_1 pipe, this rule is not true since there is no earlier pipe. When all the pipes are working together, tank gets filled in 2 h.

Q.1. What time will P_4 take to fill the tank working alone?

(a) 8 (b) 16

(c) 32 (d) Cannot be determined

Q.2. What is the difference in the time taken by P_5 and P_6?

(a) 32 (b) 64

(c) 128 (d) Cannot be determined

Q.3. How many pipes can fill the tank within 100 h working alone?

(a) 4 (b) 5 (c) 6 (d) Infinite

Q.4. Due to some technical problem, only four pipes P_1, P_2, P_3 and P_4 are in working condition. These pipes are now grouped to obtain two pairs of pipes. It is found that one pair of the pipe is taking 2/3rd of the time taken by the other pair to fill the tank independently. Which of the following is one of the two pairs?

(a) P_1, P_2 (b) P_1, P_3

(c) P_1, P_4 (d) Cannot be determined

Q.5. What time will P_2 take to fill the tank working alone?

(a) 4 (b) 8 (c) 16 (d) 32

Direction for questions 6 and 7: Read the passage below and solve the questions based on it.

There are 2 filling taps A and B and an emptying tap C which are used in a laboratory to fill or empty bottles. The three taps are attached to a computer and they function according to the commands given by the computer. The following are the commands that the computer gives:

Command	*Effect*
F(A, x, t_1)	Fill the bottle with tap A at *x* cc/min for t_1 min
F(B, y, t_2)	Fill the bottle with tap B at *y* cc/min for t_2 min
E(C, z, t_3)	Empty the bottle with tap C at *z* cc/min for t_3 min

Q.6. The following commands, in the given order, are used to fill an empty bottle and each command can be executed only after the previous command is completed. What is the final volume after these commands?

F (A, 10, 600), E (C, 50, 20)
F (A, 10, 400), F (B, 20, 200)
F (A, 50, 100), F (B, 20, 50)
E (C, 10, 50), F (B, 20, 75)

(a) 15 (b) 20
(c) 22 (d) None of these

Q.7. The commands F (A, 10, 500) and (B, 40, 500) can fill an empty bottle. Which of the following commands can now empty the bottle?
(a) E (C, 20, 500) (b) E (C, 50, 500)
(c) E (C, 100, 200) (d) None of these

Q.8. A group of workers can complete a certain job in 9 days. But it so happens that every alternate day starting from the second day, 2 workers are withdrawn from the job and every alternate day starting from the third day, one worker is added to the group. In such a way, the job is finished by the time, there is no worker left. If it takes the double time to finish the job now, find the number of workers who started the job?
(a) 5 (b) 10 (c) 15 (d) 20

Q.9. A student studying the weather for d days observed that (i) it rained on 7 days, morning or afternoon; (ii) when it rained in the afternoon, it was clear in the morning; (iii) there were five clear afternoons and (iv) there were six clear mornings. Then d equals
(a) 3 (b) 7 (c) 11 (d) 9

Q.10. Tanveer constructs a wall working in a special way and takes 12 days to complete it. If S_n is the length of the wall (in m) that he constructs on the nth day, then
$S_n = 2n,\ 0 \le n \le 4$
$S_n = 8$, for $n = 5$
$S_n = 3n - 7,\ 6 \le n \le 12$
Find the total length of the wall he constructs in the first 10 days.
(a) 31 m (b) 35 m
(c) 113 m (d) None of these

Direction for questions 11 and 12: Read the passage below and solve the questions based on it.

There are infinite number of taps that are filling a certain tank. The taps are numbered 1, 2, 3 and so on. Any tap numbered p ($p \ge 2$) can fill the tank in ($p^2 - 1$) min while the first tap does not work at all.

Q.11. If all the taps from the second tap onwards are opened simultaneously, then what is the shortest time in which the tank can be filled?
(a) 11/3 min (b) 22/3 min
(c) 2/3 min (d) 11/5 min

Q.12. The minimum number of taps that must be opened so that the tank is filled within 106 seconds is
(a) 6 (b) 3 (c) 5 (d) 4

Q.13. Alok, Mithilesh and Bimlesh started a work and after completing 1/5th of the work Bimlesh left. Alok and Mithilesh then worked for 20 days. Bimlesh then took over from Alok and Mithilesh and completed the remaining portion of the work in 12 days. If Bimlesh takes 40 days to complete the work, in how many days would Alok alone or Mithilesh alone complete the work if the efficiencies with which they work is the same?
(a) 40 (b) 60
(c) 100 (d) None of these

Q.14. Sixty-four men working 8 h a day plan to complete a piece of work in 9 days. However, 5 days later they found that they had completed only 40% of the work. They now wanted to finish the remaining portion of the work in 4 more days. How many hours per day should they need to work in order to achieve the target?
(a) 11 (b) 12
(c) 13 (d) 15

Direction for questions 15 and 16: Read the passage below and solve the questions based on it.

The employees of ABC Corporation put forward demands for a shorter working week and a better pay. They were paid by the hour and worked 5 days a week. Their demands were:

(i) the working days to be shorter by 1 h and
(ii) in the course of the shorter working day that they are demanding, payment for a working day be the same as the number of h needed to earn Rs 49.

At present, a full day's work brings in Re 1 less than that and as many rupees as it is necessary to work and earn Rs 40.

Q.15. For how many hours were the employee proposing to work?
(a) $8\frac{1}{2}$ (b) 10 (c) $10\frac{1}{2}$ (d) 9

Q.16. What was the revised rate demanded per hour?
(a) Rs $1\frac{1}{4}$ (b) Rs $2\frac{1}{3}$
(c) Rs $2\frac{3}{4}$ (d) Rs $2\frac{1}{2}$

Direction for questions 17 to 19: Read the passage given below and solve the questions based on it.

Mr Bose can do a job in two days if his four secretaries A, B, C and D are all present. His efficiency got reduced to $\frac{1}{6}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}$ respectively as A, B, C and D remain absent. In any day only one secretary remains absent. Also Mr Bose finishes his work exactly in integral number of days.

Q.17. Which of the following secretaries cannot be absent for an odd number of days?
(a) A (b) B (c) C (d) D

Q.18. If each secretary remained absent for at least one day, then in how many days will Mr Bose finish the work?

(a) 5, 6 or 7 days (b) 6, 7 or 8 days
(c) 6, 8 or 9 days (d) 7, 8 or 9 days

Q.19. If Mr Bose finished his work in 6 days with one secretary being present every day, then the secretary was

(a) A (b) B
(c) C (d) Cannot be determined

Direction for questions 20 to 22: Read the passage given below and solve the questions based on it.

A team of N workers are selected to do a certain job. Each worker can individually complete the job in 10 days. The team follows the following procedure for any job:

The work is started by one person on the first day. Then, from the next day onwards, till the work is completed, everyday one new person joins the work.

The work is such that, on any day it starts at 9.00 am and stops at 5.30 pm with a 30-min break for lunch from 1.00 pm to 1.30 pm. At whatever stage a worker joins the work, he does not leave till the job is over.

Answer each question in this set independently of the other questions.

Q.20. If the entire team works at only 50% efficiency, and the worker who is supposed to join on the last day does not report for work, then the work is completed

(a) Exactly at the end of the 6th day
(b) Before lunch, on the 6th day
(c) After lunch, before the end of the 6th day
(d) Before lunch, on the 7th day

Q.21. If the team takes up a task which is twice as time consuming as the original work, then the work will be completed on the

(a) 5th day (b) 6th day
(c) 7th day (d) 8th day

Q.22. If the team takes up a task which is twice as time consuming as the original work, then at what time on the concluding day will the work be completed ?

(a) 3.40 pm (b) 4.20 pm
(c) 5.30 pm (d) 4.10 pm

Q.23. The rate of flow of water (in litre per min) of three pipes are 2, N and 3, where $2 < N < 3$. The lowest and the highest flow rates are both decreased by a certain quantity x, while the intermediate rate is left unchanged. If the reciprocals of the three flow rates, in the order given above, are in arithmetic progression both before and after the change, then what is the quantity x (in litre per min)? (Negative flow rates indicate that the pipes act as emptying pipes instead of filling pipes.

(a) 2.6 (b) −3.2 (c) 3.8 (d) −2.6

Direction for questions 24 to 26: Read the passage below and solve the questions based on it.

Two products A and B can be made on either of the two extrusion machines M_1 or M_2. The time in min, taken by these products according to the machine used, are as given below. Assume a day has 8 working hours.

	M_1	M_2
A	8	10
B	6	6

Q.24. If A and B are produced in the ratio of 3 : 1, then what is the maximum total of the products A and B that can be produced in a day?

(a) 120 (b) 160
(c) 116 (d) None of these

Q.25. If A is produced only on Machine M_1 and B produced only on machine M_2, then what is the unutilized time on M_2 in a day, if A and B are produced in 1 : 1 ratio?

(a) 20 min (b) 60 min
(c) 150 min (d) None of these

Q.26. If each product requires working on both the machines and the time required on each machine is as indicated in the table, then what is the maximum production possible in a day, assuming only A is produced?

(a) 80 (b) 180
(c) 48 (d) None of these

Direction for questions 27 to 30: Read the passage below and solve the questions based on it.

There are three bottles of water- A, B and C whose capacities are 5 litres, 3 litres and 2 litres respectively. For transferring water from one bottle to another and to drain out the bottles, there exists a programe which is computer-controlled. The computer that controls the flow through these pipes can be fed with three types of instructions, as explained below:

Instruction type	*Explanation of the instruction*
Fill (x, y)	Fill bottle labelled x from the water in bottle labeled y, where the remaining capacity of x is less than or equal to the amount of water in y.
Empty (x, y)	Empty out the water in bottle labelled x into bottle labelled y, where the amount of water in x is less than or equal to remaining capacity of y.
Drain (x)	Drain out all the water contained in bottle labelled x.

Initially, A full of water and B and C are empty.

Q.27. First instruction: Fill (B, A), Second instruction: Fill (C, B)
What is the quantity of water in B after executing all the instructions?
(a) 1 litre (b) 2 litres
(c) 3 litres (d) None of these

Q.28. After executing a sequence of three instructions, bottle A contains one litre of water. The first and the third of these instructions are shown below
First instruction: Fill (C, A)
Third instruction: Fill (C, A)
Then, which of the following statements about the instructions is true?
(a) The second instruction is Fill (B, A)
(b) The second instruction is Empty (C, B)
(c) The second instruction transfers water from B to C
(d) The second instruction involves using the water in bottle A

Q.29. Consider the same sequence of three instructions and the same initial state mentioned in the last question. Three more instructions are added at the end of the above sequence to have A contain 4 litres of water. In this total sequence of six instructions, the fourth one is Drain (A). This is the only Drain instruction in the entire sequence. At the end of the execution of the above sequence, how much water is contained in C?
(a) 1 litres (b) 2 litres
(c) 0 (d) None of these

Q.30. Initially, A had 5 litres, B had 2 litres and C had 1 litre. What is the minimum number of times Fill (x, y) is required to be used so that A is empty, provided we can not use Empty (x, y) instruction?
(a) 1 (b) 2
(c) 3 (d) None of these

Direction for questions 31 to 35: Read the passage below and solve the questions based on it.

Modern Boutique received a large order for stitching uniforms from A. R. Academy and T. M. High School. He has two cutters who will cut the fabric, five tailors who will do the stitching, and two assistants to stitch the buttons and button-holes. Each of these 9 persons will work for exactly 10 hours a day. Each of the A. R. Academy uniform requires 20 min for cutting the fabric, 1 hour for doing the stitching and 15 min for stitching the buttons and the button holes. The T. M. High School uniform requires 30 min, 1 hour and 30 min respectively for the same activities.

Q.31. Find the maximum number of T. M. High School uniforms that Modern Boutique can complete in a day.
(a) 30 (b) 35 (c) 40 (d) 36

Q.32. On a particular day, Modern Boutique decided to complete 20 T. M. High School uniforms. How many A. R. Academy uniforms can it complete on that day?
(a) 30 (b) 35 (c) 40 (d) 36

Q.33. If Modern Boutique decided to complete 30 T. M. High School uniforms only and no other uniform on that particular day, how many total man-hours will go idle?
(a) 30 (b) 60 (c) 45 (d) 36

Q.34. If Modern Boutique hires one more assistant, what is the maximum number of A. R. Academy uniforms that can be completed in a day?
(a) 60 (b) 48 (c) 120 (d) 50

Q.35. Modern Boutique has the option to hire one more employee of any category. Whom should it hire to get the maximum increase in the production capacity, assuming that it needs to stitch only A. R. Academy uniforms on that day?
(a) One cutter
(b) One tailor
(c) One assistant
(d) Either of the tailor or the cutter

Q.36. Two poles, one 78 m in height above the ground and the other 91 m in height above the ground, are at some distance from each other. Two strings are tied, one from the top of one pole to the bottom of the other and the other from the top of the second pole to the bottom of the first. What is the height above the ground at which the string meets?
(a) 52 m (b) 86 m (c) 42 m (d) 48 m

Q.37. The daily work of 2 men is equal to that of 3 women or that of 4 youngsters. By employing 14 men, 12 women, and 12 youngsters a certain work can be finished in 24 days. If it is required to finish it in 14 days and as an additional labour, only men are available, how many of them will be required?
(a) 20 men (b) 18 men
(c) 48 men (d) 28 men

Q.38. Two Pipes A and B can fill a cistern in $12\frac{1}{2}$ and 15 hours respectively. If the pipes can be opened or closed only after every 30 min (i.e., at 6.00, 6.30 etc.) and the tank is to be filled by using both the pipes without any overflow then,
(a) Pipe B should be turned on later
(b) A should be turned on later
(c) A should be turned off after some time.
(d) Both the pipes should remain on till the tank is full

Q.39. A, B and C are assigned a piece of job which they can complete by working together in 15 days. Their efficiencies (measured in terms of rate of doing job) are in the ratio of 1:2:3. After 1/3rd of the job is

completed, one of them has to be withdrawn due to budget constraint. Their wages per day are in the ratio of 3 : 5 : 6. The number of days in which the remaining two persons can finish the job (at optimal cost) is
(a) 9 (b) 12 (c) 15 (d) 18

Direction for questions 40 and 41: Read the passage below and solve the questions based on it.

A group of 10 people, each with a different capacity to work, is assigned to complete a certain work. They work in such a way that only two people work on any given day, with no group of the same two people working on more than one day. By working in this manner, they could finish only 3/4th of the work by the time all the possible pairs of people had worked once. The remaining part of the work was then finished by all of them working together.

Q.40. What is the number of days for which all of them worked together?
(a) 135 days (b) 120 days
(c) 45 days (d) 3 days

Q.41. If on the last day before the end of the work, the work assigned to the group is doubled, then find the total number of days from the start that the group took to complete the work, assuming that all the people continue to work together?
(a) 240 days (b) 160 days
(c) 90 days (d) 60 days

Direction for questions 42 to 46: Read the passage below and solve the questions based on it.

ABC is a firm that deals with furniture. Manufacturing of a table requires three levels of assembly. The finished table is at the first level. The leg assembly and the tabletop are at the second level. The pieces that go into the leg assembly are at the third level which consist of shortrails, longrails and legs. One unit of the table requires one unit of the tabletop and one unit of the leg assembly. One unit of the leg assembly requires 2 units of shortrails, 2 units of longrails and 4 units of legs. The orders are placed just in time to minimize storage.

The lead time for activities are (Lead time is the waiting time required to complete one activity.):

Parts	*Weeks*
Assemble table	1
Finished leg assembly	1
Purchase legs	1
Purchase shortrails	1
Purchase longrails	1
Purchase table top	2

The availability of the parts at the present time

Parts	*Units*
Table	50
Leg assembly	100
Legs	150
Shortrails	50
Longrails	0
Table top	50

Demand of the finished tables:

Details	*Week 4*	*Week 5*	*Week 6*
Demand (units)	200	150	100

Q.42. For meeting the demand of 200 units of the finished table of week 4, when should the first order of the tabletops be placed?
(a) Week 1 (b) Week 3
(c) Week 5 (d) Week 5

Q.43. What is the net requirement of legs for meeting the demand of the finished table of week 4?
(a) 200 (b) 50 (c) 400 (d) 800

Q.44. When and how many units of shortrail would be placed for meeting the demand of the finished table of week 6?
(a) 100 units in week 1
(b) 200 units in week 3
(c) 300 units in week 6
(d) Data insufficient

Q.45. Due North Inc. needs a new kind of table in which 8 legs and 4 short rails would be needed for manufacturing a single table. What would be the net requirement of legs and shortrails for meeting the total demand of the finished table (including the additional demand of 200 tables for due North inc.) in week 5?
(a) 1800, 900
(b) 2200, 1100
(c) 1600, 800
(d) 800, 400

Q.46. The supplier of longrails has shifted its manufacturing units to a new location and owing to this, the delivery time of the long rail has been increased by 1 week. When should the order of longrails be placed to meet the week 5 demand of the finished table?
(a) Week 0 (b) Week 1
(c) Week 2 (d) Week 3

Q.47. Labour allocation is a very important process. A particular weaving section has 20 looms and with five labourers, loom efficiency is 75%. The production of a loom at 100% efficiency is 10 m/h. Salary of a labourer is Rs 11,000 per month. I removed one labourer due to which the efficiency came down to 70%. How much do I gain or loose due to this action? (Assume that the profit on 1 m cloth is Rs 4 and the looms are working for 30 days in a month and 10 hours per day.)

(a) Rs 1,000 profit
(b) Rs 1,500 loss
(c) Rs 1,500 profit
(d) Rs 1,000 loss

Q.48. To make an article, it takes 40 hour for a workman who is paid Rs 1.80 per hour. 20% the material of is wasted in the course of working, which costs Rs 22.5 per kilogram. At what price must the article be sold so as to yield a profit of 33.33%, if its weight was measured to be 8 kg?

(a) Rs 360 (b) Rs 404
(c) Rs 396 (d) None of these

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(d)	(c)	(a)	(b)	(c)	(b)	(a)	(a)	(a)	(b)	(d)	(a)	(b)	(c)	(b)	(d)	(a)	(c)	(d)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(a)	(d)	(c)	(c)	(c)	(c)	(d)	(d)	(b)	(c)	(c)	(a)	(b)	(b)	(b)	(c)	(a)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(a)	(c)	(c)	(d)	(c)	(b)	(c)	(c)	(c)	(a)	(a)	(b)	(b)	(d)	(d)	(d)	(a)	(d)	(b)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(b)	(a)	(c)	(c)	(d)	(c)	(d)	(d)	(b)	(d)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(b)	(b)	(c)	(a)	(b)	(b)	(b)	(b)	(c)	(a)	(d)	(a)	(c)	(d)	(b)	(c)	(c)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(b)	(d)	(c)	(c)	(b)	(a)	(b)	(a)	(b)	(d)	(a)	(a)	(b)	(d)	(c)	(a)	(a)	(b)	(c)
Q. No.	41	42	43	44	45															
Answer	(d)	(b)	(d)	(a)	(b)															

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(b)	(c)	(b)	(d)	(b)	(a)	(d)	(c)	(a)	(c)	(c)	(d)	(d)	(b)	(b)	(b)	(d)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(d)	(a)	(a)	(c)	(d)	(a)	(b)	(c)	(b)	(c)	(a)	(a)	(d)	(b)	(c)	(a)	(a)	(b)	(d)
Q. No.	41	42	43	44	45	46	47	48												
Answer	(d)	(a)	(b)	(b)	(b)	(b)	(d)	(c)												

HINTS AND SOLUTIONS

Moderate

1. Ratio of efficiency of Mayank and Shishu = 3/2
So ratio of time taken by Mayank and Shishu = 2/3
So if Shishu takens 30 hours, then Mayank will take 20 hours
Shishu in 6 hours = 1/5th work.
Remaining work = 1 – 1/5 = 4/5th work, 1 1 1
Shishu and Mayank together $= \frac{1}{20} + \frac{1}{30} = \frac{1}{12}$
So required time $= \frac{4/5}{1/12} = 9.6$ hours

2. 1 men = 2 women and 1 women = 2 boys.
So 1 men = 2 women = 4 boys.
Alternatively, assume work done by a child = x units/day, then work done by a woman and a man will be equal to $2x$ units per day and $4x$ units per day.
Now 16 men are completing the work in 12 days, so total work done = 4x × 16 × 12. Equate this with the supplied work force to obtain the answer.

4. Let the total amount of work = 180 unit
A does = 2 units/day
B does = 4.5 units/day
C does = 15 units/day
To finish 180 units of work, every body has to work for 9 days.
In 9 days A will do = 9 × 2 = 18 units work.
So A's contribution $= \ldots = \frac{18}{180} \ldots \frac{1}{10}$
So out of Rs 240, A's share = 240 × 1/10 = 24

6. First three pumps are on for 4.5 hours and two additional pumps for 2 hours. Let any one of the first three pipes takes x hours to fill the tank and it is given that any one of the two additional pumps take 40 hours to fill the tank
So according to question,
$$\frac{4.5}{x} + \frac{4.5}{x} + \frac{4.5}{x} + \frac{2}{40} + \frac{2}{40} = 1$$

10. Water filled into the tank by all pipes/hour
$$\frac{1}{10} + \frac{1}{20} + \frac{1}{25} - \frac{1}{100}$$

11. If emptying pipe is closed, then it takes 100/19 hours to fill the tank.
But, only 2/5th of 50,000 = 20,000 litres is filled.
So in 100/19 hours, 30,000 litres water drained
Hence in 1 hour, 5700 litres water drained.
So rate at which each house gets water.
$= \frac{5700}{2000} = 2.85$ litres/hours

12. For the second time, the total amount of water needed.
= 40 × 2000– 50,000 = 30,000 liters, this amount in 3/4th of total
Let pipes A, B and C are opened for x hours.
So, x (1/10 + 1/20 + 1/25) = 3/5 = > x = 60/19

15. Solve using options.

16. Lets assume that digging one well = 40 units work.
So, A does = 4 units/day, B does = 8 units/day.
Now efficiency is falling by 10%, so it is a case of Geometric progression.
$4, \frac{18}{5}, \frac{81}{25}, \ldots$ Where common ratio = 9/10
Now sum of an infinite GP = a/(1 − r)
$= \frac{4}{1 - 9/10} = 40$

Now it is clear that 'A' takes infinite time, so he will never finish the work.

17. B does = 8 units/day

$$\text{Using}\ \frac{4\left[1-(9/10)^2\right]}{1-9/10}=40 \Rightarrow n \cong 7$$

18. A and B together do = 4 + 18 = 12 units/day
Now proceed the same way, as in the previous question.

19. Let's assume that time to read one passage = x
So according to the question, $3x + 21x / 12 - 13 = 6x + x/12 \Rightarrow x = 4$
So total time taken for answering all questions = $4x\ 4 + 4x\ 27/12$ = 25 minutes
Time spent on first passage = $4 + 4x\ 5/12 = 17/3$ minutes
Required percentage = 22.6%

20. As we have found in the previous question that total time spent = 25 minutes.
If total time spent is cut down by 20%, then time = 20 minutes.
Now, let's assume increased reading speed = x
So $20 = 4x + 4x\ 27/12 \Rightarrow x = 11/4$
Hence percentage increase = 45.45%

25. Let us assume that
A finishes the work alone in a days
B finishes the work alone in b days
C finishes the work alone in c days
So according to the question,

$$\frac{1}{a}=\frac{1}{2}\left(\frac{1}{b}+\frac{1}{c}\right) \qquad \text{(i)}$$

$$\frac{1}{c}=\frac{1}{3}\left(\frac{1}{c}+\frac{1}{b}\right) \qquad \text{(ii)}$$

$$\text{nd}\ \frac{1}{a}+\frac{1}{b}+\frac{1}{c}=\frac{1}{15} \qquad \text{(iii)}$$

Using these options, C = 60 days

26. Let's assume the cistern's capacity = 60 litres
So A supplies = 3 litres/min, B supplies = 2 litres/min.
Now, due to closing of valve,
A supplies = = 3 × 4/5 = 12/5 litres/min.
B supplies = = 2 × 5/6 = 5/3 litres/min.
In last 5 minutes cistern is filled at full capacity of pipes, and it is filled by = 5 × 5 = 25 litres.
Remaining capacity = 60 – 25 = 35 litres
So required time ≈ 9 minutes

27. Since the amount of water flowing through each pipe is proportional to square of its diameter so if efficiency of longest pipe (3 cm) = 1/49
Then efficiency of pipe (2 cm) = 4/(49 × 9)
and efficiency of pipe (1 cm) = 1/ (49 × 9)

Now let cistern is filled by all three pipes in x minutes.

$$\text{So,}\ \frac{1}{49}+\frac{4}{49\times 9}+\frac{1}{49\times 9}=\frac{1}{x}$$

$\Rightarrow x$ = 31.5 minutes

28. In four hours, A + B + C = 144 – (i)
And given, A – B = B – C => A + C – 2B = 0 – (ii)
From (i) and (ii), B = 48, Hence A = 56 and C = 40
So B–C = 48 – 40 = 8 hours.
It is for 4 hours so per hours = 8/4 = 2 hours

32. Ratio of efficiency, P = 2Q – (1)
And P + Q = 3R – (2)
From (1) and (2), Q = R
P = 2Q = 2R
Hence P : Q : R = 2:1:1

34. Pranesh takes 30 days to finish the work.
Since Sagar is 20% more efficient than Pranesh, he will take 16.66% less time.
So Pranesh takes = 25 days
So work done by both of them in one day = (1/30) + (1/25)

35. Let's assume the total amount of work = 32 units
So Manoj and Harvinder does = 2 unit/day
Harvinder does = 1 unit/day
So Manoj does = 1 unit/day
So 32 units of work will be completed in 32 days.

36. To maximize the production, four men will work in each shift.
2 men will work with machines and 2 men work alone.
Total cost incurred in one hour

$$=\frac{2\times 20+(2\times 20+2\times 15)}{6}=110/6$$

So total cost incurred in 8 hour (per die) = (110/6) × 8 = 146.66

37. Let Suresh take x days to complete the work. So in one day Suresh does = $1/x$.
Given total efficiency of Ramesh and Suresh in one day = 1/P

$$\text{So Ramesh} = \frac{1}{P}-\frac{1}{x}$$

$$\text{According to question} = \frac{q}{P}+r(1/x)=1$$

$$\text{So,}\ x=\frac{Pr}{P-q}$$

$$\text{So, Suresh takes} = \frac{Pr}{P-q}$$

$$\text{So, Ramesh takes} = \frac{Pr}{r-P-q}$$

38. Let B take x hours to fill the tank,
So according to question, time required by each
$A = 2x$
$B = x$
$C = 2x/3$
$D = 2x/4$
$E = 2x/5$
A and B together fills the tank in 4 hours
$$\frac{1}{2x}+\frac{1}{x}+=\ldots\frac{1}{4}\Rightarrow x=6$$
In first case, when A, C and E act as input pipes and B and D as output pipes,
Total time = 1/12 + 1/4 + 5/12 – 1/6 – 1/3 = 4 hours
In 2nd case, when C, D and E act as input pipes and A and B as output pipes,
Total time = 1/4 + 1/3 + 5/12 – 1/12 – 1/6 = 4/3 hours

40. Solve using options.

42. Assume total amount of work = 100 units
A does = 10 units/day, B does = 5 units/day and C does = 4 units/day
Possible Pairs:
A + B = 15 units/day, A + C = 14 units/day, B + C = 9 units/day
To minimize time, we will use the first two pairs.
So, 15 + 14 + 15 + 14 + 15 + 14 + 15 = 102 Units
So, 7 days are required.

45. Let Anuj do the work in x days,
So, Bhanu = $3x$ days, Chandu = $9x$ days and Dodo = $27x$ days
Now, using options,
Anuj + Chandu = $1/x + 1/9x = 10/9x$, So, they take $9x/10$ days
Bhanu + Dodo = $1/3x + 1/27x = 10/27x$, So, they take $27x/ 10$ days
Now, 1/3 of $27x/ 10 = 9x /10$, So, Anuj and Chandu is first pair.

Advanced

[illegible]

Let efficiency of Pipe P1 – e
Let efficiency of Pipe P2 – e/2
Let efficiency of Pipe P3 – e/4
Let efficiency of Pipe P4 – e/8 and so on.
Now e, e/2, e/4, e/8… are in Geometric progression
So total efficiency = $\frac{e}{2}=e\frac{e}{4}+\ldots\frac{e}{8}+\ldots+\ldots+\ldots\infty$
All pipes together means can fill the tank in 2 hours.
So pipe P1 with efficiency e will take = 2 × 2 =4 hours

Similarly P2 = 8 hours
P3 = 16 hours
P4 = 32 hours
P5 = 64 hours
P6 = 128 hours

[illegible]

6.
F(A,10,600), E(C,50,20) = + 6000 – 1000
F(A,10,400), E(B,20,200) = + 4000 + 4000
F(A,50,100), F(B,20,50) = + 5000 + 1000
E(C,10,50), F(B,20,75) = – 500 + 1500
Final Volume = 20,000

7. F(A, 10, 500), F(B,40,500) = 5000 + 20,000
= 25000
Now option (b) E(C, 50,500) gives same amount 25000 to be emptied.

9. Let x = Number of days it rained in the morning and had clear afternoons.
y = Number of days it rained in the afternoon and had clear mornings.
z = Number of days it rained in the morning or afternoon
So according to question, $x + y = 7$
$x + z = 5$
$y + z = 6$
Adding all three equations, $x + y + z = 9$
So, d = 9 days

10. Let us make a table of the units of work everyday:

Day	1	2	3	4	5	6	7	8	9	10
Length	2	4	6	8	8	11	14	17	20	23

Total length = 113 m

13. Ratio of efficiency of Sanjay : Balram : Pranesh = 1/16 : 1/24 : 1/32 = 6 : 4 : 3
Work finished in 24 hours, so in 1 hour = 1/24 part of work
Pranesh's contribution = 3/13 × 1/24 = 1/104 Part of work
So Pranesh will take 104 days
Answer: Option (c)

14.

Men	*Working Hours*	*Day*	*Work done*
64	8	5	40%
64	n (say)	4	60%

40% work in 40 hours → 60% work in 60 hours.
Hence, working hours = 60/4 = 15 hours.

[illegible]

17. Combined efficiency of four secretaries = 1/2

$$\frac{1}{A}+\frac{1}{B}+\frac{1}{C}+\frac{1}{D}=\frac{1}{2}$$

As it is given in the question that boss's efficiency reduces respectively according as A, B, C and D remain absent, it means their efficiencies are in descending order by trial and error.

[illegible]

20. A worker in a day completes 10% work.
Since entire team works at their 50%

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*
5%	10%	15%	20%	25%	25%

(Day 6 = 25% because that day worker is absent)
Total = 5 + 10 + 15 + 20 + 25 + 25 = 100%

21. Amount of work = 200%

Day 1	*Day 2*	*Day 3*	*Day 4*	*Day 5*	*Day 6*
10%	20%	30%	40%	50%	60%

Total = 10 + 20 + 30 + 40 + 50 + 60 = 210%
So work is finished on 6th day

22. We can use the data of previous question. On the last day 50% work is required but efficiency to finish 60% work is there so 60% work in 8 hours.
50% work in (8/60) × 50 = 6 hours 40 minutes
They start working at 9.00 am and work till 1.00 pm
So 4 hours passed till 1.00 pm.
Now 2 hours 40 minutes is left, they again started working at 1.30
So 1.30 + 2 hours 40 minutes

23. According to the question, 1/2, 1/a, 1/3 are in A.P.
So 'a' can be calculated.
Now, $\frac{1}{2-x}, \frac{P}{a}, \frac{1}{3-x}$
So x can be calculated.
$x = 2.6$.

36. Let the height above the ground = EF = h.
$\Delta ECF \sim \Delta ACB$

$$\text{So, } \frac{h}{78}=\frac{FC}{BF} \qquad \dots (1)$$

$\Delta BEF \sim \Delta BCD$

$$\text{So, } \frac{h}{91}=\frac{FC}{BF+FC} \qquad \dots (2)$$

$\Delta ECF \sim \Delta ACB$

$$(1) \div (2) \dots = \frac{FC}{BF} \Rightarrow \frac{91}{78} \quad 7BF = 6FC \qquad \dots (3)$$

By putting values of equation (3) in equation (1), h = 42 m.

[illegible]

40. Total numbers of possible pairs = $^{10}C_2 = 45$
So each is working 9 times to finish 3/4th of the work.
In one day, all of them working together will do = 1/9 of 3/4 = 1/12th work
Remaining work = 1/4
So required days = 3 days

41. As we have found in a previous problem that working together, they do 1/12th work in a day i.e., they take 12 days to complete the work working together.
Now till the last day they have worked for 45 + 3 = 48 days.
Now for the same work again they will take 12 more days working together.
Hence, total number of days required from start = 48 + 12 = 60 days.

42. Fifty table and 50 tabletops are available so we need to purchase 100 more tabletops. Lead time for purchasing table top in 2 week and lead time for assemble table is 1 week so order should be placed 3 weeks in advance, i.e., Week 1.

47. At 75% efficiency loom works 7.5 m/hour
So, 20 looms @ 10 h/day in 30 days will make = 7.5 × 20 × 10 × 30 = 45000 m cloths
At 70% efficiency loom works = 7 m/hour
So, 20 looms @ 10 h/day in 30 days will make = 7 × 10 × 20 × 30 = 42000 m cloths
So, production of cloths decreases by 45000 – 42000 = 3000 m
So, Loss = 3000 × 4 = Rs 1200000
Saving on a labourer by removing him = 11000

8

CHAPTER

Time, Speed and Distance

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Basic relationship between time, speed and distance
- Different situations like Motion in a straight line, Trains, Races, Boats and Streams, Circular Track, Clocks
- Concepts attached to these motions
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

Time, speed and distance (TSD) is one of the most popular topics in the CAT. Almost 5%–10% questions in the paper are asked from this chapter every year. The concept of time, speed and distance is also used extensively for questions relating to different areas in various other examinations. Some of these areas are: motion in a straight line, boats and streams/escalators, races, circular motion, trains, clocks. Students should try to understand the inter-relationship between the factors time, speed and distance.

RELATIONSHIP BETWEEN TIME, SPEED AND DISTANCE

As we know, Distance = Speed × Time

It means that if a person is running at a speed of 20 km/h and he runs for 2 h, he will be covering a total distance of 40 km.

Distance = 20 × 2 = 40 km

Distance

When an object is moving with a certain speed in a particular time, the displacement made by an object is called the distance.

Unit of Distance Kilometre (km) and metre (m) is usually taken as the unit of distance. Sometimes, mile or feet, etc., can be found as the unit.

Time

Time is defined as a quantity, which governs the order or sequence of an occurrence. In the absence of time, the actual sequence of any occurrence or incident would be lost. If we did not have the concept of time, we would not be able know in what period or in what order something took place.

So, time could be seen as a big building with a number of floors where all the floors are designated according to the occurrence of incidents/events on the respective floors. In our case time shall be seen as the duration of happening of any event.

Unit of Time Hour and second are mostly taken as the unit of time. However, day or minute are also taken as units.

Speed

Speed is defined as the distance covered per unit time. In other words, it is the rate at which the distance is covered.

Unit of Speed Though we commonly take km/hour and metre/sec as the units of speed. Yet, any unit of distance upon any unit of time can be taken as the unit of speed. For example, mile/h, feet/s, mile/s, feet/h, etc.

Conversion from m/s to km/h and vice versa

If speed is given in m/s and it is required to convert it into km/h, then we multiply it by $\frac{18}{5}$ and when speed is given in km/h and we have to convert it into m/s, we multiply it by $\frac{5}{18}$.

$$36 \text{ km/h} = 36 \times \frac{5}{18} = 10 \text{ m/s}$$

$$20 \text{ m/s} = 20 \times \frac{18}{5} = 72 \text{ km/h}$$

MOTION IN A STRAIGHT LINE

This is one of the primary areas of application of time, speed and distance. By using the basic relationship between time, speed and distance the following three different cases are possible:

Case 1 When S (Distance) is constant

$$V \alpha \frac{1}{T}$$

So, $V_1/V_2 = T_2/T_1$

It is read as V is inversely proportional to T.

It can be understood in terms of the reciprocal-ratio-multiplication relationship between V and T.

Suppose distance = 1000 km and Speed = 100 km/h

Speed = 100 km/h	Time = 10 h
Speed = 100 × 2	Time = ½ × 10
Speed = 100 × 3	Time = 1/3 × 10
Speed = 100 × ½	Time = 2 × 10

To simplify it, the product stability ratio can be further used. So, the more the speed, the lesser is the time taken and the lesser the speed, more will be the time taken.

Example 1 A man cycles with a speed of 10 km/h and reaches his office at 1 p.m. However, when he cycles with a speed of 15 km/h, he reaches his office at 11 a.m. At what speed should he cycle so that he reaches his office at 12 noon? **(CAT 2004)**

Solution Using the product stability ratio,

The speed is being increased by 50%, so the time taken will reduce by 33.33%.

So, 33.33 % of Time = 2 h

Hence, total time = 6 h

So, distance = 10 × 6 = 60 km

This distance is to be covered in 5 h.

So, speed = 60/5 = 12 km/h

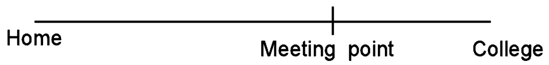
Alternatively, it can be seen that time taken in three (Given) situations are in AP. Hence speeds will be in HP.

Required Speed = Harmonic Mean of two speeds.

So, Required Speed = $\frac{2 \times 15 \times 10}{15 + 10}$ = 12 km/h

Example 2 Siddharth goes by a bike to pick up his girlfriend everyday from college and then drops her at her house. College timings are till 5 p.m. daily, but today the college at 4 p.m. His girlfriend, not finding Siddharth at the college gate, starts walking towards her house. Siddharth, unaware of this fact, leaves his house as usual meets his girlfriend on way, picks her up and drops her at her house. At night, Siddharth realizes that he had saved 40 min that day. What is the ratio of the speed of Siddharth to that of his girlfriend? (Both of them live in the same building).

Solution Let us see the following schematic representation:



The usual route of Siddharth is home-college-home

His route today is – home – meeting point – home.

And, in this way, 40 min are saved. So, he takes 20 min to cover the distance between the meeting point and the college. It can be further concluded that he usually reaches college at

5 P.M., but today he reached at 4:40 (20 min. are saved) and his girlfriend took 40 min. (she starts at 4 P.M.) to cover the distance between her college to the meeting point.

The ratio of time of Siddharth and his girlfriend = 20:40 = 1:2

The ratio of the speed of Siddharth and his girlfriend = 2:1.

Case 2 When T (Time) is constant

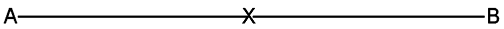
$S \propto V$

So, $S_1/S_2 = V_1/V_2$

The higher is the speed, the more will be the distance covered and the lower the speed, the lesser will be the distance covered.

We'll see that T constant is a situation specific to meeting point cases.

Example 3 Distance between two points AB = 110 km. Manoj starts running from A at a speed of 60 km/h and Ravi starts running from B at a speed of 40 km/h at the same time. They meet at a point X, somewhere on the line AB. What is ratio of *AX* to B*X*?



Solution Since both Manoj and Ravi are running for the same time, T is constant. Hence, the ratio of the distance covered by them will be the same as the ratio of their speed.

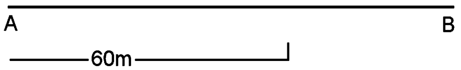
So, AX/BX = Speed of Manoj/Speed of Ravi = 60/40 = 3:2

Some typical meeting point cases

When two persons are running between the ends of a linear track for infinite time:

Example 4 Two robots Mango and Bango start from the opposite ends A and B of a linear track respectively and keep running between the ends for infinite time. They meet for the first time at a point 60 m from A. If AB = 100 m, which point is their point of 4th meeting?

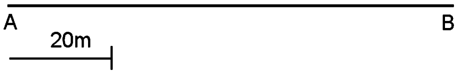
Solution 1st meeting point



The ratio of the speed of Mango and Bango is 60:40 = 3:2. Now Mango is moving towards B and Bango is moving towards A.

For the 2nd meeting, Bango reaches A after covering 60 m, Mango must have covered 90 m in the same time. So, he is at the mid-point of A and B. Now, the distance between Mango and Bango is 50 m. They will cover this distance of 50 m in the ratio of 3:2.

So, the point of their 2nd meeting is



It can be seen here that the sum of the distance covered by both Mango and Bango from the starting till now is 300 m, and the difference between the distance covered between the 1st meeting and the 2nd meeting is 200 m.

So, again they will meet for the 3rd time when they have covered a total distance of 200 m together. Mango and Bango will cover this distance of 200 m in the ratio of 3:2.

Distance covered by Mango = 120 m and distance covered by Bango = 80 m.

So, the 3rd meeting point is point B.

To have a 4th meeting, they will again have to cover a total distance of 200 m. So, the point of their 4th meeting is 20 m from A.

So, we can now generalize the above situation as:

Distance covered by both of them for the 1st meeting = 100 m

Distance covered by both of them for the 2nd meeting = 100 m + 200 m = 300 m

Distance covered by both of them for the 3rd meeting = 300 m + 200 m = 500 m

Distance covered by both of them for the 4th meeting = 500 m + 200 m = 700 m

The ratio of the speed of Mango and Bango is 3:2, so the distance covered by Mango = 420 m

Hence they will meet at 20 m from A.

Example 5 Two persons, Ram and Mohan, start from the same end A of a linear track AB and keep running to and fro for infinite time. They meet for the first time at a point 20 m from B. If AB = 100 m, which point is their point of 4th meeting?

Solution Using the above generalization, distance covered by both of them for the 1st meeting = 200 m

Distance covered by both of them for the 2nd meeting = 200 m + 200 m = 400 m

Distance covered by both of them for the 3rd meeting = 400 m + 200 m = 600 m

Distance covered by both of them for the 4th meeting = 600 m + 200 m = 800 m

The ratio of the speed of Ram and Mohan is 3:2, so the distance covered by Ram = 480 m

Hence they will meet at 80 m from A.

Limitation of above generalization For the meeting to occur after every 200 m, the ratio of the speed of the two runners should be less than 2. If it is more than or equal to 2, then the problems can only be evaluated on the basis of actual calculation.

Case 3 When V (Speed) is constant

$S \alpha T$

So, $S_1/S_2 = T_1/T_2$

In layman terms, if a person is running with a speed of 20 km/h, then the ratio of the distance covered in one hour to the distance covered in two hours will be 1:2.

BOATS AND STREAMS/ESCALATOR

Boats and streams should be ideally seen as just a logical extension of the motion in a straight Line, with distance being constant.

As we know, if the distance is constant then $V \alpha 1/T$.

Basic Terminology

Downstream movement When the direction of the movement of a river and a boat is the same, their collective movement is known as the downstream movement. And the distance covered by boat is known as downstream distance.

If the speed of the River = R and the speed of the boat = B, then Downstream Speed = B + R

Upstream movement When the direction of the movement of the river and a boat is opposite, they are said to be in upstream movement. The distance covered in this case is known as upstream distance.

If the speed of the river = R and the speed of the boat = B, then upstream speed = B – R (Conventionally the speed of one boat is taken more the than speed of the river otherwise the boat would not be able to go back.)

Now, speed of boat = ½ (downstream speed + upstream speed) = ½ (B + R + B – R) = B

And speed of river = ½ (downstream speed – upstream speed) = ½ (B + R – B + R) = R

Hence, if downstream speed and upstream speed are given as 20 km/h and 10 km/h respestively, then the speed of the boat = 15 km/h and speed of the river = 5 km/h.

In most of the cases of boats and streams, the distances covered downstream and upstream are the same. In those cases, the ratio of the time taken becomes inverse of the ratio of the speeds.

Time taken downstream: Time taken upstream = upstream speed: downstream speed

Example 6 The speed of the boat in still water is 6 km/h and the speed of the river is 1.2 km/h. Boat takes a total of 10 h to go to a place and come back. What is the total distance covered in the whole process?

Solution Let us assume D is the distance.

Upstream Speed = 4.8 km/h

Downstream speed = 7.2 km/h

According to the question, D/4.8 + D/7.2 = 10

So, D = 28.8 km and hence the total distance = 57.6 km

Alternatively, the ratio of downstream speed: upstream speed = 3:2

Ratio of the downstream time: upstream time = 2:3

The time taken in the downstream movement = 4 h and the time taken in the upstream movement = 6 h

So, the distance covered = 4 × 7.2 = 6 × 4.8 = 28.8 km

Hence the total distance = 57.6 km

In the case of escalators, moving staircase works like an external agent as the river works for boats and streams. The speed of an escalator and the person will be added when the staircase is going up and the person walking up with it have the same direction of the movement.

Now if the direction of the movement of an escalator and the person are opposite, then the resultant speed (or, the relative speed) will be equal to the speed of the person – to the speed of an escalator.

Example 7 A man can walk up in a moving escalator (upwards) in 30 s. The same man can walk down this moving 'up' escalator in 90 s. Assume that this walking speed is the same both upwards and downwards. How much time will he take to walk up the escalator when it is not moving?

Solution Let us assume that the speed of the man = m steps/s and the speed of the escalator = e steps/s

Distance covered while going up = 30 m + 30 e

Distance covered while going down = 90 m – 90 e

Now, these two are equal.

So, 30 m + 30 e = 90 m – 90 e

Or, 60 m = 120 e, Hence, 1 m = 2 e

So, the total length of escalator = 45 m

So, the time taken by the man to cover the whole escalator = Distance/Speed = 45m/m = 45 s

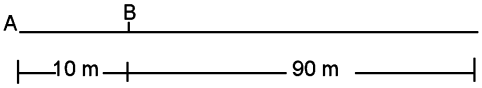
Alternatively, Answer would be Harmonic Mean of the given time = $\frac{2 \times 30 \times 90}{30 + 90}$ = 45 secs.

[See the solution of Example 1 on last page].

RACES

Basic Statements

1. A gives a start of 10 m to B → When B has already run 10 m, then A starts running.

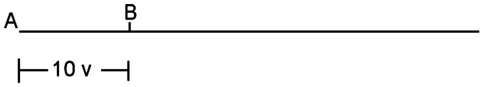


Example 8 In a race of 100 m, A gives a start of 10 m to B. Despite this, A wins the race by 20 m. What is the ratio of the speed of A and B?

Solution Time taken by A to cover 70 m = Time taken by B to cover 100 m

Since the distance is constant, the ratio of speed of A and B = 10:7

2. A gives a start of 10 secs to B → B has already run for 10 secs, now A starts running.



Where *v* m/s is the speed of B.

Example 9 In a 100 m race, Tom runs at a speed of 1.66 m/s. If Tom gives a start of 4 m to Jerry and still beats him by 12 s, what is the speed of Jerry?

Solution Time taken by Tom to cover 100 m = 60 s

Now, since Tom beats Jerry by 12 s, time taken by Jerry = 72 s

And the distance covered by Jerry = 96 m

So, speed = 96/72 = 1.33 m/s

Example 10 Karan and Arjun run a 100-m race where Karan beats Arjun by 10 m. To do a favour to Arjun, Karan starts 10 m behind the starting line in a second 100 m race. They both run at their earlier speeds. Which of the following is true in connection with the second race?

1. Karan and Arjun reach the finishing line simultaneously.
2. Arjun beats Karan by 1 m.
3. Arjun beats Karan by 11 m.
4. Karan beats Arjun by 1 m.

Solution Situation (I)

In whatever time Karan covers a distance of 100 m, Arjun covers 90 m in the same time.

Situation (II)

Karan is 10 m behind the starting point. Once again to cover 100 m from this new point Karan will take the same time as before. In the same time Arjun will cover only 90 m. This means that both of them now will be at the same point, which is 10 m away from the finish point. Since both of them are required to cover the same distance of 10 m and Karan has a higher speed, he will beat Arjun. There is no need for calculations as option (4) is the only such option.

CIRCULAR MOTION

In the case of races and motions in straight line, we have observed that if the two bodies or persons are moving with different speeds in a straight line in one direction, then they will never meet. This is due to the fact that with the passage of time, the distance between them is increasing constantly.

Circular motion should be seen as a logical extension of Races where runners are running on a circular track. Since it an enclosed track (by virtue of it being circular), runners are bound to meet at some point or the other.

Case 1 When two or more than two persons are running around a circular track in the same direction.

Example 11 When will they meet for the first time anywhere on the track?

Solution To understand the situation completely, let us assume that there are two persons A and B. Speed of A = 20 m/s, speed of B = 10 m/s, length of the track is 1000 m and they are running in the same direction. It can be seen in figure 1 that initially both of them are at the same point i.e., the starting point.

They will be meeting for the first time only if the faster runner A has taken one more round of the track than the slower runner B. This can be interpreted as—A will have to cover 1000 m more than B.

It is understood with the figures given above that the distance will keep on increasing between them with the passage of time. And the moment distance between them becomes equal to 1000 m, they will be at the same point.

So, the time taken = distance/relative speed = $\frac{1000}{10}$ = 100 s

Or, this can be done by using unitary method also:

Distance of 10 m is created in 1 s

So, the distance of 1000 m will be created in 100 s

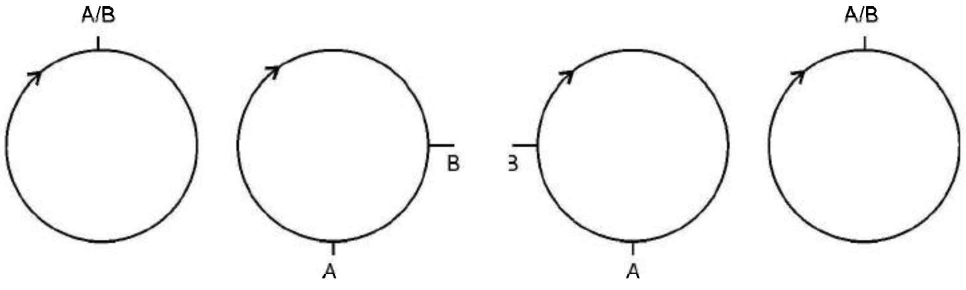
Now, let us assume that there are three persons A, B and C running with following speeds in the same direction:

Speed of A = 30 m/s

Speed of B = 20 m/s

Speed of C = 10 m/s

To calculate when will they meet for the first time, we are required to find the time taken by the fastest runner to take one round over the other runners.



Starting After 25 s After 75 s After 100 s

Time taken by A to take one round over B = $t_{A\text{-}B}$ = 1000/10 = 100 s

Time taken by A to take one round over C = $t_{A\text{-}c}$ =1000/20 = 50 s

Now, the LCM of these two values $t_{A\text{-}B}$ and $t_{A\text{-}c}$ will give us the time after which all of them will be meeting at the same place.

LCM = (100, 50) = 100 s

It can also be seen that they will be meeting after every 100 s.

Example 12 When will they meet for the first time at the starting point?

To calculate this, we will use the concept of LCM (Usage of LCM and HCF, chapter 2, case 2)

Find the time taken by each individual to take one round and then calculate LCM of these values.

Assume there are three persons A, B and C with a respective speed of 30 m/s, 20 m/s and 10 m/s running in the same direction. Length of the circular track is 1000 m.

Time taken by A to take one round = t_1 = 1000/30 = 33.33 s

Solution Time taken by B to take one round = t_2 = 1000/20 = 50 s

Time taken by C to take one round = t_3 =1000/10 = 100 s

LCM of t_1, t_2, t_3 = 100 sec.

Example 13 At how many different points of the track will they be meeting?

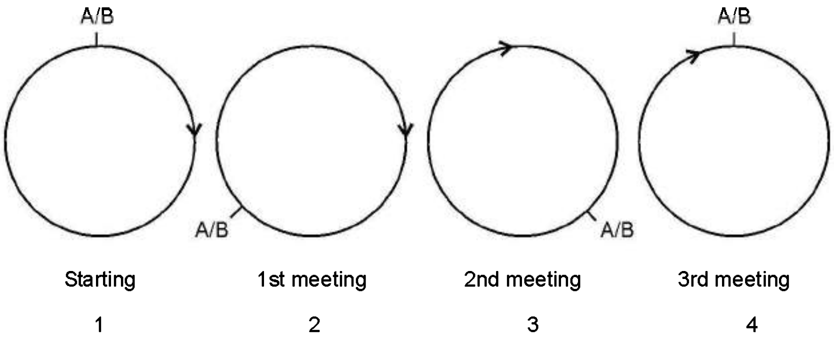
Solution Let us assume that the speed of A = 25 m/s and the speed of B = 10 m/s and the length of the track =1000 m

They will be meeting for the first time after a time-gap of 1000/15 = 66.66 s

Till this time, A has covered 1666.66 m and B has covered 666.66 m. This point is 666.66 m from the starting point. Now, this point can be assumed to be the starting point.

So, they will meet at a distance of 666.66 m from here. This is the second meeting point, at a distance of 333.33 m from the starting point. Next meeting point will be 666.66 m from here. This point will be nothing but the starting point again (Fig.1 and Fig.4 are same).

This can be seen through the figures given below:



So, there are a total of 3 distinct meeting points on the track.

In general, number of meeting points = difference of ratio of the speed the of A and B in its simplest form.

Ratio of speed of A and B = 5 : 2

So, the number of different meeting points = 5 – 2 = 3 points

Case 2 When two or more than two persons are running around a circular track in the opposite direction.

Here again there are two persons A and B with a speed of 20 m/s and 10 m/s respectively, and length of track is 1000 m.

Example 14 When will they meet for the first time anywhere on the track?

Solution Since they are running in the opposite direction, relative speed =10 + 20 = 30 m/s

So, time taken = distance/relative speed = 1000/30 = 33.33 s

Example 15 When will they meet for the first time at the starting point?

First, we will calculate the time taken by each individual to take one round and then calculate the LCM of those values.

Time taken by A to take one round = t_1 = 1000/20 = 50 s

Time taken by B to take one round = t_2 = 1000/10 = 100 s

LCM of (t_1, t_2) = 100 s

Example 16 At how many different points of the track will they be meeting?

Solution They are meeting after 33.33 s for the first time. Till this time, A has covered 666.66 m and B has covered 333.33 m. So, obviously they are meeting at a distance of 666.66 m from starting point in the direction of A. Next point will be again 666.66 m ahead of this point. And, the next point will be another 666.66 m ahead of this point, which will be the starting point.

So, a total of 3 points will be there.

In general, number of distinct meeting points = addition of the ratio of the speed of A and B in its simplest form.

The ratio of speed of A and B = 2 : 1

So, the number of different meeting points = 2 + 1 = 3 points

Example 17 Anup and Shishir start running from the same point of a circular track at the same time. After how much time will Anup and Shishir, who are running with a speed of 35 m/s and 40 m/s, respectively, meet at diametrically opposite point?

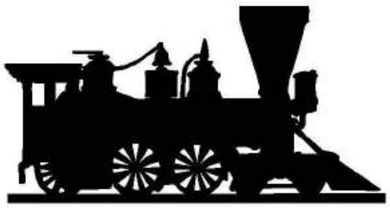
Solution The simplest ratio of speed of Anup and Shishir = 7:8

So, if they are running in the same direction, they will meet at 1 point and if they are running in the opposite direction, they will meet at 15 different points.

Now, for them to meet at a diametrically opposite point, there should be atleast two meeting points or the number of meeting points should be a multiple of 2.

Since, they would meet either at 1 point or at 15 different points, depending on the direction of their movement, they will, therefore, not meet at a diametrically opposite point.

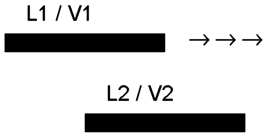
TRAINS



We know that when the direction of the movement of a boat and a river is the same, the relative speed is obtained by adding the speeds of both, the boat and the river. But if two trains are moving in the same direction, then what is the relative speed?

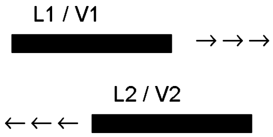
Let us see some cases:

1. When two trains of length L_1 and L_2 and speed V_1 m/s and V_2 m/s respectively are crossing each other:
   i. The direction of the movement of both the trains are the same:



Relative speed = $| V_1 - V_2 |$
Total distance covered = $L_1 + L_2$

ii. The direction of the movement of both the trains are opposite:



Relative speed = $| V_1 + V_2 |$
Total distance covered = $L_1 + L_2$

2. When a train is crossing a stationary object:
   i. When the train is crossing a pole or a stationary human being:

(Fig. 1)

(Fig. 2)

Let us assume that A is a pole. In figure 1, the front of the train is about to cross the pole and in figure 2, the tail of the train has just crossed the pole. It is understood here that the train has crossed its whole length with respect to the pole. So, when the train is crossing any stationary object of negligible width, total distance covered is its own length.

Relative speed = $V_1 + V_2$, since $V_2 = 0$, then, the relative speed = V_2

Total distance covered = $L_1 + L_2$, since L_2 (width of the pole) is negligible with respect to L_2 (Length of the train), so we do not consider it while calculating the quantities. Thus, distance = L_1

However, it should be remembered that this is mathematically not correct and all the solutions are on the assumption that the width of the pole is zero, which is obviously not true.

ii. When the train is crossing a platform or a standing train:

Relative speed = $V_1 + V_2$,

Where V_1 is the speed of the moving train and V_2 is the speed of the standing train or the platform.

Since $V_2 = 0$, so the relative speed = V_1

Total distance covered = $L_1 + L_2$

Where L_1 is the length of the moving train and L_2 is the length of the standing train or the platform.

Example 18 A train takes 10 s to cross a pole and 20 s to cross a platform of length 200 m. What is the length of train?

Solution The train takes 10 s to cross its own length and 20 s to cross its own length and length of the platform. So, it is infered that the train takes 10 s to cross the platform and 10 s to cross its own length.

Since the time taken to cross the platform = time taken to cross its own length

So, length of the platform=length of the train=200 m

Example 19 Speed of a train is 36 km/h. It takes 25 s to cross a pole. What is the length of this train?

Solution Speed of train = 10 m/s (36 × 5/18)
Distance covered = 10 × 25 = 250 m
So, length of train = 250 m

Some Special Cases

Case 1 Two trains are moving in an opposite direction with a speed of V_1 and V_2. Their lengths are L_1 and L_2. Now, see the whole situation from the point of view of a person sitting on the window seat of the 1st train.

Relative speed = $V_1 + V_2$ (This person can be assumed to be running with a speed of V_2.)

Relative distance = L_2

Case 2 A train is running with a speed V_1 and a person X is running inside the train with a speed of V_2 in the direction of the movement of train. Now if a person Y is watching this from the outside of the train, then the relative speed of Y with respect to X = $V_1 + V_2$.

Speed of person X with respect of another person Z who is sitting in the train = V_2

There is also a person P who is outside the train and is moving with a speed of V_3 in the opposite direction of train.

Relative speed of P with respect to person X = $V_1 + V_2 + V_3$

Had this person P been running in the same direction as that of the train, then the relative speed of P with respect to person X = $|V_1 + V_2 - V_3|$

CLOCKS

A clock is a typical example of a circular motion where length of the track is equal to 60 km (Assume 1 min = 1 km). Now on this track, two runners i.e., hour hand and minute hand are running with a speed of 5 km/h and 60 km/h respectively. Since the direction of their movement is the same, so the relative speed = 55 km/h.

Example 20 When will the hour hand and the minute hand of a clock be together between 1 and 2?

Solution Hands have to be together in between 1 o'clock to 2 o'clock.

At 1 o'clock, the distance between hour and minute = 5 km

And the relative speed = 55 km/h

Time = 5/55 h = 1/11 h = 60/11 min = 5 5/11 min. = 5300/11 s = 5 : 27.27s

So, the hour hand and the minute hands will be together at 1:05:27.27 s

Students can learn this value 5 min 27.27 s as a standard result.

So, both the hands will meet at

1:05:27.27 – Between 1 o'clock and 2 o'clock
2:10:54.54 – Between 2 o'clock and 3 o'clock
3:16:21.81 – Between 3 o'clock and 4 o'clock
and so on.

Example 21 How many times in a day will the hands of a clock be together?

Solution Using the data from the above question, hands of a clock meet at a regular interval of 5 min 27.27 s. So, the number of times they will the meet = $\frac{60}{5:27.27}$ = 11.3 times.

So, the hands will meet for a total of 11 times.

However, it can also be observed that the hands of a clock meet once every hour except in between 11–1. They meet just once in between 11–1. So, they are meeting for 11 times.

Degree Concept of Clocks

Total angle subtended at the centre of a clock = 360°.

Angle made by an hour hand at the centre per hour = 30° per hour, or, 0.5° per min

Angle made by the minute hand at the centre per hour = 360° per hour, or, 6° per min

Example 21 solving 20 by this method,

Angle between an hour hand and the minute hand at 1 o'clock = 30°

Relative speed (in terms of angle) = 5.5°/h

So, time taken = 30°/5.5° = 60/11 min

Example 22 Mr Binod Kumar Roy goes to a market between 4 P.M. and 5 P.M. When he comes back, he finds that the h hand and the min hand have interchanged their positions. For how much time was he out of his house?

Solution Since hands are interchanging their position, minute hand is taking the place of an hour hand and an hour hand is taking the place of min hand. So sum of the angles formed by h hand and min hand = 360°

Let us assume that he was out of house for 't' min.

So, the angle formed by min hand = $6 \times t$ and by hour hand = $0.5 \times t$

So, $0.5 \times t + 6 \times t = 360$

Or, $6.5 \times t = 360$

So, $t = 55.38$ min

Important Derivations

→The number of times hands of a clock are in a straight line (either at 0° or at 180°) in 24 h = 44

→The number of times hands of a clock are at a right angle (at 90°) in 24 h = 44

→Both the hands of clock are together after every $65\frac{5}{11}$ min

(So if both the hands of the clock are meeting after every 65 min or anything less than $65\frac{5}{11}$ mins, then the clock is running fast and if both the hands of the clock are meeting after every 66 min or anything more than $65\frac{5}{11}$ min, then clock is running slow.)

PRACTICE EXERCISES

WARM UP

Q.1. A man is running at the speed of 36 km/h, then how many m does he cover in 40 s?
(a) 400 m (b) 300 m
(c) 2,400 m (d) 100 m

Q.2. The speed of a train is 20 m/s. The time it takes to cross a telephone pole is 10 s. What is the length of the train?
(a) 100 m (b) 1.5 km
(c) 150 m (d) 200 m

Q.3. A train having a length of 400 m, travelling at a speed of 15 m/s crosses a platform in one min. What is the length of the platform?
(a) 2000 m (b) 500 m
(c) 400 m (d) Cannot be determined

Q.4. Speed of a train is 88 km/h. What time will it take to cross a platform 180 m long if the length of the train is 150 m?
(a) 11½ s (b) 10½ s (c) 13½ s (d) 14½ s

Q.5. A certain distance is covered at a speed V km/h. If half of the same distance is covered in double the time, then the ratio for the former speed to that of the latter is
(a) 4:1 (b) 1:4 (c) 2:1 (d) 1:2

Q.6. A person can row a distance of one km upstream in ten min and downstream in four min. What is the speed of the stream?
(a) 4 km/h (b) 9 km/h
(c) 5.6 km/h (d) 4.5 km/h

Q.7. Anil calculated that it will take 45 min to cover a distance of 60 km by his car. How long will it take to cover the same distance if the speed of his car is reduced by 15 km/h (approximate)?
(a) 36 min (b) 55 min
(c) 48 min (d) 40 min

Q.8. A car travels from Patna to Jehanabad at a speed of 65 km/h in one hour. If the speed is reduced by 15 km/h then, how much more time will the car take to cover the same distance?
(a) 12 min (b) 16 min
(c) 18 min (d) 44 min

Q.9. In a kilometer race, A beats B by 10 m. In a two kilometers race, A beats B by
(a) 10 m (b) 20 m (c) 40 m (d) 25 m

Q.10. A train, 225 m long, is running at 45 km/h. In what time does it cross a man running at 18 km/h in the same direction?
(a) 20 s (b) 30 s (c) 25 s (d) 15 s

Q.11. Two trains started from a station, at the same time in the same direction, at speeds of 70 km/h and 60 km/h. What will be the distance between the two trains after 6 h?
(a) 42 km (b) 54 km
(c) 60 km (d) 84 km

Q.12. A man can row half a km against the current in 12 min and returns in 6 min. Find the speed of the current.
(a) 1.25 km/h (b) 1.5 km/h
(c) 2.5 km/h (d) 3 km/h

Q.13. In a kilometer race, Amit beats Bahadur by 100 m and Bahadur beats Chandra by 200 m. By how many metres does Amit beat Chandra in the same race?
(a) 100 m (b) $83^1/_3$ m
(c) 68 m (d) 280 m

Q.14. On a river, Q is the mid-point between two points P and R on the same bank of the river. A boat can go from P to Q and back in 12 hours, and from P to R in 16 hours 40 min. How long would it take to go from R to P?
(a) $3\frac{1}{3}$ h (b) 5 h
(c) $6\frac{2}{3}$ h (d) None of these

Q.15. Two trains are running on a parallel line in the same direction at a speed of 50 km/h and 30 km/h respectively. Given that the faster train crosses a man in the slower train in 18 s, what is the length of the faster train?
(a) 170 m (b) 100 m
(c) 98 m (d) 85 m

Q.16. In a kilometre race, Ajay beats Bijay by 100 m and Bijay beats Chand by 100 m. By how many m does Ajay beat Chand in the same race?
(a) 100 m (b) 200 m
(c) 190 m (d) 119 m

Q.17. Two persons are walking with the speed A and B respectively. If the first person takes ten min less to cover a distance, what is the time taken by the second person to cover the same distance provided A : B = 3:2?

(a) 20 min (b) 30 min
(c) 10 min (d) 50 min

Q.18. A train of length 200 m crosses a pole in 15 s and crosses a platform in 60 s. What is the length of the platform?

(a) 450 m (b) 600 m
(c) 800 m (d) None of these

Q.19. Hands of a clock are meeting after every 65 min. We can conclude that the clock is running

(a) Fast
(b) Slow
(c) Cannot be determined

Q.20. In 12 hours, how many times the hour and the minutes hands of a clock will be at 0° to each other?

(a) 11 (b) 12
(c) 13 (d) None of these

FOUNDATION

Q.1. When the speed of a train is increased by 20%, it takes 20 min less to cover the same distance. What is the time taken to cover the same distance with the actual speed?

(a) 2 h (b) 1 h (c) 1.5 h (d) 2.5 h

Q.2. Difference between the time taken in covering a certain journey decreases by 45 minutes when the speed is increased from 10 km/h to 20 km/h. What is the difference between the time taken when the same distance is covered at a speed of 40 km/h and 25 km/h?

(a) 12.5 min (b) 13.5 min
(c) 18 mins (d) None of these

Q.3. Shramjeevi express goes to Patna from New Delhi at a speed of 60 km/h. At what speed (in km/h) should Shramjeevi express return from Patna to New Delhi so that the average speed during the whole journey is 120 km/h?

(a) 180 (b) 240
(c) 120 (d) This is not possible

Q.4. Sona travels 600 km to his house partly by train and partly by car. He takes 8 hours if he travels 120 km by train and the rest by car. He takes 20 min more if he travels 200 km by train and the rest by car. Find the speed of the train in km/h.

(a) 45 (b) 60
(c) 75 (d) None of these

Q.5. Sama traveled the first half of the distance from her house to school at a speed of 10 km/h and for exactly half of the remaining time she travelled at 15 km/h and the rest of the time at 20 km/h. Find her average speed.

(a) 13 km/h (b) $12\frac{8}{11}$ km/h
(c) 15 km/h (d) None of these

Q.6. Shilu can go 30 km upstream and 44 km downstream in 10 h. Also, she can go 40 km upstream and 55 km downstream in 13 h. What is her speed in still water?

(a) 6 km/h (b) 12 km/h
(c) 8 km/h (d) 16 km/h

Q.7. A train travels a distance of 192 km at an average speed of 80 km/h from station P to another station Q. It then travels back to P taking 5 hours 36 min. Find the average speed of the train on the return journey to P.

(a) 72 km/h (b) 36 km/h
(c) 20 km/h (d) 60 km/h

Q.8. Two persons, 27 km apart, start at the same time and are together in 9 h if they walk in the same direction. But, if they walk in the opposite directions they are together in 3 h. What are their speeds (in km/h)?

(a) 5, 4 (b) 4, 3 (c) 6, 3 (d) 8, 7

Q.9. In a race of 300 m, Abhishek beats Bijay by 30 m while Bijay beats Chandan by 50 m. By what distance should Abhishek beat Chandan in the same 300 m race?

(a) 80 m (b) 40 m (c) 160 m (d) 75 m

Q.10. What is the speed of a train, which overtakes a cow walking at a speed of 5 km/h in 30 s, if the train is 274 m long?

(a) 51.88 km/h (b) 27.88 km/h
(c) 37.88 km/h (d) 21.67 km/h

Q.11. In eleven hours C walks 12.5 km less than what D does in twelve h; and in five h D walks 3.25 km less than what C does in seven h. How many km does each walk per hour?

(a) 3 and 4 (b) 3.5 and 4.25
(c) 2.33 and 3.4 (d) None of these

Q.12. The ship Vikrant starts from a point P towards a point Q at noon and at 1.00 pm ship, Viraat starts from Q towards P. If ship Vikrant is expected to complete the voyage in 6 h and ship Viraat is moving at a speed of

2/3rd of that of ship Vikrant, at what time are the two ships expected to meet?
(a) 4 pm (b) 4.30 pm
(c) 3 pm (d) 2.30 pm

Q.13. Amit Kumar starts driving from Patna to Quillon at 9 a.m and Bhartendu starts driving from Quillon to Patna at 10 a.m Bhartendu is 50% faster than Amit Kumar. At what time will they meet if Patna and Quillon are 300 km apart and Amit Kumar's speed is 50 km/h?
(a) 12.30 pm (b) 12 noon
(c) 11 am (d) 11.30 am

Q.14. A ship, 156 km away from the seacoast springs a leak, which admits $2\frac{1}{3}$ metric tons of water in $6\frac{1}{2}$ minutes, but the pumps throw out 15 metric tons of water in 1 h. 70 metric tons would be sufficient to sink the ship. What should be the average speed (in km/h) of the ship so that it may just reach the shore as she begins to sink?
(a) 14.5 (b) 15 (c) 18 (d) 20

Q.15. A man rows a boat a certain distance upstream and then returns to the same place. If the time taken by him in going upstream is twice the time taken in rowing downstream, find the ratio of the speed of the boat in still water and the speed of the stream.
(a) 2:1 (b) 3:2 (c) 5:3 (d) 3:1

Q.16. Excluding stoppages, the speed of a bus is 54 km/h and including stoppages it is 45 km/h. What is the stoppage time of the bus (in minutes) per h?
(a) 10 (b) 12 (c) 9 (d) 20

Q.17. For covering a distance of P km, a man takes t h. He takes 40 min less if he moves 3 km/h faster, But if he moves slower by 2 km/h, he takes 40 min more. Then P is equal to.
(a) 22.5 km (b) 32.5 km
(c) 42.5 km (d) None of these

Q.18. Two trains start at the same time from A and B and proceed towards B and A at 36 km/h and 42 km/h respectively. When they meet, it is found that one train has moved 48 km more than the other. Then the distance between A and B (in km) is
(a) 624 (b) 636 (c) 544 (d) 460

Q.19. Anand can beat Bidhan by 5 m in a 100 m race and Bidhan can beat Chandan by 10 m in a 200 m race. Then in a race of 400 m, Anand can beat Chandan by
(a) 40 m (b) 39 m (c) 15 m (d) 10 m

Q.20. A man can swim with the stream at the rate of 3 km/h and against the stream at the rate of 2 km/h. How long will he take to swim 7 km in still water?
(a) 3 h (b) 2.8 h (c) 2.6 h (d) 3.2 h

Q.21. Points P and Q are 120 km apart. A motorcyclist starts from P to Q along a straight road P–Q with a speed of 30 km/h. At the same time a cyclist starts along a road perpendicular to road PQ, with a speed of 10 km/h. After how much time will the distance between them be the least?
(a) 3 h (b) 3.4 h
(c) 4 h (d) Cannot be determined

Q.22. A and B start at the same time from P and Q (55 km apart) to Q and P at 3 and 2½ km/h respectively. They meet at R and then reach Q and P to return immediately and meet again at S. The distance from R to S is
(a) 10 km (b) 12 km
(c) 15 km (d) 16 km

Q.23. A runs 1.75 times as fast as B. If A gives B a start of 60 m, how far should the winning post be so that A and B reach there at the same time?
(a) 140 m (b) 70 m
(c) 210 m (d) 175 m

Q.24. X, Y and Z walk at 6, 12, and 18 km/h respectively. They start from Jhumritalaiya towards Delhi at 2, 5, and 7 p.m. respectively, when Y catches up with X, Y sends X back with a message to Z, when will Z get the message?
(a) 9.45 pm (b) 9 pm
(c) 8.45 pm (d) 9.15 pm

Q.25. A train 75 meters long overtook a person who was walking at the speed of 6 km/h in the opposite direction and passed him in $7\frac{1}{2}$ s. Subsequently, it overtook a second person, walking in the same direction as the first person and passed him in $6\frac{3}{4}$ s. At what rate was the second person travelling?
(a) $11\frac{3}{4}$ km/h (b) 10 km/h
(c) 8 km/h (d) $4\frac{1}{2}$ km/h

Q.26. Alok rows a boat against a stream flowing at 2 km/h for a distance of 9 km, and then turns around and rows back with the current. If the whole trip lasts 6 h, find Alok's rowing speed in still water.
(a) 4 km/h (b) 3 km/h
(c) 2 km/h (d) 5 km/h

Q.27. A man walked 12 km at a certain rate, and then 6 km farther at a rate of $\frac{1}{2}$ km/h faster. If he had walked the whole distance at a faster rate, his time would have been 20 min less. How long did he really take to walk the distance of 18 km?
(a) 4.5 h (b) 5.33 h
(c) 5.66 h (d) None of these

Q.28. Shramjeevi Express leaves Patna for New Delhi at 10.10 am and reaches New Delhi at 3.30 pm. Magadh

Express leaves New Delhi at 9.48 am and reaches Patna at 2.36 pm. If routes of both the trains are the same, then at what time will both the trains meet each other?

(a) 12.52 pm (b) 1.12 pm
(c) 1.32 pm (d) None of these

Q.29. A can give a start of 20 m to B and B can give a start of 10 m to C in a race of 200 m. By how many metres can A beat C in the same race?

(a) 21 m (b) 28 m (c) 29 m (d) 31 m

Q.30. Find the distance between 2 towns when by increasing the speed to 7 km/h a train finishes the journey in 1 h less, and when the speed is reduced to 5 km/h, the train finishes the journey in 1 h more.

(a) 210 km (b) 180 km
(c) 240 km (d) 164.4 km

Q.31. Preeti and Uma stay next to each other and study in the same school. Preeti and Uma start walking towards their school at the speed of 2 km/h and 3 km/h respectively. The faster of the two reaches the school first, turns around and starts walking back. If Preeti and Uma meet 200 m away from the home, what is the distance between their residences and the school?

(a) 260 m (b) 240 m (c) 200 m (d) 250 m

Q.32. Vinit has to cover a certain distance of 7 km in 1 h 12 min. If he walks some distance at 4 km/h and rides a horse for some distance at 10 km/h, he reaches his destination in time. How far should he walk on foot?

(a) 14/3 km (b) 11/3 km
(c) 13/3 km (d) 10/3 km

Q.33. A starts driving from P to Q at 9 a.m. and B starts driving from Q to P at 10 a.m. B is 50% faster than A. What is the time when they meet if P and Q are 300 km apart and A's speed is 50 km/h.

(a) 12.30 pm (b) 12 noon
(c) 11.00 am (d) 11.30 am

Q.34. Binod and Neeraj start from one end of a 1000 m track while Deleep starts from the other end. Binod is 50% faster than Neeraj and Deleep is 150% faster than Neeraj. If Binod meets Deleep in 25 seconds, how long does Neeraj take to meet Deleep?

(a) 30 s (b) 35 s
(c) $28^{4}/_{7}$ s (d) $26^{3}/_{7}$ s

Q.35. Vinod and Kaurvaki start travelling in the same direction at 8 km/h and 13 km/h respectively. After 4 hours, Vinod doubled his speed and Kaurvaki reduced his speed by 1 km/h and reached the destination together. How long did the entire journey last?

(a) 3 h (b) 6 h (c) 9 h (d) 12 h

Q.36. A bus B1, left a town T1 for another town T2 at 6.00 am at 7.30 am. Another bus B2 also left T1 for T2 and the speed of this bus was 5 km/h more than that of B1. At 10.30 pm on the same day the buses were 21 km apart. Find the speed of bus B1.

i. 36 ii. 64 iii. 72

(a) i or ii only (b) i or iii only
(c) Either i or ii or iii (d) None of these

Q.37. Two men leave places A and B simultaneously. One of them left A for B and the other B for A. Both travel at their own uniform velocity. Having arrived at their destination, they turn back without stopping and travell to their starting points. For the first time they met at 12 km from B, the second time at 6 km from A. Find the distance AB.

(a) 72 km (b) 30 km
(c) 45 km (d) 60 km

Q.38. P and Q start running simultaneously – one from point A to B and the second from point B to A. P's speed is 6/5th of Q's speed. If after crossing Q, P takes 21/2 hours to reach B, how much time does Q take to reach A after crossing P?

(a) 3.36 min (b) 3.48 min
(c) 4.12 min (d) None of these

Q.39. A beats B by 20 m in a 100 m race and B beats C by 20 m in a 100 m race. How much start should A give to C in a 100 m race so that both of them reach the winning post at the same time?

(a) 30 m (b) 44 m (c) 32 m (d) 36 m

Q.40. Mahima beats Kareena in 1000 m race by 50 s and Bhumika by 450 m. If Kareena and Bhumika run 1000 m race, Kareena wins by 40 s. How much time does Bhumika take to run 1000 m?

(a) 210 sec (b) 225 sec
(c) 180 sec (d) 200 sec

Q.41. Two trains have respective lengths as 230 m and 190 m. They cross each other completely in 21 s, if they are travelling in the opposite direction and in 42 s, if they are travelling in the same direction. Find the ratio of the speeds of the two trains.

(a) 3:1 (b) 4:1
(c) 3:2 (d) None of these

Q.42. A car after travelling a distance of 110 km develops a problem in the engine and proceeds at 3/4th of its former speed and arrives at the destination 60 min late. Had the problem developed 30 km further on, the car would have arrived 12 min sooner. Find the original speed of the car.

(a) 45 km/h (b) 60 km/h
(c) 50 km/h (d) None of these

Q.43. A stone, allowed to fall under gravity falls $5t^2$ m in 't' seconds. When a stone is dropped into a well, the sound of the stone striking the water was heard 65/9 sec after it was dropped. Find the depth of the well if sound travels 324 metres in a second?

(a) 360 m
(b) 180 m
(c) 210 m
(d) 240 m

Q.44. A watch which gains uniformly is 5 min slow at 8.00 am on sunday and is 5 min 48 s fast at 8.00 pm on the following sunday. When did it show the correct time?

(a) 7.20 pm Thursday
(b) 7.20 am Wednesday
(c) 7.20 pm Wednesday
(d) 7.20 am Thursday

Q.45. Anil and Dheeraj run a 4 km race on a circular course 400 m long in the same direction. If their speeds are in the ratio of 3:2, how many times does Anil pass Dheeraj?

(a) 2 times
(b) 3 times
(c) 4 times
(d) 5 times

Q.46. At what time between 8 and 9 O'clock will the hours and min hand make an angle of 72° with each other?

(a) 8 h 306/11
(b) 8 h 568/11
(c) 1 or 2
(d) None of these

Q.47. At what time between 2 and 3 O'clock will the hands of a clock be in a straight line facing away from each other?

(a) $2{:}42{:}\frac{9}{11}$
(b) $2{:}42{:}\frac{49}{77}$
(c) $2{:}43{:}\frac{9}{11}$
(d) $2{:}43{:}\frac{49}{77}$

Q.48. A man can row a distance of 2 km upstream in 15 min and returns the same distance in 10 min. How much time will he take to row the same distance upstream if due to a tide the speed of the current gets doubled?

(a) 25 min
(b) 45 min
(c) 20 min
(d) None of these

Q.49. Two points A and B are diametrically opposite points on a 6 km long circular road. A cyclist started from A and made two rounds. He made the first round with a certain uniform speed and then decreased his speed by 3 km/h. The interval between his two passages through point B is 50 min. Find the speed with which the cyclist made the first round.

(a) 6 km/h
(b) 9 km/h
(c) 12 km/h
(c) 16 km/h

Q.50. Minute hand of a clock overtakes the hour hand at the intervals of 63 min of the correct time. How much does the clock lose or gain in a day?

(a) $56\frac{24}{77}$ min
(b) $55\frac{8}{77}$ min
(c) $56\frac{8}{77}$ min
(d) None of these

MODERATE

Q.1. A train of 300 m is travelling with the speed of 45 km/h when it passes point A completely. At the same time, a motorbike starts from point A with the speed of 70 km/h. When it exactly reaches the middle point of the train, the train increases its speed to 60 km/h and motorbike reduces its speed to 65 km/h. How much distance will the motorbike travel while passing the train completely?

(a) 2.52 km
(b) 2.37 km
(c) 2 km
(d) None of these

Q.2. Two persons A and B are standing diametrically opposite on a circular track of with a radius of 3.5 units. They start running in an opposite direction on the circular track and their speeds are 4 units/min and 7 units/min respectively. On meeting each other they take rest for half a min and then start running with twice the original speed. In five min how many times will they take rest together?

(a) 6 times
(b) 7 times
(c) 10 times
(d) None of these

Q.3. Two riders on the horseback with a gun and a bullet proof shield were moving towards each other at a constant speed of 20 km/h and 5 km/h respectively. When they were 100 km apart, they started firing bullets at each other at the speed of 10 km/h. When a bullet of rider 1 hits the shield of rider 2, rider 2 fires a bullet and the process continues vice versa. Neglecting the time lag at the instant when the bullet hits the shield and the rider fires the shot, find the total distance covered by all the bullets shot by both the riders.

(a) 50 km
(b) 40 km
(c) 25 km
(d) None of these

Q.4. A group of soldiers are marching with a speed of 5 m/s. The distance between the first and the last row of soldiers is 100 m. A dog starts running from the last row and moves towards the first row, turns and comes back to the last row. If the dog has travelled 400 m, the speed of the dog is

(a) $5\sqrt{2}$ m/s
(b) $3\sqrt{5}$ m/s
(c) $6\sqrt{5}$ m/s
(d) $6\sqrt{2}$ m/s

Q.5. A train approaches a tunnel PQ which is 16 m long. Two rabbits A and B are standing at points which are 12 m and 8 m inside the tunnel with respect to the entrance P. When the train is x m away from P, A starts running towards P and B towards Q. Difference between the ratios of the speed of A to that of the train and the ratio of the speed of B to that of train is 1/8. How much can the distance x be, if both of them get caught at the ends of the tunnel?

(a) 30 m (b) 16 m
(c) 48 m (d) None of these

Q.6. Pranesh wants to travel to Pune. He has three options.

Route-1 Travel by a train from Chiplun to Mumbai and then from Mumbai to Pune. The trains take 4.5 h and 3 h respectively.

Route-2 Travel by a bus via Bhor. The distance covered by this route is 300 km at 40 km/h and there is a half an hour break in between.

Route-3 Travel by a bus via Koyana. The distance covered by this route is 500 km at 60 km/h. If Pranesh goes by train there is a time span of 30 min between the respective trains. Which route should he take to save his time?

(a) Route 1 (b) Route 2
(c) Route 3 (d) [1] or [2]

Q.7. The numbers expressing (in metres) the length of a fast and a slow trains are three times the numbers expressing (in km/h) the speed of the slow and the fast train respectively. The speed of the slow train and that of the fast train consists of the same two digits, but in a reverse order. The time taken by the two trains to pass each other, when traveling in an opposite direction on parallel lines will be

(a) 10.8 s (b) 5.4 s
(c) 9.77 s (d) 5/18 s

Q.8. Michael Johnson is a great sprinter. He can climb 12 stairs of a staircase in 3 s. But after climbing 480 stairs his speed reduces to 8 stairs in 3 s. The empire state building in NY has 90 floors and the lift takes 4 s per floor. To get to the next floor one has to climb 12 stairs. If Michael is on the ground floor how should he reach the 90th floor to be the first?

(a) By lift
(b) Running
(c) Run till 50th floor and then take the lift
(d) None of these

Q.9. A car goes 20 miles on a gallon of gas driving at 60 miles/h. If the car is driven at 75 miles/h it goes as far as 70%. How many gallons of gas will be required to travel 210 miles/h.

(a) 6.17 (b) 22.5 (c) 12 (d) 15

Q.10. Rakesh and Siddharth row on a river simultaneously from the same point. Rakesh rows downstream and Siddharth rows upstream. In 15 min, they are 2.25 km apart. Rakesh then turns to follow Siddharth and after 30 min from the beginning, the boats have rowed together 3.5 km. If the speeds of Rakesh and Siddharth and the stream are constant at how many km/hour does the stream flow?

(a) 3 km (b) 2.5 km
(c) 2 km (d) 5 km

Direction for questions 11 and 12: Refer to the data below and answer the questions that follow.

There is a race between Sagar and Sapna. Both of them Delhi. Sagar started on a bike with a speed of 40 km/h and Sapna has started in a car with a speed of 60 km/h from Mumbai to Delhi. After 3½ h of the journey there was a snag in the car. She tried to repair the car but in vain. After half an hour she got a lift for another 500 km in a truck, which was travelling with a speed of 45 km/h. From there Delhi was at a distance of 200 km on road, instead, Sapna took a shorter route which was only 100 km away from Delhi. She started running at the speed of 30 km/h on the shorter route to reach Delhi.

Q.11. How much time did Sapna take to reach Delhi?

(a) 15.5 h (b) 16.5 h
(c) 17.5 h (d) None of these

Q.12. If there was no snag in the car, by how much distance Sapna would have defeated Sagar?

(a) 252 km (b) 264 km
(c) 303 km (d) 321 km

Q.13. A 200 m-long train passes a 350 m long platform in 5 s. If a man is walking at a speed of 4 m/s along the track and the train is 100 m away from him, how much time will it take to reach the man?

(a) Less than 1 s
(b) 1.04 s
(c) More than 2 s
(d) Data insufficient

Q.14. A man waiting at a point P has to reach point Q at some distance from P. He is considering three plans of action.

A. Wait at P itself and catch the first taxi going towards Q.
B. Start walking towards Q and catch the first taxi going towards Q.
C. Start walking away from Q and catch the first taxi going towards Q.

He walks at a constant rate of 5 km/h and taxi travel at a constant rate of 50 km/h. The arrival time of the taxi is totally random. Which of the following statement is then true? (Assume that he is the only person who is looking for a taxi on that road.)

(a) He will always reach Q first if he chooses plan A.
(b) He will always reach Q at the same time regardless of which plan he chooses.

(c) He will sometimes reach Q at the earliest if he chooses plan B.
(d) He will always reach Q at the earliest if he chooses plan C.

Q.15. Three cockroaches John, Mac and Roe decide to take a race from one corner of the room to the diagonally opposite corner of the room. John can fly, Mac can walk and Roe can walk only along the edges. If all of them reach the destination at the same time by taking the smaller paths, what is the ratio of the speeds of John, Mac and Roe assuming that the room is cubical in shape?
(a) $\sqrt{3}+1:3:\sqrt{3}$ (b) $\sqrt{3}:5:3$
(c) $\sqrt{3}:\sqrt{2}:3$ (d) None of these

Q.16. Four men start to move from a particular point. Two of them go in the direction opposite to each other at an equal speed. The other two also move in the opposite directions at an equal speed, but in a direction that is perpendicular to that of the original two. All of them stop at the same time, if the final positions of the men are joined, then it forms.
(a) A parallelogram (b) A rectangle
(c) A rhombus (d) A square

Q.17. Saket runs twice on a circular track at a uniform speed. Bidhan starts at the same time as Saket and after Bidhan has completed one lap, he stops and Sunny starts and completes the second lap at a uniform speed. It is given that they start the race exactly at 2.30 pm and the track is 500 m in circumference. Sunny runs at a speed of 5 km/h greater than that of Bidhan and Saket's speed is 2 km/h greater than that of Bidhan. When does Saket finish the 2nd lap if Sunny also finishes at the same time?
(a) 2.32 pm (b) 2.35 pm
(c) 2.33 pm (d) 2.36 pm

Q.18. Two persons A and B are running between two points P and Q to and from infinitely. A is starting from P and B is starting from Q. They meet for the first time at 0.6 D from point P, where D is the distance between the points AB. At how many distinct points will they meet till their 10th meeting?
(a) 10 (b) 3 (c) 4 (d) 6

Direction for questions 19 and 20: Refer to the data below and answer the questions that follow.

When BMW introduced its most advanced engine, it was found that the engine of the car followed certain mileage characteristics in a speed limit of 25 km/h to 80 km/h. The car can travel a distance of 15 km in one litre of petrol while speeding at 50 km/h. From this point onwards for every increase of 5 km/h in the speed of the car the mileage is increased by 1 km per litre. Similarly, for every decrease of 5 km/h in the the speed of the car, the mileage is decreased by 1 km per litre. Assume that the cars operate in this speed limit only.

Q.19. Rahul Ghosh has filled exactly 10 litres of petrol in his car. He drives at a speed of 45 km/h for first two hours. What is the maximum distance that he can cover before his tank goes dry?
(a) 80 km (b) 67.37 km
(c) 75 km (d) None of these

Q.20. In the previous question, what is the maximum span of time for which Rahul Ghosh can enjoy the drive?
(a) 24/7 h (b) 15/4 h
(c) 37/15 h (d) None of these

Q.21. Two rabbits A and B are running a race in which they have to go upto 50 m mark and then come back to the starting point. A runs in a sequence of 3 jumps in which the distance traveled in the first one is twice that of the other two and B runs in a sequence of 3 jumps in which the distance travelled in the first jump is 1.5 that of the distance travelled in the 2nd jump and distance travelled in the second jump is twice the distance travelled in the third jump. Distance travelled by A in 2 jumps and by B in 3 jumps equal to 6 m. In the 2nd race they interchange their individual sequence of jumps. Who will win the 1st and the 2nd match if on completing the 50 m mark they turn back and start from the 1st jump?
(a) A, A (b) B,B
(c) A,B (d) None of these

Q.22. There are two clocks. One of them gains 2 min in 12 h and another one loses 2 min in 36 h. Both are set right at 12 noon on Tuesday. What will be the correct time when both of them show the same time for the next time?
(a) 12 night (b) 1.30 am
(c) 10.30 pm (d) 12 noon

Q.23. A train's journey is disrupted due to an accident on its track after it has travelled 30 km. Its speed then comes down to 4/5th of its original and consequently it runs 45 min late. Had the accident taken place 18 km farther away, it would have been 36 min late. Find the original speed of the train.
(a) 25 km/h (b) 36 km/h
(c) 30 km/h (d) 20 km/h

Q.24. A passenger train departs from Ahmadabad at 6 pm for Bombay. At 9 p.m. an express train, whose average speed exceeds that of the passenger train by 15 km/h, leaves Bombay for Ahmedabad. Two trains meet each other mid-route. At what time do they meet, given that the distance between the cities is 1080 km?
(a) 4 pm (b) 2 pm
(c) 12 midnight (d) 6 am

Q.25. In a race on a circular track, there were 12 laps of 150 m each. Akhil and Bharat were very close till the end of the third lap, but thereafter Bharat went on to win the race by 81 m. If Bharat gained uniformly over Akhil in all the remaining laps, the ratio of the speed of Akhil and Bharat for each of other laps was

(a) 129:150 (b) 150:141
(c) 150:129 (d) 141:150

Q.26. Magadh Express and Vikramshila Express run between Patna and New Delhi. Every night each of these train leaves Patna for Delhi. Likewise every night each of the trains leaves Delhi for Patna. The train journey lasts 36 h and the trains are identical in all aspects. If the train that leaves in the night is like the one that arrived in the morning, what is the minimum number of trains required?

(a) 4 (b) 8
(c) 16 (d) None of these

Q.27. An athlete runs to and fro between points A and B at a speed of 10 km/h. A second athlete simultaneously runs from point B to A and back at a speed of 15 km/h. If they cross each other 12 min after the start, after how much time will they cross each other?

(a) 18 min (b) 24 min
(c) 36 min (d) 48 min

Q.28. There are two swimmers A and B who start swimming towards each other from opposite banks of the lake. They meet at a point 900 ft from one shore for the first time. They cross each other, touch the opposite bank and return. They meet each other again at 300 ft from the other shore. What is the width of the lake?

(a) 2400 ft (b) 1800 ft
(c) 2700 ft (d) 3600 ft

Q.29. DMRC is plying trains between Vishwavidyalaya and Central Secretariat, which are at a distance of 120 km. There are three more stations in between Vishwavidyalaya and Central Secretariat and each train stops at these two stations for 2 minutes. What is the minimum numbers of trains DMRC should put in this route, if the trains are travelling at a speed of 40 km/h and it is desirable to have a regular interval of 6 minutes between the departures of two trains from both the stations?

(a) 32 (b) 63
(c) 64 (d) None of these

Q.30. The front wheels of a tractor make 36 revolutions per min while the rear wheels make 18 revolutions per min. After how many seconds will the two wheels be in the same position relative to each other as they were when they started moving?

(a) 2.22 (b) 3.33 (c) 1.66 (d) 4.44

Q.31. A train and a car start at the same time, parallel to each other in the same direction. The car starts its journey from the rear end of the train. The car reaches the front end of the train and comes back to the back end of the train. In the meantime, the mail train travels a distance of 1 km. If the speed and the length of the mail train is 1 km/h and 1 km respectively then what is the distance covered by the car?

(a) 2 km (b) $1+\sqrt{2}$ km
(c) $2+\sqrt{2}$ km (d) Cannot be determined

Direction for questions 32 to 34: Refer to the data below and answer the questions that follow.

A, B and C participate in a triathlon. The triathlon consists of three parts. The first part consists of running, the second part cycling and the third part swimming. A can cycle thrice as fast as he can run and swim one-third as fast as he can run. B can cycle four times as fast as he can run and swim one-fourth as fast as he can run. C can swim half as fast as he can run and cycle four times as fast he can run. A can run 10 percent faster than B and 20% faster than C. The ratio of the length of the running track to that of the cycling is ½ and to that of swimming tracks is 2.

Q.32. Who is leading after the 2nd part is over?

(a) A (b) B
(c) C (d) Cannot be determined

Q.33. Who will win the triathlon?

(a) A (b) B
(c) C (d) Cannot be determined

Q.34. C beats B by 100 m finally. What is the length of the cycling tracks given that the length of the swimming track is more than 100 m?

(a) 580 m (b) 622 m
(c) 518 m (d) 576 m

Q.35. Pramod can row at 3 km/h in still water. He wants to cross a river that is flowing at the speed of 4 km/h in the shortest possible time. Find the ratio of the distance between the point he leaves the bank on one side and the point that he reaches on the other side to the distance between the shortest possible distance to cross the river?

(a) 1:1 (b) 1:2
(c) 2:3 (d) None of these

Q.36. Vinit's rowing at a speed of 4 km/h in still water. He has to reach a destination that is 130 km downstream of a river. The river flows at a speed of 2 km/h but due to tides every two h the velocity doubles and then after another two h it halves. If Vinit starts at the change of the tide when the river was flowing at 2 km/h, then after how much time will he reach his destination?

(a) 8¼ h (b) 18 ½ h
(c) 9 ½ h (d) None of these

Q.37. Ramesh and Somesh are competing in a 100 m race. Initially, Ramesh runs at twice the speed of Somesh for the first fifty m. After the 50 m mark, Ramesh runs at 1/4th his initial speed while Somesh continues to run at his original speed. If Somesh catches up with

Ramesh at a distance of 'N' m from the finish line, then N is equal to

(a) 35 (b) 10
(c) 45 (d) None of these

Direction for questions 38 and 39: Read the passage given below and solve the questions based on it.

Two cars A and B start simultaneously from two different cities P and Q and head for the cities Q and P respectively. As soon as car A reaches the city Q, it turns and starts for city P and as soon as it reaches P again, it leaves for city Q and so on. Similarly car B, travels from Q to P, from P to Q and so on. The speeds of the car A and B are in the ratio of 3 : 4.

Q.38. If car B travelled 1040 m when the two cars meet for the seventh time, what is the distance between the two cities P and Q?

(a) 140 m (b) 280 m
(c) 182 m (d) Cannot be determined

Q.39. If car A travelled 1860 m, how many times did the cars A and B meet? (Take the distance PQ as calculated in the previous question.)

(a) 14 (b) 15
(c) 16 (d) Cannot be determined

Q.40. The Howrah Mail leaves Trivandrum everyday at 12:00 noon and reaches Howrah exactly three days later. Also, the Trivandrum Mail leaves Howrah at 12:00 noon everyday and reaches Trivandrum exactly three days later. Not counting the trains just starting or just leaving the platform when a train leaves or arrives, how many Howrah Mails will one cross when travelling by Trivandrum Mail?

(a) 2 (b) 3 (c) 5 (d) 9

Q.41. A person starts writing natural numbers, starting from 1, on a blackboard, at the rate of 60 digits per minute. At the same time, another person begins to erase the digits from 1 onwards, at the rate of 40 digits per minute. Find the difference between the leftmost digit and the rightmost digit on the blackboard immediately after one hour.

(a) 3 (b) 5
(c) 2 (d) None of these

Q.42. Two persons Prabhat and Vinay are walking around a circular park of the length 960 m. Prabhat walks at the rate of 80 m/min while Vinay walks at the rate of 60 m/min. If both of them start from the same starting point at the same time in the same direction, when will they be together?

(a) 24 min (b) 48 min
(c) 96 min (d) 120 min

Q.43. A 100 m-long train is approaching an unmanned railway crossing. The train is running at a uniform speed of 90 km/h and is 1 km away from the crossing. At the same time, a packed jeep, also approaching the crossing, is 700 m away from it. Assuming that the jeep is also moving at a uniform speed of N, what is the range of N, for which the jeep will not collide with the train?

(a) 59 km/h < N < 61 km/h
(b) 57 km/h < N < 63 km/h
(c) 55 km/h < N < 65 km/h
(d) 53 km/h < N < 67 km/h

Q.44. A road from A to B is 11½ km long, first goes uphill, then crosses a plain, and then goes downhill. A person walking from A to B covered this road in 2 h 54 min, and the return journey took him 3 hours 6 min. His speed uphill is 3 km/h, on the plain is 4 km/h and downhill is 5 km/h. What is the length of the plain part of the journey?

(a) 4 km (b) 6 km
(c) 4½ km (d) None of these

Q.45. Two cyclists start from the same place to ride in the same direction. Aflatoon starts at noon with a speed of 8 km/h and Bablajoon starts at 2 pm with a speed of 10 km/h. At what times Aflatoon and Bablajoon will be 5 km apart?

(a) 7.30 pm same day and 1.30 am on the next day
(b) 7.30 pm same day and 12.30 am on the next day
(c) 8.30 pm same day and 1.30 am on the next day
(d) 8.30 pm same day and 12.30 am on the next day

Q.46. Mrs and Mr Sinha travel from Patna to Lucknow and break their journey at Buxer. Somewhere between Patna and Lucknow, Mrs Sinha asks, "How far have we travelled?" Mr Sinha replies, "Half as far as the distance from here to Lucknow." Somewhere between Buxer and Lucknow, exactly 300 km from the point where she asked the first question, Mrs Sinha asks, "How far have we to go?" Mr Sinha replies, "Half as far as the distance from Buxer to here." The distance between Patna to Lucknow is

(a) 750 km (b) 600 km
(c) 450 km (d) None of these

Q.47. At his normal speed, Rishu can go 18 km downstream in a flowing river in 9 h less than what he takes to go the same distance upstream. The downstream journey would take 1 h less than what the upstream journey would take provided he doubles his rate of rowing. Find the speed of the river (in km/h)?

(a) $6\frac{2}{3}$ km (b) 8 km

(c) $9\frac{1}{2}$ km (d) 12 km

Q.48. Umesh and Harish are running on a circular track of length 1200 m in opposite direction. Everytime they meet, they exchange their speed as well as their directions. What is the shorter distance along the track between their 13th and 23rd meeting points, if the ratio of their oringinal speeds is 1:5?

(a) 200 m (b) 400 m

(c) 800 m (d) 1000 m

Q.49. Two friends Manoj and Sagar start running simultaneously in an opposite direction on a circular track of the length 2100 m with speed of 7 m/s and 3 m/s respectively. With every meeting they exchange their speed and they meet for a total of ten times. How much distance did Sagar cover in total?

(a) 1400 m (b) 700 m

(c) 1050 m (d) None of these

Q.50. A and B start simultaneously at one end of a swimming pool whose length is 50 m. The swimming race is a race of 1000 m. If A beats B and meets him 17 times during the course and A's speed is 5 m/s then the speed of B could be

(a) 1 m/s (b) 3 m/s

(c) 4 m/s (d) 6 m/s

ADVANCED

Direction for questions 1 to 5: Read the passage given below and solve the questions based on it.

There are 8 days in a week from Sunday to Saturday and another day called Funday on planet North. There are 36 h in a day and each hour has 90 min while each minute has 60 s.

Q.1. Find the approximate angle between the hands of a clock on North when the time is 16.50 am?

(a) 189 (b) 131

(c) 320 (d) 165

Q.2. At what time between 14 o'clock and 15 o'clock will the two hands of the clock be at an angle of 60°?

(a) 14:89 $^{16}/_{17}$ (b) 14:37 $^{13}/_{17}$

(c) 14:58 $^{14}/_{17}$ (d) 14:55 $^{5}/_{17}$

Q.3. Bubby, an inhabitant of the planet North leaves home between 7 am and 8 am and returns between 1 pm and 2 pm to find that the minutes and hours hands have interchanged their positions. How long was bubby out of the house?

(a) 6 h 28 min (b) 12 h 39 min

(c) 12 h 28 min (d) 12 h 22 min

Q.4. A watch which is correctly set at 16 o'clock in the morning shows 10 min after 13 o' clock in the evening when the correct time is 14 o'clock. What is the time when the watch indicates 15 o'clock in the evening?

(a) 16:00 (b) 16:32 $^{5}/_{7}$

(c) 16:16:$^{16}/_{17}$ (d) 16:33 $^{6}/_{17}$

Q.5. A watch on planet North which loses time uniformly is 15 min fast at 14 am on Sunday and is 45 min slow at 8 pm on the next Sunday. When did the clock show the correct time?

(a) 6 am on Saturday

(b) 8 am on Funday

(c) 17 am on Tuesday

(d) 11 am on Funday

Q.6. In the shooting of the movie 'AKS', Manu, the actor, is running towards a vertical mirror with a speed of 10 m/s on a line which is perpendicular to the plane of the mirror. The mirror is moving towards Manu with the speed of 6 m/s along the same line. Find the speed of Manu's image with respect to Manu (the image of Manu forms on the other side of the mirror such that the distance between the image of Manu and the mirror is equal to the distance between Manu and the mirror).

(a) 15 m/s (b) 25 m/s

(c) 30 m/s (d) None of these

Q.7. A rabbit, a tortoise and a mongoose decide to run a closed circuit race. If the rabbit overtakes the mongoose 2 times per round, with the 2nd overtake being at the starting point and the mongoose overtakes the tortoise 4 times per round, the 4th overtake being at the starting point, then find the ratio of the speed of the rabbit to that of the tortoise.

(a) 15:1 (b) 8:1 (c) 6:1 (d) 10:1

Q.8. Anil left his house for school S min late but he traveled at 4/3th of his usual speed and reached school S min early. Next day he left home S min early than the previous day and travelled at 8/11th of his usual speed. He reached his office

(a) S min late (b) 2S min late

(c) Exactly on time (d) Cannot be determined

Direction for questions 9 and 10: Read the passage given below and solve the questions based on it.

There are two cities Nagpur and Pune along a straight line 120 km apart. A and B start at the same time from Nagpur and Pune respectively with the speed of 40 km/h and 60 km/h respectively. They travel towards each other, and after they meet for the first time, they reverse directions and also interchange their speeds. After reaching their respective starting points, they reverse their directions and start proceeding towards each other again.

Q.9. How many hours from the beginning do they meet for the second time?

(a) 2 h (b) $2^1/_5$ h

(c) $2^1/_3$ h (d) $3^3/_5$ h

Q.10. How much distance had B covered till the time he met A for the second time?

(a) 216 km (b) 180 km

(c) 174 km (d) 168 km

Direction for questions 11 and 12: Read the passage below and solve the questions based on it.

A number of runners, numbered 1, 2, 3,…,N and so on, start simultaneously at the same point on a circular track and keep on running continuously in the same direction, around the track. They run in such a way that the speed of the runner numbered N (N>1) is N times that of the runner numbered 1.

Q.11. If there are exactly six runners, then at how many distinct points on the track is the runner numbered 1 overtaken by any of the other five runners?

(a) 15 (b) 11 (c) 9 (d) 10

Q.12. If there are exactly four runners, then at how many distinct points on the track do two or more runners meet?

(a) 3 (b) 4 (c) 5 (d) 6

Direction for questions 13 and 14: Read the passage given below and solve the questions based on it.

Amit intended to travel a certain distance at a certain uniform speed. But after one hour, he increased his speed by 25%. As a result, in the remaining part of the time that he originally planned for the journey, he could now cover as much distance as he initially thought he would be able to cover.

Q.13. What is the total time taken for the journey?

(a) 4 hrs (b) 5 hrs

(c) 6 hrs (d) None of these

Q.14. After Amit increased his speed, if he decided to terminate his journey after covering the distance he initially intended to cover and not cover the extra distance as given in the data, what is the total time taken for the journey?

(a) 4 h 12 min (b) 5 h 24 min

(c) 3 h 36 min (d) None of these

Direction for questions 15 and 16: Read the passage given below and solve the questions based on it.

A train started at 9.00 am from station X with a speed of 72 km/h. After two hours, another train started from station Y towards X with a speed 90 km/h. The two trains are expected to cross each other at 1.30 pm. Owing to a signal problem arising at 12 noon, the speed of each of them was reduced by the same quantity and they crossed each other at 4.30 pm.

Q.15. What is the new speed of the train that started from station X?

(a) 18 km/h (b) 36 km/h

(c) 45 km/h (d) 54 km/h

Q.16. If the signal problem had occurred at 1.00 pm instead of 12 noon, at what time would the two trains cross each other?

(a) 2.30 pm (b) 3.00 pm

(c) 2.00 pm (d) 2.30 pm

Direction for questions 17 to 19: Read the passage given below and solve the questions based on it.

Bihar Couriers (BC) sends a person on motorcycle every day to the airport to collect the courier. This person reaches the airport exactly when the plane lands at its scheduled time. One day BC came to know that the flight would be late by 50 min. So, the BC person also started from his office 50 min later than the usual time. However, the plane landed earlier than was anticipated and thus the airport authorities dispatched the courier through a cyclist to BC. The cyclist meets the BC personnel after 20 min and hands over the courier to the motorcyclist. As a result, the BC person reached office 10 min earlier than expected. (Assume that everybody travels at their own uniform speeds.)

Q.17. What was the delay in the landing of the plane when compared to the normal schedule?

(a) 30 min (b) 35 min

(c) 20 min (d) 25 min

Q.18. If the cyclist covered a distance of 2.5 km before he met the motorcyclist, what is the speed of the motorcyclist?

(a) 60 km/h (b) 45 km/h

(c) 30 km/h (d) Cannot be determined

Q.19. If on that particular day when the cyclist covered 2.5 km before he met the motorcyclist, the motorcyclist spent a total of 30 min for his complete journey, what is the distance from BC's office to the airport?

(a) 20 km	(b) 15 km
(c) 10 km	(c) Cannot be determined

Q.20. My digital clock is peculiar. It counts 10 s of a normal clock as 1 min and 60 such min of itself as 1 h. It has also a display which shows the day. It was at par with the normal clock at 12 noon on Monday. At 3 pm of the same day (Actual time), I just read the time display of my weird clock. I read

(a) 6.00 pm Monday
(b) 6.00 pm Tuesday
(c) 6.00 am Tuesday
(d) 4.00 am Tuesday

Direction for questions 21 and 22: Read the passage below and solve the questions based on it.

Every morning Ravi and Beena start walking from A and B respectively towards each other. They meet at C. Ravi always starts at 5.30 am and Beena starts at 4.45 am. Ravi's speed is 6 km/h, whereas Beena's speed is 5 km/h.

Q.21. One fine morning, Beena started late by a few min and hence met Ravi who started on time at D in between CB. CD = 2.5 km. By how many min was Beena late in starting on that day?

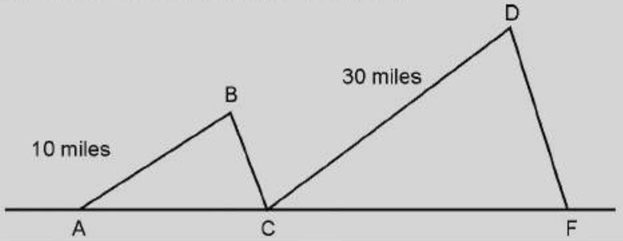
(a) 25 min	(b) 30 min
(c) 55 min	(d) Data insufficient

Q.22. One morning Ravi got delayed but Beena was on time and that day the two met at 7.00 am at E which was 2.5 km away from C. What is the distance between A and B?

(a) 20.25 km	(b) 17.25 km
(c) 14.75 km	(d) Data insufficient

Direction for questions 23 and 24: Read the following passage and solve the questions based on it.

Given below is the structure of roads from A to F. ABC and CDF are two similar mountains with ideal straight lines AB and CD as their slant heights. ∠ BAC = ∠ DCF = 30° and ∠ BCA = ∠ DFC = 60°. Speed of Manoj Kumar uphill is 20 miles/h and downhill is 40 miles/h.



Q.23. What is the total time taken by him to travel from A to F ?

(a) 3 h 12 min	(b) 2 h 21 min
(c) 2 h 35 min	(d) Cannot be determined

Q.24. Jonathan Livingston Seagull is flying between the points B and D. What is the distance traveled by Jonathan?

(a) $10\sqrt{\frac{28}{3}}$ miles	(b) $10\sqrt{\frac{7}{3}}$ miles
(c) $\frac{10}{3}$ miles	(d) None of these

Q.25. Ram and Rahim left Bombay simultaneously towards Pune. Speed of Ram is 15 km/h and the speed of Rahim is 12 km/h. Half an hour later, Mohan started from Bombay towards Pune on the same road in the same direction. After some time he overtook Rahim and 90 min further, he overtook Ram. What is Mohan's speed?

(a) 18 km/h	(b) 24 km/h
(c) 32 km/h	(d) 36 km/h

Direction for questions 26 and 27: Read the passage below and solve the questions based on it.

Ankit and Sanket are traveling by bus from Ahmedabad to Kolkatta via Bangalore in a bus which leaves Ahmedabad at 8.00 am. The bus reaches Bangalore at 10.00 am, just in time for them to catch the last bus to Kolkata. However, Ankit decides to get off earlier (at point P_1) and travels to Bangalore by car. Because of this he reaches 15 min late. If he had got off at a distance 16 km ahead (at point P_2), he would have reached only 7 min late. Sanket also gets off at a point 16 kms from Bangalore, where as the bus continues at its normal speed of 40 km/h and the ratio of speeds of Ankit and Sanket is 15:16.

Q.26. If the bus takes 45 minutes to reach Bangalore from point P_1, Sanket will reach

(a) Early by 6 min	(b) Late by 6 min
(c) On time	(d) Cannot be determined

Q.27. Had Sanket got off at P_2, at what time would he reach Bangalore?

(a) 10.05.15	(b) 10.05.05
(c) 9.54.45	(d) Cannot be determined

Q.28. A candidate misinterpreted the timings of an interview and reached the venue 35 min before the scheduled time. After 25 min, he presumed that the interview might have been cancelled and so he started walking back home at the speed of 1 km/h. But then he realized that he had left his bag there. So he comes back with the same speed and enters the office along with the interviewer, who is 5 min late. How much distance had he moved away from the office?

(a) 7.5 km (b) 250 m
(c) 125 m (d) 750 m

Q.29. Two cyclists are travelling at a speed of 3 km/h and 10 km/h around a circullar track. What is the ratio of number of meeting points that thye would meet on that track if they travel in the same direction first and in the opposite direction next?
(a) 3:13 (b) 7:13
(c) 3:7 (c) Cannot be determined

Direction for questions 30 and 31: Read the passage below and solve the questions based on it.

A jetfighter moving 3 km above the ground with a speed of 360 km/h tries to cross the border. A radar station notices it at distance of 6 km from the Jet. But it can fire anti-craft missile only when the plane enters in its border. The border is at a distance of $\sqrt{3}$ km from radar station. (Assume that the fighter moves in a straight line in the same plane with the same speed).

Q.30. If it fires the missile at the fighter as soon as it notices the jetfighter such that it will hit the fighter as soon as it enters in the borderline, then at what speed should the be missile fired?
(a) 360 km/h (b) 240 km/h
(c) $200\sqrt{3}$ km/h (d) None of these

Q.31. The radar station can fire 10 missiles simultaneously in such a way that the first missile will meet the fighter exactly at the border if the fighter moves with its original speed. The other missiles will reach up to the line of movement of the jet at same time but at a distance of 100 m from each other such that one of them will hit the fighter even if the fighter increases its speed. If jetfighter has to escape the attack, at what increased speed it should move?
(a) 400 km/h (b) 545 km/h
(c) 464 km/h (d) 380 km/h

Q.32. Two trains going on a parallel line in opposite directions take 10 s to cross each other. But if they are going in the same direction the longer train crosses the shorter train in 30 s. If the length of the longer train is decreased by 50%, the time taken to cross the shorter train while going in the same direction decreases by 8 s. Find the time taken by the longer train to cross a tunnel twice its length, if the difference between the length of the trains is 25 m?
(a) 30 s (b) 24 s (c) 40 s (d) 36 s

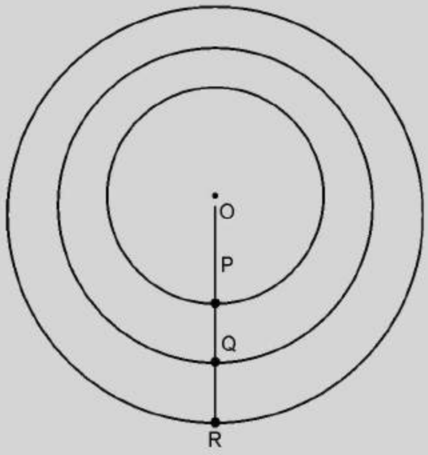
Q.33. A runner, Ajay runs twice on a circular track at a uniform speed. Vijay start at the same time as Ajay and after Vijay has completed one lap, he stops and Sanjay starts and completes the second lap at a uniform speed. It is given that they start the race exactly at 2.30 pm and the track is 500 m is circumference. Sanjay runs at a speed of 5 km/h greater than that of Vijay and Ajay's speed is 2 km/h greater than that of Vijay. When does Ajay finish the 2nd lap, if Sanjay also finishes at the same time?
(a) 2.32 pm (b) 2.33 pm
(c) 2.35 pm (d) 2.36 pm

Q.34. A man travels 1200 km from A to B by air, train and ship. The distance travelled by ship is twice the distance travelled by train. The average speed of the aircraft is twice the overall speed and the time he was on the ship was thrice the time he was on the train. If the man travels 10 h by train out of the total travel time of 50 h, then what is the total distance covered by the ship?
(a) 720 km (b) 630 km
(c) 480 km (d) 570 km

Q.35. Three friends Rupesh, Nishit and Vijay planned to race with each other on their bikes. Rupesh has a lot of riding experience, while Nishit had just learned how to ride. The race seemed unfair. Therefore, they decided that the excellent biker Rupesh will form a team with Nishit. They would bike against the other rider Vijay, each riding half the total distance of 50 kms. Also Nishit can ride only half as fast as Vijay and Rupesh travels four times as faster as Nishit. What is the ratio of the time taken by the winner to that of the looser?
(a) 4:5 (b) 5:2 (c) 2:5 (d) 4:3

Direction for questions 36 and 37: Read the passage below and solve the questions based on it.

Three persons A, B and C start running simultaneously on three different concentric circular tracks from three collinear points P, Q and R respectively which are collinear with the centre O and are on the same side of the centre as shown. The speeds of A, B and C are 5 m/s, 9 m/s and 8 m/s respectively. The lengths of the tracks on which A, B and C are running are 400 m, 600 m and 800 m respectively.



Q.36. If A and B run in a clockwise direction and C in an anti-clockwise direction, after how much time will the positions of A, B and C be collinear with the centre (and on the same side of the centre), for the first time after they start?

(a) 200 s (b) 400 s
(c) 600 s (d) 800 s

Q.37. If A runs in the clock wise direction and C in the anti-clockwise direction, after how much time will the positions of A and C be collinear with the centre, (and on the same side), for the fourth time after they start?

(a) $177\frac{7}{9}$s (b) 160 s
(c) $145\frac{5}{11}$s (d) 1

Direction for questions 38 and 39: Read the passage below and solve the questions based on it.

A train started at 9.00 am from station X with a speed of 72 km/h. After two hours, another train started from station Y towards X with a speed of 90 km/h. The two trains are expected to cross each other at 1.30 pm. Owing to a signal problem arising at 12 noon, the speed of each of them was reduced by the same quantity and they crossed each other at 4.30 pm.

Q.38. What is the new speed of the train that started from station X?

(a) 18 km/h (b) 36 km/h
(c) 45 km/h (d) 54 km/h

Q.39. If the signal problem had occurred at 1.00 pm instead of 12 noon, at what time would the two trains have crossed each other?

(a) 3.30 pm (b) 3.00 pm
(c) 2.00 pm (d) 2.30 pm

Direction for questions 40 to 42: Read the passage below and solve the questions based on it.

P, Q and R are three villages, located in the same order, along a highway. An absent minded person, Andrews, intending to reach R, starts from Q and travels at a speed of 60 km/h in the direction of QP instead of QR. At the same time as Andrews starts, Bill leaves R to reach Q and travels at the speed of 90 km/h in the direction of RQ. After travelling for 1 h in the direction of QP, Andrews realizes his mistake and reverses his direction and increases his speed to 90 km/h. After reaching Q, Bill reverses his direction and proceeds back to R at a reduced speed of 60 km/h. After reaching R, Andrews starts back towards Q with no change in speed. The distance PQ and QR are 162 km and 108 km respectively.

Q.40. At what distance from Q, do Andrews and Bill meet for the first time?

(a) 72 km (b) 90 km
(c) 84 km (d) 96 km

Q.41. At what distance from R, do the two meet for the second time?

(a) 9.6 km (b) 2.4 km
(c) 4.8 km (d) 7.2 km

Q.42. Which of the two persons travelled more distance up to the point of their second meeting, and by how much?

(a) Bill by 33.6 km
(b) Bill by 31.2 km
(c) Andrews by 16.8 km
(d) Andrews by 21.6 km

Q.43. In a certain race, the winner beats the other 6 contestants by 10, 20, 30, 40, 50, 60 m respectively. Find by what distance the 3rd runner-up beats the 4th runner-up, if he was 11.11 m ahead of the 4th runner-up, when the fist runner-up finished the race.

(a) 14.14 m (b) 14.28 m
(c) 16.66 m (d) None of these

Q.44. Distance between Lucknow and Patna is 300 km. Mayank leaves at a speed of x km/h from Lucknow towards Patna. After three hours Sharat leaves at the speed of $(x + 10)$ km/h from Lucknow towards Patna. If x and the number of hours taken to meet after Sharat starts are integers, how much distance can Mayank cover before they meet?

(a) 174 km (b) 60 km (c) 150 km (d) 180 km

Q.45. Two champion swimmers start a two-length swimming race at the same time, but from the opposite ends of the pool. They swim at constant but different speeds. They first pass at a point 18.5 m from the deep end. Having completed one length, each swimmer takes a rest at the edge of the pool for 45s. After setting off on the return length, the swimmers pass for the second time just 10.5 m from the shallow end. Thus, the length of the pool is

(a) 90 m (b) 45 m
(c) 26.5 m (d) Data insufficient

Q.46. A tortoise lays eggs on the beach and goes back to the sea where its mate is waiting. One way to reach the mate is going down 12 km on a straight line perpendicular to the sea, turning 90° and swimming for 5 dm on a straight line. On its way down to the sea, the tortoise can cut the water at any point. The speed of the tortoise on land is 1 km/h and in water is 2 km/h. If the tortoise takes the described route, at what distance from the point where it had laid the eggs will the tortoise cut the water, if the total time taken is 12 h?

(a) 7 km (b) 8 km (c) 5 km (d) 4 km

Direction for questions 47 and 48: Read the passage below and solve the questions based on it.

Trains X and Y start towards each other at the same time from points A and B for destinations B and A respectively that are 300 km apart. They meet each other 120 km from point A.

Q.47. Another express train Z starts from point A to point B, 1 h after X starts. This express train meets train Y 1.5 h after Z starts. If this express train is at least 20 km/h faster than the faster of X and Y, which of the following is true? (Ignore train lengths in calculations.)
(a) The minimum possible speed of Y is 60 km/h
(b) The minimum possible speed of X is 45 km/h
(c) The maximum possible speed of Y is 60 km/h
(d) The maximum possible speed of X is 45 km/h

Q.48. What is the minimum speed for train Z to overtake train X, before it meets train Y? (use the data from previous question, if necessary)
(a) 45 km/h
(b) 50 km/h
(c) 60 km/h
(d) Train Z will always overtake train X, before it meets train Y

Q.49. A, B and C start running a race from the same starting point at the same time in the same direction. A's speed around a path which is an equilateral triangle. B's path is a square and C's path is a regular hexagon. One edge of the triangular path, square path and hexagonal path completely overlaps with each other. If all of them complete one round at the same time then which of the following is true?
(a) Speed of C is twice that of the speed of B.
(b) Speed of A is half that of C.
(c) Speed of B is 50% more than that of A.
(d) None of these

Q.50. On a river there is no current form A to B but a current from B to C. A man rows down a stream from A to C in 3 h, and up stream from C to A in $3\frac{1}{2}$ h; had there been the same current in all the ways as from B to C his journey down stream would have occupied $2\frac{3}{4}$ h; find the length of the time, his return journey would have taken under the same circumstances.
(a) 4 h (b) 4.2 h
(c) $3\frac{17}{20}$ h (d) 2 h 24 mins

TRUE/FALSE

1. If the ratio of speed of two trains and ratio of their lengths are given, we can find out the time taken by them to cross each other while passing in the same direction.

   State whether true or false.

2 Sum of lengths of two trains are given. Speed of the longer train is less than the speed of the smaller train and both the speeds are given. We can find out the time taken by these trains to cross each other while passing in the opposite direction.

State whether true or false.

3. Two runners are running in opposite direction on a circular track starting from the same point in the same direction same time. Length of track is given. We can find out the distinct meeting points of the runners if the ratio of their speeds are given.

   State whether true or false.

4. Two runners A and B are running on a linear track. Length of linear track is given. B gives a start of 10 seconds to A and both of them finish the race in dead heat. With the information given we can find out the ratio of speed of A and B.

   State whether true or false.

5. There are 5 runners running on a circular track starting from the same point in the same direction same time. If we know the speeds of each of the runners and the distances between each of the possible distinct meeting points between any two of them is given, we can find out the length of track.

   State whether true or false.

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(d)	(b)	(c)	(a)	(d)	(b)	(c)	(b)	(b)	(c)	(a)	(d)	(d)	(b)	(c)	(b)	(b)	(a)	(a)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(d)	(b)	(b)	(c)	(d)	(c)	(d)	(c)	(b)	(a)	(b)	(b)	(d)	(a)	(d)	(a)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(a)	(a)	(c)	(b)	(a)	(d)	(d)	(c)	(a)	(d)	(d)	(b)	(c)	(c)	(a)	(b)	(a)	(d)	(d)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(a)	(c)	(b)	(c)	(b)	(c)	(d)	(d)	(b)	(c)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(d)	(b)	(a)	(c)	(d)	(a)	(d)	(d)	(c)	(d)	(c)	(a)	(c)	(d)	(a)	(b)	(b)	(c)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(c)	(c)	(d)	(b)	(c)	(a)	(c)	(b)	(b)	(b)	(c)	(c)	(d)	(d)	(d)	(a)	(c)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(d)	(b)	(b)	(a)	(b)	(d)	(a)	(b)	(d)	(b)										

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(c)	(a)	(b)	(d)	(a)	(b)	(a)	(a)	(d)	(b)	(b)	(a)	(a)	(a)	(d)	(c)	(d)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(c)	(c)	(a)	(a)	(b)	(a)	(c)	(b)	(a)	(c)	(b)	(c)	(c)	(a)	(b)	(a)	(a)	(d)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(c)	(d)	(d)	(b)	(b)	(a)	(d)	(d)	(b)	(c)										

■■■ True/False

1. False 2. True 3. True 4. False 5. True

HINTS AND SOLUTIONS

Moderate

1. Speed of train while passing point A = 70 × (5/18) m/s = V1
   Speed of bike initially = 70 × (5/18) m/s = V2
   Time taken by the bike to reach at the mid-point of the train = 150/(V2 – V1)
   Again find out the new speeds of train and bike, and calculate the time taken by the bike to cover the rest 150 m distance relative to the train.

3. This question has actually got nothing to do with bullets initially. We can see that it takes them 4 hours to reach each other. And this is the same time for which bullets will cover some distance.
   So, the total distance covered by the bullet = 4 × 10 = 40 km

4. Form the equations first and then use the options.

5. Go through the actual calculation method.

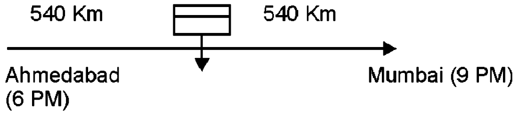
12. Total distance from Mumbai to Delhi = 910 km
    Time taken by Sapna to cover this distance, had there been no snag = 910/60
    Since their speeds are in the ratio 3 : 2, Manoj would have covered [910 – (910 × 40)/60] km less than Sapna.

13. The train can cover (200 + 350) m distance in five seconds which means the speed of the train is 110 m/s. Relative speed of man and train is 114 m/s. To cover the distance of 100 metre, it will take less than one second.

14. Go through the options.

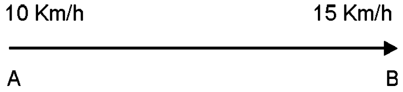
23. Let the original speed be X km/h
    According to the question, $18/(4/5x) - 18/x = 9/60$ hr
    $x = 30$ km/h

24.



Now using options can get us the result.
Take the option 6 A.M. which means the train from Ahmedabad takes 12 hours to cover 540 km. In this way, the speed will be 45 km/h and train from Mumbai takes 9 hours to cover 540 km which means the speed is 60 km/h. It is written in the question that the difference between the speed of the train from Ahmedabad and that from Mumbai is 15 km/h. Hence, this is the answer.

27.



Both the trains are crossing each other after 12 minutes which means the distance between them is 5 km. It will be easy to go through the ratio of the speed which is 2 : 3. The answer is 36 minutes.

28. Let us assume that the width of the lake = x. So, when one of the runners A covers 900 m, the other one B is covering $(x - 900)$ m. To meet next time, A will be covering $(x - 900 + 300)$ m whereas B will be covering $(900 + x - 300)$ m.
    Now, $900/(x - 900) = (x - 900 + 300)/(x + 900 - 300)$
    Now use options to find the answer.

36. Go through the options.

37. This question gives us the freedom to assume any value of speeds of Ramesh and Somesh. Let us assume the initial speed of Somesh = 20 m/s, then the initial speed of Ramesh = 40 m/s. Till 50 m they are running with this speed only. Time taken by Ramesh in covering 50 m = 1.25 sec. In the same time Somesh is covering 25 m.
    After this stage, speed of Somesh is 20 m/s, whereas speed of Ramesh = 10 m/s. Now relative speed = 10 m/s and distance = 25 m. At 75 m from the starting, both of them will be meeting.

[illegible]

38. For seventh meeting, both of them together must have covered a total of 13 D distance, where D is the distance between P and Q. Now, ratio of their speeds are 3 : 4, so distance covered will also be in the same ratio.

    Hence, distance covered by B = $13D \times \frac{4}{7} = 1040$

    Hence, D = 140 m

39. If A covers 1860 m, then in the same time B covers 2480 m (ratio of speed 3 : 4). So, total distance covered = 4340 m = 140 m + 30 × 140 m = 31 × 140 m, this is the distance covered for 16th meeting.

40. Assume that the Howrah mail is leaving on 13th November at 12 noon. It will be reaching its destination on 16th November at 12 noon. Now any train which leaves Trivandrum between the 10th and 16th November (excluding extremes) will be meeting Howrah mail on the track. These are five trains.

42. Relative speed of Vinay and Prabhat will be 20 m/minute to cover the track of 960 m. It will take 48 minutes.

43. The train has to cover 1100 m @ 90 km/h and to cover that it will take 44 seconds to cover the distance 700 m by jeep and not to collide with each other the jeep has to take either less than 44 seconds or more than that.

45. At 2 O' clock Aflatoon has already covered 16 km @ 8 km/h, Bablajoon starts running in the same direction @ 10 km/h. The relative speed is 2 km/h. They will be 5 km apart at 7.30 p.m. the same day and 12.30 a.m. on the next day.

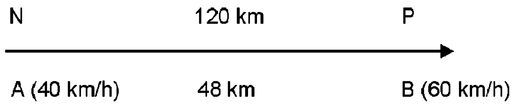
Advanced

[illegible]

1. Since there are 36 hours in a day, 18 hours dial will be there in the clock.
At 16:50, the hour hand will be in between 10 and 11 of the actual clock and the minute hand will be in between 6 and 7 of the actual clock. Now we can eliminate the options.

8. Assume any value of S and solve.

[illegible]



Ratio of relative speed 2:3
Distance travelled by A [2/5]×120= 48 km
Total distance travelled by B [3/5]120 = 72 km

9. According to the question:
First time meet at 48 km and reverse the direction with interchanging speed
T = [48 km/40] × 60 = 72 minutes (Reverse T)
And second time = [7260]/60 = 72

10. First time B covers 72 km. In reverse B covers (72 + 72) km. Hence, the total distance is 216 km.

11. Assume that the track length is 1000 m.
Now, runner 1 and runner 2 will meet at one point, i.e., the starting point.
Runner 1 and runner 3 will meet at two points, at 500 m and at the starting point.
Runner 1 and runner 4 will meet at three points, at 333.33 m, at 666.66 m and at the starting point.
Runner 1 and runner 5 will meet at four points, at 250 m, 500 m, 750 m and at starting point.
Runner 1 and runner 6 will meet at five points, at 200 m, at 400 m, at 600 m, at 800 m and at the starting point.
These are 10 distinct points.

12. Do as done in 11.

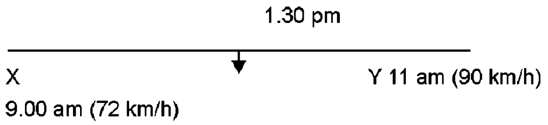
[illegible]

13. If Amit would have increased his speed by 25% in the beginning, he would have saved one hour in covering the actual planned distance. So, 1/5 T = 1 hr. (Where T is the actual planned time). Hence, T = 5 hours.

14. Saving in time would be in the later 4 hours part. Since speed is increase by 25%, time taken to cover the same distance would reduce by 20%. Hence, he would save 48 minutes.

[illegible]

15.



They cross each other at 1.30 p.m. which means the distance is 549 km.
At 12 noon, a signal was down, distance travelled by x and y at 12 noon is 306 km. Rest of the distance will be covered in 4.5 hours which mean the relative speed is 54 km. After reducing an equal speed from the speed of both the trains, the equation is like this:
$72 - p + 90 - p = 54$, so, $p = 54$
Hence, the answer is 18 km/h.

16. Suppose that the signal problem occured at 1 p.m., the distance travelled by them is $(72 \times 4 + 19 \times 2)$ km. Rest of the distance will be covered by reduced speed.

36. It does not create any difference whether they are running in the clockwise direction or anti-clockwise direction.

44. One of the ways of solving this question is going through equations. But after a certain stages we will be required to start assuming the values because all the data are not given.
Another way of doing this problem is: Start working by assuming some values. Let us assume the speed of Mayank = 10 km/h. In three hours he has covered 30 km. Now Sharat starts with a speed of 20 km/h. He will take 3 hours to meet Mayank. Till that time, the total distance covered by Mayank = 60 km.

BENCHMARKING TEST 2

This test paper contains 30 questions of 4 marks each. One-third of the marks allotted to a particular question will be deducted in case of wrong answer.

Time Given: 70 Minutes **Total Marks: 120** **Qualifying Marks: 55 Marks**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
88+	99+
83	98.5+
78	98
55	92
40	85
30	80

Direction for questions 1 and 2: Read the passage given below and solve the questions based on it.

A pedestrian having travelled from A to B found that if his rate had been one km/h quicker, he would have completed his journey in $1\frac{1}{2}$ h less and if it had been one km/h slower, he would have taken 2 hours more to complete it.

Q.1. The distance from A to B is
(a) 12 km (b) 20 km
(c) 26 km (d) 30 km

Q.2. The rate at which, the pedestrian covered the distance is
(a) 2 km/h (b) 3 km/h
(c) 4 km/h (d) 6 km/h

Q.3. There were 500 seats in PVR placed in similar rows. After the reconstruction of the hall the total number of seats became 450. The number of rows was reduced by 5 but each row contained 5 seats more than the earlier setting. What is the total number of rows and total numbers of seats in a row initially in the hall?
(a) 30 and 15 (b) 25 and 20
(c) 20 and 25 (d) 50 and 10

Q.4. Krishna owns several cows, some black and some white. He finds that 4 black cows and 3 white cows provide the same amount of milk in 5 days as 3 black cows and 5 white cows provide in 4 days. What is the ratio of milk provided by a black cow in a day to that by a white cow in a day?
(a) 8 : 5 (b) 5 : 8 (c) 3 : 5 (d) 5 : 3

Q.5. Two cities P and Q are 60 km apart. A college is to be built to serve 150 students in city P and 50 students in city Q.If the total distance to be travelled by all 200 students is to be as small as possible, then the college should be built at
(a) City P
(b) City Q
(c) 45 km from City P
(d) 45 km from City Q

Q.6. When ACP Rathod was going to Wahed from New Delhi to arrest Mirchi Seth, he observed few surprising things through his rear-view mirror

i. In nth hour he saw inscribed on a milestone—New Delhi—*xy* kms, where *x* and *y* are the tens and units place digit of the distance from that place to New Delhi (remember, he was observing it in rear-view mirror)

ii. In (n + 1)th hour, he saw inscribed on a milestone—New Delhi—*yx* kms, where *x* and *y* are representing the same digits.

What is the speed of ACP Rathod?

(a) 9 km/h (b) 12 km/h
(c) 18 km/h (d) Cannot be determined

Q.7. A and B start from the same point and in the same direction at 7 a.m. to walk around a rectangular field 400 m × 300 m. A and B walk at the rate of 3 km/h and 2.5 km/h respectively. How many times shall they cross each other, if they continue to walk till 12.30 p.m.?

(a) Not even once (b) Once
(c) Twice (d) Thrice

Q.8. Left pan of a faulty balance weighs 100 g more than its right pan. A shopkeeper keeps the weight measure in the left pan while buying goods but keeps it in the right pan while selling his goods. He uses only 1 kg weight measure. If he sells his goods at cost price, what is his gain percentage?

(a) $1\frac{1}{2}$ (b) $\frac{100}{11}\%$

(c) $\frac{1000}{9}\%$ (d) $\frac{200}{9}\%$

Q.9. Four different candles, which can last for 5 h, 4 h, 3 h and 2 h of burning, respectively, are lit in a room at the same instance and allowed to burn till the time the three candles in the room get extinguished. The cost of burning each one of these candles is 75 paise per h. What is the total cost involved?

(a) Rs 2.75 (b) Rs 3.75
(c) Rs 9.75 (d) None of these

Q.10. Two ladies simultaneously leave cities A and B connected by a straight road and travel towards each other. The first lady travels 2 km/h faster than the second lady and reaches B one hour before the second lady reaches A. The two cities A and B are 24 km. apart. What is the difference in their speeds/h?

(a) 4 km/h (b) 3 km/h
(c) 2 km/h (d) 1 km/h

Q.11. In a company, 605 of the employees are men. Of these 40% are drawing more than Rs 50,000 per year. If 36% of the total employees of the company draw more than Rs 50,000 per year, what is the percentage of women who are drawing less than Rs 50,000 per year?

(a) 70 (b) 60
(c) 40 (d) None of these

Q.12. The following table shows the percentage change in the consumption of electricity by five towns P, Q, R, S and T from 1986 to 1988:

Town	*Percentage change*	
	From 1986 to 1987	*From 1987 to 1988*
P	+8	–18
Q	–15	+11
R	+6	+9
S	–7	–5
T	+13	–6

In town T were 50,000 units consumed in 1986, how much did it consume in 1988?

(a) 37100 units (b) 53110 units
(c) 55110 units (d) 57100 units

Q.13. Water is filled in a container in such a manner that its volume doubles after every five minutes. If it takes 30 minutes for the container to be full, in how much time will it be one-fourth full?

(a) 7.30 min (b) 10 min
(c) 20 min (d) 25 min

Q.14. Amar, Akbar and Anthony are friends, being looked after by a matron Farah, Amar weighs 50% more than Akbar and Anthony weighs 25% less than Amar. Farah weighs a third of the combined weight of the three boys. All four together weigh 232 kg. What is the correct sequence of the persons in the ascending order of their weights?

(a) Anthony, Akbar, Farah, Amar
(b) Anthony, Akbar, Amar, Farah
(c) Akbar, Anthony, Amar, Farah
(d) Akbar, Anthony, Farah, Amar

Q.15. In a family, a couple has a son and a daughter. The age of the father is three times that of his daughter and the age of the son is half of his mother. The wife is nine years younger to her husband and the brother is seven years older than his sister. What is the average age of the mother and son?

(a) 40 years (b) 45 Years
(c) 50 years (d) 60 years

Direction for questions 16 and 17: Read the passage below and solve the questions based on it.

Several runners, numbered 1, 2, 3,... and so on, start running simultaneously from the same point on a circular track and run continuously, in the same direction. They run such that the speed of the runner numbered n ($n > 1$) is n times that of the runner numbered 1 and so on.

Q.16. If there are exactly six runners, then at N distinct points on the track is the runner numbered 1 overtaken by any of the other five runners. What is the value of N?
(a) 15 (b) 11 (c) 9 (d) 10

Q.17. If there are exactly four runners, then at how many distinct points on the track do two or more runners meet?
(a) 3 (b) 4 (c) 5 (d) 6

Q.18. Sharat is walking down a descending escalator and takes 40 steps to reach the bottom. Chandra starts simultaneously from the bottom, taking two steps for every one step taken by Sharat. Time taken by Sharat to reach the bottom from the top is the same as the time taken by Chandra to reach the top from the bottom. How many more steps did Chandra take than Sharat before they crossed each other on the escalator?
(a) 20 steps (b) 25 steps
(c) 40 steps (d) 60 steps

Q.19. A hare and a tortoise decide to run around a pond, in opposite directions from the same point at the same time. The hare meets the tortoise at a certain pine tree for the first time. The second time, they meet at a banyan tree and the third time at a mango tree. The fourth time, the hare meets the tortoise exactly at the first pine tree. How many of the following statements is/are true?
i. One of them runs three times as fast as the other.
ii. One of them is twice as fast as the other.
iii. One of them is six times as fast as the other.
iv. The starting points and starting times are required to comment about the ratio of their speeds.
(a) 0 (b) 1
(c) 2 (d) Cannot be determined

Q.20. Chunmun is running at a uniform speed from Rajender Nagar to Patna Junction along the railway tracks. After every 12 min, a train moving from Rajender Nagar to Patna Junction overtakes him. Also, after every 4 min, he meets a train travelling from Patna Junction to Rajender Nagar. Trains leave from Rajender Nagar for Patna Junction and Patna Junction for Rajender Nagar at a regular interval of 'S' min. What is the value of 'S'?
(a) 4 (b) 6 (c) 8 (d) 12

Q.21. Two brothers Kumar Kalyan and Amar Kumar are walking around a circular track of length 1800 m, with a speed of 40 m/min and 50 m/min respectively. Both of them start in the same direction simultaneously, but Amar Kumar reverses his direction every time he completes one round. After how much time (in min) from the start will they meet for the first time?
(a) 120 (b) 40 (c) 80 (d) 160

Q.22. Trains are coming from Patna to New Delhi and New Delhi to Patna at a regular interval of 1 h. They take 5 h to complete the journey. How many trains coming from New Delhi will cross a train coming from Patna that started at 10 p.m.?
(Given that the trains start from both the stations at the same time.)
(a) 5 (b) 6 (c) 10 (d) 11

Q.23. Two runners are running in a clockwise direction on a circular race-track of n ($n > 1$) km long. They meet for the second time at a point C which is 200 m from their starting point in the anticlockwise direction on the circular track. If both of the runners started at the same time, then the location of their first meeting point is
i. 100 m from the starting point in the anti-clockwise direction.
ii. 100 m from the point diagonally opposite to the starting point in anti-clockwise direction.
(a) Definitely 1 (b) Definitely 1
(c) Either 1 or 2 (d) Neither 1 nor 2

Q.24. In a 4000 m race around a circular stadium having a circumference of 1000 m, the fastest runner and the slowest runner reach the same point at the end of the 5th min, for the first time after the start of the race. All the runners have the same starting point and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of the slowest runner, what is the time taken by the fastest runner to finish the race?
(a) 5 min (b) 10 min
(c) 15 min (d) None of these

Q.25. In a race of 1000 m, A beats B by 100 m or 10 sec. If they start a race of 1000 m simultaneously from the same point and if B gets injured, after running 50 m less than half the race length and due to which his speed gets halved, then by how much time will A beat B?
(a) 45 s (b) 50 s
(c) 60 s (d) 65 s

Q.26. A train, 100 m long is approaching an unmanned railway crossing. The train is travelling at a uniform speed of 90 km/h and is 1 km away from the crossing.

At the same time, a bus, also approaching the crossing, is 700 m away from it. Assuming the bus is also travelling at a uniform speed of B, for what range of values of N will the bus collide with the train?
(a) 59 km/h < N < 61 km/h
(b) 57 km/h < N < 63 km/h
(c) 55 km/h < N < 65 km/h
(d) 53 km/h < N < 67 km/h

Q.27. Two friends Karan and Arjun start running around a circular track in the same direction. Karan and Arjun take one full round in 7 min and 4 min respectively. After how much time will they meet at diametrically opposite point of their starting point?
(a) 120 s
(b) 240 s
(c) They will never meet
(d) None of these

Q.28. The analysis of the recently held CBSE exams shows that Chennai zone, Delhi zone and Ajmer zone have performed the best. The average marks of the Ajmer zone and the Delhi zone put together is 71%. The average marks of the Chennai zone and the Delhi zone put together is 76% and the average marks of the Ajmer zone and the Chennai zone put together is 79%. What is the range of the average marks of all the three zones (Assume N) put together?
(a) 71% < N < 78%
(b) 70.5% < N < 77.5%
(c) 73.5% < N < 77.5%
(d) 73% < N < 79%

Q.29. At the end of the year 1998, a shepherd bought nine dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year, where $p > 0$ and $q > 0$. If shepherd had nine dozen goats at the end of the year 2002, after making the sales for that year, which of the following is true?
(a) $p = q$ (b) $p < q$ (c) $p > q$ (d) $p = q/2$

Q.30. Two fuel tanks, one shaped like a cylinder and other like a cone, contain fuel. The cylindrical tank holds 500 litres more fuel than the conical fuel. After 200 litres of fuel have been taken out from each tank, the cylindrical tank contains twice the quantity of fuel in the conical tank. How many litres of fuel did the cylindrical tank have when it was full?
(a) 700 (b) 1000 (c) 1100 (d) 1200

ANSWERS

■■■ Benchmarking Test 2

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(c)	(b)	(b)	(a)	(a)	(b)	(a)	(c)	(c)	(a)	(b)	(c)	(b)	(a)	(d)	(d)	(a)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(b)	(d)	(c)	(b)	(d)	(b)	(c)	(c)	(c)	(d)										

Module 3

X+2 Maths

- Basics of Algebra
- Equations
- Squence and Series
- Function
- Graphs and Maxima Minima
- Logarithm
- Permutation and Combination
- Probability

9

C H A P T E R

Basics of Algebra

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Basic terms involved in Algebra
- Different types of solving the equations
- System of equations
- Kinds of questions in the CAT
- Methods of solving questions

Introduction to the topic

Algebra is important for CAT in terms of weightage of the questions asked from this area. In this chapter, I have tried to brush up the basics of Algebra of those students who have ‘passed’ through it at some point of time in their life, but have not much of idea currently. Initially students are advised to see this chapter without any connection with the other chapters/concepts of the algebra. Once one has completed this chapter thoroughly, one will start seeing the connection between the basics given in this chapter with the concepts given ahead.

WHAT IS ALGEBRA?

Algebra is a branch of mathematics that substitutes letters for numbers Þ It simply means that while in Arithmetic we were having the statements like–"Ram is having 5 apples", in Algebra we will be having the statements like–"Ram is having x apples" where value of x can be deduced from the given conditions.

Moving from Arithmetic to Algebra will look something like this: Arithmetic: 3 + 4 = 3 + 4. In Algebra it would look like: $x + y = y + x$

While in Arithmetic, we mostly deal with the numbers (which have a constant fixed value always), in case of Algebra, we mostly deal with the variables (which don't have a fixed value and can take different values according to the conditions governing.) For example, if we say that "Ram is having Rs 100", we get a clear picture that how much money Ram is having. And even if next day Ram is having 100 apples instead of Rs 100, the number "100" has the same meaning in this different situation.

Now replace "100" by "x" $\Rightarrow$ "Ram is having Rs x", this does not give us a clear picture that how much money Ram is having. And if next day, Ram is having x apples, the letter "x" may not have the same value in a different situation.

ALGEBRAIC NOTATIONS AND OPERATIONS

i. Notations

Algebraic Notations	Meaning	Example
$x = y$	x is equal to y	$1 = 1$
$x \neq y$	x is not equal to y	$1 + 1 \neq 1$
$x < y$	x is less than y, or, y is greater than x	$2 < 5$
$x > y$	x is greater than y, or, y is less than x	$4 > 3$
$x \leq y$	y is greater than or equal to x, or, x is less than or equal to y	
$x \geq y$	x is greater than or equal to y, or, y is less than or equal to x	
$x \approx y$	x is approximately equal to y	$2.99999 \approx 3$

ii. Additions/Subtractions

Statement 1 – Ram is having 5 more apples than Shyam.

$\Rightarrow$ Number of apples with Ram = Number of apples with Shyam + 5

Assume that number of apples with Shyam = x, then number of apples with Ram = $x + 5$

Alternatively, if we assume that the number of apples with Ram = y, then number of apples with Shyam = $y - 5$

iii. Multiplication

When there are two or more equal numbers to be added together, the expression of their sum may be abridged. For example,

$x + x = 2 \times x = 2x$
$x + x + x = 3 \times x = 3x$
$x + x + x + x = 4 \times x = 4x$

In this manner, we may form an idea of multiplication; and it is to be observed that,

$2 \times x$ signifies 2 times x or twice x
$3 \times x$ signifies 3 times x or thrice x

Further, we can multiply such products again by other numbers; for example:

$2y \times 5 = 10y$
$2y \times 5z = 10yz$
$2y \times 6y = 12y^2$

iv. Linear Equation

An algebraic equation, such as $y = 2x + 7$ or $3x + 2y - z = 4$, in which the highest degree term in the variable or variables is of the first degree. The graph of such an equation is a straight line if there are two variables.

The general form of linear equation in two variables x and y is $ax + by + c = 0$, $a \neq 0$, $b \neq 0$, and a, b and c real numbers. Here a and b are known as coefficient of x and y respectively and c is a constant. A solution of such an equation is a pair of values, one for x and the other for y, which makes LHS and RHS of the equation equal.

For a linear equation in two variables x and y:

$$ax + by + c = 0 \Rightarrow by = -ax - c \Rightarrow y = \frac{-ax - c}{b}$$

In the above equation, for every real x, there exists a real number y corresponding to x.

Therefore every linear equation in two variables has infinitely many solutions, i.e., infinitely many pairs (x,y). For example, the equation $2x + 3y = 10$ will have infinite solution.

All these solutions are represented by points on a certain line. Due to this fact only this equation is called LINEAR because the graph of the equation on the x–y Cartesian plane is a straight line.

For example, consider the equation

$$3x + 2y = 8 \Rightarrow 2y = 8 - 3x \Rightarrow y = \frac{8 - 3x}{2} \qquad (1)$$

To find a solution of linear equation in two variables, we assign any value of one of the two variables and determine the value of the other variable, from the given equation (1).

Thus, taking $x = 1$, we get corresponding value of $y = \frac{5}{2}$.

Similarly, taking $x = 0$, we get $y = 4$; and so on.

The following table lists six possible values for x and the corresponding values for y, i.e. six solutions of the equation:

x	-2	-1	0	1	2	8/3
y	7	11/2	4	5/2	1	0

Taking any two pairs of given equation, we plot corresponding points, say, P and Q. The line PQ through these points is related with the given equation in the following manner:

(i) Every solution $x = p$ and $y = q$ of the given equation determines a point (p, q) that lies on this line.

(ii) Every point (x_i, y_1) lying on the line PQ, determines a solution $x = x_p\ y = y_i$ of the given equation.

The line PQ is said to be the graph of the given equation. It is worth noting that:

(a) We can add or subtract any number on both sides of the equation without affecting the equation and its solution.

(b) We can multiply or divide both sides of an equation by a non-zero number without affecting the equation and its solution.

If we plot the solutions of the equation $3x + 2y = 8$, which is represented in the table above then we notice that they all lie on the same line. We call this line the graph of the equation since it corresponds precisely to the solution set of the equation.

SYSTEM OF TWO LINEAR EQUATIONS IN TWO UNKNOWNS

A system of two linear equations in two unknowns is a system of two equations of the form

$$a_1 x + b_1 y = c_1$$
$$a_2 x + b_2 y = c_2$$

where $a_1, a_2, b_1, b_2, c_1, c_2$ are arbitrary real numbers.

The solution set of the system of linear equation in two unknowns is a pair of real numbers (x_0, y_0) which satisfies each of the equations of the system. In general,

$$\Rightarrow x = \frac{c_1 b_2 - c_2 b_1}{a_1 b_2 - a_2 b_1}$$

x is defined if $a_1 b_2 - a_2 b_1 \neq 0$ (i.e., for the existence of a unique solution of the system of linear equations)

In other words, the system of linear equations has a unique solution if $(a_1 b_2 - a_2 b_1) \neq 0$

$$\Rightarrow \frac{a_1}{a_2} \neq \frac{b_1}{b_2} or \Rightarrow \frac{a_1}{b_1} \neq \frac{a_2}{b_2}$$

Now let us consider the geometric interpretation of the discussing given above. We know that the set of points in a plane whose coordinates satisfy an equation of the form $ax + by = c$, where either a or b are non-zero, constitutes a straight line. And to solve a system in which every equation has at least one unknown means to find the common point of two straight lines. Therefore, a system of linear equations has a unique solution if the lines intersect.

E.g., 1. Consider the system of linear equations.

$$4x + 6y = 10 \text{ and } 4x - 2y = 2$$

Point (1,1) is the intersection of two straight lines. Hence $x = 1$, $y = 1$ is the unique solution of system of linear equations.

We can say that the system of linear equations has a unique solution if $a_1 b_2 - a_2 b_1 \neq 0$.

What if $a_1b_2 - a_2b_1 = 0$?

If $a_1 b_2 - a_2 b_1 = 0$

or $a_1 b_2 = a_2 b_1$

or $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, then there are two situations:

Situation (i)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

If situation (i) exists, then the system of linear equations has an infinite number of solutions.

Situation (ii)

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

If condition (ii) exists, then the system of linear equations has no solutions.

For example, consider following equations:

$$x + y = 10 \qquad (1)$$
$$2x + 2y = 25 \qquad (2)$$

In the given equations, $a_1/a_2 = b_1/b_2$, hence unique solution is not possible. To have a better understanding, it can be seen that LHS of equation (1) × 2 will give us the LHS of equation (2). Hence there will not be any point of intersection of these two graphs drawn on X–Y axis. In other words lines will be parallel.

Summarizing the whole discussion:

[illegible]	[illegible]
(i) The lines intersect	System of linear equations has a unique solution, i.e. $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$ (known as system is determinate)
(ii) The lines are	The system is inconsistent, i.e., parallel the system of linear equations has no solution, i.e.,
(iii) The lines are	System of linear equations has coincident infinitely many solutions, i.e., $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ (known as system is indeterminate)

SOLVING A SYSTEM OF LINEAR EQUATIONS

(i) Graphical Method

We plot the straight lines correspondence to each of the given linear equations in two variables. The point of intersection is the solution for x and y (i.e., two variables) in the system of linear equation.

Example 1 Solve the two equations $2x - y = 1$ and $2x + 3y = 5$.

Solution We will draw the graph of both the equations. We can see that the straight lines intersect at the point (1, 1). Hence the set of solutions of the given equations is (1, 1).

(ii) Elimination Method

Step 1 Multiply the coefficients of the equations by suitable numbers so that the coefficients of one of the variables becomes same in both the equations.

Step 2 Add or subtract to get one of the variables (which has become same in step 1) cancelled. This will lead to obtaining the value of other variable.

Step 3 Substitute this value in one of the equations to obtain the value of the other variable.

Example 2 Solve $3x - 5y = 1$

$x + 4y = 6$

Solution

Given equations are $3x - 5y = 1$ (1)

$x + 4y = 6$ (2)

Step 1 – If we multiply equation (2) by 3, then co-efficient of x will become same in both the equations. Alternatively, if we multiply equation (1) by 4 and equation (2) by 5, co-efficient of y will become same in both the equations. We are making co-efficient of x same in both the equations.

$$3x - 5y = 1 \quad (3)$$

$$3x + 12y = 18 \quad (4)$$

Step 2- Subtracting (3) from (4), we get

$(3x + 12y) - (3x - 5y) = 18 - 1$

$\Rightarrow 17y = 17 \Rightarrow y = 1$

Step 3 – Substitute value of y in equation 1.

$3x - 5 \times 1 = 1$ Þ $x = 2$

Alternative Method:

Suppose we have to solve the system of equations

$$a_1x + b_1y = c_1$$

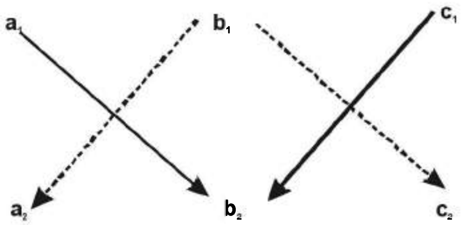
$$a_2x + b_2y = c_2$$

Write the coefficients of x, y and the constants in the following manner (taking notice of the signs, i.e., positive and negative values, of the coefficients).

I		II
a_1	b_1	c_1
a_2	b_2	c_2

Now, do the cross-multiplication in the part I and part Ii as given below:

I II



Note that there are two types of cross-multiplication in each of the Part I and Part II - expressed by bold lines and the expressed by dotted lines.

Now, subtract the value obtained by the cross-multiplication of dotted lines from that by the cross-multiplication of bold lines in each of the parts. Suppose the values obtained from part II is m and the value obtained from part I is n; then the value of the variables corresponding to coefficient a_1 and a_2 is $\frac{m}{n}$;

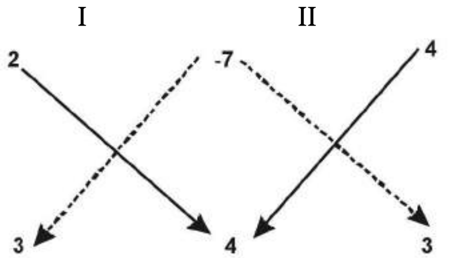
i.e., $x = \frac{m}{n}$.

Let us see an example:

Given equations are

$2x - 7y = 4$

$3x + 4y = 3$



$8 - (-21) = 29 \qquad 16 - (-21) = 37$

can be obtained by putting this value of x in any of the equations [$y = (2x - 4)/7$]

(iii) Substitution Method

Assume that equations are in terms of x and y.

Step 1 – Express x in terms of y in one of the equations.

Step 2 – Put the value of x obtained from step 1 in the 2nd equation. Calculate the value of 'y' now.

Step 3 – Substitute the value of 'y' in any of the equations to find the value of x.

Example 3 Solve $x + 7y = 15$

$7x - 3y = 1$

Solution We have $x + 7y = 15$ (1)

$7x - 3y = 7$ (2)

From equation (1), $x = (15 - 7y)$

Substituting this value of x in equation (2),

$7 \times (15 - 7y) - 3y = 1$

$\Rightarrow y = 2$ and corresponding value of $x = 1$.

REMAINDER AND FACTOR THEOREM

(i) Remainder Theorem

Remainder theorem gives us a method for finding the remainder without actual division.

Let us understand this with the examples:

Example 4 Let $p(x) = x^4 - 3x^2 + 2x + 5$. Find the remainder when $p(x)$ is divided by $(x-1)$.

Solution Rather going for the actual division, Let us compute $p(1)$, ie value of $p(x)$ when x is replaced by 1, we have, $p(1) = 1 - 3 + 2 + 5 = 5$.

We find that the remainder when $p(x)$ is divided by $(x-1)$ is equal to $p(1)$, i.e., the value of $p(x)$ at $x = 1$.

Example 5 Find the remainder when

$p(y) = y^3 + y^2 + 2y + 3$ is divided by $y+2$

Solution Compute $p(-2)$ = Value of $p(y)$ when y is replaced by -2, we have,

$P(-2) = (-2)^3 + (2)^2 + 2(-2) + 3 = -8 + 4 - 4 + 3 = -5$

It follows from these two examples that the remainder obtained when $p(x)$ is divided by $(x-a)$ is equal to $p(a)$ i.e. the value of $p(x)$ at $x = a$.

The above result is stated as remainder theorem (Look in chapter 2 – Number system for more reference.)

Note:

1. If a polynomial $p(x)$ is divided by $(x + a)$, the remainder is the value of $p(x)$ at $x = -a$, i.e., $p(-a)$.
2. If a polynomial $p(x)$ is divided by $(ax-b)$, the remainder is the value of $p(x)$ at $x = b/a$, i.e., $P(b/a)$.
3. If a polynomial $p(x)$ is divided by $(ax + b)$, the remainder is the value of $p(x)$ at $x = -b/a$, i.e., $p(-b/a)$.
4. If a polynomial $p(x)$ is divided by $(b - ax)$, the remainder is equal to the value of $p(x)$ at $x = b/a$, i.e., $p(b/a)$.

Example 6

Determine the remainder when the polynomial $p(x) = x^4 - 3x^2 + 1$ is divided by $x-1$.

Solution

By remainder theorem, the required remainder is equal to $p(1)$.

Now, $p(x) = x^4 - 3x^2 + 2x + 1$

$\Rightarrow p(1) = (1)^4 - 3 \times 1^2 + 2 \times 1 + 1 = 1 - 3 + 2 + 1 = 1$

Hence, required remainder $= p(1) = 1$.

(ii) Factor Theorem

If $g(x)$ divides $f(x)$, we say that $f(x)$ is divisible by $g(x)$ or $g(x)$ is a factor of $f(x)$.

Factor theorem gives us a method to determine whether a polynomial $g(x)$ is a factor of a polynomial $f(x)$ or not without actual division.

Let $f(x)$ be a polynomial of degree greater than or equal to 1 and 'a' be a real number such that $f(a) = 0$, then $(x - a)$ is a factor of $f(x)$, Conversely, if $(x - a)$ is a factor of $f(x)$ then $f(a) = 0$.

Note:

1. $(x + a)$ is a factor of a polynomial $f(x)$ if $f(-a) = 0$
2. $(ax-b)$ is a factor of a polynomial $f(x)$ if $f(b/a) = 0$.
3. $ax + b$ is a factor of polynomial $f(x)$ if $f(-b/a)=0$
4. $(x-a)(x-b)$ is a factor of a polynomial $f(x)$ if $f(a) = 0$ and $f(b) = 0$.

Example 7

Show that $(x - 3)$ is a factor of the polynomial $x^3 - 3x^2 + 4x - 12$

Solution

Let $f(x) = x^3 - 3x^2 + 4x - 12$ be the given polynomial. By factor theorem, $(x-a)$ is a factor of a polynomial $f(x)$ if $f(a) = 0$. Therefore, in order to prove that $(x-3)$ is a factor of $f(x)$, it is sufficient to show that $f(3) = 0$.

Now, $f(x) = x^3 - 3x^2 + 4x - 12$

$\Rightarrow f(3) = 3^3 - 3\times3^2 + 4\times3 - 12 = 27 - 27 + 12 - 12 = 0$

Hence, $(x-3)$ is a factor of $f(x)$.

Example 8

Find out if $(x - 1)$ is a factor of (i) and (ii) .

Solution

Let $f(x) = x^{10} - 1$ and $g(x) = x^{11} - 1$.

To find out that $(x-1)$ is a factor of both $f(x)$ and $g(x)$, it is sufficient to show that $f(1) = 0$ and $g(1) = 0$.
Now, $f(x) = x^{10} - 1$ and $g(x) = x^{11} - 1$
$\Rightarrow f(1) = 1^{10} - 1 = 0$ and $g(1) = 1^{11} - 1 = 0$
$\Rightarrow (x-1)$ is a factor of both $f(x)$ and $g(x)$

More worked out problems:

Example 9

Solve the equation $3x - 2y = 7$.

Solution

Let us first find one particular solution and then we will extend that. Its not too difficult to find one solution using hit and trial. One such solution is (3, 1). Alternatively, we can find it using $[x = (7 + 2y)/3]$

To find the next solution, let us re–write the equation as $3x = 2y + 7$.

Using the concepts of remainder, above written equation can be explained as:

LHS = A multiple of 3

RHS = A number that gives remainder 7 when divided by 2 which means a number that gives remainder 1 when divided by 2.

Obviously the next number will be obtained after the LCM (2,3) = 6. To move ahead by 6, we will cover 2 multiples of 3 (2 × 3) and 3 multiples of 2 (3 × 2). Hence the next solutions are (5, 4) (7,7) (9, 10) (11, 13) etc.

Note– Since it is a linear equation, there will be obviously infinite solution.

Example 10

Find all integral roots of the equation $21x + 48y = 5$.

Solution

LHS is divisible by 3, but RHS is not divisible by 3. Hence no integral solution is possible.

Statement I

(i) $1990x - 173y = 11$ has no solution in integers for x and y.
(ii) $3x - 12y = 7$ has no solution in integers for x and y.

Which of the following is true?

(1) Statement I is wrong and statement II is right.
(2) Statement I is right and statement II is wrong.
(3) Statement I and II both are right.
(4) Statement I and II both are wrong.

Statement II

The coefficients in the equation are large enough to find a particular solution. However, we can see that the numbers 1990 and 173 are relatively prime and this helps us reach the conclusion.

Hence we can find the solution of $1990x - 173y = 11$ in integers for x and y.

Hence statement I is wrong.

Statement II

In the linear equation $3x - 12y = 7$; coefficients of x and y, i.e. 3 and 12, are not relatively prime. HCF of (3, 12) = 3. Also, constant 7 is not exactly divisible by 3. In other words, LHS is divisible by 3, but RHS is not divisible by 3.

Hence $3x - 12y = 7$ has no solution for x and y in integers.

Hence, statement II is wrong.

Therefore option (4) is true.

PRACTICE EXERCISES

Q.1. If $x + y + z = 0$, then $x^3 + y^3 + z^3$ is equal to

(a) 0
(b) $3xyz$
(c) $\dfrac{xy + yz + zx}{xyz}$
(d) $xyz\,(xy + yz + zx)$

Q.2. If $x - \dfrac{1}{x} = 2$, then the value of $x^4 + \dfrac{1}{x^4}$ is

(a) 4
(b) 8
(c) 12
(d) 34

Q.3. If $\left(x+\frac{1}{x}\right)=3$, then the value of $\left(x^6+\frac{1}{x^6}\right)$ is

(a) 927 (b) 414
(c) 364 (d) 322

Q.4. Let $f(x)=x^2-27x+196$. If $(x)=x$, then what is the value of x?

(a) 28 (b) 14
(c) 7 (d) 4

Q.5. Consider the following statement: The equations $ax+by=1$ and $ax-by=1$ have a unique solution for

(a) $a=1, b=0$ (b) $a=0, b=1$
(c) $a=0, b=0$ (d) $a=1, b=1$

Q.6. Find the numerical value of 'c', if the expression $xy-3x+5y+c$ can be factorized.

(a) 9 (b) –15
(c) 15 (d) –9

Q.7. If x is less than 2, then which of the following statements is always true?

(a) x is negative
(b) $2x$ is greater than or equal to x
(c) x^2 is greater than or equal to x
(d) None of these

Q.8. If $x-y=8$, then which of the following must be true?

I Both x and y are positive.
II if x is positive, y must be negative
III If x is negative, y must be negative

(a) I only (b) II only
(c) I and II (d) III only

Q.9. Five tables and eight chairs cost Rs 7,350. Three tables and five chairs cost Rs 4,475. What is the price of one table?

(a) Rs 950 (b) Rs 325
(c) Rs 925 (d) Rs 350

Q.10. The cost of 3 apples and 2 guavas is Rs 23 and that of one apple and four guavas of the same type is Rs 21. What is the cost of 5 guavas?

(a) Rs 4 (b) Rs 20
(c) Rs 12 (d) Rs 25

Q.11. If the two digits of the age of Mr Manoj Sagar are reversed then the new age so obtained is the age of his wife. If the sum of their ages is equal to the difference between their ages. If Mr Manoj Sagar is elder than his wife then find the difference between their ages.

(a) 9 years (b) 10 years
(c) 8 years (d) Cannot be determined

Q.12. Find the remainder when the polynomial $f(x)=2x^4-6x^3+2x^2-x+2$ is divided by $x+2$.

(a) 102 (b) –102
(c) 92 (d) –92

Q.13. Find the remainder when $f(x)=4x^3-12x^2+14x-3$ is divided by $g(x)=x-1/2$

(a) 2/3 (b) –2/3
(c) 3/2 (d) –3/2

Q.14. Find the remainder when $f(x)=x^3-6x^2+2x-4$ is divided by $g(x)=3x-1$.

(a) 107/27 (b) – 107/27
(c) – 4 (d) 4

Q.15. Find the remainder when $f(x)=x^3-6x^2+2x-4$ is divided by $g(x)=1-3x$.

(a) 107/27 (b) – 107/27
(c) – 4 (d) 4

Q.16. If the polynomials ax^3+4x^2+3x-4 and x^3-4x+a leave the same remainder when divided by $(x-3)$, find the value of a.

(a) 1 (b) 0
(c) – 1 (d) None of these

Q.17. The polynomials ax^3+3x^2-13 and $2x^3-5x+a$ are divided by $x+2$. If the remainder in each case is the same, find the value of a.

(a) 5/9 (b) – 5/9
(c) 2/7 (d) – 2/7

Q.18. What is the value of a, if $(x-a)$ is a factor of (x^3-a^2x+x+2)?

(a) 2 (b) – 2
(c) 0 (d) 1

Q.19. Find the values of $(a+b)$ if the polynomial $x^3-ax^2-13x+b$ has $(x-1)$ and $(x+3)$ as factors.

(a) 2 (b) 15
(c) 18 (d) None of these

Q.20. What is the value of p, if $x+3$ is a factor of $3x^2+px+6$?

(a) 9 (b) – 9
(c) 11 (d) – 11

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(d)	(d)	(b)	(d)	(b)	(d)	(d)	(a)	(b)	(a)	(c)	(c)	(b)	(b)	(c)	(a)	(b)	(c)	(c)

HINTS AND SOLUTIONS

1. $x + y = -z \Rightarrow (x + y)^3 = (-z)^3$

   $\Rightarrow x^3 + y^3 + 3xy(x + y) + z^3 = 0.$

   So, $x^3 + y^3 + z^3 = 3xyz$

2. $x - \frac{1}{x} = 2 \Rightarrow \left[x - \frac{1}{x}\right]^2 = 4 \Rightarrow x^2 + 1/x^2 = 6$

   Or, $[x^2 + 1/x^2]^2 = 36$

   Hence $x^4 + \frac{1}{x^4} = 34 = 34$

4. $f(x) = x^2 - 27x + 196 = x$, or, $x^2 - 28x + 196 = 0$

   $\Rightarrow (x - 14)(x - 14) = 0$

   So, $x = 14$

5. $a_1x + b_1y = 1$

   $a_2x - b_2y = 1$

   For unique solution we know $\frac{a_1}{a_2} \neq \frac{b_1}{-b_2}$

   Now go through the options *a* and *b* cannot be zero hence options (*a*) (*b*) (*c*) rejected. Only option (4) is answer.

6. $xy - 3x + 5y + c$ can be factorized (given) as

   $x(y - 3) + 5(y - 3) \Rightarrow xy - 3x + 5y - 15$

   After comparing we get ($c = -15$)

7. Statement 1 is obviously not always true.

   Statement 2 is not true for negative values of x.

   Statement 3 is not true for $0 < x < 1$.

8. If $x = 10$ and $y = 2 \therefore x - y = 8$ so II rejected.

   If $x = 6$ and $y = -2 \therefore x - (-2) = 6 + 2 = 8$ so (I) rejected

   If $x = -$ve, for $x - y = +$ve, y must be $-$ve as

   $x = -2$ and $y = -10 \Rightarrow x - y = -2 - (-10) = -2 + 10 = 8$

   So III is true.

9. 5T (Table) + 8C (chair) = 7350 (given) –(1)

   3T + 5C = 4475 (given) –(2)

   5 × (1) – 8 × (2)

   ⇒ 25T + 40C = 36,750

   $\underline{24T + 40C = 35,\ 800}$

   T = 950 Rs.

10. Assume that price of an apple = Rs x and price of a guava = Rs y

    Given is $3x + 2y = 23$ and $x + 4y = 21$

    Solving we get $y = 4$, Hence cost of 5 guavas = Rs 20

12. We have, $x + 2 = x-(-2)$. So, by remainder theorem, when $f(x)$ is divided by $\{x - (-2)\}$ the remainder is equal to $f(-2)$.

    Now, $f(x) = 2x^4 - 6x^3 + 2x^2 - x + 2$

    $\Rightarrow f(-2) = 2(-2)^4 - 6(-2)^3 + 2(-2)^2 - (-2) + 2$

    $\Rightarrow f(-2) = 2\times16 - 6 \times -8 + 2 \times 4 + 2 + 2$

    $\Rightarrow f(-2) = 32 + 48 + 8 + 2 + 2 = 92$

    Hence, required remainder = 92.

13. Using remainder theorem, we know that $f(x)$ when divided by $g(x)$ = gives a remainder equal to $f\left(\frac{1}{2}\right)$.

    Now, $f(x) = 4x^3 - 12x^2 + 14x - 3$

    $\Rightarrow f\left(\frac{1}{2}\right) = \frac{3}{2}$

    Hence, required remainder $= f\left(\frac{1}{2}\right) = \frac{3}{2}$

14. We have, $g(x) = 3x - 1 = 3\left(x - \frac{1}{3}\right)$. Therefore, by remainder theorem, when $f(x)$ is divided by $g(x) = 3\left(x - \frac{1}{3}\right)$, the remainder is equal to $f\left(\frac{1}{3}\right)$.

Now, $f(x) = x^3–6x^2–2x–4$

$$\Rightarrow f\left(\frac{1}{3}\right)=\left(\frac{1}{3}\right)^3-6\left(\frac{1}{3}\right)^2+2\left(\frac{1}{3}\right)-4$$

$$=\frac{1}{27}-\frac{6}{9}+\frac{2}{3}-4=-\frac{107}{27}$$

Remainder = $-\frac{107}{27}$

15. Using remainder theorem, when $f(x)$ is divided by $g(x) = 1–3x$, the remainder is equal to $f(1/3)$.

Now, $f(x) = x^3 – 6x^2 + 2x – 4$

$$f\left(\frac{1}{3}\right)=\frac{1}{27}-\frac{2}{3}+\frac{2}{3}-4=\frac{-107}{27}$$

Hence, required remainder

16. Let $f(x) = ax^3 +4x^2 + 3x – 4$ and $g(x) = x^3– 4x + a$ be the given polynomials. The remainders when $f(x)$ and $g(x)$ are divided by $(x–3)$ are $f(3)$ and $g(3)$ respectively.

According to the question, $f(3) = g(3)$

$\Rightarrow 27a + 36 + 9 – 4 = 27 – 12 + a$

$\Rightarrow 26a + 26 = 0 \Rightarrow a = – 1$

17. Let $f(x) = ax^3 + 3x^2– 13$ and $g(x)=2x^3– 5x + a$ be the given polynomials. The remainder when $p(x)$ and $q(x)$ are divided by $(x + 2)$ are $f(– 2)$ and $g(– 2)$ respectively.

Given that $f(– 2) = g(– 2)$

$\Rightarrow a(– 2)^3 + 3(– 2)^2 – 13 = 2 (– 2)^3 – 5(– 2) + a$

$\Rightarrow –8a + 12 – 13 = – 16 + 10 + a$

$\Rightarrow – 8a – 1 = a – 6$

$\Rightarrow a = \frac{5}{9}$

18. Let $f(x) = x^3– a^2x + x + 2$ be the given polynomial. By factor theorem, $(x – a)$ is a factor of $f(x)$ if $f(a) = 0$.

Now, $f(a) = 0 \Rightarrow a^3– a^2 \times a + a + 2 = 0$

$\Rightarrow a + 2 = 0 \Rightarrow a= – 2.$

Hence, $(x–a)$ is a factor of the given polynomial, if $a = –2$

19. Let $f(x) = x^3 – ax^2 – 13x + b$ be the given polynomial. If $(x–1)$ and $(x–3)$ are factors of $f(x)$, then

$f(1) = 0$ and $f(–3) =0$

$\Rightarrow 1^2– a \times 1^2– 13 \times 1 + b = 0$

and $(– 3)^2 – 13 \times (–3) + b = 0$

$\Rightarrow 1– a – 13 + b = 0$

and $– 27 – 9a + 39 + b = 0$

$\Rightarrow – 12 – a + b = 0$

and $– 9a – b = 12$

Solving the equations, we get $\Rightarrow a = 3$

Putting $a = 3$ in $a – b = – 12$, we get $b = 15$

Hence, $a = 3$ and $b = 15$

20. $f(– 3) = 0$

$\Rightarrow 3(– 3)^2+ p(– 3) + 6 = 0$

$\Rightarrow 27 – 3p + 6 = 0 \Rightarrow p = 11$

10

CHAPTER

Equations

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Polynomials and different equations
- Definitions and properties of those equations
- Concepts attached to the these equations and their maxima-minima
- Inequations
- Kinds of questions asked in the CAT
- Methods of solving problems

Introduction to the topic

Quadratic equations is one of the popular topics for the CAT. Almost every year, 1–3 questions are asked from this topic. Normally, the questions are asked using the simple concepts of quadratic equations only, but it is desirable to have a good knowledge of the cubic equation and other polynomials also. Most of the questions asked from this topic are logical in nature, i.e., they can be solved with the help of alternative methods such as using options, or assuming some value.

DEFINITIONS

Polynomials

Assume that a_1, a_2, a_3, a_4,… are real numbers and x is a real variable. Then $f(x) = a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ldots + a_{n-1}x + a_nx$ is called a polynomial.

For example, $5x^5 + 3x^4 + \ldots + x$ is a polynomial in x, where x is a real variable.

Degree of a Polynomial

Polynomial $f(x) = a_1x^n + a_2x^{n-1} + a_3x^{n-2} + \ldots + a_{n-1}x + a_nx$ is a polynomial of degree n, where $a_1 \neq 0$.

For example, $5x^5 + 3x^4 + \ldots + x$ is a polynomial of degree 5.

$2x^4 + x^3 + 4x^2 + 2x + 10$ is a polynomial of degree 4.

$4x^3 + 4x^2 + 2x + 10$ is a polynomial of degree 3.

$4x^2 + 2x + 10$ is a polynomial of degree 2.

Remember that

- The degree of a polynomial is defined for real as well as complex polynomials.
- The degree of a polynomial cannot be a fraction.

Polynomial Equation

If $f(x)$ is a real or complex polynomial, then $f(x) = 0$ is the corresponding equation. Simply put, when we equate any polynomial with zero, then it becomes equation.

For example, $5x^5 + 3x^4 + \ldots + x = 0$ is an equation.

$2x^4 + x^3 + 4x^2 + 2x + 10 = 0$ is an equation.

Polynomial Inequation

If $f(x)$ is a real or complex polynomial, then $f(x) \neq 0$ is the corresponding inequation. Simply put, if any polynomial is not equal to zero, i.e., either greater than zero or less than zero, then it is known as an inequation.

For example, $2x^4 + x^3 + 4x^2 + 2x + 10 > 0$ is an inequation.

$4x^2 + 2x + 10 < 0$ is an inequation.

Roots of an Equation

All the values of a variable satisfying the equation are known as the roots of the equation. It is known that an equation of degree n will have n roots, real or imaginary. Roots can be the same or distinct. When the roots are the same, they are known as repeated roots.

Quadratic Equation

Any equation of degree 2 is known as a quadratic equation.

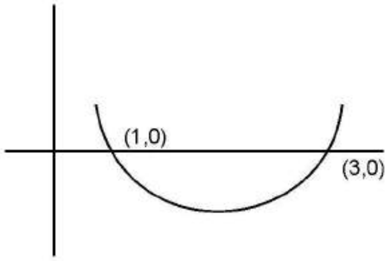
$ax^2 + bx + c = 0$ is known to be the standard equation of quadratic equation. This equation will have two roots either real or imaginary.

Geometrical Meaning of Roots

For any given equation $y = f(x) = 0$, the number of times the graph of this equation cuts X-axis is equal to the distinct real roots of this equation.

For example, $y = (x-1)(x+2)(x-2) = 0$ will intersect X-axis at three distinct points, namely $x = 1, -2$ and 2.

For any quadratic equation $f(x) = (x-1)(x-3)$, the graph of this equation (as can be seen below) will intersect X-axis at two distinct points, namely $x = 1$ and $x = 3$.



So, the equation $f(x) = x^2 - 4x + 3 = 0$ will have two roots, i.e., two values of $x = 1$ and $x = 3$, which will satisfy this equation.

If we say that the two roots 'p' and 'q' of the quadratic equation $f(x) = ax^2 + bx + c = 0$ are such that $f(2) \times f(-2) > 0$, then either both the roots 'p' and 'q' of this equation will lie inside -2 and 2 or both the roots 'p' and 'q' will lie outside the range of -2 and 2.

Similarly, if we say that the two roots 'p' and 'q' of the quadratic equation $f(x) = ax^2 + bx + c = 0$ are such that $f(2) \times f(-2) < 0$, then one of the two roots of this equation will lie inside this range and another will lie outside the range of -2 and 2.

FACTORIZATION OF A QUADRATIC EQUATION

When factorizing quadratic equations, there is generally nothing that is common in every term. In this case, we cannot pull a constant value out and rewrite the problem.

You should know that your answer would look like. $(y + a) \times (y + b)$. The trick is knowing how to find a and b.

For example, $y^2 + 4y - 12$

Start with the constant term. In this particular case, the constant term is -12. Factorize -12 into groups. Each group is a set of two numbers that when multiplied together will give -12. With the number -12, these can be the following groups:

$\{-1, 12\}$, $\{-2, 6\}$, $(-3, 4\}$, $\{1, -12\}$, $\{2, -6\}$, $\{3, -4\}$

Now you have six different groups. The next step is to find out the coefficient of the 'y' term and the number to be multiplied by 'y' in the quadratic equation. In this case, that number is 4. Now, we have to find out which one of the group will give us 4, when the two numbers of each group are added together.

If you add -1 and 12, you get 11. This is not correct, So, try again.

If you add −2 and 6, then you get 4. This is the required group.

The numbers that you choose in the group (in this case −2 and 6) will be the a and the b that we have been looking for. Which one you chose for a and which you chose for b does not matter. I will just use $a = -2$ and $b = 6$,

$[y + (-2)] \times (y + 6)$

$= (y - 2)(y + 6)$

Properties of the Roots

1. A polynomial equation of degree n will have n roots, real or imaginary.
2. Complex roots or surds always occur in pair.
   For example, if $2 + \sqrt{5}$ is a root of any equation, then $2 - \sqrt{5}$ will also be a root of that equation.
3. Every equation of an even degree with coefficient of the highest degree term positive and the constant term negative will have at least two real roots. Out of these two real roots, one will be positive and the other one will be odd.
4. Any equation with the sum of all of its coefficients equal to zero will have 1 as one of its roots.
   For example, $x^2 - 5x + 4 = 0$ will have 1 as one of its roots.
5. If all the terms of an equation are +ve and it does not involve any odd powers of x, then this equation will not have any real root.
   For example, $f(x) = x^4 + x^2 + 1 = 0$ will not have any real root. As $x^4 \geq 0$, $x^2 \geq 0$, either of the numbers can become the a or b, here we will take 1 and 1 is positive. And we know that sum of three positive number. can never be equal to zero for any real value of x.

Example 1 How many real roots will be there of the quadratic equation $f(x) = x^2 + 5|x| + 6 = 0$?

Solution If $x > 0$, then $f(x) = x^2 + 5x + 6 = 0$

Or, $x^2 + 5x + 6 = (x + 2)(x + 3) = 0$

So, $x = -2$ and $x = -3$

But as we have taken $x > 0$, the negative values of x are not admissible.

Now, if $x < 0$, then $f(x) = x^2 - 5x + 6 = 0$

Or, $x^2 - 5x + 6 = (x - 2)(x - 3) = 0$, So, $x = 2$ and $x = 3$

Now as we have taken $x < 0$, so the positive values of x are not admissible.

And $x = 0$ is not possible.

So, no real value of x can be obtained.

Alternatively, it is worth observing that all the terms of $f(x)$, viz., x^2 and $5|x|$ and 6 are positive. So, the sum of these three terms cannot be equal to zero for any real value. Hence, no real value of x can be obtained.

Descartes' Rule of Signs of Roots

Descartes' rule tells us how to find the maximum number of positive roots in any equation. The maximum number of positive roots in $f(x)$ is equal to the change of signs from positive (+ve) to negative (–ve) and from negative (–ve) to positive (+ve).

With the help of this, we can also find out the maximum number of real roots. To find out the negative roots, we apply the same procedure with $f(-x)$.

For example, let us see the roots of the quadratic equation $x^2 + x - 2 = 0$

Signs of different terms: x^2 has a positive sign, x has a '+' sign, the constant term (2) has a negative sign.

$f(x) = x^2 + x - 2$, and their signs are

$+ \quad + \quad -$

As there is only one sign change, we can conclude that this equation will have one positive real root.

Now, $f(-x) = (-x)^2 + (-x) - 2 = x^2 - x - 2$

Again, there is just one sign change, so there will be one negative root. Also there are no positive or negative imaginary roots and only real numbers can be negative or positive.

Example 2 How many real roots will be there for the equation $x^4 + x^2 + 1 = 0$?

Solution To find out the positive roots:

$f(x) = x^4 + x^2 + 1$, and signs are

$+ \quad + \quad +$

There is no sign change between any two terms, so, no roots will be positive.

To find out the negative roots:

$f(-x) = (-x)^4 + (-x)^2 + 1 = x^4 + x^2 + 1$

Again, there is no sign change, so, the equation will have no negative roots.

And $x = 0$ is also not a root of this equation.

So, no real roots are possible which can satisfy this equation.

It can also be inferred, that all the four roots of this equation will be negative.

Alternatively, we have done this question while seeing the properties of roots (v) also.

ROOTS OF A QUADRATIC EQUATION AND THEIR NATURE

Given is the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$.

Assuming that a and b are the roots of this equation.

$$\text{Then } \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ and } \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

It can be seen that the nature of both these two roots depends upon the value of $b^2 - 4ac$.

($b^2 - 4ac$) is also known as **Discriminant (D)** of the quadratic equation $ax^2 + bx + c = 0$.

If D = 0, then $\sqrt{b^2 - 4ac} = 0$.

So, the roots will be real and equal.

If D > 0, then $\sqrt{b^2 - 4ac} > 0$.

So, the roots will be real and distinct.

If D < 0, then $\sqrt{b^2 - 4ac}$ is not real.

So, the roots will not be real.

If D is a perfect square (including D = 0) and a, b and c are rational, then the roots will also be rational.

Example 3 Given is the quadratic equation $ax^2 + bx + 1 = 0$, where a, b (1, 2, 3, 4). For how many set of values of (a, b), the quadratic equation $ax^2 + bx + 1 = 0$ will have real roots? **(CAT 2003)**

Solution For the roots to be real, D ≥ 0.

$D = b^2 - 4a \geq 0$.

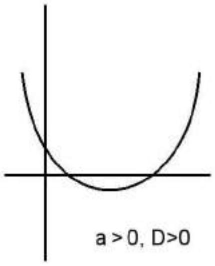
Forming the table for the above-written condition:

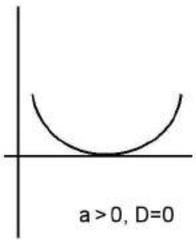
b	*a*
1	Not possible
2	1
3	1
3	2
4	1
4	2
4	3
4	4

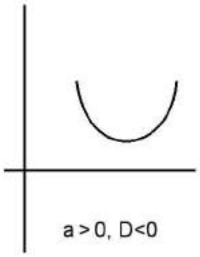
So, there are seven set of values for (a, b).

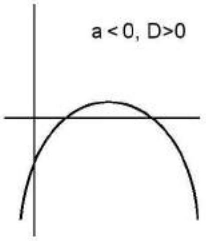
GRAPH OF A QUADRATIC EQUATION

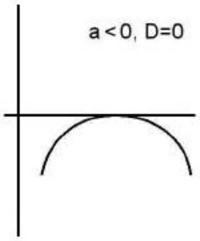
$f(x) = ax^2 + bx + c = 0$, where $a \neq 0$, and b and c are real coefficients.

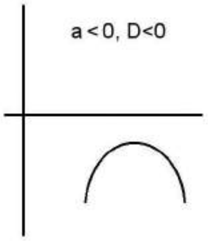












As is obvious from the above graphs, when D > 0, there are two points at which this curve is intersecting X-axis. And we also know that there are two distinct values of x which satisfy $f(x)$.

When D = 0, there is only one point at which this curve is intersecting X-axis, and there is only one distinct value of x, which satisfies $f(x)$.

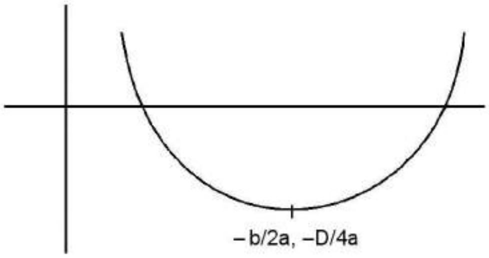
When D < 0, there is no point at which this curve is intersecting X-axis. So, there are no real values of x which can satisfy $f(x)$.

MAXIMA AND MINIMA OF A QUADRATIC EQUATION

$f(x) = ax^2 + bx + c$, if $a > 0$.

Finding Minima

Graphical method



At $x = -b/2a$, $f(x)$ will have the minimum value = – D/4a

Equation method

In this method, the objective is to break the equation into a format of $(P)^2 \pm Q$. Now, as we know that any perfect square can have a minimum value of 0, so the minimum value of $f(x) = \pm Q$

Let us assume $f(x) = x^2 - 5x + 6$

$$f(x) = \left[x^2 - 2\times\frac{5}{2}\times x + \left(\frac{5}{2}\right)^2\right] - 0.25 = \left(x - \frac{5}{2}\right)^2 - 0.25$$

Since $\left(x - \frac{5}{2}\right)^2$ can have a minimum value of zero, so the minimum value of $f(x)$ here is (– 0.25).

Using the graphical method, at $x = -b/2a = 5/2$, the minimum value of $f(x) = -D/4a = -1/4 = (-0.25)$

Finding maxima As the value of x is increased, the value of y also increases. So, the maximum value of y is $+\propto$.

It is obvious from the graph that it is going upwards till infinity.

$f(x) = ax^2 + bx + c$, if $a < 0$.

Finding minima As the value of x is increased, the value of y decreases (since $a < 0$). So, the minimum value of y is $-\propto$.

It is obvious from the graph that it is going downward till negative of infinity.

Finding maxima The same method of finding minima is applied as in the earlier case ($a > 0$).

ALGEBRAIC CALCULATIONS ON ROOTS

$f(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots + K$

The sum of the roots taking one at a time = (Coefficient of x^{n-1}/Coefficient of x^n) × $(-1)^n$, where n is the number of roots taken at a time.

The sum of the roots taking two at a time = (Coefficient of x^{n-2}/Coefficient of x^n) × $(-1)^n$, where n is the number of roots taken at a time.

The sum of the roots taking three at a time = (Coefficient of x^{n-3}/Coefficient of x^n) × $(-1)^n$, where n is the number of roots taken at a time.

The product of roots, taking all at a time = (Constant term/Coefficient of x^n) × $(-1)^n$, where n is the total number of roots.

Quadratic Equation

$f(x) = ax^2 + bx + c = 0$

Assume that the roots are α and β.

$\alpha + \beta$ = (Coefficient of x^{n-1}/Coefficient of x^n) × $(-1)^n$= $\frac{b}{a}(-1)^n = -\frac{b}{a}$

$\alpha\,\beta$ = (Constant term/Coefficient of x^n) × $(-1)^n$ $= \frac{c}{a}(-1)^2 = \frac{c}{a}$

Cubic Equation

$f(x) = ax^3 + bx^2 + cx + d = 0$

Assume that the roots are α, β and γ.

$\alpha + \beta + \gamma$ = (Coefficient of x^{n-1}/Coefficient of x^n) × $(-1)^n$= $\frac{b}{a}(-1)^1 = -\frac{b}{a}$

$\alpha\,\beta + \gamma\,\alpha + \beta\,\gamma$ = (Coefficient of x^{n-2}/Coefficient of x^n) × $(-1)^n$= $\frac{c}{a}(-1)^2 = \frac{c}{a}$

$\alpha\,\beta\,\gamma$ = (Constant term/Coefficient of x^n) × $(-1)^n$ $= \frac{d}{a}(-1)^3 = -\frac{d}{a}$

Bi-quadratic Equation

$f(x) = ax^4 + bx^3 + cx^2 + dx + e = 0$

Assume that the roots are α, β, γ and δ.

$\alpha + \beta + \gamma + \delta$ = (Coefficient of x^{n-1}/Coefficient of x^n) × $(-1)^n$= $\frac{b}{a}(-1)^1 = -\frac{b}{a}$

$\alpha\beta + \gamma\delta + \alpha\delta + \beta\gamma + \alpha\gamma + \delta\beta$ = (Coefficient of x^{n-2}/ Coefficient of x^n) × $(-1)^n$ = $\frac{e}{a}(-1)^4 = \frac{e}{a}$

$\alpha\,\beta\,\gamma + \delta\,\alpha\,\beta + \gamma\,\delta\,\alpha + \beta\,\gamma\,\delta$ = (Coefficient of x^{n-3}/ Coefficient of x^n) × $(-1)^n$ = $\frac{d}{a}(-1)^3 = -\frac{d}{a}$

$\alpha\,\beta\,\gamma\,\delta$ = (Constant term/Coefficient of x^n) × $(-1)^n$ $= \frac{e}{a}(-1)^4 = \frac{e}{a}$

Example 4 If the polynomial $ax^4 + bx^3 + cx^2 + dx + e$ has the property that the product of all the roots, taken at a time, is 1/3rd of the sum of the product of the roots, taking two at a time. What is the relationship between e and c?

Solution The product of all the roots, taken at a time = $\frac{e}{a}$

The sum of the product of the roots, taking two at a time $= \frac{c}{a}$

Now, $\frac{e}{a} = 1/3\ \frac{c}{a}$

So, $c = 3 \times e$

Example 5 If a, b and c are the roots of the equation $x^3 - 3x^2 + 2x + 1 = 0$, then what is the value of $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$?

Solution $\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{ab + bc + ca}{abc}$

$ab + bc + ca = 2$ and $abc = -1$

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{ab + bc + ca}{abc} = -2$$

FORMATION OF QUADRATIC EQUATION FROM ITS ROOTS

Case 1 If α and β are the roots of the equation $ax^2 + bx + c = 0$, then we can write

$ax^2 + bx + c = x^2 - (\alpha + \beta)x + \alpha\,\beta = x^2 -$ (sum of the roots) x + product of the roots = 0

Or, $ax^2 + bx + c = a(x - \alpha)(x - \beta) = 0$

In general, if $x_1, x_2, x_3, \ldots, x_n$ are the roots of the equation $f(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots + K = 0$

Then $f(x) = ax^n + bx^{n-1} + cx^{n-2} + \ldots + K = a(x - x_1)(x - x_2)(x - x_3)\ldots(x - x_n)$

Case 2 p and q are the roots of the equation $ax^2 + bx + c = 0$, and the quadratic equation with $(1/p)$ and $(1/q)$ as the roots is to be found:

Put $x = 1/p$

$a(1/p)^2 + b(1/p) + c = 0$

or, $cp^2 + bp + a = 0$

So, the quadratic equation is: $cx^2 + bx + a = 0$

Case 3 p and q are the roots of the equation $ax^2 + bx + c = 0$, and the quadratic equation with $(-p)$ and $(-q)$ as the roots is to be found:

Put $x = -p$

$a(-p)^2 + b(-p) + c = 0$

Or, $cp^2 - bp + a = 0$

So, the quadratic equation is: $cx^2 - bx + a = 0$

Example 6 What will be the quadratic equation having the roots opposite in sign as that of the quadratic equation $x^2 + 5x + 6 = 0$?

Solution Putting $(-x)$ at the place of x will give us the solution.

Desired equation is $(-x)^2 + 5(-x) + 6 = 0$

So, equation is: $x^2 - 5x + 6 = 0$

Alternatively, the roots of the quadratic equation $x^2 + 5x + 6 = 0$ are -2 and -3.

So, the roots of the desired equation should be 2 and 3.

So, the quadratic equation is $(x-2)(x-3) = 0$, or, $x^2 - 5x + 6 = 0$

Case 4 p and q are the roots of the equation $ax^2 + bx + c = 0$, and the quadratic equation with $(p)^2$ and $(q)^2$ as the roots is to be found:

Let x be a root of the original equation and y be a root of the transformed equation.

$y = x^2$, so we will put $x = \sqrt{y}$

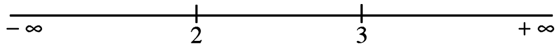
$a(\sqrt{y})^2 + b(\sqrt{y}) + c = 0,$

Solving this will give us the desired equation.

Solution of Inequations

Case 1 $f(x) = x^2 - 5x + 6 \geq 0$

$f(x) = x^2 - 5x + 6 = (x-2)(x-3) \geq 0$



We have got three intervals of values here: $-\infty$ to 2, 2 to 3 and 3 to $+\infty$

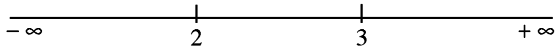
To check the values of x satisfying the above written inequation, any one value of x is taken from any of the intervals. If this value satisfies the inequation, then inequality will be satisfied in the alternate intervals.

Taking $x = 10$, $f(x) = (x-2)(x-3) = (10-2)(10-3) = +\text{ve}$

So, inequality is satisfied in this interval. Hence, it will not be satisfied in the interval $2 \leq x \leq 3$ and again will be satisfied in the interval $-\infty < x \leq 2$

So, the values of x satisfying the above written inequality lies outside 2 and 3 i.e., $x \leq 2$ or $x \geq 3$

Case 2 $f(x) = x^2 - 5x + 6 \leq 0$



It can similarly be observed that the values of x are lying in between 2 and 3

Hence, $2 \leq x \leq 3$

In general, if $(x-a)(x-b) \geq 0$, then $a \leq x \leq b$

And if $(x-a)(x-b) \leq 0$ and $a < b$, then $x \leq a$ or $x \geq b$

Example 7 What values of x will satisfy the quadratic inequation $f(x) = -x^2 + 3x + 4 > 0$?

Solution $f(x) = -x^2 + 3x + 4 > 0$, or, $x^2 - 3x - 4 < 0$

Or, $(x + 1)(x - 4) < 0$

So the values of x satisfying $f(x) = -1 < x < 4$

Worked-out Problems

Example 8 If the roots of the quadratic equation $x^2 - 4x - \log_3 N = 0$ are real, then find the minimum value of N?

Solution Since the roots of the given equation are real, so $D \geq 0$

Or, $16 + 4 \log_3 N \geq 0$, or, $\log_3 N \geq -4$

Or, $N \geq 3^{-4}$

So, the least value of N = 1/81

Example 9 Find the real values of N for which the quadratic equation $2x^2 - (N^3 + 8N - 1)x + N^2 - 4N = 0$ will have the roots of an opposite sign.

Solution The roots of the given equation will be of opposite signs only if they are real and the product of the roots is negative.

Or, $D \geq 0$, and the product of the roots < 0

Or, $(N^3 + 8N - 1)^2 - 8(N^2 - 4N) \geq 0$ and $\frac{N^2 - 4N}{2} < 0$

Or, $N^2 - 4N < 0$. Hence $0 < N < 4$

Example 10 If p and q are the roots of the quadratic equation $2x^2 + 6x + N = 0$ $(N < 0)$, then what is the maximum value of $\frac{p}{q} + \frac{q}{p}$?

Solution $p + q = -3$, and $pq = N/2$

Now, $N < 0$, hence $D = 36 - 4N > 0$. So, p and q are real.

Again, $\frac{p}{q} + \frac{q}{p} = \frac{(p+q)^2}{pq} - 2 = \frac{18}{N} - 2$

So, the maximum value of $\frac{p}{q} + \frac{q}{p} = -2$

Example 11 For which value of k does the following pair of equators yield a unique solution of x such that the solution is positive? (CAT 2005)

$x^2 - y^2 = 0$

$(x - k)^2 + y^2 = 1$

(a) 2 (b) 0 (c) $\sqrt{2}$ (d) $-\sqrt{2}$

Solution (3) $y^2 = x^2$

$2x^2 - 2kx + k^2 - 1 = 0$

$D = 0$

$\rightarrow 4k^2 = 8k^2 - 8$

$\rightarrow 4k^2 = 8$

$\rightarrow k = \sqrt{2}$

Example 12 Let

$$x = \sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\text{...to infinity}}}}}$$. Then x equals

(a) 3 (b) $\left(\frac{\sqrt{13}-1}{2}\right)$

(c) $\left(\frac{\sqrt{13}+1}{2}\right)$ (d) $\sqrt{3}$ **(CAT 2005)**

Solution (c) $x = \sqrt{4+\sqrt{4-x}} \Rightarrow x^2 - 4 + \sqrt{4-x}$

$$\left(x^2-4\right) = \sqrt{4 = x}$$

Now, put the values from the options.

Only the 3rd option satisfies the condition.

Alternatively, the value of x will be more than 2, which is given only in option 1 and option 3. Since it is only slightly more than 3, option 2 will be the answer.

Example 13 In a quadratic equation $ax^2 + bx + c = 0$, a, b and c are positive. Graph of this equation cuts X-axis at two distinct points. If both the roots of this equation are real, what can be said about the signs of both the roots?

Solution **Method 1**

Let us assume p and q are the roots of the equation.

Product of the roots = $p\,q = c/a$ = + ve [Both c and a are positive]

Hence, p and q are both positive, or both are negative.

Sum of the roots = $p + q = -\,b/a$ = – ve

Hence, p and q are both negative.

Method 2

Since the roots are real and distinct, D > 0

$$p = \left[-b-\sqrt{b^2-4ac}\right]/2a$$

Obviously p is negative.

$$q = \left[-b+\sqrt{b^2-4ac}\right]/2a$$

In this case, we need to compare b and $\left[\sqrt{b^2-4ac}\right]$.

$b > \left[\sqrt{b^2-4ac}\right]$, so q will also be negative.

Method 3

Using Descartes' Theorem, sign of $f(x)$ = + + +, so there will be no positive roots and $x = 0$ is not a solution of this equation.

So, it can be concluded that both the roots will be negative.

PRACTICE EXERCISES

■■■

WARM UP

Q.1. What is the value of k when the equation $x^2 - (k + 6)\,x + 2\,(2k - 1) = 0$ has the sum of the roots equal to half of their product?
(a) –2 (b) 7 (c) 9 (d) 12

Q.2. What is the value of P when the sum of the squares of the roots of the equation $x^2 - (p - 2)x - p - 1 = 0$? Assume the least value.
(a) –2 (b) 7 (c) 9 (d) 12

Q.3. If the equation $x^2 + 2\,(p + 1)\,x + 9p - 5 = 0$ has only positive roots, then which of the following is true?
(a) $p \geq 6$ (b) $p \leq 0$
(c) $p \leq -6$ (d) None of these

Q.4. The quadratic equations $x^2 - ax - 21 = 0$ and $x^2 - 3ax + 35 = 0$, where $a > 0$ have a common root. What is the value of a?
(a) 0 (b) 1 (c) 4 (d) 6

Q.5. A, B and C are real values such that $A + B + C = 2$, $A^2 + B^2 + C^2 = 6$ and $A^3 + B^3 + C^3 = 8$, then find the value of $A^4 + B^4 + C^4$?
(a) 6 (b) 12 (c) 18 (d) 24

Q.6. What is the sum of the roots of the quadratic equation $3x^2 + 2x - 1 = 0$?
(a) 1/3 (b) 2/3 (c) 3 (d) –2/3

Q.7. What is the product of the roots of the quadratic equation $3x^2 + 2x - 1 = 0$?
(a) –1/3 (b) 2/3 (c) 3 (d) –2/3

Q.8. What is the sum of the roots of the cubic equation $x^3 + 2x^2 + x - 1 = 0$?
(a) –2 (b) 2/3 (c) –1 (d) 2

Q.9. Which of the following quadratic equations will not have real roots?

(a) $3x^2 + 2x - 1 = 0$ (b) $x^2 + 2x + 1 = 0$
(c) $3x^2 + 2x + 5 = 0$ (d) $2x^2 + 3x + 1 = 0$

Q.10. Roots of the quadratic equation $ax^2 + bx + c = 0$ are 1 and 2, for some certain value of a, b and c. What will be the sum of the roots of the equation having the roots opposite in sign of the equation $ax^2 + bx + c = 0$?
(a) 3 (b) 6
(c) −3 (d) None of these

Q.11. The roots of the equation $a^2x^2 + abx = b^2$, where $a \neq 0$, are
(a) Real but unequal
(b) Real but equal
(c) Not real
(d) None of these

Q.12. Which of the following satisfies the equation:
$-x^2 + 6x - 8 > 0$
(a) $3 < x < 5$ (b) $2 < x < 4$
(c) $x < 2$ and $x > 4$ (d) $x < 3$ and $x > 5$

Q.13. For what value of K, the equations $2x + 3y = 5$ and $4x + Ky = A$ will not have any solution?
(a) 2
(b) −2
(c) 6
(d) More than one value is possible

Q.14. Five burgers, six pizzas and seven cold drinks cost Rs 178 and six burgers, four pizzas and two cold drinks cost Rs 124. What is the cost of 3 (pizzas + burgers + cold drinks)?
(a) Rs 60 (b) Rs 62.5
(c) Rs 90 (d) Cannot be determined

Q.15. A and B are the roots of the equation $x^2 - 13x + N = 0$. If $A - B = 3$, what is the value of N?
(a) 20 (b) 27
(c) 36 (d) None of these

Q.16. If $x < 0$. then what is the maximum value of $\frac{9}{x} + \frac{x}{9}$?
(a) 2 (b) −2
(b) +∝ (d) None of these

Q.17. If the coefficient of x^2 and the constant term of a quadratic equation are interchanged, then which of the following will not get changed?
(a) Sum of the roots
(b) Product of the roots
(c) Roots of the equation
(d) None of these

Q.18. If $(x + 2)^2 = 9$ and $(y + 3)^2 = 25$, then what is the maximum value of $\frac{x}{y}$?
(a) −5/8 (b) 5/8
(c) 3/7 (d) −3/7

Q.19. If one of the roots of the quadratic equation $2x^2 - 7x + q = 0$ is 3, what is the other root of this equation?
(a) 1/3 (b) 1/2 (c) 1 (d) 2

Q.20. What is the remainder when the polynomial $x^4 - 3x^2 + 7x - 10$ is divided by $x - 2$?
(a) 12 (b) 1 (c) −1 (d) 8

FOUNDATION

Q.1. How many real roots will be there of the equation $x^4 + x^2 = 0$
(a) 0 (b) 2
(c) 4 (d) None of these

Q.2. If $(x) + (1/x) = 2$, then what is the value of $(x)^3 + (1/x)^3$?
(a) 1 (b) −2 (c) −1 (d) 2

Q.3. Given that $as^4 + bs^3 + cs^2 + ds + e = 0$ is a bi-quadratic equation in s and $a \neq 0$. What is the value of $(1-s_1)(1-s_2)(1-s_3)(1-s_4)$?
(a) 1 (b) $(a+b+c+d+e)/a$
(c) 0 (d) $a+b+c+d+e$

Q.4. If $(x + 2)$ is a factor of $x^4 - 4x^2 + 2ax + 3 = 0$, then what is the value of a?
(a) −2 (b) 3 (c) $\frac{3}{4}$ (d) −3

Q.5. $x^2 - 5x + 6 = 0$ and $x^2 + mx + 3 = 0$ have a root in common. What is the value of m?
(a) 4 (b) −7/2
(c) Both 1 and 2 (d) None of these

Q.6. Find the number of integral roots of the equation $x3 - x^2 + 2x - 17 = 0$.
(a) 1 (b) 2
(c) 3 (d) None of these

Q.7. The number of real solutions of $x - \frac{1}{x^2-4} = -2$ $\frac{1}{x^2-4}$ is
(a) 0 (b) 1 (c) 2 (d) Infinite

Q.8. The number of real solutions of the equation $e^x = x$ is

(a) 1 (b) 2
(c) 0 (d) None of these

Q.9. The sum of the real roots of the equation $x^2 + |x| - 6 = 0$ is
(a) 4 (b) 0
(c) −1 (d) None of these

Q.10. If one root of the equation $(k^2 + 1)\,x^2 + 13x + 4k = 0$ is reciprocal of the other then k has the value
(a) $-2 + \sqrt{3}$ (b) $2 - \sqrt{3}$
(c) 1 (d) None of these

Q.11. If α, β are roots of the equation $(x - a)(x - b) = c, c \neq 0$, then the roots of the equation $(x - \alpha)(x - \beta) + c = 0$ are
(a) a, c (b) b, c
(c) a, b (d) $a + c, b + c$

Q.12. If the sum of the roots of the quadratic equation $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals then $\dfrac{b^2}{ac} + \dfrac{bc}{a^2}$ is equal to
(a) 2 (b) −2 (c) 1 (d) −1

Q.13. If a, b are roots of $x^2 + px + q = 0$ and g, d are the roots of $x^2 + px - r = 0$ then $(a - g)(a - d)$ is equal to
(a) $q + r$ (b) $q - r$
(c) $-(q + r)$ (d) $-(p + q + r)$

Q.14. If α, β, γ be the roots of the equation $x(1 + x^2) + x^2(6 + x) + 2 = 0$ then the value of $\alpha^{-1}+\beta^{-1}+\gamma^{-1}$ is
(a) −3 (b) $\dfrac{1}{2}$
(c) $-\dfrac{1}{2}$ (d) None of these

Q.15. If the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root then
(a) $a = 2, b - 7$ (b) $a = \dfrac{7}{2}, b = 1$
(c) $a = 4, b = -14$ (d) None of these

Q.16. If $(x - 1)^3$ is a factor of $x^4 + ax^3 + bx^2 + cx - 1$ then the other factor is
(a) $x - 3$ (b) $x + 1$
(c) $x + 2$ (d) None of these

Q.17. If $x^2 - bx + c = 0$ has equal integral roots then
(a) b and c are integers
(b) b and c are even integers
(c) b is an even integer and c is a perfect square of a positive integer
(d) 1 and 3 both

Q.18. Find the number which exceeds its reciprocal by 35/6.
(a) 4 (b) 5 (c) 6 (c) 7

Q.19. If a and b are the roots of the equation $2x^2 - 3x - 2 = 0$. Find the value of $(a^3 + b^3)$ without finding the roots of the equation.
(a) $\dfrac{8}{3}$ (b) $\dfrac{64}{3}$ (c) $\dfrac{63}{8}$ (d) 16

Q.20. If the roots of the equation $3ax^2 + 2bx + c = 0$ are in the ratio of 2:3, then
(a) $8ac = 25b$ (b) $8ac = 9b^2$
(c) $8b^2 = 9ac$ (d) $8b^2 = 25ac$

Q.21. In writing a quadratic equation of the form $x^2 + bx + c = 0$, a student writes the coefficient of x incorrectly and finds the roots as 7 and 8. Another student makes a mistake in writing the constant term and gets the roots as 8 and −3. Find the correct quadratic equation.
(a) $x^2 - 5x + 56 = 0$ (b) $x^2 - 5x + 21 = 0$
(c) $x^2 - 21x + 56 = 0$ (d) $x^2 - 7x + 21 = 0$

Q.22. The quadratic equation, whose roots are reciprocal of the roots of $1 - 3x - 2x^2 = 0$, is
(a) $2 - 3x - 2x^2 = 0$ (b) $2x^2 + 3x - 1 = 0$
(c) $x^2 - 3x - 2 = 0$ (d) $x^2 - 2x - 3 = 0$

Q.23. Find the value of K in the quadratic equations $Kx^2 - 3x + 2 = 0$, such that one root is double the other.
(a) −1 (b) 1 (c) −2 (d) 2

Q.24. The denominator of a positive fraction is 1 more than the square of the numerator. If 4 is added to both the numerator and the denominator, the fraction becomes ½. Find the original fraction.
(a) 1/3 (b) 2/5 (c) 4/17 (c) 3/10

Q.25. Ramesh bought a certain number of chairs for Rs 2,400. If the price of each chair is reduced by Rs 20, then ten more chairs can be purchased for the same amount. Find the number of chairs he purchased initially.
(a) 20 (b) 30 (c) 40 (d) 50

Q.26. If $x^2 - kx + 5 = 0$ has 3 as a root, then the value of K is
(a) 17/5 (b) 13/2 (c) 14/3 (d) 16/5

Q.27. If a, b and c are real numbers such that $a+b+c = 0$, then the value of $a^3 + b^3 + c^3$ is
(a) 0 (b) $ab^2 + bc^2 + ca^2$
(c) $3abc$ (d) None of these

Q.28. One root of the equation $x^3 - ax^2 + 11x - 6 = 0$ is 2. Find other two roots.
(a) 0, 1 (b) 1, 3 (c) 2, 3 (d) 1, 2

Q.29. If $x^4 + \dfrac{1}{x^4} = 47$, then find the value of $x^3 + \dfrac{1}{x^3}$.
(a) 9 (b) 18
(c) 12 (d) 27

Q.30. If the roots of the equation $px^2 + rx + r = 0$ are in the ratio of $a : b$, then the value of $\sqrt{b/a} + \sqrt{a/b}$ is

(a) $\sqrt{r/p}$ (b) $-\sqrt{r/p}$
(c) $\sqrt{1/p}$ (d) $\sqrt{1/r}$

Q.31. How many numbers in the set {−4, −3, 0, 2} satisfy the conditions $|y-4| \leq 6$ and $|y+4| < 5$?
(a) 3 (b) 1
(c) 2 (d) None of these

Q.32. If $x^3 - ax^2 + bx + 10$ is perfectly divisible by $(x + 5)$ and $x^4 + x^3 + bx^2 - ax + 42$ is perfectly divisible by $(x - 3)$, find the value of $(a - 3b)$.
(a) $\frac{450}{7}$ (b) 50
(c) 100 (d) None of these

Q.33. If a and b are the roots of $x^2 + x + 2 = 0$, then $\dfrac{\alpha^{10}+\beta^{10}}{\alpha^{-10}+\beta^{-10}}$ is
(a) 4096 (b) 2048
(c) 1024 (d) 512

Q.34. A man born in 1900s realized that in 1980 his age was the square root of the year of his birth. What is his birth year?
(a) 1929 (b) 1949 (c) 1936 (d) 1946

Q.35. In solving a problem on quadratic equation, one student makes a mistake in the constant term of the equation and gets −3 and −2 for the roots. Another student makes a mistake in the coefficient of the first degree term and finds −1 and −2 for the roots. The correct equation was.
(a) $x^2 - 5x + 2 = 0$ (b) $x^2 + 5x + 2 = 0$
(c) $x^2 + 5x - 2 = 0$ (d) $x^2 - 5x - 2 = 0$

Q.36. What is the solution set of the inequation $x^3 - x^2 + x - 1 > 0$?
(a) $x < -1$ (b) $-1 < x < 1$
(c) $x > 1$ (d) $x > 0$

Q.37. The number of distinct real roots of the equation $|V^2 + V - 6| - 3V + 7 = 0$ is
(a) 0 (b) 4 (c) 2 (d) 3

Q.38. In solving an equation of the form $ax - b = 0$ (a and b having only 1 as the common factor), A made a mistake of copying 'b' and got $\frac{7}{3}$ as the root whereas B made a mistake of copying 'a' and got $\frac{8}{5}$ as the root. The correct root is
(a) $\frac{7}{5}$ (b) $\frac{3}{8}$
(c) $\frac{8}{3}$ (d) Cannot be determined

Q.39. If one of the roots of the cubic expression $x^3 - ax^2 + 11x - 6$ is 3, what are the other roots?
(a) 6 and 2 (b) −1 and 2
(c) 1 and 2 (d) −2 and −1

Q.40. If m and n are natural numbers, then
(a) $m^{100} + n^{100} > (m + n)^{100}$
(b) $m^{100} + n^{100} \geq (m + n)^{100}$
(c) $m^{100} + n^{100} < (m + n)^{100}$
(d) $m^{100} + n^{100} \leq (m + n)^{100}$

MODERATE

Q.1. How many real roots will be there of the equation $= x^2 + 5\,|x| + 6 = 0$
(a) 0 (b) 2
(c) 4 (d) None of these

Q.2. If $s = 2 + 2^{1/3} + 2^{2/3}$, then what is the value of $s^3 - 6s^2 + 6s$?
(a) (b) 2 (c) (d)

Direction for questions 3 and 4: Read the passage below and solve the questions based on it.

x_1, x_2 and x_3 are the roots of two distinct quadratic equations (with coefficient of x^2 being 1 in both the equations) in which x_1 is a common root.

Q.3. If x_1, x_2 and x_3 are in AP and the coefficient of x in the equation with x_1 and x_2 as its root is −4, then one of the roots is
(a) 1 (b) 3
(c) 5 (d) Cannot be determined

Q.4. If x_1, x_2 and x_3 are in AP and the coefficient of x in the equation with x_1 and x_3 as its root is −6, then one of the roots is
(a) 1 (b) 3
(c) 5 (d) Cannot be determined

Direction for questions 5 and 6: Read the passage below and solve the questions based on it.

One of the +ve roots of a quadratic equation $ax^2 + mx + c = 0$, where $a = 1$, is twice that of the other. The sum of the coefficient of x and constant term is 2.

Q.5. What is the value of m in the equation?
(a) −4 (b) 4 (c) −6 (d) 6

Q.6. If x_1 and x_2 are the roots of this equation, then find the value of $|x_1 - x_2|$?
(a) 2 (b) 4
(c) 6 (d) None of these

Q.7. The two roots of the quadratic equation $ax^2 + bx + c = 0$ are both two-digit integers with the same units digit, but D/a^2 (where D is the discriminant given as $b^2 - 4ac$) is not a multiple of 100. Which of the following is a possible value of the unit digit of $b/2a$?
(a) 4 (b) 5 (c) 6 (d) 7

Q.8. The polynomial $(ax^2+bx+c)\,(ax^2-dx-c)$, $ac \neq 0$, has
(a) Four real zeros
(b) At least two real zeros
(c) At most two real zeros
(d) No real zeros

Q.9. The number of real solutions of $1 + |e^x - 1| = e^x(e^x - 2)$ is
(a) 0 (b) 1 (c) 2 (d) 4

Q.10. If $y \neq 0$ then the number of values of the pair (x, y) such that $x+y+\frac{x}{y}=\frac{1}{2}$ and $(x+y)\frac{x}{y}=-\frac{1}{2}$, is:
(a) 1 (b) 2
(c) 0 (d) None of these

Q.11. The equation has $\sqrt{x+1}-\sqrt{x-1} = \sqrt{4x-1}$
(a) No solution (b) One solution
(c) Two solutions (d) More than two solutions

Q.12. The number of real solutions of the equation $\frac{6-x}{x^2-4}=2+\frac{x}{x+2}$ is
(a) Two (b) One
(c) Zero (d) None of these

Q.13. If $x \in Z$ (the set of integers) such that $x^2 - 3x < 4$ then the number of possible values of x is
(a) 3 (b) 4
(c) 6 (d) None of these

Q.14. If x is an integral satisfying $x^2-6x+5 \leq 0$ and $x^2-2x > 0$ then the number of possible value of x is
(a) 3 (b) 4 (c) 2 (d) Infinite

Q.15. If l, m are real and $l \neq m$ then the roots of the equation $(1 - m)\,x^2 - 5\,(1 + m)\,x - 2\,(10\text{ m}) = 0$ are
(a) Real and equal (b) Nonreal complex
(c) Real and unequal (d) None of these

Q.16. The number of solutions of $\log_4(x-1) = \log_2(x-3)$ is
(a) 2 (b) 1 (c) 3 (d) 0

Q.17. If $a > 1$, roots of the equation $(1 - \text{a})\,x^2 + 3ax - 1 = 0$ are
(a) One positive and one negative
(b) Both negative
(c) Both positive
(d) Both nonreal complex

Q.18. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are less than 3 then
(a) $a < 2$ (b) $2 \leq a \leq 3$
(c) $3 < a \leq 4$ (d) $a > 4$

Q.19. The number of integral values for which $x^2-(a-1)\,x+3 = 0$ has both the roots positive and $x^2+3x+6-a = 0$ has both the roots negative, is
(a) 0 (b) 1 (c) 2 (d) Infinite

Q.20. If the equations $ax^2 + bx + c = 0$, $cx^2 + bx + a = 0$, $a \neq c$ have a negative common root then the value of $a - b + c$ is
(a) 0 (b) 2
(c) 1 (d) None of these

Q.21. The number of values of k for which $(x^2 - (k - 2)\,x + k^2)\,(x^2 + kx + (2k - 1))$ is a perfect square, is
(a) 1 (b) 2
(c) 0 (d) None of these

Q.22. If $x^2 + ax + b = 0$ and $x^2 + bx + a = 0$, $a \neq b$, have a common root a then, which of the following is true?
(a) $a + b = 1$ (b) $a + 1 = 0$
(c) $a = 0$ (d) $a + b + 1 = 0$

Q.23. If 2 and 4 are two roots of the expression $x^4 + 2x^3 + mx^2 + nx + 3 = 0$ (m and n are constants) then what is the value of $2m + 3n$?
(a) 106 (b) −106 (c) 53 (d) −53

Q.24. If $p+q+r = 1$, $p^2 + q^2 + r^2 = 9$ and $p^3 + q^3 + r^3 = 1$, then what is the value of $\frac{1}{p}+\frac{1}{q}+\frac{1}{r}$?
(a) 1 (b) −4 (c) −1 (d) −8

Q.25. If the roots of the equation $x^2 + (3k - 36)\,x + k^2 - 24\,k + 144 = 0$ are reciprocal to each other, then find the value of k.
(a) K = 11 or K = 13 (b) K = −11 or K = −13
(c) K = 12 (d) K = −12

Q.26. The number of common roots of $x^2 + 3x - 4 = 0$ and $x^3 - 4x = 0$ is
(a) 0 (b) 1 (c) 2 (d) 3

Q.27. If $Z+\frac{1}{Z}=1, Z^{64}+\frac{1}{Z^{64}}$ is equal to
(a) 0 (b) 1 (c) −1 (d) −2

Q.28. Find the maximum and the minimum values of the function $\frac{x^2-x+1}{x^2+x+1}$ for real values of x.
(a) 3 and −3 (b) $\frac{1}{3}$ and $-\frac{1}{3}$
(c) 3 and $\frac{1}{3}$ (d) None of these

Q.29. Consider the equation of the form $x^2 + bx + c = 0$. The number of such equations that have real roots and have coefficients b and c in the set $\{1, 2, 3, 4, 5, 6\}$, (b may be equal to c), is

(a) 20 (b) 18 (c) 17 (d) 19

Q.30. If a, b, c, d are four consecutive terms of an increasing AP then the roots of the equation $(x - a)(x - c) + 2(x - b)(x - d) = 0$ are

(a) Real and distinct
(b) Nonreal complex
(c) Real and equal
(d) Integers

Q.31. If $2a^{-1/3} + 2a^{1/3} = 5$, then find the value of a.

(a) 8 or $\frac{1}{8}$ (b) 2 or $\frac{1}{2}$
(c) 3 or $\frac{1}{3}$ (d) None of these

Q.32. If α, β and γ are the roots of the equation $9x^3 - 7x + 6 = 0$, then the equation whose roots are $3a + 2$, $3b + 2$ and $3\gamma + 2$ is

(a) $x^3 - 6x^2 + 5x + 24 = 0$
(b) $9x^3 - x + 16 = 0$
(c) $2x^3 - 27x - 8 = 0$
(d) None of these

Q.33. If the roots of equation $(q - r)x^2 + (r - p)x + p - q = 0$ are equal then p, q, r are in

(a) AP (b) GP
(c) HP (d) None of these

Q.34. Both the roots of the equation $(x-b)(x-c) + (x-a)(x-c) + (x-a)(x-b) = 0$ are always

(a) +ve (b) –ve
(c) Real (d) None of these

Q.35. If $a(b-c)x^2 + b(c-a)xy + c(a-b)y^2$ is a perfect square, then, a, b, c are in

(a) AP (b) GP
(c) HP (d) None of these

Q.36. $x^{3^n} + y^{3^n}$ is divisible by $x + y$ if

(a) n is any integer ≥ 0
(b) n is an odd positive integer
(c) n is an even positive integer
(d) n is a rational number

Q.37. The condition that both the roots of quadratic equation $ax^2 + bx + c = 0$ are positive is

(a) a and c have an opposite sign that of b
(b) b and c have an opposite sign that of a
(c) a and b have an opposite sign that of c
(d) None of these

Q.38. The condition that both the roots of the equation $ax^2 + bx + c = 0$ are zero is

(a) $b \neq 0$
(b) $c \neq 0$
(c) Either b or c (not both) = 0
(d) $b = c = 0$

Q.39. There is a quadratic equation of the form $ax^2 + bx + c = 0$ along with 3 conditions.

I. If only the value of a is correct in the equation, the roots are $\frac{1}{3}$ and 1.
II. If only the value of b is correct, the roots are 6 and 5.
III. If only the value of c is correct, the roots are -3 and -2.

What is the equation of a quadratic equation opposite in sign to the roots of the given quadratic equation?

(a) $3x^2 + 11x + 6 = 0$ (b) $3x^2 - 7x - 6 = 0$
(c) $x^2 - 11x + 30 = 0$ (d) $x^2 - 6x - 1 = 0$

Q.40. Quadratic equations $(2p - 1)z^2 + (2p + 1)z + c = 0$ and $(q + 1)y^2 + (4q + 1)y + 3c = 0$ have the same pair of roots. Given that $c \neq 0$, what is the value of $(p + q)$?

(a) 3 (b) 4
(c) 2 (d) Cannot be determined

ADVANCED

Q.1. Quadratic equation $x^2 + bx + c =$ has roots a and b, such that LCM of $(a, b) = 24$ and HCF of $(a, b) = 2$. What is the total number of such quadratic equations if a and b are natural numbers?

(a) 1 (b) 2 (c) 3 (d) 4

Q.2. $(p^2 + q^2)x^2 - \sqrt{2}(p + q)x + 1 = 0$. What must be the relation between q and p if the equation has equal and real roots?

(a) $p = q$ (b) $p > q$
(c) $p < q$ (d) None of these

Q.3. How many positive integers p, q and r exist such that $p/q + q/r + r/p = 2$?

(a) 1 (b) 3
(c) 15 (d) None of these

Q.4. If $x^2 + y^2 + z^2 = 1$ then the value of $xy + yz + zx$ lies in the interval of

(a) $\left[\frac{1}{2}, 2\right]$ (b) $[-1, 2]$
(c) $\left[-\frac{1}{2}, 1\right]$ (d) $\left[-1, \frac{1}{2}\right]$

Q.5. If $a \in$ R, $b \in$ R then the factors of the expression a $(x^2 - y^2) - bxy$ are
(a) Real and different (b) Real and identical
(c) Complex (d) None of these

Q.6. If the sum of the roots of the quadratic equations $ax^2 + bx + c = 0$ is equal to the sum of the squares of their reciprocals, then $\frac{b^2}{ac} + \frac{bc}{a^2} =$
(a) 0 (b) −1 (c) 1 (d) 2

Q.7. If equations $x^2 + 3x + 10 = 0$ and $ax^2 + bx + c = 0$ have a common root, then $a:b:c =$
(a) 2:3:10 (b) 1:3:5
(c) 1:3:10 (d) None of these

Q.8. Sum of all the real roots of the equation $|x - 2|^2 + |x - 2| - 2 = 0$ is...
(a) 0 (b) −4 (c) 4 (d) 2

Q.9. If $f(x) = \sum_{r=1}^{x} \frac{1}{r}, f(10) = \frac{a}{b}, f(20) = \frac{c}{d}$ and $f(30) = \frac{m}{n}$, where HCF of a and b = HCF of c and d = HCF of m and n = 1, then the roots of the equation $ax^2 + cx + m = 0$ are
(a) Imaginary (b) Rational
(c) Cannot be rational (d) Integers

Q.10. If pth, qth and rth term of a GP be 27, 8 and 12 respectively, then the equation $px^2 + 2qx - 2r = 0$ has
(a) Only one root in (0, 1)
(b) No root in (0, 1)
(c) Both roots in (0, 1)
(d) Imaginary roots

Q.11. If a, b be the roots of equation $x^2 + ax + b = 0$ and m, n, p, q are distinct integers in AP such that $a < m < n < p < q < b$ and $q = m^2 + n^2 + p^2$, then which of the following is true?
(a) $b + 1 + |a| > 0$ (b) $b + 1 + |a| = 0$
(c) $b + |a| = 0$ (d) $1 + b + |a| < 0$

Q.12. For $a, b, c \in$ R if $ax^2 + bx + c$ is real for real values of x and imaginary for imaginary values of x, then
(a) $a > 0$ (b) $a < 0$
(c) $a = 0$ (d) Nothing can be said

Q.13. If $x + \frac{1}{x} = 1$ and $p = x^{4000} + \frac{1}{x^{4000}}$ and q be the digit at units place in the number $2^{2^n} + 1$, n being a natural number greater than 1, then $p + q =$
(a) 2 (b) 4
(c) 6 (d) None of these

Q.14. If the roots of $ax^2 + bx + b = 0$ be in the ratio $p:q$, then $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{b}{a}} =$
(a) 1 (b) −1 (c) 2 (d) 0

Q.15. There are two quadratic expressions $a_1x^2 + b_1x + c_1$ and $a_2x^2 + b_2x + c_2$. Both of them have the same roots. If the ratio of a_1 to a_2 is 1 : 2, what is the ratio of the maximum values of the two quadratic expressions? (a_1, $a_2 < 0$)
(a) 1:3 (b) 3 : 1
(c) 1:5 (d) None of these

Q.16. If the two equations $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ have a common root, then how many of the following statements can be true?
I. $a = c$
II The common root is 1
III. $a - b + c = 0$
(a) 0 (b) 1 (c) 2 (d) 3

Q.17. Given that P and Q are the roots of $Ax^2 - 4x + 1 = 0$ and R and S are the roots of the equation $Bx^2 - 6x + 1 = 0$. What is the value of A so that P, Q, R and S are in HP?
(a) 1 (b) 2 (c) 3 (d) 4

Q.18. If $a < b < c < d$, then which of the following is true about the roots of the quadratic equation $(x-a)(x-c) + K(x-b)(x-d) = 0$, for the real value of K?
(a) Both the roots of this equation will have a positive sign
(b) Both the roots of this equation will have a negative sign
(c) Both the roots of this equation will be real
(d) Cannot be determined

Q.19. If $a + b + c = 0$ and a, b, c are rational, then which of the following is true about the roots of the quadratic equation $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$?
(a) Both the roots of the equation are rational
(b) Both the roots of the equation are irrational
(c) One of the roots of the equation is rational and the other is irrational
(d) None of these

Q.20. Find all the values of p for which one root of the equation $x^2 - (p + 1)x + p^2 + p - 8 = 0$, is greater than 2 and of the other root is smaller than 2.
(a) $-2 < p < 3$ (b) $-3 < p < 2$
(c) $-3 < p$ (d) $p < 2$

Q.21. For how many values of a, $a > 0$, both the roots of $ax^2 - (a + 1)x + (a - 2) = 0$ are greater than 3?
(a) 0 (b) 2 (c) 5 (d) Infinite

Q.22. The number of real values of the parameter k for which $(\log_{16}x)^2 - \log_{16} x + \log_{16} k = 0$ with real coefficients will have exactly one solution is:

(a) 2 (b) 1
(c) 4 (d) None of these

Q.23. If the roots of the equation $(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0$, (where a, b, c are real numbers) are equal, then:

(a) $b^2 - 4ac = 0$ (b) $a = b = c$
(c) $a + b + c = 0$ (d) None of these

Q.24. If the equation $ax^2 + bx + c = 0$ has a root less than -2 and a root greater than 2, and $a > 0$, then which of the following is true?

(a) $4a + 2|b| + c < 0$
(b) $4a + 2|b| + c > 0$
(c) $4a + 2|b| + c = 0$
(d) None of these

Q.25. The equations $x^2 + x + a = 0$ and $x^2 + ax + 1 = 0$.

(a) Cannot have a common real root for any value of a
(b) Have a common real root for exactly one value of a
(c) Have a common real root for exactly two values of a
(d) Have a common real root for exactly three values of a

ANSWERS

Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(a)	(c)	(c)	(d)	(a)	(a)	(c)	(c)	(d)	(b)	(c)	(c)	(d)	(b)	(d)	(b)	(b)	(d)

Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(d)	(b)	(c)	(b)	(d)	(a)	(c)	(b)	(b)	(c)	(a)	(c)	(c)	(c)	(b)	(d)	(c)	(c)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(c)	(b)	(d)	(b)	(c)	(a)	(d)	(b)	(b)	(b)	(b)	(c)	(c)	(b)	(c)	(a)	(d)	(c)	(c)

Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(d)	(d)	(c)	(d)	(b)	(b)	(b)	(b)	(a)	(b)	(b)	(a)	(c)	(b)	(c)	(a)	(c)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(d)	(a)	(a)	(a)	(a)	(c)	(c)	(d)	(a)	(a)	(d)	(a)	(c)	(c)	(a)	(a)	(d)	(a)	(a)

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(a)	(b)	(c)	(a)	(d)	(c)	(c)	(d)	(a)	(d)	(c)	(c)	(d)	(d)	(d)	(c)	(c)	(a)	(a)
Q. No.	21	22	23	24	25															
Answer	(a)	(b)	(b)	(a)	(b)															

HINTS AND SOLUTIONS

Warm up

1. $K + 6 = 2K - 1$

   $K = 7$

2. Let roots = α, β

   So, $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta = (P-2)^2 + 2(p+1)$

   $= p^2 - 4p + 4 + 2p + 2 = (P-1)^2 + 5$

   So, value of p for the least value = 1

3. $\alpha+\beta=-2(P+1)$; $\alpha\beta = 9p - 5$

   $\therefore\ \alpha < 0$ and $\beta < 0$

   $\therefore\ \alpha+\beta<0 \Rightarrow -2(p+1)<0 \Rightarrow p>-1$

   $\therefore\ \alpha\beta>0 \Rightarrow 9P-5>0 \Rightarrow p>5/9$

   Hence option (*d*) is the answer.

4. Let common root = α

   $\therefore\ \alpha^2 - a\alpha - 21 = 0$

   $\alpha^2 - 3a\alpha + 35 = 0$

   Solving the two equations, we get $a = 4$

5. Taking the values of A, B and C as 1, 2 and –1.

   We get $A^4 + B^4 + C^4 = 18$

6. Sum of roots = $-2/3(-b/a)$.

7. Product of roots = $-1/3(c/a)$

8. In a cubic equation $ax^3+ bx^2 + cx + d= 0$, Sum of roots $= -b/a$

   $\therefore$ Here sum of roots = –2

9. Calculating $D = b^2 - 4ac$ for each of the given option and for a non–real root, D should be less than 0.

10. For roots with opposite sign replace x with $-x$ in the given equation.

    We get, $ax^2 - bx + c = 0$

    Now the sum of roots: b/a.

    For the new equation sum of roots = –3

11. $a^2x^2 + abx = b^2$

    $$\left(ax+\frac{b}{2}\right)^2 = \frac{5b^2}{4}$$

    $$ax + b/2 = \pm\frac{\sqrt{5}}{4}b$$

    If $b = 0$, then real and equal roots, else real and unequal.

    Hence option *d* is the answer.

13. $2x + 3y = 5$

    $4x + ky = A$

    For no solution

    $$\frac{2}{4}=\frac{3}{k}\neq\frac{5}{A}$$

    $\therefore\ K = 6$

14. $5b + 6p + 7c = 178$ ---(1)

    $6b + 4p + 2c = 124$ ---(2)

    Multiplying equation (1) by 2 and adding to equation (2):

    $16b + 16p + 16c = 480$

    $b + p + c = 30$

    $\therefore\ 3b + 3p + 3c = 90$

15. $x^2 - 13x + N = 0$ with roots A and B

    $\therefore\ A + B = 13$

    and $A - B = 3$

    $\therefore\ A = 8$ and $B = 5$

    $\therefore\ N = AB = 40$

16. let $x = -a$ where $a > 0$

    For any 2 positive numbers $\frac{9}{a}, \frac{a}{9}$

AM ≥ GM

$$\Rightarrow \frac{\frac{9}{a}+\frac{a}{9}}{2} \geq \sqrt{\frac{9}{a}\times\frac{a}{9}}$$

$$\Rightarrow \frac{9}{a}+\frac{a}{9} \geq 2$$

$$\Rightarrow -\left(\frac{a}{9}+\frac{9}{a}\right) \leq -2$$

17. Take a quadratic equation: $x^2 + 3x + 2 = 0$

$\therefore$ sum of roots = –3

products of roots = 2

roots = –1, –2

Now new quadratic equation: $2x^2 + 3x + 1 = 0$

$\therefore$ sum of roots = –3/2

product of roots: 1/2

roots: –1, –1/2

18. $(x + 2)^2 = 9$, $(y + 3)^2 = 25$

$\therefore x = 1, -5; y = 2, -8$

$\therefore$ maximum value of $\frac{x}{y} = \frac{5}{8}$

19. Sum of roots $= \frac{7}{2}$ and one of the roots = 3

So other root $= \frac{7}{2} - 3 = 1/2$

20. Using remainder theorem; put $x = 2$ in the main polynomial:

$\therefore p(2) = 16 - 12 + 14 - 10 = 8$

$\therefore$ Remainder = 8

Foundation

1. $x^4 + x^2 = 0$ can be written as $\Rightarrow x^2(x^2 + 1) = 0$

So, either $x^2 = 0$ or $(x^2 + 1) = 0$

Case 1 - If $x^2 = 0$, then $x = 0, 0$ (two solutions)

Case 2 - $(x^2 + 1) = 0 \Rightarrow x^2 = -1$

Minimum value of $x^2 = 0$, and $x^2 = -1$ is not possible.

Hence a total of 2 real solutions are possible.

2. Method 1 - Using the formula $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$

$(x^3 + 1/x^3) = (x + 1/x)^3 - 3\,(x).(1/x)\,(x + 1/x) = 2^3 - 3 \times 1 \times (2) = 2$

Method 2 - $(x + 1/x) = 2$

Or, $(x + 1/x) - 2 = 0$, or, $x^2 - 2x + 1 = 0$

$\Rightarrow (x - 1)^2 = 0 \Rightarrow x = 1$

So, $(x^3 + 1/x^3) = 2$

Method 3:

$x + \frac{1}{x} = 2$. Going by hit and trial, we can see that $x = 1$.

$\therefore \quad x^3 + \frac{1}{x^3} = 1 + 1 = 2$

3. $(1 - S_1)(1 - S_2)(1 - S_3)(1 - S_4)$

$= 1 - (S_1 + S_2 + S_3 + S_4) + (S_1S_2 + S_2S_3 + S_3S_4 + S_4S_1)$

$- (S_1S_2S_3 + S_2S_3S_4 + S_3S_4S_1 + S_4S_1S_2) + S_1S_2S_3S_4$

$= 1 + \frac{b}{a} + \frac{c}{a} + \frac{d}{a} + \frac{e}{a}$

$= \frac{a + b + c + d + e}{a}$

Alternate solution:

$as^4 + bs^3 + cs^2 + ds + e = 0$

$\Rightarrow a(s - s_1)(s - s_2)(s - s_3)(s - s_4) = 0$ (because S_1, S_2, S_3 and S_4 are roots)

Now, putting $s = 1$ in L.H.S

$\Rightarrow P(1) = a(1 - s_1)(1 - s_2)(1 - s_3)(1 - s_4)$

$\Rightarrow (1 - s_1)(1 - s_2)(1 - s_3)(1 - s_4) = (a + b + c + d + e)/a$

4. Since $(x + 2)$ is a factor of $P(X)$, so $p(-2) = 0$

$\Rightarrow 16 - 16 - 4a + 3 = 0 \Rightarrow a = \frac{3}{4}$.

5. $x^2 - 5x + 6 = 0 \Rightarrow$ Roots = 2, 3

When 2 is the common root then $p(2) = 0 \Rightarrow 2^2 + 2m + 3 = 0$

$\Rightarrow m = -7/2$

When 3 is the common root then $P(3) = 0 \Rightarrow 3^2 + 3m + 3 = 0$

$\Rightarrow m = -4$

6. $x^3 - x^2 + 2x - 17 = 0$

let roots be α, β and γ

$\therefore \alpha\beta\gamma = 17$

Now, 17 is having only two factors 1 and 17, so the only possible integral roots = 1 and 17. Checking for these two values we find no integral root possible.

7. $x - \frac{1}{x^2 - 4} = 2 - \frac{1}{x^2 - 4}$

$\Rightarrow x = 2$. Although we can see that for $x = 2$, $\frac{1}{x^2 - 4}$ will not hold.

Hence no value of x is possible.

8. Draw the graphs of both L.H.S and R.H.S to find there are no intersection points.

9. $x^2 + |x| - 6 = 0 \Rightarrow x^2 + x - 6 = 0$ where ≥ 0 $\therefore$ root : 2

Else $x^2 - x - 6 = 0$ if $x < 0$ $\therefore$ root: –2.

So, sum of roots = 0

10. Product of roots = 1

$$\Rightarrow \frac{4k}{k^2+1} = 1 \Rightarrow k = 2 \pm \sqrt{3}$$

11. $(x - a)(x - b) = c$

$\Rightarrow x^2 - (a + b)x + ab - c = 0$; roots: α, β

$(x - \alpha)(x - \beta) + c = 0$

$\Rightarrow x^2 - (a + \beta)x + \alpha\beta + c = 0$

$\Rightarrow x^2 - (a + b) + ab - c + c = 0$

(putting values of $(\alpha + \beta)$ and $\alpha\,\beta$ from previous equation)

Hence new equation = $x^2 - (a + b) + ab = 0$. So the roots are a and b.

12. $$\alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{\alpha^2 + \beta^2}{\alpha^2\beta^2} \Rightarrow \frac{-b}{a} = \frac{(\alpha+\beta)^2 - 2\propto\beta}{\alpha^2\beta^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{(b^2/a^2) - \frac{2c}{a}}{c^2/a^2}$$

$$\Rightarrow \frac{b^2}{ac} + \frac{bc}{a^2} = 2$$

14. $x(1 + x^2) + x^2(6 + x) + 2 = 0$

$\Rightarrow 2x^3 + 6x^2 + x + 2 = 0$

For roots: α, β, γ

$$\alpha + \beta + \gamma = -\left(\frac{6}{2}\right) = -3$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{1}{2}$$

$$\alpha\,\beta\,\gamma = -\left(\frac{2}{2}\right) = -1$$

$$\therefore \frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma} = \frac{\alpha\beta + \beta\gamma + \gamma\alpha}{\alpha\beta\gamma} = \frac{1/2}{-1} = -1/2$$

15. let 'x' be the common root

$\therefore$ $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$

$\Rightarrow 4x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$

$\therefore$ Subtracting the two equation

$(a - 4)x^2 + (b + 14)x = 0 \Rightarrow (a - 4)x^2 + (b + 14)x = 0$ {because $x \neq 0$}

Now go through the options.

Alternatively, for common roots between the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$, value of constant term in 2nd equation is twice the constant term in 1st equation. Hence value of a and b in 2nd equation also should be twice than that of 1st equation.

Hence $a = 4$, $b = -14$.

16. As, $(x - 1)^3$ is a factor of the polynomial, so, 1 is a repeated root (3 times) of the given equation.

Let the fourth root be x, so, $1.1.1.\,x = -1$

$\Rightarrow x = -1. \Rightarrow -1$ is another root

Hence $(x + 1)$ is a factor.

17. Let 'α' be the equal integral root,

$\therefore$ $x^2 - bx + c = 0$

$(x - \alpha)^2 = 0$

$\Rightarrow x^2 - 2\alpha x + \alpha^2 = 0$

Now comparing the two equations

$\therefore$ $2\alpha = b$ and $c = \alpha^2$

$\therefore$ b is even integer and c is an integer as well as a perfect square

18. $$x - \frac{1}{x} = \frac{35}{6}$$

$\therefore$ $x = 6$

19. $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$

$$= \left(\frac{3}{2}\right)^2 + 3 \times 1 \times \frac{3}{2} = \frac{63}{8}$$

20. Assume that roots of the equation $3ax^2 + 2bx + c = 0$ are α, β.

$$\alpha + \beta = -\frac{2b}{3a},\ \alpha\beta = \frac{c}{3a} \text{ and } \frac{\alpha}{\beta} = \frac{2}{3} \text{ (given)}$$

$$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{2}{3} + \frac{3}{2}$$

$$\frac{\alpha^2 + \beta^2}{\alpha\beta} = \frac{13}{6} \Rightarrow \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{13}{6}$$

Now putting the values of $(\alpha + \beta)$ and $\alpha\beta$ and then solving,

We get, $8b^2 = 25ac$.

21. Coefficient of $x^2 = 2$

In first case the product of the roots = $8\times7 = 56$ which is the constant term. So $c = 56$.

In the second case the sum of the roots = $-b$ = 8–3 = 5

$\Rightarrow b = -5$

$\therefore$ The correct quadratic equation is $x^2 - 5x + 56 = 0$

22. For reciprocal roots, replace x with $1/x$ in L.H.S of

$1 - 3x - 2x^2 = 0$

$\Rightarrow x^2 - 3x - 2 = 0$

23. Let the roots be α and 2α

So, Sum of roots = $\alpha + 2\alpha = 3\alpha = \frac{3}{k} \Rightarrow \alpha = \frac{1}{k}$

and product of roots $\Rightarrow 2\alpha^2 = \frac{2}{k} \Rightarrow \frac{2}{k^2} = \frac{2}{k}$

Hence $k = 1$.

24. Go through options.

25. Let the number of chairs bought initially = n

$$\therefore \frac{2400}{n} - \frac{2400}{n+10} = 20$$

$$\Rightarrow 120\left[\frac{10}{n(n+10)}\right] = 1$$

$\therefore\ n(n + 10) = 1200 \Rightarrow n = 30$

26. Here sum of roots = k and product of roots = 5

One root is 3 the other root is $\frac{5}{3}$

$\therefore k = 3 + \frac{5}{3} = \frac{14}{3}$

27. We know

$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

As, $a + b + c = 0$

$\therefore\ a^3 + b^3 + c^3 = 3abc$

28. Let the roots be 3, α and β

$\therefore 3 + \alpha + \beta = a;\ 3\alpha + 3\beta + \alpha\beta = 11\ ;\ 3\alpha\beta = 6$

So, $\alpha\beta = 2$ and $3(\alpha + \beta) + 2 = 11$

$\Rightarrow 3(\alpha + \beta) + 2 = 11 \Rightarrow \alpha + \beta = 3$

$\therefore \alpha, \beta = (1,2)$

29. $x^4 + \frac{1}{x^4} = 47$

$$\Rightarrow \left(x^2 + \frac{1}{x^2}\right)^2 = 47 \Rightarrow x^2 + \frac{1}{x^2} = 7$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 - 2 = 7$$

$$\Rightarrow \left(x + \frac{1}{x}\right)^2 = 9$$

$$\Rightarrow x + \frac{1}{x} = 3 \Rightarrow \left(x + \frac{1}{x}\right)^3 = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right) = 27$$

$$\Rightarrow x^3 + \frac{1}{x^3} + 3 \times 3 = 27 \Rightarrow x^3 + \frac{1}{x^3} = 18$$

30. Let roots be ak and bk.

$\therefore (a + b)k = -r/p$

$\therefore abk^2 = r/p$

now, $\sqrt{\frac{b}{a}} + \sqrt{\frac{a}{b}} = \frac{a+b}{\sqrt{ab}} = \frac{(a+b)k}{\sqrt{abk^2}} = -\frac{r/p}{\sqrt{r/p}}$

$= -\sqrt{r/p}$

31. $|y - 4| \le 6 \Rightarrow -2 \le y \le 10$

$|y + 4| < 5 \Rightarrow -9 < y < 1$

$\therefore$ Combining the two region we get.

$-2 \le y < 1$

$\therefore$ Only one element satisfies the condition i.e. = 0

Alternative solution:

Go through the options.

32. $p(x) = x^3 - ax^2 + bx + 10$; since it is divisible by $(x + 5)$

$\therefore\ p(-5) = 0$

$\Rightarrow (-5)^3 - 25a - 5b + 10 = 0$

$\Rightarrow\ 5a + b = -23$ ---- (1)

$\Rightarrow\ Q(x) = x^4 + x^3 + bx^2 - ax + 42 = 0$

$\therefore\ Q(3) = 0$

$\Rightarrow\ 81 + 27 + 9b - 3a + 42 = 0$

$\Rightarrow\ a - 3b = 50$

33. $x^2 + x + 2 = 0$

$\therefore \alpha + \beta = -1$

$\alpha\beta = 2$

$$\text{Now, } \frac{\alpha^{10} + \beta^{10}}{\alpha^{-10} + \beta^{-10}} = \frac{\alpha^{10} + \beta^{10}}{\frac{\alpha^{10} + \beta^{10}}{(\alpha\beta)^{10}}} = (\alpha\beta)^{10} = 2^{10} = 1024$$

34. Go through the options to find

For option (c) $44^2 = 1936$ and $1980 - 44 = 1936$.

35. In the correct equation;

Sum of roots = –3–2 = –5 and the product of roots = $(-1) \times (-2) = 2$

So, correct equation = $x^2 + 5x + 2 = 0$

36. $x^3 - x^2 + x - 1 > 0$

$x^2(x - 1) + (x - 1) > 0$

$(x^2 + 1)(x - 1) > 0$

As, $x^2 + 1 > 0$ for all x, so, $x > 1$.

38. $ax - b = 0$

$\therefore \quad x = \frac{b}{a}$

Since 'b' and a have only 'a' have only '1' as a common factor

$\therefore$ we can directly equate with a rational number in its empirical form.

$\therefore$ For $A; X = \frac{7}{3}$ with incorrect 'b'

$\therefore$ Correct value of 'a' = 3 or any multiple of 3.

Similarly from 2nd statement, $x = \frac{8}{5} \Rightarrow$ we get correct value of 'b' = 8 or any multiple of 8.

Since we don't get any unique value of a and b, we cannot find unique value of $\frac{b}{a}$.

39. Let the roots be 3, α, β

$\therefore 3\,\alpha\,\beta = 6 \Rightarrow \alpha\,\beta = 2$

and $3x + 3\,\beta + \alpha\,\beta = 11$

$\Rightarrow 3(\alpha + \beta) + 2 = 11$

$\therefore \alpha + \beta = 3$

$\therefore \alpha = 1, \beta = 2$

40. $(m + n)^{100} = m^{100} + {}^{100}c_1\, m^{99}.\, n + {}^{100}c_2\, m^{98}.\, n + \ldots.. + n^{100}$

$= m^{100} + n^{100} + k$

[where $k = {}^{100}C_1\, m^{99}n + {}^{100}C_2\, m^{98}.\, n + \ldots.. + {}^{100}C_{99}\, mn^{99}$]

$\therefore \quad k > 0$ for m and n belonging to natural no.

$\therefore \quad (m + n)^{100} > m^{100} + n^{100}$

Moderate

1. $x^2 + 5\,|\,x\,| + 6 = 0$

$\because$ All the terms in LHS are positive

Hence no real roots possible.

2. $S - 2 = 2\frac{1}{3} + 2^{2/3} \Rightarrow (S - 2)^2 = 2^{2/3} + 2.2^{1/3} + 2.2$

Now, required $= 2^{2/3} - 2.2^{1/3} + 2.2 - (2.2^{1/3} + 2.2^{2/3}) - 2 = 2$

5. Let the roots be α, 2α. Where $\alpha > 0$

$\therefore \alpha + 2\alpha = -m \Rightarrow m = -3\alpha$

and, $2\alpha^2 = C$

Now, since $m + c = 2 \Rightarrow 2\alpha^2 - 3\alpha = 2$

$\alpha = -1/2, 2$

$\because \alpha > 0$

$\therefore \alpha = 2 \therefore m = -3\alpha = -6$

6. In the above question the roots are α and 2α i.e. (2, 4)

$\therefore |x_1 - x_2| = |\alpha - 2\alpha| = 2$

7. Method 1–

Let p, q be the roots with same unit digit.

$\therefore \quad \frac{D}{a^2} = \frac{b^2 - 4ac}{a^2} = \left(\frac{b}{a}\right)^2 - \frac{4c}{a} = (p + q)^2 - 4pq$

$= (p - q)^2$

Now, given that 'p' and 'q' have same unit digit still $(p - q)^2$ is not divisible by 100.

Which means p and q have to be of opposite signs.

Now $\frac{b}{2a}$ is nothing but $= \frac{-1}{2}(p + a)$

Now, since p and q are of opposite signs with same unit digit.

Thus unit digit of $p + q = 0$

Thus, unit digits of $\frac{1}{2}(p + q)$ will be either '0' or '5'.

Out of the given options, 2nd option is the answer.

Method 2 -

Let the roots be –21 and 11

Equation is $X^2 + 10X - 231 = 0$

$(X + 21)(X - 11) = 0$

$\frac{b^2 - 4ac}{a^2} = \frac{100 + 924}{1} = 1024$ (not multiple of 100)

$b/2a = 10/2 = 5$

8. $(ax^2 + bx + c)\,(ax^2 - dx - c) = 0$

$\therefore$ either $ax^2 + bx + c = 0$ or, $ax^2 - dx + c = 0$ or both.

$\therefore$ roots of $ax^2 + bx + c = 0$ will be real if

$b^2 - 4ac > 0$

Similarly for $ax^2 - dx - c$ roots will be real if

$d^2 + 4ac > 0$

Now, at least one of the two conditions will hold true since either $4ac$ will be greater than zero or less than zero or equal to zero.

$\therefore$ At least 2 real zeroes will be there.

9. $1 + |e^x - 1| = e^x(e^x - 2)$

When $x \geq 0$

$1 + e^x - 1 = e^x(e^x - 2) \Rightarrow e^x - e^x(e^x - 2) = 0$

$\Rightarrow e^x(1 - e^x + 2) = 0$

$\Rightarrow e^x = 0\ e^x = 3$

for $x \geq 0$ $e^x = 0$ is not possible

$\therefore e^x = 3$ is one root

When $x < 0$

$1 - e^x + 1 = e^x(e^x - 2)$

$\Rightarrow 2 - e^x + (2 - e^x)e^x = 0$

$(2 - e^x)(1 + e^x) = 0 \Rightarrow e^x = 2 \qquad e^x = -1$

for $x < 0$ $\quad e^x = 2$ is not possible and e^x can never be equal to -1

$\therefore$ no. of roots = 1

10. $(x+y) + \left(\dfrac{x}{y}\right) = \dfrac{1}{2}$ and $(x+y)\dfrac{x}{y} = \dfrac{-1}{2}$

Solving these two equations the values of

$(x+y)$ and $\left(\dfrac{x}{y}\right)$ will be $(1, -1/2)$

When $x + y = 1$ and $\dfrac{x}{y} = -1/2$

$(x, y) = (2, -1)$

When $x + y = -1/2$ and $\dfrac{x}{y} = 1$

$(x, y) = \left(\dfrac{-1}{4}, -\dfrac{1}{4}\right).$

$\therefore$ No. of possible pairs = 2.

11. $\sqrt{x+1} - \sqrt{x-1} = \sqrt{4x-1}$

$\Rightarrow (x + 1) + (x - 1) - 2\sqrt{x^2 - 1} = 4x - 1$

$\Rightarrow -2\sqrt{x^2-1} = 2x - 1$

$\Rightarrow 4(x^2 - 1) = 4x^2 + 1 - 4x$

$\Rightarrow x = \dfrac{5}{4}$ which when put in the main equation does not satisfy it.

Hence no solution is possible.

12. $\dfrac{6-x}{x^2-4} = \dfrac{2(x+2)+x}{x+2}$ with $x + 2 \neq 0$ and $x - 2 \neq 0$

$\Rightarrow \dfrac{6-x}{x-2} = 3x + 4$

$\Rightarrow 6 - x = (3x+4)(x-2)$

$\Rightarrow 3x^2 - x - 14 = 0$

$\Rightarrow x = -2$ or $x = 7/3$

But $(x + 2) \neq 0$, hence only one root is possible.

13. $x^2 - 3x - 4 < 0$

$(x - 4)(x + 1) < 0$

$\therefore$ Possible values = (0, 1, 2, 3)

Hence there are 4 values.

14. $x^2 - 6x + 5 \leq 0 \Rightarrow (x - 5)(x - 1) \leq 0 \Rightarrow x \in (1, 5)$

$x(x - 2) > 0 \Rightarrow (-\infty, 0) \cup (2, \infty)$

Combining these two regions will give following integers –3, 4, 5.

Hence there are three values.

16. $\text{Log}_4(x - 1) = \log_2(x - 3)$

$\Rightarrow \dfrac{1}{2}\log_2(x-1) = \log_2(x-3)$

$\Rightarrow (x - 1) = (x - 3)^2$

$\therefore x = 5, 2$

Now $x = 2$ is not possible as $\log(x - 3) = \log(-1)$ is not possible

17. $\alpha + \beta = \dfrac{3a}{a-1}, \alpha\beta = \dfrac{1}{a-1}$ where α and β are the roots.

$\because a > 1$

$\therefore \alpha + \beta > 0$ and $\alpha\beta > 0$

$\therefore \alpha > 0$ and $\beta > 0$. Hence both the roots are positive.

18. $x^2 - 2ax + a^2 + a - 3 = 0$

$\because$ Roots are less than 3.

$\therefore$ Their sum will be less than 6.

$\therefore 2a < 6 \Rightarrow a < 3$. Out of the given options only option 1 satisfies it.

20. $ax^2 + bx + c = 0$ and $cx^2 + bx + a = 0$ have roots which are reciprocal to each other.
i.e. if the roots of $ax^2 + bx + c = 0$ are α, β then the roots of

$cx^2 + bx + c = 0$ are $\dfrac{1}{\alpha}, \dfrac{1}{\beta}$.

Now it is given that $\alpha < 0$ and $\alpha = \dfrac{1}{\alpha}$

$\therefore \ a = -1$

$\therefore$ The common root is –1.

$\therefore$ putting $x = -1$ in any of the equations we get, $a - b + c = 0$.

21. $(x^2 - (K - 2) + K^2)(x^2 + Kx + (2K - 1))$

for the expression to be a perfect square there can two possible ways:

i) when both the quadratic expression are perfect square for a particular value

   For that to happen, $K - 2 = 2K \Rightarrow K = -2$

   Now for $K = -2$ in the 2nd expression we get, $x^2 - 2x - 5$

   which is not a perfect square.

ii) The other is when both the quadratic equations are same.

$\therefore -(K-2) = K \Rightarrow K = 1$
$+K^2 = 2K - 1 \Rightarrow K = 1$
$\therefore$ at $K = 1$ the expression is a perfect square.

22. Let the common root be x
$\therefore \alpha^2 + \alpha x + b = 0$
and $\alpha^2 + b\alpha + a = 0$
$\Rightarrow (a-b)\alpha - (a-b) = 0$
$\Rightarrow (a-b)(\alpha - 1) = 0 \Rightarrow \alpha = 1 \; \{\because a \neq b\}$
$\therefore$ putting $\alpha = 1$ is any of the equations.
$a + b + 1 = 0$

23. $P(x) = x^4 + 2x^3 + mx^2 + nx + 3$
Now, $P(2) = 0$
$\Rightarrow 16 + 16 + 4m + 2n + 3 = 0$ ……………… (1)
$\Rightarrow 4m + 2n + 35 = 0$
and, $P(4) = 0$
$\Rightarrow 256 + 128 + 16m + 4n + 3 = 0$
$\Rightarrow 16m + 4n + 387 = 0$ ………………… (2)
Multiplying 5 in equation (1) and then subtracting from eq. (2)
$4m + 6n - 212 = 0$
$\therefore 2m + 3n = 106$

24. Putting $p = 2$, $q = -2$, $r = 1$; we can get the value of
$\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$

25. $x^2 + (3K - 36)x + K^2 - 24K + 144 = 0$
Since, roots are reciprocal, product of roots = 1
$\Rightarrow K^2 - 24K + 144 = 1$
$\therefore K = 11, 13.$

27. $Z + \frac{1}{Z} = 1$
$\Rightarrow Z^2 - Z + 1 = 0$
$\Rightarrow (Z+1)(Z^2 - Z + 1) = 0 \; \{\text{if } z \neq -1\}$
$\Rightarrow (Z^3 + 1) = 0$
$\therefore Z^3 = -1$ and $Z \neq -1$
Now, $Z^{64} + \frac{1}{Z^{64}} = Z^{63}.Z + \frac{1}{Z^{63}.Z}$
$\Rightarrow (Z^3)^{21}.Z + \frac{1}{(Z^3)^{21}.Z} = -Z - \frac{1}{Z} = -1$

29. $X^2 + bx + c = 0$
for real roots $b^2 - 4c \geq 0$
$\Rightarrow b^2 \geq 4c$
Now for each values of 'b' from the given set checking the no. of values of 'c' possible in the same set.
$b = 1$ no. of values of $c = 0$
$b = 2$ no. of values of $c = 1(1)$
$b = 3$ no. of values of $c = 2(1, 2)$
$b = 4$ no. of values of $c = 4(1, 2, 3, 4)$
$b = 5$ no. of values of $c = 6(1, 2, 3, 4, 5, 6)$
$b = 6$ no. of values of $c = 6\ (1, 2, 3, 4, 5, 6)$
Therefore, total possible sets = $1 + 2 + 4 + 6 + 6 = 19$

30. Let $a, b, c, d, = -1, 0, 1, 2$ (respectively)
$\therefore$ Given expression comes out to be $(x-1)(x+1) + 2(x-2)x = 0$
$\Rightarrow 3x^2 - 4x - 1 = 0$; $\therefore D = 4^2 + 12 = 28$
Hence it will have Real and distinct roots.

31. $2\left[a^{1/3} + \frac{1}{a^{1/3}}\right] = 5$
$\Rightarrow \; 2a^{2/3} - 5a^{1/3} + 2 = 0$
$\Rightarrow \; (a^{1/3} - 2)(2a^{1/3} - 1) = 0$
$\therefore \; a^{1/3} = 2, a^{1/3} = 1/2$
$\Rightarrow \; a = 8, a = 1/8$

32. Since each of the roots are changing symmetrically from
$\alpha \rightarrow 3\alpha + 2$
Thus, to get the required equation, just replace x with $\frac{x-2}{3}$ in the given equation.
Thus we get
$9\left(\frac{x-2}{3}\right)^3 - 7\left(\frac{x-2}{3}\right) + 6 = 0$
$\Rightarrow x^3 - 6x^2 + 5x + 24 = 0$

33. Since the roots are equal, so, $D = 0$
$\Rightarrow (r-p)^2 - 4(q-r)(p-q) = 0$
$\Rightarrow [(r-q) + (q-p)]^2 - 4(r-q)(q-p) = 0$
$\Rightarrow [(r-q) - (q-p)]^2 = 0$
$\Rightarrow r - q = q - p$
$\Rightarrow p + r = 2q$ i.e. p, q, r are in A.P.

34. $(x-b)(x-c) + (x-c) + (x-a)(x-b) = 0$
$\Rightarrow 3x^2 - 2(a+b+c)x + ab + bc + ca = 0$
$\therefore X = 4(a+b+c)^2 - 4(ab+bc+ca) \times 3$
$\Rightarrow D = 4[a^2 + b^2 + c^2 - (ab + bc + ca)]$
Now we know $(a-b)^2 > 0$
$\Rightarrow a^2 + b^2 > 2ab$
Similarly, $b^2 + c^2 > 2bc$
and, $c^2 + a^2 > 2ca$
$\therefore a^2 + b^2 + c^2 > ab + bc + ca$
$\Rightarrow a^2 + b^2 + c^2 - (ab + bc + ca) > 0$
$\Rightarrow 4(a^2 + b^2 + c^2 - (ab + bc + ca)) > 0$

$\Rightarrow D > 0$

$\therefore$ The roots are Real.

35. Applying $D = 0$ in the given equation

$b^2(c-a)^2 = 4ac(a-b)(b-c)$

$$\Rightarrow \frac{b^2(c-a)^2}{ac} = 4(a-b)(b-c)$$

$$\Rightarrow b^2ac\left(\frac{c-a}{ac}\right)^2 = 4\left(\frac{a-b}{ab}\right)\left(\frac{b-c}{bc}\right).ab^2c$$

$$\Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)^2 = 4\left(\frac{1}{b}-\frac{1}{a}\right)\left(\frac{1}{c}-\frac{1}{b}\right)$$

$$\Rightarrow \left[\left(\frac{1}{b}-\frac{1}{b}\right)+\left(\frac{1}{b}+\frac{1}{c}\right)\right]^2 - 4\left(\frac{1}{a}-\frac{1}{b}\right)\left(\frac{1}{b}-\frac{1}{c}\right) = 0$$

$$\Rightarrow \left[\left(\frac{1}{a}-\frac{1}{b}\right)-\left(\frac{1}{b}-\frac{1}{c}\right)\right]^2 = 0$$

$$\Rightarrow \frac{1}{a}-\frac{1}{b} = \frac{1}{b}-\frac{1}{c}$$

$\Rightarrow \frac{2}{b} = \frac{1}{c}+\frac{1}{a}$. So, a, b, c are in H.P.

36. We know $(x+y)$ is a factor of $x^p + y^p$ if p is odd. Thus here $p = 3^n$ which is odd integer when 'n' is an integer ≥ 0

37. For both then roots : (α, β) to be positive

$\alpha + \beta > 0$ and $\alpha\beta > 0$

$\Rightarrow \frac{-b}{a} > 0$ and $\frac{c}{a} > 0$

i.e. b and a are of opposite sign and c and a are of same sign.

38. $ax^2 + bx + c = 0$

for both roots to be zero, sum of roots $= \frac{-b}{a} = 0$

$\therefore b = 0$

and product of roots $= \frac{c}{a} = 0 \ \therefore c = 0$

39. $ax^2 + bx + c = 0$

Now in first case when root is 1/3 and 1

The equation is $(3x-1)(x-1) = 0$

$\therefore$ correct value of $a = x\ 3 \times 1 = 3$

Similarly in second case the equation is $(x-6)(x-5) = 0$

$\therefore$ correct value of $b = -(5+6) = -11$

and in third case the equation is $(x+3)(x+2) = 0$

$\therefore$ Correct equation is $3x^2 + 11x + 6 = 0$

40. For the equations to have same pair of roots

$$\frac{2p-1}{q+1} = \frac{2p+1}{4q+1} = \frac{c}{3c}$$

$\therefore 3(2p-1) = q+1 \Rightarrow 6p - q = 4$

and, $3(2p+1) = 4q+1 \Rightarrow 6p - 4q = -1$

Solving two equation $q = 2$ and $p = 1$

$\therefore (p+q) = 3$

Advanced

1. HCF of α, $\beta = 2$, so we can assume $\alpha = 2x$ and $\beta = 2y$.

So, $2x \times 2y = 24 \times 2$, or, $xy = 12$

Different values of x and y possible are (12, 1), (4, 3). and hence different values of roots of the equation will be (24, 2), (8, 6).

Now, since we are talking about LCM and HCF of α, β, roots of the equation has to be non-negative and real.

So, 2 such quadratic equations are possible.

This is a question on Number System, and not on Quadratic Equation.

2. If roots are real and equal, then $D = 0$

$D = [\sqrt{2}(p+q)]^2 - 4(p^2+q^2) \times 1 = 2(p^2+q^2+2pq) - 4(p^2+q^2) = -2(p^2+q^2-2pq) = -2[(p-q)^2] = 0$

So, $p = q$

4. Due to symmetry, we can say that the maximum value of $xy + yz + zx$ will be at $x = y = x$

Now $x^2 + y^2 + z^2 = 1$

$\Rightarrow x = y = z = 1/\sqrt{3}$

$\therefore xy + yz + zx \leq 1$ which is present in only one option.

5. $ax^2 - bxy - ay^2 = 0$

$D = b^2y^2 + 4a^2y^2 = y^2\{b^2 + (2a)^2\} > 0$ but not a perfect square. So, factors are real and different.

6. Let the roots be α and β.

$$\therefore \alpha + \beta = \frac{1}{\alpha^2} + \frac{1}{\beta^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2} \Rightarrow \frac{-b}{a} = \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha^2\beta^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{(b/a)^2 - \frac{2c}{a}}{c^2/a^2}$$

$$\Rightarrow \frac{-b}{a} = \frac{b^2 - 2ca}{c^2} \Rightarrow b^2a + bc^2 = 2ca^2$$

$$\Rightarrow \frac{b^2}{ac} + \frac{bc}{a^2} = 2$$

7. $ax^2 + bx + c = 0$

$x^3 + 3x + 10 = 0$

For common roots $a = 1$, $b = 3$, $c = 10$ (ratio)

$a : b : c = 1 : 3 : 10$

8. Taking $(x - 2) > 0$ we get $x = 0, 3$. So, $x = 3$

Taking $(x - 2) < 0$ we get $x = 1, 4$. So, $x = 1$

Hence sum of roots = 4

Alternatively,

Assuming $|x - 2| = Z$, this equation is now $z^2 + z - 2 = 0$

Or, $(z + 2)(z - 1) = 0$, Hence $z = -2$ or 1.

Taking $z = -2$ is not possible since $z = |x - 2|$

Taking $z = 1$, or, $|x - 2| = 1$, so $x = 3$ and $x = 1$.

And these are the only real roots of this equation. So sum of real roots = 1 + 3 = 4

12. It is possible only if the given quadratic equation becomes linear equation. So, $a = 0$

13. $x + \frac{1}{x} = 1 \Rightarrow x^2 - x + 1 = 0$

Now as $x \neq -1 \Rightarrow (x + 1)(x^2 - x + 1) = 0$

$\Rightarrow x^3 + 1 = 0$

$\therefore x^3 = -1 \Rightarrow x^{4000} = (x^3)^{1333}.\, x = -x$

$$\therefore P = x^{4000} + \frac{1}{x^{4000}}$$

$$= -x - \frac{1}{x} = -1 \Rightarrow P = -1$$

Now let $n = 2$

$\therefore$ p = unit digit of 17 i.e. 7. So, $p + q = 7 - 1 = 6$

14. Let the roots of the equation $ax^2 + bx + b = 0$ be pk and qk.

$$\therefore (p + q)K = \frac{-b}{a} \text{ and } pqk^2 = \frac{b}{a}$$

$$\therefore \sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{b}{a}} = \frac{p+q}{\sqrt{pq}} + \sqrt{\frac{b}{a}}$$

$$= \frac{(p+q)k}{\sqrt{pak^2}} + \sqrt{\frac{b}{a}} = \frac{-b/a}{\sqrt{b/a}} + \sqrt{b/a} = 0$$

15. $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$

For same pair of roots $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{1}{2}$ (given)

Expression (2) $= a_2x^2 + b_2x + c_2 = 2(a_1x^2 + b_1x + c_1)$

$= 2$(expression (1))

Then the ratio of the maximum value is 1:2

16. We have to check that if the statements can be true.

Checking Statement I, if $a = c$, then obviously both the equation will have common root.

Checking Statement II, if the common root is 1, then both the equations are giving $a + b + c = 0$.

Checking Statement III, If the common root is -1, then the given condition is true.

So, all the three statements are true.

18. Solving equation we get $(1 + k)X^2 - \{a + c + (b + d)k\}x + (ac + bdk) = 0$

Here $D \geq 0$, so roots of equation will be real.

19. Any operation of rational nos. will always result in rational numbers.

20. Let the roots of the given equation be α, β

Now for roots $(\alpha - \beta)$, $(\beta - 2)$ the equation can be deduced by replacing x with $(x + 2)$

$\therefore$ The deduced equation would be

$\Rightarrow (x + 2)^2 - (p + 1)(x + 2) + p^2 + p - 8 = 0$

$\Rightarrow x^2 + (3 - p)x + p^2 - p - 6 = 0$

$\Rightarrow x^2 + (p - 3)x + (p + 2)(p - 3) = 0$

Now, $\alpha > 2$ and $\beta < 2$

$\therefore$ $(\alpha - 2) > 0$ and $(\beta - 2) < 0$

$\therefore$ $(\alpha - 2)(\beta - 2) < 0 \Rightarrow (p + 2)(p - 3) < 0$

$\therefore$ $(\alpha - 2)(\beta - 2) < 0 \Rightarrow (p + 2)(p - 3) < 0$

$\therefore$ $-2 < p < 3$.

21. Solving eqn we get $D = -5a^2 + 10a + 1$

Hence we can conclude that no value of a fulfills the equation.

22. If there is only one solution of any quadratic equation, then it means both the roots of the equation are equal or D = 0.

Alternatively, Put $x = 4$ and $k = 2$.

23. Solving eqn we get $3x^2 - 2(a + b + c)x + ab + bc + ca = 0$

For roots to be equal, $a = b = c$.

24. Let the eqn be $x^2 - x - 12 = 0$ $(x = -3, 4)$

Here $a = 1$, $b = -1$, $c = -12$

25. Putting $a = 1$ we get one common real roots for both equation.

11

CHAPTER

Sequence and Series

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- The concept of sequences and series
- Progression and their types
- Definitions and properties of progressions
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

Sequence and series is a mathematical concept that draws majorly from the basic number system and the simple concepts of arithmetic. This is the reason that makes it an important topic for this exam. On an average, 1–3 questions have been asked from this topic in the CAT almost every year in the last 12 years. Besides the CAT, this topic is important for other exams like IIFT, SNAP, XAT, MAT. One good thing about the problems from this chapter is that they can be solved simply by the application of logic or some very simple concepts of calculation.

SEQUENCE AND SERIES

Let us consider the following series:

- 1, 4, 9, 16, ...
- 2, 6, 12, 20,...

It can be observed here that each of these two series shares some or the other common property:

Series (i) is → $1^2, 2^2, 3^2, 4^2 \ldots$

Series (ii) is → $1^2 + 1, 2^2 + 2, 3^2 + 3, 4^2 + 4 \ldots$

With this, any term or in general t_n, for either of the two series can be very easily found out.

For series (i), $t_{10} = 10^2$

For series (i), $t_{10} = 10^2 + 10$.

If the terms of a sequence are written under some specific conditions, then the sequence is called a *progression*.

With respect to preparation for the CAT, we will confine ourselves only to the following standard series of progression:

1. Arithmetic Progression
2. Geometric Progression
3. Harmonic Progression

1. Arithmetic Progression

A succession of numbers is said to be in arithmetic progression (AP) if the difference between any term and the previous term is constant throughout. In other words, the difference between any of the two consecutive terms should be the same. This difference which is common between any two consecutive terms is known as common difference of this AP and is denoted by *'d'*.

For example → Series (i) 1, 2, 3, ...

Series (ii) → 2, 5, 7, 10,...

Series(iii) → $a, a + d, a + 2d, \ldots$

Common Difference(d) of series(i) = 1

Common Difference(d) of series(ii) = 3

Common Difference(d) of series(iii) = d

nth term of an arithmetic progression

First term $t_1 = a = a + (1 - 1)d$

Second term $t_2 = a + d = a + (2 - 1)d$

Third term $t_3 = (a + d) + d = a + 2d = a + (3 - 1)d$

Fourth term $t_4 = a + 3d = a + (4 - 1)d$

nth term $t_n = a + (n - 1)d$, where a is the first term, d is the common difference and n is the number of terms.

Important points

- t_n is also known as the general term of AP.
- If in any question, some particular term is given (like t_4 or t_{10}), then we should assume those terms in the form of t_n. However, if the total number of terms are given then we should assume the terms in the following way:

If three terms or any odd number of terms are involved, then we should assume these terms as $a - d, a, a + d$ and so on.

If four terms or any even number of terms are involved, then we should assume these terms as $a - 3d, a - d, a + d, a + 3d$ and so on.

Example 1 The sum of three numbers in an AP is 27 and the sum of their squares is 293. Find the numbers.

Solution Let the numbers be $a - d, a, a + d$.

Given is $(a - d + a + a + d) = 27$

So, $a = 9$

Also, $(a - d)^2 + a^2 + (a + d)^2 = 293$

$\Rightarrow d^2 = 25$

$\Rightarrow d = \pm 5$

When $d = + 5$, then the terms are 4, 9, 14.

When $d = - 5$, then the terms are 14, 9, 4.

Alternatively, this question can be worked out very easily with the help of options.

Example 2 If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

(a) 0	(b) −1
(c) 1	(d) Not unique

Solution Given

$t_1 + t_2 + \ldots + t_{11} = t_1 + t_2 + \ldots + t_{19}$ (for an AP)

$$\Rightarrow \frac{11}{2}[2a + (11-1)d] = \frac{19}{2}[2a + (19-1)d)]$$

$\Rightarrow 22a + 110d = 38a + 342d$

$\Rightarrow 16a + 232d = 0$

$\Rightarrow 2a + 29d = 0$

$$\Rightarrow \frac{30}{2}[2a + (30-1)d] = 0$$

$\Rightarrow S_{30\text{ terms}} = 0$

Properties of AP

If $a, b, c, d, \ldots$ are in AP, then

1. $a + k, b + k, c + k, d + k \ldots$ will be in AP, where k is any constant.
2. $a - k, b - k, c - k, d - k \ldots$ will be in AP, where k is any constant.

   In the above two cases, the common difference will be the same as earlier.
3. $ak, bk, ck, dk \ldots$ will be in AP, where k is any constant.

   In this case, new common difference will be k times the earlier common difference.
4. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \frac{d}{k}$ will be in AP, where $k \neq 0$.

   In this case, new common difference will be $\frac{1}{k}$ times the earlier common difference.

Example 3 If a, b, c are in AP, then $b + c, c + a, a + b$ will be in

(a) AP
(b) GP
(c) HP
(d) Cannot be determined uniquely

Solution *a, b, c* are in AP, then $a - (a + b + c)$, $b - (a + b + c)$, $c - (a + b + c)$ will be in AP.

$\Rightarrow -(b + c), -(a + c)$ and $-(a + b)$ will be in AP.

$\Rightarrow (b + c), (a + c)$ and $(a + b)$ will also be in AP.

Alternatively, let us assume *a, b, c* to be 1, 2, 3. Then $(b + c) = 5$, $(a + c) = 4$ and $(a + b) = 3$, which are obviously in AP.

Example 4 If *x, y, z* are in GP, then $1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ will be in

(a) AP
(b) GP
(c) HP
(d) Cannot be determined uniquely

Solution Let us go through the options

Checking option (a), the three will be in AP if the 2nd expression is the average of the 1st and the 3rd expressions. This can be mathematically written as

$$2/(1 + \log_{10}y) = 1/(1 + \log_{10}x) + 1/(1 + \log_{10}z)$$

$$= \frac{[1 + (1 + \log_{10} x) + 1 + (1 + \log_{10} z)]}{[(1 + \log_{10} x)(1 + \log_{10} z)]}$$

$$= [2 + \log_{10}xz]/(1 + \log_{10}x)\,(1 + \log_{10}z)$$

Obviously this will not give us the answer.

Checking option (b),

$$[1/(1 + \log_{10}y)]^2 = [1/(1 + \log_{10}x)]\,[1/(1 + \log_{10}z)]$$

$$= [1/(1 + \log_{10}(x + z) + \log_{10}xz)]$$

Again, no solution is found.

Checking option (c),

$1/(1+\log_{10}x)$, $1/(1+\log_{10}y)$ and $1/(1+\log_{10}z)$ are in HP then $1+\log_{10}x$, $1+\log_{10}y$ and $1 + \log_{10}z$ will be in AP.

So, $\log_{10}x$, $\log_{10}y$ and $\log_{10}z$ will also be in AP.

Hence, $2 \log_{10}y = \log_{10}x + \log_{10}z$

$\Rightarrow y^2 = xz$ which is given.

So, (c) is the answer.

Alternatively, we can also apply the following process:

Assume $x = 1$, $y = 10$ and $z = 100$ as *x, y, z* are in GP.

So, $1 + \log_{10}x = 1$, $1 + \log_{10}y = 2$ and $1 + \log_{10}z = 3$.

$\Rightarrow$ Thus we find that since 1, 2 and 3 are in AP, we can assume that

$1 + \log_{10}x$, $1 + \log_{10}y$ and $1 + \log_{10}z$ are in AP.

Hence, by definition of an AP we have that

$1/(1 + \log_{10}x)$, $1/(1 + \log_{10}y)$ and $1/(1 + \log_{10}z)$ are in AP.

So, option (c) is the answer.

Sum of n terms of an arithmetic progression

$S_n = \frac{n}{2}[2a + (n - 1)d]$, where n = number of terms, a = 1st term and d = common difference.

Example 5 Find the sum of AP 3, 5, 7,... 50 terms and find its sum.

Solution Here $n = 50$, $d = 2$ and $a = 3$.

Using formula $S_n = \frac{n}{2}[2a + (n - 1)d]$

$= 25 \times [2 \times 3 + (50 - 1)2)] = 25 \times 104 = 2600$

However, we can find out the sum of any AP in a better way through average also.

The last term of this series = 101, so the average

$$= \frac{3 + 101}{2} = 52$$

So, sum = average × number of numbers

$= 52 \times 50 = 2600$

Hence, $S_n = \frac{F + L}{2} \times n$, where F is the first term, L is the last term and *n* is the number of terms of that AP.

Example 6 What is the sum of all the two-digit numbers which when divided by 7 gives a remainder of 3? (CAT 2003)

Solution This series is like → 10, 17, 21,... 94.

Here $n = 13$, $d = 7$ and $a = 10$

Using the formula for the sum

$S_n = \frac{n}{2}[2a + (n - 1)d]$, sum = 676

Alternatively, using the average method,
average = (1st number + last number)/2

$$\text{Average} = \frac{10 + 94}{2} = 52$$

So, the sum = average × number of numbers
$= 52 \times 13 = 676$

Example 7 Find the value of the expression

$1 - 4 + 2 - 5 + 3 - 6 + \ldots$ to 100 terms.

(a) −250 (b) −500
(c) −450 (d) −300

Solution We can write the given expression ($1 - 4 + 2 - 5 + 3 - 6 + \ldots$ to 100 terms) as:

($1 + 2 + 3 + \ldots$ to 50 terms) − ($4 + 5 + 6 + \ldots$ to 50 terms)

Both of these are AP's with different values of '*a*' and '*d*'.

$a = 1$, $n = 50$ and $d = 1$ for 1st series and $a = 4$, $n = 50$ and $d = 1$ respectively.

Using the formula for the sum of AP

$= \frac{n}{2}[2a + (n-1)d]$, we get

$$\frac{50}{2}[2.1 + (50 - 1).1] - \frac{50}{2}[2.4 + (50 - 1).1]$$

$= 25(2 + 49) - 25(8 + 49)$

$= 25(51 - 57) = -150$

Alternatively, logically this question can be done a bit faster by assuming (1 – 4), (2 – 5), etc as one unit.

$1 - 4 = 2 - 5 = \ldots = -3.$

Thus the above series is equivalent to a series of fifty (–3)'s added to each other.

So, $(1 - 4) + (2 - 5) + (3 - 6) + \ldots$ 50 terms $= -3 \times 50 = -150$

2. Geometric Progression

A succession of numbers is said to be in a geometric progression if the ratio of any term and the previous term is constant. This constant ratio which is common to any of the two terms is known as the common ratio and is denoted by 'r'.

Example i. 1, 2, 4, 8,.,.,.,.,.,.,.,.,.,.,
ii. 20, 10, 5,.,.,.,.,.,.,
iii. $a, ar, ar^2, \ldots\ldots$

Common ratio of series (i) is 2.
Common ratio of series (ii) is 0.5.
Common ratio of series (iii) is r.

nth term of a geometric progression

First term $t_1 = a = ar^{1-1}$
Second term $t_2 = ar = ar^{2-1}$
Third term $t_3 = ar^2 = ar^{3-1}$
Fourth term $t_4 = ar^3 = ar^{4-1}$
nth term $t_n = ar^{n-1}$

where a is the first term, r is the common ratio and n is the number of terms.

Important points

- t_n is also known as the general term of GP.
- In any question, if some particular term is given like t_4 or t_{10}, then we should assume those terms in the form of t_n. However, if the total number of terms are given then we should assume the terms in the following way:

If three terms or any odd number of terms are involved, then we should assume these terms as $\frac{a}{r}$, a, ar and so on.

Example 8 The seventh term of a GP is 8 times the fourth term and the 5th term of the same GP is 48. Find the 6th term of this GP.

Solution Given $t_7 = 8 \times t_4$,

Or, $ar^6 = 8 \times ar^3$

$\Rightarrow r^3 = 8$, or, $r = 2$

Now, $ar^4 = 48$, So, $a = 3$

So, 6th term $= ar^5 = 3 \times 2^5 = 96$

Properties of GP

If $a, b, c, d, \ldots$ are in GP, then

1. ak, bk, ck, dk… will be in GP, where k is any non-zero constant.
2. $\frac{a}{k}, \frac{b}{k}, \frac{c}{k}, \frac{d}{k}$ will be in GP, where k is any non-zero constant.

   In the above two cases, the common ratio will be the same as the earlier.
3. If a GP of any even number of terms is given, then its common ratio will be the same as the ratio of the sum of all the even terms and the sum of all the odd terms.

Sum of n terms of a geometric progression

$$S_n = \frac{a(1-r^n)}{1-r} \text{ when } r \neq 1$$

$S_n = na$ when $r = 1$

where n = number of terms, a = 1st term and d = common difference.

Sum of infinite geometric progression

So far, we have done the summation of n terms of a GP. Now there is also a need of a separate expression for the sum of infinite GP. In case of AP, since either of the terms are always decreasing and going till $-\infty$ or are always increasing and going till $+\infty$. So, the summation of the infinite terms in AP will be either $-\infty$ or $+\infty$. However, the case is not the same in GP.

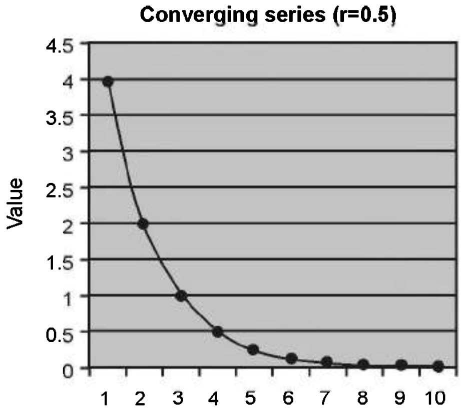
The need of a formula for infinite GP can be seen with the following example:

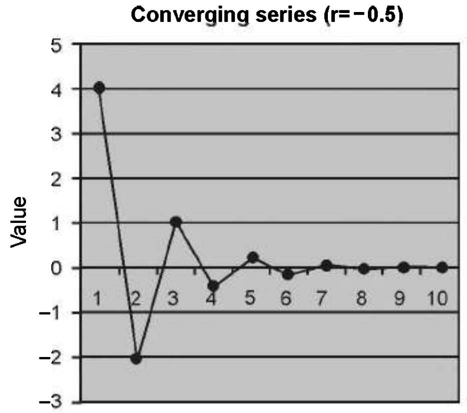
Find the sum of the series → $2 + 1 + \frac{1}{2} + \frac{1}{4} + \ldots$ till infinite terms.

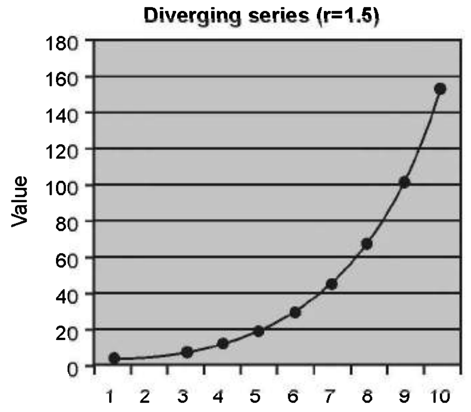
In the above-written expression, the number of terms is not given, so we cannot find out the sum using the formula for finding the sum of n terms of a GP.

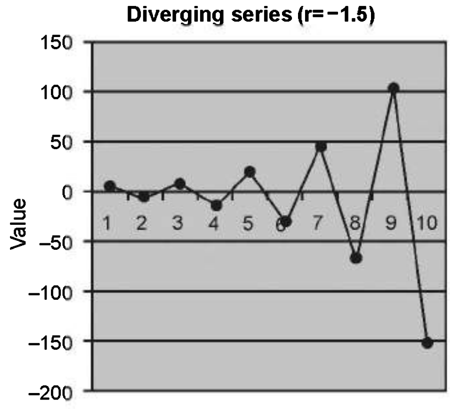
This can be further seen with the help of the following graphs:

If $\boldsymbol{r}$ is outside the range of -1 to 1, the terms of the series get bigger and bigger (even if they change the sign), and the series ***diverges***. If $\boldsymbol{r}$ is within the range of -1 to 1, the terms get smaller and smaller (closer to 0) and the series ***converges***.


Converging series (r=0.5)


Converging series (r=−0.5)


Diverging series (r=1.5)


Diverging series (r=−1.5)

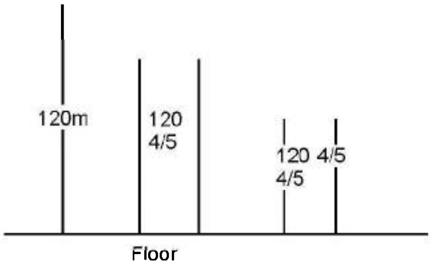
Sum of Infinite GP = $\frac{a}{1-r}$, where $-1 < r < 1$.

Example 9 What is the sum of the following series:
$1 + 2 + 4 + \dots$ till infinity.

Solution It is very obvious that the sum is going to be $+\infty$. This formula is applicable only for $-1 < r < 1$.

Example 10 After striking the floor, a ball rebounds to 4/5th of the height from which it has fallen. What is the total distance that it travels before coming to rest if it is gently dropped from a height of 120 m?

Solution The distance covered before the first rebound = 120 m



And then the ball bounces back to a height of $120 \times \frac{4}{5}$ and then falls from the same height. Next time the ball will go up by $120 \times \frac{4}{5} \times \frac{4}{5}$ m and then it will fall from the same height.

So, the total distance covered

$$= 120 + 2 \times 120 \times \frac{4}{5} + 2 \times 120 \times \frac{4}{5} \times \frac{4}{5} + \dots \infty.$$

$$= 120 + 2 \times 120 \left(\frac{4}{5} + \frac{4}{5} \times \frac{4}{5} + \dots \infty \right)$$

$$= 120 + 240 \left[\frac{\frac{4}{5}}{1 - \frac{4}{5}} \right] = 1080 \text{ m}$$

Alternatively, if the ball rebounds to $\frac{a}{b}$ th of the original height H, then the total distance covered = $H \times \frac{a+b}{a-b}$.

In the above example, the total distance covered $= 120 \times \frac{4+5}{5-4} = 1080$ m

Example 11 On 1 January 2004, two new societies S_1 and S_2 are formed, each of n numbers. On the first day of each subsequent month, S_1 adds b members while S_2 multiples its current numbers by a constant factor r. Both the societies have the same number of members on 2 July 2004. If $b = 10.5n$, what is the value of r? **(CAT 2004)**

(a) 2.0 (b) 1.9 (c) 1.8 (d) 1.7

Solution There will be an increase of 6 times. The number of members in S_1 will be in an AP.

On 2 July 2004, S_1 will have $n + 6b$ members

$= n + 6 \times 10.5\, n, = 64n$

The number of members in S_2 will be in a GP.

On 2 July 2004, the number of members in $S_2 = nr^6$

They are equal, hence $64\, n = nr^6$

$\Rightarrow 64 = r^6 \Rightarrow r = 2$

3. Harmonic Progression

Unequal numbers $a, b, c, \dots$ are said to be in a harmonic progression HP if reciprocals of these terms i.e., $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}, \dots$ are in an AP. It is noteworthy, that no term of a HP can be equal to zero.

nth term of a harmonic progression

nth term of HP = 1/(nth term of the corresponding AP)

Example 12 If a, b and c are in HP, then $\frac{a}{b+c}, \frac{b}{c+a}, \frac{c}{a+b}$ are in

(a) AP
(b) GP
(c) HP
(d) Cannot be determined uniquely

Solution a, b, c are in HP, so, $\frac{1}{a}, \frac{1}{b}$ and $\frac{1}{c}$ will be in AP.

Or, $\frac{a+b+c}{a}, \frac{a+b+c}{b}$ and $\frac{a+b+c}{c}$ will be in AP.

Or, $1+\frac{b+c}{a}, 1\frac{a+c}{b}$ and $1+\frac{a+b}{c}$ will be in AP.

Hence $\frac{b+c}{a}, \frac{a+c}{b}$ and $\frac{a+b}{c}$ are in AP.

So, $\frac{a}{b+c}, \frac{b}{c+a}$ and $\frac{c}{a+b}$ will be in HP.

Alternatively, these kinds of problems can also be done by assuming values.

Let us take 1, ½ , 1/3 (which are in HP)

$$\frac{a}{b+c} = \frac{6}{5}, \quad \frac{b}{a+c} = \frac{3}{8}, \quad \frac{c}{a+b} = \frac{2}{9}$$

Now, when we check these values for AP, GP and HP, we find that $\frac{8}{3}$ is the AM of $\frac{5}{6}$ and $\frac{9}{2}$.

So, obviously the given terms are in HP.

Properties of HP

If a, b, c and d are in HP, then

1. $a + d > b + c$
2. $ad > bc$

Sum of n terms of a harmonic progression

There is no standard formula for finding the sum of n terms of a HP.

ARITHMETICO GEOMETRIC SERIES

A series is said to be in arithmetico geometric series if each of its term is the product of the corresponding terms of an AP and a GP.

For example, $1, 2x, 3x^2, 4x^3, \dots$

In the above series, the first part of this series is in an AP (1, 2, 3, 4, …) and the 2nd part is ($x^0, x^1, x^2, x^3, \dots$.) in a GP.

Sum of *n* terms of any arithmetico geometric series (AGS)

The sum of n terms of any AGS a, $(a + d)$r, $(a + 2d)$r^2,… is given by

$$S_n = \frac{a}{1-r} + dr\frac{(1-r^{n-1})}{1-r} - \frac{[a+(n-1)d]}{1-r} \cdot r^n, \text{ if } r \neq 1.$$

$$S_n = \frac{n}{2}[2a + (n-1)d], \text{ if } r = 1$$

Sum of infinite terms of any arithmetico geometric series (AGS)

$$S_\infty = \frac{a}{1-r} + \frac{d \times r}{(1-r)^2}, |r| < 1$$

However, I would suggest students to desist from using these formulae. They should use the standard process to find out the sum of any AGS which is given below:

Let N be the sum of the arithmetico geometric series. Then each term of the series is multiplied by r (the common ratio of GP) and is written by shifting each term one step rightward, and then by subtracting rN from N to get (1–r) N. Thus N is finally obtained.

Example 13 What is the sum of the following series till infinity: $1 + 2x + 3x^2 + 4x^3 + \dots, |x| < 1$

Solution Assume $S = 1 + 2x + 3x^2 + 4x^3 +$ … (1)

Multiplying S by x, $x.\ S = x + 2x^2 + 3x^3 + 4x^4 +$ … (2)

Subtracting (2) from (1)

$$S - xS = 1 + (x + x^2 + x^3 + \ldots \infty)$$

$$S(1 - x) = 1 + (x + x^2 + x^3 + \ldots \infty) = 1 + \frac{x}{1-x}$$

$$S = \frac{1}{1-x} + \frac{x}{(1-x)^2} = \frac{1}{(1-x)^2}$$

MEANS

Arithmetic Mean

It two numbers a and b are in AP then their arithmetic mean (AM) = $\frac{a+b}{2}$

In general, if $a, b, c, \ldots n$ terms are in AP, then their AM = $\frac{a+b+c+\ldots n \text{ terms}}{n}$

Its imperative to mention here is that, in simple terms, AM of n terms is nothing but the average of n terms.

Hence, the sum of n AMs between P and Q

$$= \left(\frac{P+Q}{2}\right)n$$

Example 14 If a, b, c, d, e and f are the AMs between 2 and 12, then find the value of $a + b + c + d + e + f$?

Solution The Sum of n AMs between P and Q

$$= \left(\frac{P+Q}{2}\right)n$$

Hence the sum = $\left(\frac{2+12}{2}\right)6 = 42$

Inserting a given number of arithmetic means between two given quantities

Let p and q be the given quantities and n be the number of means which is to be inserted. After inserting n means, the total number of terms including the extremes will be equal to $n + 2$. Now we have to find a series of $n + 2$ terms in AP, of which p is the first, and q is the last term.

Let d be the common difference;

then q = the $(n + 2)$th term $= p + (n + 1)\, d$;

Hence, $d = \frac{(q-p)}{(n+1)}$

And the inserted means are:

$$p + \frac{(q-p)}{(n+1)}, p + 2\,\frac{(q-p)}{(n+1)}, \ldots, p + n\,\frac{(q-p)}{(n+1)}.$$

Geometric Mean

If two numbers a and b are in GP, then their geometric mean (GM) = $\sqrt{ab}$.

In general, if $a, b, c, \ldots n$ terms are in GP, then their GM = $\sqrt[n]{a \times b \times c \times \ldots n \text{ terms}}$.

So, if three terms a, b and c are in GP, then their GM = $b = \sqrt[3]{a \times b \times c}$.

Inserting a given number of geometric means between two given quantities

Let p and q be the given quantities and n be the required number of geometric means to be inserted between p and q. In all, there will be $n + 2$ terms, so we have to find a series of $n + 2$ terms in a GP of which p is the first term and q is the last term.

Let us assume that r is the common ratio.

Then q is the $(n + 2)$th term.

$q = pr^{n+1}$;

So, $r^{n+1} = \frac{q}{p}$; $r = \left(\frac{q}{p}\right)^{\frac{1}{n+1}}$

Hence, the required means are $p\left(\frac{q}{p}\right)^{\frac{1}{n+1}}, p\left(\frac{q}{p}\right)^{2\frac{1}{n+1}}, \ldots, p\left(\frac{q}{p}\right)^{n\frac{1}{n+1}}$.

The above written means are nothing but $pr, pr^2, \ldots, pr^n$, where $r = \left(\frac{q}{p}\right)^{\frac{1}{n+1}}$

Harmonic Mean

If two numbers a and b are in HP then their harmonic mean (HM) = $\frac{2ab}{a+b}$

The process of inserting n HMs between two given numbers is quite similar to the process of inserting AM's between two given numbers.

Relationship among AM, GM and HM

Now we know that for any two given numbers a and b,

$$AM = \frac{a+b}{2}$$

$$GM = \sqrt{ab}$$

$$HM = \frac{2ab}{a+b}$$

1. AM, GM and HM will be in a GP.
   So, GM is the geometric mean of this series.
   And, GM = $\sqrt{AM \times HM}$ [True only for two terms]
2. AM $\geq$ GM $\geq$ HM [Always true for any no. of terms]
3. The equation having a and b as its roots is $x^2 - 2Ax + G^2 = 0$.

Sum of *n* terms of some special series

In this part of progression, we shall see the sum of some other special sequences.

1. The sum of the 1st n natural numbers:

$$S_n = \frac{n(n+1)}{2}$$

Hence $\sum_{S=1}^{n} S = 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$

2. The sum of the squares of the 1st n natural numbers:

$$S_n = \frac{n(n+1)(2n+1)}{6}$$

Hence $\sum_{S=1}^{n} S^2 = 1^2 + 2^2 + 3^2 + \dots + n^2$

$$= \frac{n(n+1)(2n+1)}{6}$$

3. The sum of the cubes of the 1st n natural numbers:

S_n = (Sum of the 1st n natural numbers)^2

$$= \left(\frac{n(n+1)}{2}\right)^2$$

It can be seen that $(1 + 2 + 3)^2 = 1^3 + 2^3 + 3^3$

Hence, $\sum_{S=1}^{n} S^3 = 1^3 + 2^3 + 3^3 + \dots + n^3$

$$= \left(\frac{n(n+1)}{2}\right)^2$$

4. $2^2 + 4^2 + 6^2 + \dots + n$ terms

$$= 1/4\ \frac{n(n+1)(2n+1)}{6}$$

Worked-out Problems

Example 15 An investor starts with \$ 500 in an investment account, and each month it earns a constant interest of \$ 32. After how many months will the sum exceed \$ 700 in the account?

(a) 2 (b) 3 (c) 6 (d) 8

Solution Here, we know that $a + (n - 1)\,d = 700$ where $a = 500$ and $d = 32$. The reason for not using the sum formula is that the amount in the account, after each successive month, represents each term of the series and not the added up terms.

The equation becomes $500 + (n - 1) \times 32 = 700$. This solution gives $n = 7.25$, which is rounded up to 8. After eight months the account will hold in excess of \$ 700.

So, option (d) is the answer.

Example 16 The sum of an AP, consisting of 12 terms, is 354. The ratio of the sum of the odd terms to the sum of the even terms is 27 : 32. What is the common difference of this AP?

(a) 2 (b) 3 (c) 4 (d) 5

Solution Let the common difference be 'd' and the terms are $T_1, T_2, T_3 \dots$

So, $T_2 = T_1 + d$

And $T_4 = T_3 + d$ and so on.

So, $T_2 + T_4 + T_6 + T_8 + T_{10} + T_{12} = T_1 + T_3 + T_5 + T_7 + T_9 + T_{11} + 6d$ ...(1)

Sum of the even terms : Sum of the odd terms = 32 : 27 and their sum = 354

So, the sum of the even terms = 192 and of the odd terms = 162, Using (1), d = 5

Example 17 Find the sum of n terms of the series 11 + 103 + 1005 + ...

(a) $10/9(10^n - 1) - 1$ (b) $100/99(10^n - 1) + n^2$

(c) $10/9(10^n - 1) + n^2$ (d) None of these

Solution Ideally, in these kinds of problems, instead of going by the mathematical process of solving, we should use options.

Checking option (1), Put $n = 1$.

$10/9\,(10^n - 1) - 1 = 9$, so it is not correct.

Checking option (2), Put $n = 1$.

$100/99\,(10^n - 1) + n^2$ is not equal to 11, so this is also not correct.

Checking option (c), Put $n = 1$.

$10/9(10^n - 1) + n^2 = 11$. But just because this option satisfies $n = 1$, it should not be assumed to be correct. Let us check it for $n = 2$.

Option (c) gives us 104. So, this is the answer.

Normally, in these cases, checking the options till $n = 2$ guarantees the answer, but sometimes we need to check it till $n = 3$.

PRACTICE EXERCISES

■■■

WARM UP

Q.1. Find the sum of the first 15 terms of the series whose nth term is $(4n + 1)$.

(a) 485 (b) 495

(c) 505 (d) 630

Q.2. What is the sum of the first 7 terms of the series 1/3, 1/2, 3/4, ...?

(a) 2059/164 (b) 2050/192

(c) 2059/164 (d) None of these

Q.3. In a GP, the first term is 7 and nth term is 448, and the sum of the n terms is 889. What is the common ratio of this GP?

(a) 2 (b) 4 (c) 1 (d) 3

Q.4. The sum of 15 terms of an AP is 600, and the common difference is 5. Find the first term.

(a) 4 (b) 5

(c) 3 (d) None of these

Q.5. A man arranges to pay off a debt of Rs 3,600 by 40 annual installments which form an AP. When 30 of the installments were repaid he died leaving a third of the debt unpaid. What is the value of the first installment?

(a) 50 (b) 51 (c) 52 (d) 53

Q.6. An AP has 23 terms, the sum of the middle three terms is 144, the sum of the last three terms is 264. What is the 16th term?

(a) 102 (b) 64

(c) 82 (d) None of these

Q.7. Find A, B and C between 2 and 18 such that (i) their sum is 25, (ii) 2, A and B are consecutive terms of an AP and (iii) B, C and 18 are the consecutive terms of a GP.

(a) 5, 8, 14 (b) 4, 8, 12

(c) 5, 8, 12 (d) None of these

Q.8. If the sum of an AP is the same for p terms as for the q terms, find the sum for $(p + q)$ terms.

(a) 2 (b) 0

(c) 4 (d) None of these

Q.9. The sum of the three numbers in GP is 70. If the two extreme terms are multiplied by 4, and the middle term by 5, the resultants are in an AP. Find the numbers.

(a) 10, 25, 35 (b) 10, 20, 40

(c) 15, 30, 35 (d) 12, 24, 34

Q.10. Find the first term of the infinite GP whose first two terms add up to 5, and each term is three times the sum of the all terms that follow it.

(a) 1 (b) 2

(c) 3 (d) 4

Q.11. If $x = 1 + a + a^2 + a^3 + \ldots$ to ∞ $(|a| < 1)$,
$y = 1 + b + b^2 + b^3 + \ldots$ to ∞ $(|b| < 1)$,
Then find $1 + ab + a^2b^2 + a^3b^3 + \ldots$ to ∞

(a) $x + y/x + y - 1$ (b) $xy/x + y$

(c) $xy/x + y - 1$ (d) None of these

Q.12. If the $(n + 1)$th term of a harmonic progression is twice the $(3n + 1)$th term, find the ratio of the first term to the $(n + 1)$th term.

(a) 1 (b) 2 (c) 3 (d) 4

Q.13. If the sum of the reciprocals of the first seven terms of a harmonic progression is 70, find the 4th term of the HP.

(a) 2/15 (b) 1/10 (c) 3/7 (d) 5/12

Q.14. If the mth term of a HP is n and the nth term is m, what is the value of the $(m + n)$th term?

(a) $m/(m + n)$ (b) $mn/m + n$

(c) $n/m + n$ (d) None of these

Q.15. If $(x^{n+1} + y^{n+1})/(x^n + y^n)$ is the harmonic mean of x and y, find the value of n.

(a) 1 (b) -1 (c) 2 (d) 4

Q.16. A series is simultaneously in an AP and a HP. Which of the following is true?

(a) The set of common ratios of all the possible GPs can have only one element.

(b) The set of common ratios of all the possible GPs can have more than one element but there will be a finite value.

(c) The set of common ratios of all the possible GPs can have infinite elements.

(d) None of these

Q.17. A series is simultaneously in an AP and a HP. Which of the following is true?

(a) The set of common differences of all the possible APs can have only one element.
(b) The set of common differences of all the possible APs can have more than one element but there will be a finite value.
(c) The set of common differences of all the possible APs can have infinite elements.
(d) None of these

Q.18. Find the sum of all the numbers divisible by 6 in between 100 to 400.

(a) 12,550
(b) 12,450
(c) 11,450
(d) 11,550

FOUNDATION

Q.1. Let t_r denote the *r*th term of an AP. If $t_m = \frac{1}{n}$ and $t_n = \frac{1}{m}$ then t_{mn} equals

(a) $\frac{1}{mn}$ (b) $\frac{1}{m}+\frac{1}{n}$
(c) 1 (d) 0

Q.2. Let *x, y, z* be three positive prime numbers. The progression in which $\sqrt{x}$, $\sqrt{y}$, $\sqrt{z}$ can be three terms (not necessarily consecutive) is

(a) AP (b) GP
(c) HP (d) None of these

Q.3. Let $f(x) = 2x + 1$. Then the number of real values of *x* for which the three unequal numbers $f(x)$, $f(2x)$ and $f(4x)$ are in a GP is

(a) 1 (b) 2
(c) 0 (d) None of these

Q.4. If *a, b* and *c* are in a GP then $a + b$, $2b$, $b + c$ are in

(a) AP (b) GP
(c) HP (d) None of these

Q.5. If *a, b* and *c* are in an AP then $\frac{a}{bc}, \frac{1}{c}, \frac{2}{b}$ are in

(a) AP (b) GP
(c) HP (d) None of these

Q.6. In the sequence 1, 2, 2, 4, 4, 4, 4, 8, 8, 8, 8, 8, 8, 8, 8,..., where *n* consecutive terms have the value *n*, the 1025th term is

(a) 2^9 (b) 2^{10} (c) 2^{11} (d) 2^8

Q.7. If the pth, qth and rth terms of an AP are in a GP then the common ratio of the GP is

(a) $\frac{p+q}{r+q}$ (b) $\frac{r-1}{q-p}$
(c) $\frac{p-r}{p-1}$ (d) None of these

Q.8. The sum of an infinite geometric series is 4 and the sum of the cubes of the terms of the same GP is 192. The common ratio of the original geometric series is

(a) $\frac{1}{2}$ (b) $-\frac{1}{2}$ (c) $\frac{1}{4}$ (d) $\frac{1}{6}$

Q.9. The 10th common term between the series 3 + 7 + 11 +... and 1 + 6 + 11 +...is

(a) 191 (b) 193
(c) 211 (d) None of these

Q.10. If *a, b, c* and d are four numbers such that the first three are in an AP while the last three are in a HP then

(a) $bc = ad$ (b) $ac = bd$
(c) $ab = cd$ (d) None of these

Q.11. The ratio of the sum of n terms of two AP's is $(3n - 13) : (5n + 21)$.
Find the ratio of their 24th terms.

(a) 1:2 (b) 1:3 (c) 2:3 (d) 2:5

Q.12. In a GP of even number of terms, the sum of all the terms is 5 times the sum of the odd terms. The common ratio of the GP is

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$
(c) 4 (d) None of these

Q.13. If *a, b* and *c* are in an AP then $a+\frac{1}{b}, b+\frac{1}{ca}, c+\frac{1}{ab}$ are in

(a) AP (b) GP
(c) HP (d) None of these

Q.14. The ratio of HM and GM for two positive numbers is 4:5. What is the ratio of the numbers?

(a) 4:1 (b) 3:2 (c) 3:4 (d) 2:3

Q.15. In a GP of alternatively positive and negative terms, any term is the AM of the next two terms. Then the common ratio is

(a) –1 (b) –3 (c) –2 (d) $-\frac{1}{2}$

Q.16. The 288th term of the series *a, b, b, c, c, c, d, d, d, d, e, e, e, e, e,*... is

(a) *u* (b) *v* (c) *w* (d) *x*

Q.17. The product of n positive numbers is 1. Their sum is

(a) A positive integer

(b) Divisible by n

(c) Equal to $n+\frac{1}{n}$

(d) Greater than or equal to n

Q.18. If t_n denotes the nth term of the series 2 + 3 + 6 + 11 + 18 + ... then t_{50} is

(a) $49^2 - 1$ (b) 49^2

(c) $50^2 + 1$ (d) $49^2 + 2$

Q.19. If $a^x = b^y = c^z$ and x, y, z are in GP then $\log_c b$ is equal to

(a) $\log_b a$ (b) $\log_a b$

(c) $\frac{z}{y}$ (d) None of these

Q.20. The numbers 1, 4 and 16 can be three terms (not necessarily consecutive) of

(a) No AP

(b) Only one GP

(c) Infinite number of APs

(d) More than one but finite number of GPs

Q.21. If a, b, c and d are four positive numbers then which of the following is not true?

(a) $\left(\frac{a}{b}+\frac{b}{c}\right)\left(\frac{c}{d}+\frac{d}{e}\right) \geq 4.\sqrt{\frac{a}{e}}$

(b) $\left(\frac{a}{b}+\frac{c}{d}\right)\left(\frac{b}{c}+\frac{d}{e}\right) \geq 4.\sqrt{\frac{a}{e}}$

(c) $\frac{a}{b}+\frac{b}{c}+\frac{c}{d}+\frac{d}{e}+\frac{e}{a} \geq 5$

(d) $\frac{b}{a}+\frac{c}{b}+\frac{d}{c}+\frac{e}{d}+\frac{a}{e} \geq \frac{1}{5}$

Q.22. Find the 45th term of the series 12, 13½, 15

(a) 57 (b) 77.5 (c) 74 (d) 78

Q.23. The 5th term of an AP is 15 and the 9th term is 23. Find the 14th term.

(a) 31 (b) 33 (c) 35 (d) 37

Q.24. If 9 times the 9th term in an AP is equal to 15 times the 15th term in the AP what is the 24th term?

(a) −1 (b) −3 (c) 0 (d) 1

Q.25. Find the sum of an AP of 14 terms whose first and the last terms are 8 and 99 respectively.

(a) 707 (b) 749 (c) 789 (d) 747

Q.26. A gets as many marks less than B as B gets less than C. If the total marks secured together by them is 270 and C got 90 marks more than A, find the marks secured by them individually.

(a) 30, 60, 180 (b) 45, 90, 135

(c) 40, 80, 150 (d) 35, 70, 165

Q.27. Find the sum of all the 2-digit numbers that leave remainder 1 when divided by 5.

(a) 963 (b) 968 (c) 967 (d) 960

Q.28. Find the common ratio of GP whose first term is 3, the last is 3072 and the sum of the series is 4095.

(a) 2 (b) 3 (c) 4 (d) 6

Q.29. The least value of n, for which $1 + 4^2 + 4^4 + \ldots$ to n terms is greater than 1000, is

(a) 5 (b) 4 (c) 3 (d) 2

Q.30. The number of terms in an AP is odd. If the sum of all the terms is 45 and the number of terms is 4 more than the middle term, find the middle term.

(a) 9 (b) 4 (c) 6 (d) 5

Q.31. If $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the arithmetic mean between a and b then n is

(a) 0 (b) 1

(c) 1/2 (d) None of these

Q.32. In an infinite GP, every term is equal to the sum of all the terms that follow. Find the common ratio.

(a) 1/2 (b) 1/3 (c) 1/4 (d) 1/6

Q.33. The difference between the two numbers is four and the AM between them is six. The product of the numbers is

(a) 24 (b) 12 (c) 32 (d) 48

Q.34. Which of the following statements is always true? (statements are related to AP and the terms are having their general notations.)

(a) $t_5 + t_8 = t_1 + t_{12}$ (b) $t_8 = a + 8d$

(c) $t_1 + t_2 = t_3$ (d) Only a and b

Q.35. The number of terms common between the series 1 + 2 + 4 + 8 + ... to 100 terms and 1 + 4 + 7 + 10 + ... to 100 terms is

(a) 6 (b) 4

(c) 5 (d) None of these

Q.36. If the geometric mean of two non-negative numbers is equal to their harmonic mean, then which of the following is necessarily true?

I. One of the numbers is zero

II. Both the numbers are equal

III. One of the numbers is one

(a) I only (b) Either I or II

(c) II only (d) I or III

Q.37. Consider three numbers in an AP. Which of the following will definitely divide the sum of the cubes of all the three numbers?

(a) Middle number

(b) Difference of the first and last numbers

(c) 3

(d) (a) and (c)

Q.38. My neighbour, Mr Makrant Chatpatia, is a nice person. Last night he hosted a party for me. He told me that he would retire next year. He also added that when he joined his firm he was 19 years old and his basic salary was Rs 100 and after every two years there was an increase of Rs 60. What must be the basic salary of Mr Chatpatia now, if the age of retirement is 60 years?

(a) Rs 1,990 (b) Rs 1,240
(c) Rs 1,200 (d) Rs 1,300

Q.39. Given is 54 + 51 + 48 + … till n terms = 513. How many values of 'n' are possible?

(a) 1 (b) 2
(c) 0 (d) None of these

Q.40. In a decreasing AP the sum of all its terms, except the first term, is equal to –36, the sum of all its terms, except the last term, is zero, and the difference of the tenth and the sixth term is equal to –16. What will be first term of this series?

(a) 16 (b) 20 (c) –16 (d) –20

MODERATE

Q.1. A group of friends have some money which was in an increasing GP. The total money with the first and the last friend was Rs 66 and the product of the money that the second friend had and that the last but one friend had was Rs 128. If the total money with all of them together was Rs 126, then how many friends were there?

(a) 6 (b) 5
(c) 3 (d) Cannot be determined

Q.2. If the sum of the 10th, 20th and 30th terms of an AP is equal to the 58th term, what is the ratio of the sum of the 10th, 20th and 30th terms to the sum of the 5th, 10th and 15th terms; when the common difference ≠ 0?

(a) 2 : 1 (b) 20:9
(c) 19:9 (d) Cannot be determined

Q.3. Let $a_1, a_2, a_3, \ldots$ be in an AP and a_p, a_q, a_r, be in a GP. Then $a_q : a_p$ is equal to

(a) $\frac{r-p}{q-p}$ (b) $\frac{q-p}{r-q}$
(c) $\frac{r-1}{q-q}$ (d) None of these

Q.4. If a, b, c and d are non-zero real numbers such that $(a^2 + b^2 + c^2)(b^2 + c^2 + d^2) \leq (ab + bc + cd)^2$, then a, b and c and d are in

(a) AP (b) GP
(c) HP (d) None of these

Q.5. If $\log\left(\frac{5c}{a}\right), \log\left(\frac{3b}{5c}\right)$ and $\log\left(\frac{a}{3b}\right)$ are in an AP, where a, b and c are in a GP, then a, b and c, are the lengths of sides of

(a) An isosceles triangle
(b) An equilateral triangle
(c) A scalene triangle
(d) None of these

Q.6. The coefficient of x^{15} in the product $(1-x)(1-2x)(1-2^2.x)\ldots(1-2^{15}.x)$ is equal to

(a) $2^{105} - 2^{121}$ (b) $2^{121} - 2^{105}$
(c) $2^{120} - 2^{104}$ (d) None of these

Q.7. The coefficient of x^{49} in the product $(x-1)(x-3)\ldots(x-99)$ is

(a) -99^2 (b) 1
(c) –2500 (d) None of these

Q.8. The AM of two given positive numbers is 2. If the larger number is increased by 1, the GM of the numbers becomes equal to the AM of the given numbers. Then the HM of the given numbers is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$
(c) $\frac{1}{2}$ (d) None of these

Q.9. Let a and b be two positive numbers, where $a > b$ and $4 \times \text{GM} = 5 \times \text{HM}$ for the numbers. Then a is

(a) $4b$ (b) $\frac{1}{4}b$
(c) $2b$ (d) b

Q.10. The sequence P_n is in a GP with $P_2/P_4 = \frac{1}{4}$ and $P_1 + P_4 = 108$. What will be the value of P_3?

(a) 42 (b) 48 (c) 44 (d) 56

Q.11. Let $t_n = n.(n!)$. Then $\sum_{n=1}^{15} t_n$ is equal to

(a) 15! – 1 (b) 15! + 1
(c) 16! – 1 (d) None of these

Q.12. In a GP the product of the first four terms is 4 and the second term is the reciprocal of the fourth term. The sum of the GP upto infinite terms is

(i) 8 (ii) –8 (iii) $\frac{8}{3}$ (iv) $-\frac{8}{3}$

(a) Only i and ii (b) Only ii and iii
(c) Only i, ii and iii (d) i, ii, iii and iv

Q.13. Let a_n = product of the first n natural numbers. Then for all $n \in N$

(i) $n^n \geq a_n$ (ii) $\left(\frac{n+1}{2}\right)^n \geq n!$

(iii) $n^n \geq a_{n+1}$ (iv) None of these

(a) i and ii only (b) ii and iii only

(c) i, ii and iii only (d) None of these

Q.14. A GP consists of 1000 terms. The sum of the terms occupying the odd places is P_1 and the sum of the terms occupying the even places is P_2. Find the common ratio of this GP.

(a) P2/P1 (b) P_1/P_2

(c) $(P_2 - P_1)/P_1$ (d) $(P_2 + P_1)/P_2$

Q.15. The harmonic mean of the roots of the equation $(5 + \sqrt{2})x^2 - (4 + \sqrt{5})x + 8 + 2\sqrt{5} = 0$ is

(a) 2 (b) 4 (c) 6 (d) 8

Q.16. Let A, G and H be the AM, GM and HM of two positive numbers a and b. The quadratic equation whose roots are A and H is

(a) $Ax^2 - (A^2 + G^2)x + AG^2 = 0$

(b) $Ax^2 - (A^2 + H^2)x + AH^2 = 0$

(c) $Hx^2 - (H^2 + G^2)x + HG^2 = 0$

(d) (*a*) and (*c*) both

Q.17. If the roots of $x^3 - 12x^2 + 39x - 28 = 0$ are in an AP then their common difference is

(a) ± 1 (b) ± 2 (c) ± 3 (d) ± 4

Q.18. In an infinite GP, each term is equal to four times the sum of all the terms that follow. Find the common ratio.

(a) 1/4 (b) 1/5 (c) 1/8 (d) 1/7

Q.19. If the terms of one geometric progression are multiplied by the corresponding terms of another geometric progression, the sequence obtained will be in

(a) AP (b) HP

(c) GP (d) Cannot be determined

Q.20. In a network system, each person has to include four more persons under him and such a chain should continue. The person at any level would get Re 1 commission per person below him in his group. If a person earns Rs 84, find the number of persons under him, in his group earning zero amount.

(a) 32 (b) 64 (c) 128 (d) 256

Q.21. If the nth term of AP is p and the mth term of the same AP is q, then $(m + n)$th term of AP is

(a) $p + q$ (b) $p + q - a + d$

(c) $p + q - d - a$ (d) $p + q - d + a$

Q.22. Two sequences of numbers {1, 4, 16, 64 ...} and {3, 12, 48, 192,...} are mixed as follows {1, 3, 4, 12, 16, 48, 64, 192,...}. One of the numbers in the mixed series is 1048576. Then the number immediately preceding it is

(a) 7,86,432 (b) 2,62,144

(c) 8,14,572 (d) 7,86,516

Q.23. For each positive integer n consider the set S_n defined as follows

$S_1 = \{1\}$, $S_2 = \{2, 3\}$, $S_3 = \{4, 5, 6\}$ ..., and in general, S_{n+1} consists of $n + 1$ consecutive integers the smallest of which is one more than the largest integer in S_n. Then the sum of all the integers in S_{21} equals.

(a) 1,113 (b) 5,336 (c) 5,082 (d) 4641

Q.24. If $a_1, a_2, a_3, a_4 ... a_{24}$ are in an arithmetic progression and $a_1 + a_5 + a_{10} + a_{20} + a_{24} = 225$, then the sum of the series $a_1 + a_2 + a_3 + a_4 + ... + a_{24}$ is

(a) 909 (b) 75 (c) 750 (d) 900

Q.25. The harmonic mean of two positive real numbers is 4. Their arithmetic mean A and their geometric mean G satisfy the relation $2A + G^2 = 27$. Find the two numbers.

(a) 4 and 4 (b) 2 and 6

(c) 3 and 6 (d) 5 and 10/3

Q.26. Find the sum of the 37th bracket of the following series.

$(1) + (7 + 7^2 + 7^3) + (7^4 + 7^5 + 7^6 + 7^7 + 7^8) + (7^9 + 7^{10} + ... + 7^{15})....$

(a) $\frac{7^{37}}{6}\left(7^{73} - 1\right)$ (b) $\frac{\left(7^{73} - 1\right)}{6}$

(c) $\frac{7^{71}}{6}\left(7^{73} - 1\right)$ (d) None of these

Q.27. If a, b and c are positive integers, then find the product of $(a + b)(b + c)(c + a)$.

(a) $> 8\,abc$ (b) $< 8\,abc$

(c) $= 8\,abc$ (d) None of these

Q.28. The sum the of ‘n^{th}’ terms of two different arithmetic progression is zero. The sum of a certain number of terms (beginning with the first) of one of the progression is 386. What is the sum of the same number of terms (from the beginning) of the second progression?

(a) 386 (b) + 193 (c) − 193 (d) − 386

Q.29. An infinite GP has the first term ‘X’ and sum ‘5’, then X belongs to

(a) $X < -10$ (b) $-10 < X < 0$

(c) $0 < X < 10$ (d) $X > 10$

Q.30. $f(x) = 2x + 1$, where $x = 1, 2, ..., 100$. $g(x) = 3x - 2$, where $x = 1, 2, ..., 100$. For how many values of x, $f(x) = g(x)$?

(a) 31 (b) 32 (c) 33 (d) 34

Q.31. Find the sum of the series:
$1.2 + 2.2 + 3.2^2 + 4.2^3 + \ldots + 100.2^{99}$.

(a) $99 \times 2100 - 1$ (b) $99 \times 2101 + 1$
(c) $99 \times 2101 - 1$ (d) $99 \times 2100 + 1$

Q.32. The mid-points of the adjacent sides of a square are joined. Again the mid-points of the adjacent sides of the newly formed figure are connected and this process is repeated again and again. Calculate the sum of the areas of all such figures given that the diagonal of outermost square is $6\sqrt{2}$cm.

(a) 35 cm^2 (b) 44 cm^2
(c) 72 cm^2 (d) 58 cm^2

Q.33. The sum of the first n terms of an AP is $n(n - 1)$. Then the sum of the squares of these terms is

(a) $n^2(n - 1)^2$ (b) $\frac{n}{6}(n-1)(2n-1)$
(c) $\frac{2}{3}n(n-1)(2n-1)$ (d) $\frac{2}{3}n(n-1)(2n+1)$

Q.34. What is the sum of the following series?
$7 + 26 + 63 + 124 + \ldots + 999$

(a) 3014 (b) 3013
(c) 3015 (d) None of these

Q.35. In ΔABC, points P_1, Q_1 and R_1 divide the lines AB, BC and AC respectively in the ratio of 2 : 1. In $\Delta P_1Q_1R_1$, the points P_2, Q_2 and R_2 divide the sides P_1Q_1, Q_1R_1 and P_1R_1 in the ratio of 2: 1. In every such new triangle, a new triangle is generated by joining the points on the sides that divide these sides in the ratio of 2 : 1. Find the sum of the areas of all such triangles formed till infinity. (Area of ΔABC = 'A' sq. units.)

(a) $\frac{A}{2}$ sq. units (b) 2A sq. units
(c) $\frac{A}{3}$ sq. units (d) 3A sq. units

Q.36. Find the sum of the series $-1 + 1^2 - 2 + 2^2 - 3 + 3^2 + \ldots n + n^2$.

(a) $\frac{-n(n+1)}{3}$ (b) $\frac{n(n+1)(n-1)}{3}$
(c) $\frac{n(n-1)}{3}$ (d) None of these

Q.37. There are three numbers in an arithmetic progression. If the two larger numbers are increased by one, then the resulting numbers are prime. The product of these two primes and the smallest of the original numbers is 598. Find the sum of the three numbers.

(a) 45 (b) 29 (c) 42 (d) 36

Q.38. If three successive terms of a GP with the common ratio $r > 1$ form the sides of a triangle and [r] denotes the integral part of x, then $[r] + [-r] =$

(a) 0 (b) 1
(c) −1 (d) None of these

Q.39. In how many ways can we select three natural numbers out of the first 10 natural numbers so that they are in a geometric progression with the common ratio greater than 1?

(a) 2 ways (b) 3 ways
(c) 4 ways (d) 5 ways

Q.40. If m times the mth term of an AP is equal to p times the pth term, find the $(m + p)$th term.

(a) 0 (b) $a^2 - b^2$
(c) $a - b$ (d) 1

ADVANCED

Q.1. If $a_1, a_2, a_3, \ldots a_n$ $(n \geq 3)$ are in an AP then $\left(\frac{1}{a_1a_2} + \frac{1}{a_2a_3} + \ldots + \frac{1}{a_{n-1}a_{n-2}}\right)$ will be equal to

(a) $\frac{n^2}{a_1a_n}$ (b) $\frac{(n-1)}{a_1a_n}$
(c) $\frac{2n}{a_1a_n}$ (d) None of these

Q.2. Three distinct numbers x, y, z, form a GP in that order and the numbers $x + y$, $y + z$, $z + x$ form an AP in that order. Find the common ratio of the GP.

(a) 1 (b) −2
(c) 2 (d) Either (a) or (b)

Direction for questions 3 and 4: Answer the questions based on the following information.

A series of positive integers have the following terms. First term: 1; Second term: (2 + 3); Third term: (4 + 5 + 6),… so on.

Q.3. The last number in the nth term of the series is

(a) $\frac{(n^2-n+2)}{2}$ (b) $\frac{(n^2+n)}{2}$
(c) $\frac{(2n^2+3)}{4n}$ (d) None of these

Q.4. The sum of the numbers of the nth term is

(a) $n^2 + n$ (b) $\frac{n(n^2+1)}{2}$

(c) $\frac{(n+n)^2}{3n}$ (d) None of these

Q.5. An arithmetic series consists of $2n$ terms, and the first term equals the value of the common difference. If a new series is formed taking the 1st, 3rd, 5th, ... $(2n-1)$ th term of the old series, find the ratio of the sum of the new series to that of the sum of the terms of the old series.

(a) $\frac{n+1}{2(2n+1)}$ (b) $\frac{n}{2n+1}$

(c) $\frac{1}{2}$ (d) Cannot be determined

Q.6. If the sum of the first $2n$ terms of the AP 2, 5, 8 ... is equal to the sum of first n terms of the AP 57, 59, 6 ..., then what is the value of n?

(a) 7 (b) 9 (c) 11 (d) 13

Q.7. Let the positive numbers a, b, c and d be in AP. Then abc, abd, acd, bcd are

(a) Not in AP or GP or HP
(b) In AP
(c) In GP
(d) In HP

Q.8. Given that $(m + 1)$th, $(n + 1)$th and $(r + 1)$th term of an AP are in GP and m, n, r are in HP, then find the ratio of the first term of the AP to its common difference in terms of n.

(a) $2 : n$ (b) $n : 2$ (c) $2n : 3$ (d) $3 : 4n$

Q.9. Let x be the arithmetic mean and y and z be the two geometric means between any two positive numbers then the value of $\frac{y^3+z^3}{xyz} = \ldots$

(a) xy (b) 2 (c) 0 (d) xyz

Q.10. If rational numbers a, b, c and d are in GP then the roots of the equation $(a-c)^2 x^2 + (b-c)^2 x + (b-d)^2 = (a-d)^2$ are necessarily

(a) Imaginary (b) Irrational
(c) Rational (d) Real and unequal

Q.11. Let $a = 111 \ldots 1$ (55 digits),
$b = 1 + 10 + 10^2 + 10^3 + 10^4$,
$c = 1 + 10^5 + 10^{10} + 10^{15} + \ldots + 10^{50}$, then

(a) $a = b + c$ (b) $a = bc$
(c) $b = ac$ (d) $c = ab$

Q.12. If b be the pth term of a GP where $(p + q)$th and $(p - q)$th terms are a and c respectively, then for all $x \in$ R

(a) $af(x) = 0$ (b) $af(x) > 0$
(c) $af(x) \leq 0$ (d) $af(x) > 0$

Q.13. If a geometric mean of two non-negative numbers is equal to their harmonic mean, then which of the following is necessarily true?

I. One of the numbers is zero.
II. Both the numbers are equal.
III. One of the numbers is one.

(a) I and III only (b) Either I or III
(c) III only (d) None of these

Q.14. The middle points of the sides of a triangle are joined forming a second triangle. Again a third triangle is formed by joining the middle points of this second triangle and this process is repeated infinitely. If the perimeter and the area of the outer triangle are P and A respectively, find the sum of the areas of all the triangles.

(a) 4/5A (b) 4/3 A (c) 3/4A (d) 5/4A

Q.15. A series in which any term is the sum of the preceding two terms is called a Fibonacci series. The first two terms are given initially and together they determine the entire series. If the difference of the squares of the ninth and the eighth terms of a Fibonacci series is 715 then, what is the 12th term of that series?

(a) 157 (b) 142
(c) 144 (d) Cannot be determined

Q.16. In a sequence of terms, a_1, a_2, a_3 ..., the nth term, a_n (for $n > 1$), is given by $a_n = a_{n-1} + a_{n+1.}$ If $a_7 = 6$ and $a_{14} = 7$, find S_{26}/S_{13}, where S_i is the sum of all the terms from a_1 to a_i in the sequence.

(a) $\frac{1}{6}$ (b) $-\frac{1}{6}$

(c) $\frac{13}{6}$ (d) Cannot be determined

Q.17. The sum of an infinite GP is 162 and the sum of its first n terms is 160. If the inverse of its common ratio is an integer, then how many values of common ratio is/are possible, common ratio is greater than 0?

(a) 0 (b) 1 (c) 2 (d) 3

Q.18. In the above question, which of the following is the value of n?

(a) 4 (b) -2
(c) -1 (d) None of these

Q.19. The first term of an AP = the common ratio of a GP and the first term of the GP = common difference of the AP. If the sum of the first two terms of the GP is equal to the sum of the first 2 terms of the AP, then the ratio of the first term of the GP to the first term of an AP is

(a) > 1
(b) Independent of first term of AP
(c) Independent of first term of GP
(d) < 1

Direction for questions 20 to 23: Read the passage below and solve the questions based on it.

Let there be a series 'S' with it's nth term be equal to $n(x)^n$. Also S_n denotes the sum of the first n terms of the series S.

Q.20. What is $S_5 - S_4$ equal to?
(a) $3x^3$ (b) $4x^4$
(c) $5x5$ (d) None of these

Additional information for questions 21 to 23:

If the $(n + 1)$th term of S is equal to 64 and the nth term is equal to 24 then:

Q.21. What is the value of n?
(a) 2 (b) 3
(c) 4 (d) Can't say

Q.22. What is the value of x?
(a) 2 (b) 3
(c) 4 (d) None of these

Q.23. What is the value of S_3?
(a) 20 (b) 14
(c) 34 (d) None of these

Q.24. Let S_n be defined as the sum of n terms of the following series: $S_n = 2^{66} - 2^{65} - 2^{64}$. What is the sum of S_{20}?
(a) 2^{47} (b) 2^{12}
(c) 2^{19} (d) None of these

Q.25. The odd natural numbers are arranged in a pyramidical shape given below:

1
3 5
7 9 11
13 15 17 19
… … … … …

And so on.
What is the sum of the numbers in the nth row of this structure?
(a) $2n^2 - 1$ (b) n^3
(c) $(n - 1)^2 + 1$ (d) $2n^2 - 1$

Q.26. Set X is having 2000 elements in AP with the first term = 1, and the common difference = 1. What is the maximum number of elements Set Y can have such that sum of no two elements of Y is equal to 9 or a multiple of 9?
(a) 890 (b) 891
(c) 892 (d) None of these

Q.27. In an AP of even number of terms, the sum of the odd terms is 24 while that of the even terms is 30. Find the number of terms if the last term exceeds the first by 10.5.
(a) 7 (b) 9 (c) 6 (d) 8

Q.28. Find the sum of the products of the integers, 1, 2, 3,... n taking two at a time.
(a) $1/12\, n(n^2 + 1)\,(n + 1)$
(b) $1/6\, n(n^2 + 1)\,(n + 1)$
(c) $\frac{1}{12} n\,(n2 - 1)\,(3n + 2)$
(d) None of these

Q.29. The sum of the first 10 terms of an AP is equal to 155 and the sum of the first two terms of a GP is 9. How many values for the first term of an AP is possible if the first term of an AP is equal to the common ratio of a GP and the first term of a GP is equal to the common difference of a AP.
(a) 0 (b) 1 (c) 2 (d) Infinite

Q.30. The sum upto the n terms of the following series:

$$\frac{1}{1.3.5} + \frac{1}{3.5.7} + \frac{1}{5.7.9} \ldots$$

(a) $\frac{(n+2)(n+4)}{n(n+1)(n+3)}$ (b) $\frac{(n+2)(n+4)}{(2n+1)(n-2)}$
(c) $\frac{n(n+2)(n+4)}{3(2n+1)(n+3)}$ (d) $\frac{n(n+2)}{3(2n+1)(2n+3)}$

Direction for questions 31 and 32: Read the passage below and solve the questions based on it.

30 Alphonso mangoes are to be distributed among Tom, Dick and Harry. Harry's share is less than that of Dick. Further, they have agreed to divide the mangoes such that the number of mangoes with the three of them are in an arithmetic progression.

Q.31. If none of them is to receive more than 15 mangoes, in who many ways can the mangoes be distributed?
(a) 10 (b) 20 (c) 5 (d) 15

Q.32. Which of the following statements must be true?
(a) The number of mangoes Dick receives is at least 11.
(b) The number of mangoes Tom receives is at most 9.
(c) One person gets exactly 10 mangoes.
(d) More than one of the above.

Q.33. The sum of the first n terms $(n > 1)$ of an AP is 153 and the common difference is 2. If the first term is an integer, then the number of possible values of n is
(a) 2 (b) 3 (c) 4 (d) 5

Q.34. The positive numbers a, b, c and d are in a HP and $a \neq b$, then
(a) $a + d > b + c$ is always true
(b) $a + b > d + c$ is always true
(c) $a + c > b + d$ is always true
(d) None of these

Q.35. The number of ways in which three distinct numbers in an AP can be selected from 1, 2, … , 24 is

(a) 112 (b) 124

(c) 132 (d) 146

Q.36. Numbers are placed on the vertices of a polygon with n (>4) sides in such a way that each of them equals the arithmetic mean of its neighbours. Then it can be concluded that all the numbers are equal

(a) For every n (>4)

(b) Only for even n (>4)

(c) Only for odd n (>4)

(d) Only for odd prime n (>4)

Q.37. In an infinite geometric progression, the nth term is equal to three times the sum of all the terms that follow it and the sum of the first two terms is 15. What is the sum of the whole series?

(a) 32 (b) 16

(c) $+\infty$ (d) None of these

Direction for questions 38 and 39: Read the passage below and solve the questions based on it.

$N_1 + N_2 + N_3 + \ldots + N_N = P$, where $N_1, N_2, N_3 \ldots$ and N_N are $n(n>1)$ consecutive natural numbers such that $N_1 < N_2 < N_3 < N_4 \ldots < N_N$.

Q.38. If $P = 100$, then how many values of N is/are possible?

(a) 0 (b) 1 (c) 2 (d) Infinite

Q.39. If $P = 100$, then what is the highest value of N?

(a) 5 (b) 10 (c) 20 (d) 8

Q.40. An employee joined a company on 01.04.2004 in the salary grade of Rs 8,000-500-9,500-750-12,500 with a basic salary of 9,000. He is due to retire on 31.03.2007. He contributes 10% of his basic salary to an EPF scheme. His employer contributes an equal amount. If, on retirement, he gets the full amount of his share of the EPF and 50% of the employer's share, what is the amount of EPF received by him, ignoring any interest on the deposits?

(a) 34,500 (b) 17,250

(c) 51,750 (d) 62,500

TRUE/FALSE

1. If we know any two terms of an AP, we can definitely find out the remaining terms of the same AP.
   State whether true or false.
2. If we know any two terms of a GP, we can definitely find out the remaining terms of the same GP.
   State whether true or false.
3. If we know any two terms of a GP, we can definitely find out the common ratio of the same GP.
   State whether true or false.
4. If we know any two terms of an AP, we can definitely find out the common difference of the same AP.
   State whether true or false.
5. Sum of all the odd terms (viz., 1st term, 3rd term etc.) and sum of all the even terms (viz., 2nd term, 4thterm etc.) of a GP with 'n' terms are given. We can find out the common ratio of this GP with this information.
   State whether true or false.
6. If a series is in both AP and GP, then sum of 1st 10 terms will be equal to sum of any other set of consecutive 10 terms.
   State whether true or false.

ANSWERS

Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Answer	(b)	(d)	(a)	(b)	(b)	(b)	(c)	(b)	(b)	(d)	(c)	(b)	(b)	(b)	(b)	(a)	(a)	(b)

Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(a)	(c)	(d)	(b)	(b)	(b)	(a)	(a)	(a)	(c)	(a)	(a)	(c)	(c)	(d)	(d)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(b)	(c)	(b)	(b)	(a)	(c)	(b)	(d)	(a)	(a)	(c)	(a)	(c)	(b)	(d)	(d)	(b)	(a)

Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(c)	(b)	(d)	(a)	(c)	(a)	(a)	(b)	(c)	(d)	(a)	(a)	(b)	(d)	(c)	(b)	(c)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(a)	(d)	(d)	(c)	(d)	(a)	(d)	(c)	(c)	(d)	(c)	(c)	(c)	(b)	(d)	(c)	(c)	(b)	(a)

Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(d)	(b)	(b)	(b)	(c)	(d)	(b)	(b)	(c)	(b)	(d)	(d)	(b)	(c)	(c)	(c)	(a)	(c)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(a)	(c)	(a)	(b)	(b)	(d)	(c)	(c)	(d)	(d)	(c)	(d)	(d)	(c)	(a)	(b)	(c)	(d)	(c)

True/False

1. True 2. True 3. True 4. True 5. False 6. True

HINTS AND SOLUTIONS

Moderate

1. The sum of money with the first and the last friend = 66. This can be used as a hint. Let us assume the first friend was having Rs 2 and the last friend was having Rs 64. So, the money can be in the sequence 2, 4, 8, 16, 32, 64. It satisfies the given conditions.

   Alternatively, this can be done by using the formula for t_n of GP also.

2. Sum of the 10th, 20th, 30th terms of an AP is equal to the 58th term. Hence, $3a + 57d = a + 57d$

   So, $a = 0$ and d can be anything not equal to zero. Now, we have to find out the ratio $3a + 57d$ and $3a + 27d$, or, we have to find out the ratio $57d : 27d = 19 : 9$.

5. For a, b, c to be the length of the sides of the triangle, it's AM should not be 0.

8. Take two numbers such that their AM is 2 like 1 and 3. And go according to the conditions given in the question.

9. Using option (a), let $a = 2$, $b = 8$ and solve. If it does not satisfy, use the other options.

11. Solve through options.

14. Take any small sample of 4 terms of any GP viz. 1, 2, 4 and 8. Its common ratio = 2 = (8+2)/(1 +4)

    And this will be true for any number of terms as long as we are taking total number of terms as even(in this question, it is given as 1000). However, if the total number of terms is given as odd, we cannot find the common ratio in this way, because the number of odd terms ≠ number of even terms.

15. We can easily find out the values : $2ab$ and $a + b$, where a and b are the roots of the equation given.

16. Solve through options.

17. Factorize the equation and we get $(x-1)\ (x-4)\ (x-7)$

19. Take two series and solve.

20. Make the three and solve though option.

21. $T_n = p = a + (n-1)d$
    $T_m = q = a + (m-1)d$
    Add the two and solve through the options given.

22. 1048576 is divisible by 4
    so, previous number is divisible by 3.

23. Sum = $n(n^2 + 1)/2$, where n is the number of terms in S.

25. Solve through options.

26. Solve through options.

27. Put $a = 1$, $b = 2$ $c = 3$ and check through options.

28. Assume the 1st AP as 1, 1, 1,... or 2, 2, 2,...
    and the 2nd AP as −1, −1, −1 or −2, −2, −2

30. Get the terms of sets $f(x)$ and $g(x)$ and check how many terms are common.

31. Sum = $(n - 1)2^n + 1$.

32. The side of the outermost square is 6 cm. Area of the square one in G.P with first term 36 and common ratio = ½

36. Break the series in two sequence 1, 2, 3, 4,... n and 1^2, 2^2, 3^2, ... n^2 and solve

37. Factorize 598 as the product of two prime numbers and an even number.

    598 = 2 × 13 × 23.

38. Take any value of $r > 1$ and then solve.

39. Make pair of three numbers. Only sequence with 2 and 3 as the common ratio is possible.

40. Take any series for which the given condition is true. One such series can be: 4, 3, 2, 1, 0, ...

Advanced

1. Since a_1, a_2, a_3 are in AP, So $a_2 - a_1 = a_3 - a_2 = d$

$$\left[\frac{1}{a_1a_2}+\frac{1}{a_2a_3}+\ldots\frac{1}{a_{n-1}a_{n-2}}\right]$$

$$=\frac{1}{d}\left[\frac{1}{a_1}-\frac{1}{a_2}+\frac{1}{a_2}-\frac{1}{a_3}+\ldots\right]$$

2. Since x, y, z are in GP so $y/x = z/y$ and also $x + y$, $y + z$, $z + x$ are in AP so $2(y + z) = 2x + y + z$ i.e., $z = 2x - y$.
   So, $y/x\ (2x - y)/y \Rightarrow y/x = (2x/y) - 1$
   Since y/x is the ratio, assume $y/x = r$
   and solve the equation $r^2 + r - 2 = 0$

3. Go through options.

4. Go through options.

5. The series consist of $2n$ terms,
   first term = a ,common diff = a, no of terms = $2n$

$$\text{Sum of all terms} = \frac{2n}{2}[2a+(2n-1)a] \qquad \ldots\text{(i)}$$

   for the new series taking 1st, 3rd, 5th, $(2n - 1)$th term of old series.

   First term = a, common difference = $2a$, number of terms = n

$$\text{Sum of all terms} = \frac{n}{2}[2a+(2n-1)2a] \qquad \ldots\text{(ii)}$$

   Dividing (i) by (ii) gives the required ratio.

6. The equation can be written as:

$$\frac{2n}{2}[4+(2n-1)3]=\frac{n}{2}[114+(n-1)2]$$

Solving this equation gives us the value of n.

7. Divide each term by $abcd$ will give $1/d$, $1/c$, $1/b$ and $1/a$ respectively which are in HP (since a, b, c, d are in AP).

Alternatively, assume values of a, b, c and d such that they are in AP.

8. Since the $(m+1)$th, $(n+1)$th and $(r+1)$th term of an A.P. are in G.P. so,

$(a+nd)^2 = (a+md)(a+rd)$ ...(i)

[assume a and d as the first term and common difference of an AP]. Also, m, n, r are in HP so,

$$\frac{2n}{2}=\frac{m+r}{mr} \quad \text{... (ii)}$$

By solving the equation you will get

$\frac{a}{d}=\frac{n^2-mr}{m+r-2n}$, put the $m+r=2mr/n$ from equation (iii)

You will get $a/d=-n/2 \approx n:2$ [–ve sign indicates that either common difference or first term is –ve]

9. Assume any two positive numbers a, b. So, according to the question a, x, b are in AP

So, $x=\frac{a+b}{2}$, also, a, y, z, b are in GP so, $y^2=az$ i.e., $y^3=ayz$ and $z^3=byz$

Now solve $\frac{y^3+z^3}{xyz}$.

10. In a quadratic equation, $ax^2+bx+c=0$. If $a+b+c=0$ then, roots will be rational. Use this property. Also a, b, c, d are in GP i.e., $b/a=c/b=d/c$.

11. Since, $a = 1111 \ldots 1$ (55 digits)

$b=1+10+10^2+10^3+10^4=\frac{1(10^5-1)}{10-1}=\frac{10^5-1}{9}$ and

$c=1+10^5+10^{10}+10^{15}+\ldots 10^{50}=1\,\frac{\left[(10^5)^{11}-1\right]}{10^5-1}$

$bc=(10^5-1)/9\times(10^{55}-1)/(10^5-1)=(9999\ldots$ 55 digits $)/9=a$

13. Assuming two non-negative a and b, then according to the question $\sqrt{ab}=\frac{2ab}{a+b}$.

It is true only if $a=b$

14. Area of the triangle formed by joining the middle point of the triangle will be 1/4th of the original triangle so, the series will be

$A+1/4\,A+1/16\,A+1/64\,A+\ldots\infty$.

15. The series is like 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144…

The difference is 715 and the 12th term is 144.

17. $a/(1-r)=162$ and $a(1-r^n)/1-r=160$

$\Rightarrow 1-r^n=160/162 \Rightarrow r^n=1/81$

Hence, there will be only two values of r, i.e., 2 and 4.

18. See the hints of the above question.

19. $(\alpha-\beta)\,(\alpha)\,(\alpha-\beta)$ AP

(a, ar, ar^2) GP

$\alpha-\beta=r+a=\beta$

$a+ar=2\alpha-\beta$

$a+ar=2\alpha-2\beta+\beta$

$a(1+r)=2(r+a)+r+a$

$a(1+r)=3(r+a)$

$a+r=a(1+r)/3$

$\frac{a}{\alpha-\beta}=?$

$a/(a+r)=(a.3)/a(1+r)$

$=3/1+r$

This is independent of the first term of GP.

20. $S_5=x+2x^2+3x^3+4x^4+5x^5$ and $S_4=x+2x^2+3x^3+4x^4$

27. $a=0.75$, $d=1.75$

28. Go through the options.

29 Put values like (3, 6)

Alternatively this question can be done by going through options also.

30. Put $n=1$ and check the options.

31.

D	H	T	T	H	D	T	H	D
10	5	15	5	10	15	10	5	15

……………………………………………………

……………………………………………………

……………………………………………………

Total = 15

32. Assume that they are getting $a-d$, a and $a+d$ mangoes irrespective of the fact who gets what numbers.

Now, $a-d+a+a+d=30$, So, $a=10$

Regarding who gets how many mangoes, we have only one information, i.e., D > H. So, we cannot decide about the first two options.

35. … $=1\times12\times13=$ Number of ways = 132.

36. Go through the options.

37. The series is: 12, 3, 3/4, 3/16 …

38. $N_1+N_2+N_3+\ldots+N_N=100$ is possible only in two ways:

23 + 24 + 25 + 26 + 27 = 100, or, 9 + 10 + 11 + 12 + 13 + 14 + 15 + 16 = 100

12

CHAPTER

Function

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Functions and its types
- Definitions and properties
- Nature of functions
- Ways of representing a function and questions associated with it
- Methods of solving questions

Introduction to the topic

Function is an important chapter for CAT and other exams. The number of questions being asked from this topic is almost constant. This chapter is almost a nightmare for the non-math students, all due to its technical language and interpretations. But most of the questions asked in the CAT from this area can be solved with the knowledge of the basics of functions, and a bit of logic.

FUNCTION

A function is a rule which associates two or more than two variables. Functions can be understood with the help of some very basic examples:

i. Area of a circle (A) = πr^2, where r is the radius.
   So, the area of a circle is dependent upon the value of the radius of a circle. We can write this mathematically as $A = f(r)$
ii. If the distance (d) is constant, then the time taken (t) to cover that distance (d) will be dependent on the value of the speed (v). This can be written mathematically as $t = f(v)$
   Normally any function is represented in the format of $y = f(x)$

Domain and range of $y = f(x)$ In case of $y = f(x)$, values of y are dependent upon the corresponding values of x. Here y is known as the dependent variable and x is known as the independent variable.

All the real values of x for which atleast one real value of y exists, are known as the domain of this function $y = f(x)$

All the possible real values of y are known as the range of this function $y = f(x)$.

While finding out the domain, the variable generally takes a range of values unlike the case of equations where the variable will, mostly, take one value or a discrete set of values in interval notation. Hence, it is important to understand the standard notations which are used to represent the solution set of the variable as well as the interval notation.

- (a, b) read as "open interval a, b" means all real numbers between a and b excluding a and b; $a < b$.
- $[a, b]$ read as "closed interval a, b" means all real numbers between a and b including a and b; $a < b$.
- $[a, b)$ means all numbers between a and b, with a being included and b excluded; $a < b$.
- $(a, b]$ means all numbers between a and b, with a being excluded and b included; $a < b$.

Example 1 Represent all real numbers between 1 and 10 in the interval from where

(i) 1 and 10 are included,
(ii) 1 and 10 are excluded
(iii) 1 is included and 10 is excluded and
(iv) 1 is excluded and 10 is included

Solution

(i) [1, 10] i.e., $1 \le x \le 10$ and x is a real number.
(ii) (1, 10) i.e., $1 < x < 10$ and x is a real number.
(iii) [1, 10) i.e., $1 \le x < 10$ and x is a real number.
(iv) [1, 7] i.e., $1 \le x \le 7$ and x is a real number.

Process to find out the domain of $y = f(x)$

1. Denominator should not be equal to zero.
2. Expression under square root should be a non-negative.

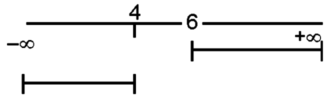
Example 2 Find the domain of the definition of function $y = f(x) = \sqrt{4-x} + \sqrt{x-6}$.

Solution We know that the expression under square root should be non-negative.

So, $4 - x \ge 0$, or, $x \le 4$

And $x - 6 \ge 0$, or, $x \ge 6$.

Arranging these values on the numberline:



So, no value of x is common. Hence the domain of $y = f(x)$ is not defined.

Example 3 The domain of $y = f(x) = \dfrac{1}{\sqrt{|x| - x}}$ is

Solution For $y = f(x)$ to be defined, $|x| - x > 0$, or, $|x| > x$.

It is true only if $x < 0$. So, the domain is $-\infty < x < 0$.

Example 4 Are the following functions identical?

$y = f(x) = x$

$y = h(x) = \dfrac{x^2}{x}$

Solution Domain of $f(x)$ = all the real values

Domain of $h(x)$ = all the real values except at $x = 0$.

Because the domains are not the same for $f(x)$ and $h(x)$, so $f(x)$ and $h(x)$ are not identical.

SOME SPECIAL FUNCTIONS

Modulus Function

$y = |x|$

It is defined as

$y = x$; if $x > 0$

$y = -x$; if $x < 0$

$y = 0$; if $x = 0$

Despite that in the above equations, we are finding a negative value of x if $x < 0$, its absolute value can never be negative. This can be seen through the following example:

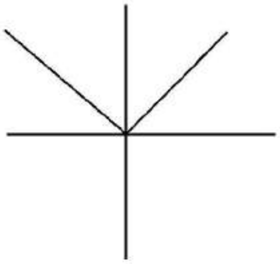
We are finding out the value of $y = |-5|$.

Assuming that $-5 = x$, so $y = |x|$

Now, since $x < 0$, so $y = -x = -(-5) = 5$

Graphical representation of modulus function

This is the graph of $y = |x|$



It is observed that for every value of x, be it +ve or –ve, the value of y cannot be negative.

Example 5 What is the value of x if $|2x + 3| = 9$?

Solution In the questions involving modulus, first the value of expression under the modulus is taken as a positive and then as a negative.

Case 1 When $(2x + 3) > 0$, or, $x > \frac{-3}{2}$, then $|2x + 3| = 2x + 3$

So, $2x + 3 = 9$ or, $2x = 6$.

So, $x = 3$

Case 2 When $(2x + 3) < 0$, or, $x < \frac{-3}{2}$, then $|2x + 3| = -(2x + 3)$

So, $-(2x + 3) = 9$, or, $-2x = 12$

So, $x = -6$

Example 6 What is the value of 'x' if $x^2 + 5|x| + 6 = 0$?

Solution Taking $x > 0$, $x^2 + 5|x| + 6 = x^2 + 5x + 6 = (x + 2)(x + 3) = 0$

Or, $x = -2$ and $x = -3$

But as we have assumed that $x > 0$, so $x = -2$ and -3 are not admissible.

Taking $x < 0$, $x^2 + 5|x| + 6 = x^2 - 5x + 6 = (x - 2)(x - 3) = 0$

Or, $x = 2$ and $x = 3$

But as we have assumed that $x < 0$, so $x = 2$ and 3 are not admissible.

So, there is no real value of x which can satisfy this equation.

Alternatively, it can be seen that x^2 and $5|x|$ and 6, are all positive values. So the sum of these three can never be equal to 0, so no real value of x is possible.

Example 7 If $|x^2 - 5x + 6| > x^2 - 5x + 6$, then find the values of x?

Solution If $|N| > N$, then $N < 0$. (It can be understood by assuming the values)

So, $x^2 - 5x + 6 < 0$

Or, $(x - 2)(x - 3) < 0$

So, $2 < x < 3$

Greatest Integer Value Function

$y = [x]$

It is defined as the largest integral value of x which is less than or equal to x.

It is given $y = [3.23]$ and we have to find the greatest integer value of y.

Taking the second part of the definition, i.e., the integer less than or equal to 3.23, we get a set of integers less than or equal to 3.23 ⇒ 3, 2, 1, 0, −1,… and so on.

The largest integer among all these integers = 3. So the greatest integer value of [3.23] = 3

Similarly, if we find the greatest integer value of $y = [-2.76]$, then all the integers less than this value $(-2.76) = \{-3, -4, -5, -6, \dots\}$.

Now the greatest integer among all these integers given in the above set = −3

It can also be seen through the tabular presentation:

x	y
0–1 (excluding at $x = 1$)	0
1–2 (excluding at $x = 2$)	1
2–3 (excluding at $x = 3$)	2

And so on …

Example 8 What is the value of x in the following expression?

$[x]^2 \le 16$?

Solution $4 \le [x] \le 4$

$-4 \le [x]$, or, $-4 \le x$

And $[x] \le 4$, or, $x < 5$.

So, the value of x is: $-4 \le x < 5$.

Logarithmic Function

$y = \log_a x$

It is known that the value of x has to be positive here. However, y can have negative values.

Since with an increase in the value of x, y always increases; so $y = \log_a x$ is an increasing function.

We will discuss more about this function in the logarithm chapter.

Exponential Function

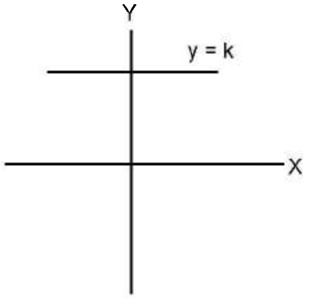
$y = e^x$

Exponential function is the inverse of a logarithmic function.

Again it can be observed that the value of y cannot be negative, whatever be the value of x in $y = e^x$

Constant Function

$f(x) = k$, (where k is any constant) is known as a constant function.



WAYS OF REPRESENTING FUNCTIONS

There are three standard methods of representing functions:

1. Equation method
2. Tabular method
3. Graphical method

1. Equation method

It is the most commonly used form of writing any function. This form of representation involves a simple equation written in terms of the variable.

Example $y = f(x) = 5x^2 + 8$.

2. Tabular method

In this form of representation a table of the values of x and y is formed, with or without mentioning the equation of functions.

Example A function $y = f(x)$ is represented by the table given below:

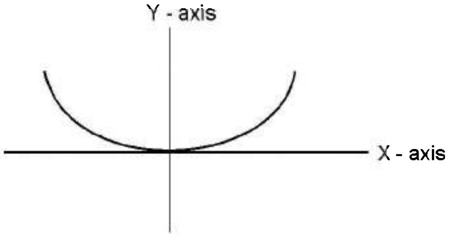
x	1	2	–1	–2
y	1	4	1	4

Obviously, the function which will satisfy the above written table is $y = f(x) = x^2$

3. Graphical method

A graph is considered to be the most-revealing-way of representing a function. Apart from giving an inkling about the function $y = f(x)$, its nature and shape, it gives us the information about the maximum and minimum value of the function also.

For example,



It is quite evident here that for both the negative and the positive values of x, y is having the same value and the shape of the graph is parabolic. So this graph is being represented by the function $y = f(x) = x^2$.

NATURE OF A FUNCTION (IN TERMS OF EQUATION)

On the basis of the symmetric nature of functions, we define the graphs to be of three natures:

Even Function

For $y = f(x)$,

Mathematically, even functions are defined as $y = f(x) = f(-x)$

In layman terms, if we replace 'x' from the given equation by '$-x$', then the equation should be the same.

For example, $y = f(x) = x^2 + 8$.

Here, $f(-x) = (-x)^2 + 8 = x^2 + 8 = f(x)$

So, $y = f(x) = x^2 + 8$ is an even function.

Properties of even function

1. The sum or the difference or the product or the division of any two even functions is an even function.
2. For $y = f(x)$, the graph of the even functions are symmetrical to Y-axis.

Odd Function

For $y = f(x)$,

Mathematically odd functions are defined as

$y = f(x) = -f(-x)$

In layman terms, if we replace 'x' from the given equation by '$-x$', then the equation should be the same with an opposite sign.

For example, $y = f(x) = x^3$.

Here $f(-x) = (-x)^3 = -(x^3) = -f(x)$

So, $y = f(x) = x^3$ is an odd function.

Properties of odd function

1. The sum or the difference of any two odd functions is an odd function.
2. For $y = f(x)$, the graph of the odd functions are symmetrical to the origin.

Neither Odd Nor Even Function

For $y = f(x)$,

If any equation is not satisfying the conditions of either an odd or an even function, then it is said to be neither odd nor even function.

In layman terms, if we replace 'x' from the given equation by '$-x$', then the equation should not be the same, either with the same sign or with an opposite sign.

For example, $y = f(x) = x^3 + 1$

Here, $f(-x) = (-x)^3 + 1 = -x^3 + 1$

It is quite clear now that $f(-x)$ is neither equal to $f(x)$ or $-f(x)$.

So, $y = f(x) = x^3 + 1$ is neither an even function nor an odd function.

Properties of neither odd nor even function

1. The sum of the difference of odd and even functions is neither an odd nor an even function.
2. To understand the mechanism of the sum/difference/product of two functions, we can take an even function as a positive number and an odd function as a negative number. For example, x^2 (Even function) × x^3 (Odd function) = Odd function
   x^2 (Even function) + x^3 (Odd function) = Neither odd nor even function
   Remember that no function can be both an even and an odd function simultaneously.

Example 9 Find the nature of the following functions:

(a) $y = f(x) = e^{-x}$

(b) $y = f(x) = x^2 + x^4$

(c) $y = f(x) = \frac{1}{x} + x^3$

(d) $y = f(x) = x^2 + x^3$

(e) $y = f(x) = \log x^2$

(f) $y = f(x) = \log_a x$

Solution (a) Neither odd nor even

(b) Even

(c) Odd

(d) Neither odd nor even

(e) Even

(f) Neither odd nor even

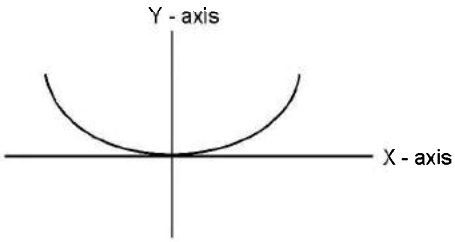
NATURE OF FUNCTION (IN TERMS OF A GRAPH)

For $y = f(x)$

Even Function

The graph of an even function will be symmetrical to Y-axis. It simply means that the graph on the left side of the Y-axis will be the same as the graph on the right side of the Y-axis.

As we have seen in the case of $y = f(x) = x^2$, the graph on the left hand side of the Y-axis is the same in shape and size with respect to the graph on the right hand side of the Y-axis.



Properties of the graph of an even function

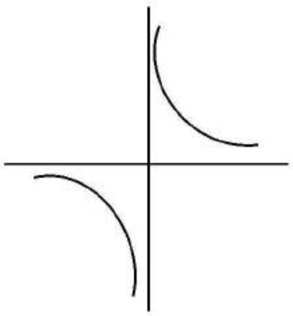
1. Even function graphs are one-fold graphs. i.e., we can make both the parts of the graphs overlap on each other, by just folding the graph paper once.
2. Whenever we fold the paper, we fold it along the axis, either on the X-axis or on the Y-axis.

Odd Function

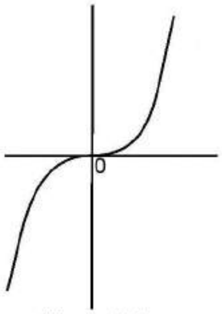
For $y = f(x)$

The graph of an odd function will be symmetrical to the origin. It simply means that the graph on one side of the origin will be same as the graph on the other side of the origin.

As we have seen in the case of $y = f(x) = 1/x$, the graph on one side of the origin is the same in shape and size with respect to the graph on the other side of the origin.



(i) $y = 1/x$



(ii) $y = 1/x^3$

Properties of the graph of an odd function

1. Odd function graphs are two-fold graphs, i.e., we can make both the parts of the graph overlap on each other, by folding the graph paper twice, once along the X-axis and then along the Y-axis.
2. Whenever we fold the paper to overlap the graph, we fold it along the axis, either on the X-axis or on the Y-axis.

Composite Function

Composite function is the kind of function which is composed into one by two or more than two functions.

For example, if $y = f(x) = 5x + 3$ and $y = g(x) = x^2$ is given, then $f(g(x))$ or $g(f(x))$ is the composite function.

In the above example, $f(g(x)) = 5\,g(x) + 3 = 5\,x^2 + 3$

Similarly, $g(f(x)) = [f(x)]^2 = (5x + 3)^2$

Example 10 If $g(x) = \frac{1}{f(x)}$ and $f(x) = x, x \neq 0$ is given. Which of the following is true?

(a) $f(f(f(g(g(f(x)))))) = g(g(f(g(f(x)))))$

(b) $f(f(g(g(g(f(x)))))) = g(g(f(g(f(x)))))$

(c) $f(g(f(g(g(f(g(x))))))) = g(g(f(g(f(x)))))$

(d) $f(f(f(g(g(f(x)))))) = f(f(f(g(f(x)))))$

Solution $f(x) = x$ and $g(x) = 1/x$

Since applying 'f' any number of times is not changing the final result, but applying 'g' is making the value reciprocal. So, LHS can be equal to RHS only if the number of 'g' on both the sides is either even or odd.

So, option (b) is the answer.

Inverse Function

If two functions f and g satisfy $g(f(x)) = x$ for every x in the domain of f and similarly $f(g((x)) = x$ for every x in the domain of g, then f is said to be the inverse of g and vice-versa. It is written as f^{-1} and g^{-1}. For example, $y = \log_e x$ and $y = e^x$ are inverse to each other.

Example 11 $y = f(x) = \dfrac{x+1}{x+2}$. Find $f^{-1}(x)$.

Solution $y = \dfrac{x+1}{x+2}$, or, $y - 1 = \dfrac{x+1}{x+2} - 1 = \dfrac{-1}{x+2}$

Or, $x + 2 = \dfrac{1}{1-y}$, or, $x = \dfrac{1}{1-y} - 2 = \dfrac{2y-1}{1-y}$

$f^{-1}(x) = \dfrac{2x-1}{1-x}$

There lies an important relationship between function and graphs. We will see more application of this in the Chapter: Graphs and Maxima Minima

Example 12 A polynomial $f(x)$ with a real coefficient satisfies the functional equation $f(x).f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)$. If $f(2) = 9$, then $f(4)$ is:

(a) 82 (b) 17

(c) 65 (d) None of these

Solution Assume $f(x) = x^n + 1$ and $f\left(\dfrac{1}{x}\right) = \dfrac{1}{x^n} + 1$

So, $f(x).\ f\left(\dfrac{1}{x}\right) = \left(x^n + 1\right)\left(\dfrac{1}{x^n}\right) + 1$

$= \left(1 + x^n\right) + \left(\dfrac{1}{x^n} + 1\right)$

Now $f(x) = x^n + 1 = 9$

Or, $x^n = 8$, or, $n = 3$

$f(x) = x^3 + 1$

Hence, $f(x) = 4^3 + 1 = 65$

Example 13 Let $f(x)$ be a function such that $f(x + 1) + f(x - 1) = f(x)$ for every real x. Then for what value of p is the relation $f(x + p) = f(x)$ necessarily true for every real x?

(a) 3 (b) 4 (c) 5 (d) 6

Solution $f(x + 1) + f(x - 1) = f(x)$ ...1

Putting $(x + 1)$ at the place of x

or, $f(x + 2) + f(x) = f(x + 1)$ ...2

Adding 1 and 2, we get $f(x + 2) + f(x - 1) = 0$

Similarly, $f(x + 3) + f(x) = 0$

Or, $f(x + 4) + f(x + 1) = 0$

Or, $f(x + 5) + f(x + 2) = 0$

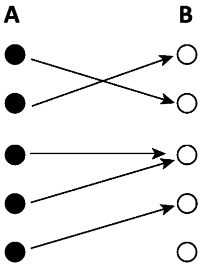
Or, $f(x + 6) + f(x + 3) = 0$

Now replacing $f(x + 3)$ by $-f(x)$, $f(x + 6) + f(x + 3) = f(x + 6) - f(x) = 0$

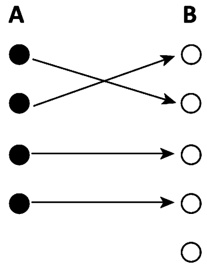
So, $p = 6$

INJECTIVE, SURJECTIVE AND BIJECTIVE FUNCTIONS

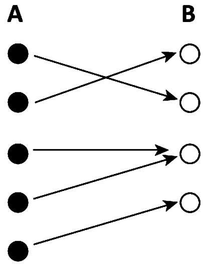
Let us consider a function from A to B and understand the types of function:



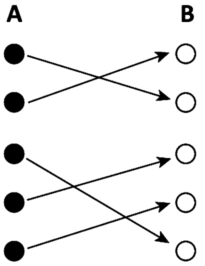
General
Function



Injective
Not Injective



Surjective
Not Surjective



Bijective
(Injective and
Surjective)

(a) General Function – We have already discussed this. Two or more elements of A can have same mapping in B. For example: $y = f(x) = x^2$. In this function, two values of x will point to same value of y. ($x = 2$ or $x = -2$ both will give $y = 4$).

(b) Injective – In this type of functions, each element in A has its unique corresponding mapping in B. B may have some elements without any correspondence in A. For example: $y = f(x) = 3x + 2$

In this case, you won't have two values of A having same mapping in B. So "many to one" is not possible. (One to many is obviously not possible because it's a function).

(c) Surjective – In this type of functions, each element in B should be mapped by one or more than one element in A.

There won't be an element in "B" left out.

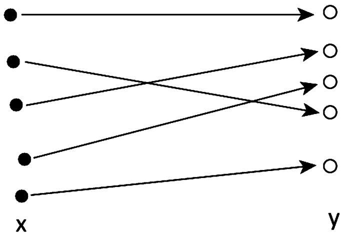
Surjective Functions are also known as Onto function.

(d) Bijective – It means both Injective and Surjective.

So each element of B will be mapped by a unique element in A.

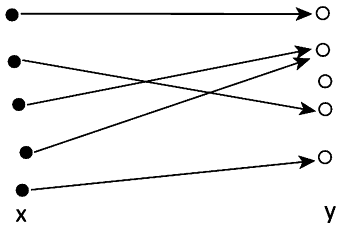
Example 14 Indentify what type of function is this (x to y)?

(a)



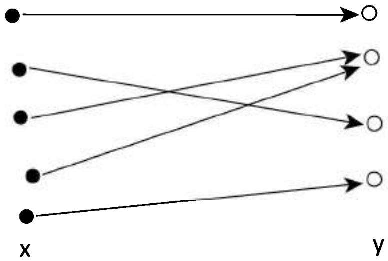
(i) A general function
(ii) Injective
(iii) Surjective
(iv) Bijective

(b)



(i) A general function
(ii) Injective
(iii) Surjective
(iv) Bijective

(c)



(i) A general function
(ii) Injective
(iii) Surjective
(iv) Bijective

Solution

(a) Bijective is the answer.
(b) A general function is the answer because many to one mapping is there.
(c) It cannot be Injective because two X are pointing to single Y. Since no Y is left out, hence Surjective is the answer.

PRACTICE EXERCISES

WARM UP

Q.1. The domain of the function $f(x) = \sqrt{|x|}$ is
(a) $x \varepsilon (-\infty, \infty)$
(b) $x \varepsilon (0, \infty)$
(c) $x \varepsilon (-\infty, 0) \cup (0, \infty)$
(d) $x \varepsilon [0, \infty)$

Q.2. The domain of the function $f(x) = |\sqrt{x}|$ is
(a) $x \varepsilon (-\infty, \infty)$
(b) $x \varepsilon (0, \infty)$
(c) $x \varepsilon [0, \infty)$
(d) None of these

Q.3. The domain of the function $f(x) = \frac{1}{\sqrt{x}}$ is
(a) $-\infty < x < \infty$
(b) $0 < x < \infty$
(c) $-\infty < x < \infty$ excluding $x = 0$
(d) $0 \le x < \infty$

Q.4. The domain of the function $f(x) = \sqrt{x^2}$ is
(a) $-\infty < x < \infty$
(b) $0 < x < \infty$
(c) $-\infty < x < \infty$ excluding $x = 0$
(d) None of these

Q.5. The domain of the function $f(x) = \sqrt{4x-3} + \sqrt{2x-6}$ is

(a) $[0, \infty)$ (b) $\left[\frac{3}{4}, \infty\right]$

(c) $\left[\frac{4}{3}, \infty\right]$ (d) $[3, \infty)$

Q.6. The domain of the function $f(x) = \log(5x - 6 - x^2)$ is

(a) $(5, 6)$ (b) $(2, 3)$

(c) $[2, \infty)$ (d) None of these

Q.7. The domain of $y = \log_x 5$ is

(a) $x > 5$

(b) $(0, \infty)$

(c) $(0, \infty)$ excluding $x = 1$

(d) $[5, \infty)$

Q.8. If $f(x) = \frac{x}{\sqrt{1+x^2}}$, then $f(f(f(x)))$ is

(a) $\frac{3}{\sqrt{1+x^2}}$ (b) $\frac{x}{\sqrt{1+3x^2}}$

(c) $\frac{x^3}{\sqrt{1+x^2}}$ (d) $\frac{x}{\sqrt{3+3x^2}}$

Q.9. Let $f(x) = \frac{x}{x+3}$, then $f(x + 1) =$

(a) $\frac{3x+2}{x+2}$ (b) $\frac{x+1}{x+4}$ (c) $\frac{x+1}{x+3}$ (d) $\frac{2x+3}{x+3}$

Q.10. If $f(x) = \frac{x+1}{x-1}$, $x \neq 1$, find $f(f(f(f)f(2))))$.

(a) 1 (b) 2 (c) 3 (d) 4

Q.11. Let $(x) = \sqrt{x^5}$, then $f(5x)$ is equal to

(a) $5x\sqrt{5x^5}$ (b) $25x\sqrt{5x^5}$

(c) $5x$ (d) None of these

Q.12. If $f(x) = 4x - 5$, $g(x) = x^2$ and $h(x) = \frac{1}{x}$, then $f(g(h(x)))$ is

(a) $\frac{4}{x-5}$ (b) $\frac{1}{(4x-5)^2}$

(c) $\left(\frac{x}{4} - 5\right)$ (d) None of these

Q.13. $f(x) = \text{In}\left(x + \sqrt{x^2+1}\right)$ is

(a) An even function

(b) An odd function

(c) Neither even nor odd

(d) None of these

Q.14. Which of the following is an even function?

(a) $\text{In}\left(\frac{1+x^3}{1-x^3}\right)$ (b) $x\,\frac{e^{ax}-1}{e^{ax}+1}$

(c) $\frac{(1+2^x)^2}{2^x}$ (d) None of these

Q.15. Which of the following function is an even function?

(a) $f(x) = \log\frac{1-x}{1+x}$

(b) $f(x) = x\,\frac{a^x+1}{a^x-1}$

(c) $f(x) = \log\left[x + \sqrt{(1+x^2)}\right]$

(d) $f(x) = \sqrt{(1+x+x^2)} - \sqrt{(1-x+x^2)}$

Q.16. The function $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$ is

(a) Even

(b) Odd

(c) Neither even nor odd

(d) Cannot be determined

Q.17. Given $f(x) = \frac{1}{(1-x)}$, $g(x) = f(f(x))$ and $h(x) = f(f(f(x)))$, then the value of $f(x).\,g(x).\,h(x)$ is:

(a) 0 (b) -1 (c) -2 (d) 2

Q.18. If $f(x) = x^2 + 2$ then $f^{-1}(x)$ is

(a) x^5 (b) $\sqrt{x+2}$

(c) $\sqrt{x} + 2$ (d) None of these

Q.19. If $f(x) = \frac{x}{x+1}$, then $f^{-1}(x)$ is

(a) $\frac{x+1}{x}$ (b) $\frac{x}{1-x}$

(c) $\frac{x-1}{x}$ (d) None of these

Q.20. If $f(x) = \frac{x-1}{x+1}$, then $f^{-1}(x)$ is

(a) $\frac{x+1}{1-x}$ (b) $\frac{x+1}{1-x}$

(c) $\frac{1}{x+1}$ (d) None of these

FOUNDATION

Q.1. Given is $f(x) = x^2 + 4x + 4$ and $g(x) = x^2 + 4x + 3$, then find the values of x such that $f(g(x)) = g(f(x))$.

(a) $x = -1$ (b) $x = -2$

(c) $x = -3$ (d) Not possible

Q.2. For $y = f(x)$

Even functions are defined as $f(-x) = f(x)$ and

Odd functions are defined as $f(-x) = -f(x)$ and

$f(x) = \log_{10}\left(x + \sqrt{x^2+1}\right)$ is

(a) An odd function

(b) An even function

(c) Both even and odd function

(d) Neither odd nor even function

Q.3. A function $f(x)$ is such that $f(x) + f(y) = f(xy)$. Which of the following could be $f(x)$?

(a) a^x (b) $\sqrt{x}$

(c) x^2 (d) $\log ax$

Q.4. $f(x) = 3x^2 - 2x + 4$ and $g(x) = x + 1$. Find $f(g(f(1)))$.

(a) 69 (b) 100

(c) 96 (d) None of these

Q.5. If $f(x) = x^2$ and $f[g(x)] = g[f(x)]$, then which of the following can never be equal to $g(x)$?

(a) $\sqrt{x}$ (b) x^2 (c) x (d) $2x$

Q.6. Let $f(x) = |x - 1|$, then

(a) $f(x^2) = [f(x)]^2$ (b) $f(x+y) = f(x) + f(y)$

(c) $f(|x|) = |f(x)|$ (d) None of these

Direction for questions 7 to 9: Read the passage below and solve the questions based on it.

For any $y = f(x)$

If $f(-x) = f(x)$, then this function is known as an even function.

And if $f(-x) = -f(x)$, then this function is known as an odd function.

Q.7. $f(x) = |x|^3$ is

(a) An odd function

(b) An even function

(c) Neither odd nor even function

(d) Cannot be determined

Q.8. The sum of the two odd functions is always

(a) An odd function

(b) An even function

(c) Neither odd nor even function

(d) Nothing can be said

Q.9. The sum of an odd function and an even function is always

(a) An odd function

(b) An even function

(c) Neither odd nor even function

(d) Nothing can be said

Q.10. $f(x) = c.x + 1$ and $g(x) = 3x + 2$. If $f(g(x)) = g(f(x))$, then what is the value of c?

(a) 1 (b) 2 (c) 3 (d) 4

Q.11. If $f(x) = 2^x - 2^{-x}$, then the value of $2f(x) - 5f(x-1) + 2f(x-2)$ is

(a) 1 (b) −3

(c) 15 (d) None of these

Direction for questions 12 to 14: Read the passage below and solve the questions based on it.

If $(x, y) = x^{y^2}$

$g(x, y) = y^{y^2}$

$h(x, y)$ = Remainder when x is divided by y.

$i(x, y)$ = Quotient when x is divided by y.

Q.12. The value of g[h {f {i (16, 3), 2}, 7}, 3] is

(a) 16 (b) 25 (c) 49 (d) 81

Q.13. The value of $\dfrac{\text{h[g\{i(76,28),2\},3]}}{\text{h[f\{i(16,3),2\},7]}}$ is

(a) 1 (b) $\dfrac{1}{4}$ (c) $\dfrac{1}{3}$ (d) 1/2

Q.14. The value of f[h {g (2,2), 3}, 2] −g[h[f (5,2), 7}, 3] is

(a) −100 (b) −90 (c) −85 (d) −80

Q.15. If $(x) = 2x + 3$ then the value of $f[f\{f(x)\}]$ is

(a) $-6x + 7$ (b) $7x + 8$

(c) $8x + 21$ (d) $9x + 10$

Q.16. If $f(x) = e^x + e^{-x}$, then $f(x)$ is

(a) An odd function

(b) An even function

(c) Neither odd nor even

(d) None of these

Q.17. If $f(x) = \dfrac{x^2 - 2x}{1+x}$, then the value of $f\left(\dfrac{1}{-x}\right)$ is

(a) $\dfrac{x^2-2}{1+2x}$ (b) $\dfrac{1-2x^2}{1+x}$

(c) $\dfrac{1+2x}{x(x-1)}$ (d) None of these

Q.18. If one root of the equation $x^2 - 3ax + f(a) = 0$ is double of the other, then which of the following is equal to $f(x)$?

(a) x^2 (b) $2x$

(c) $2x^2$ (d) $\frac{x^2}{2}$

Q.19. If $f(x, y) = 3x^2 - 2xy - y^2 + 4$, then $f(f(2, 3), f(-1, 1)) = ?$
(a) −68 (b) 95 (c) 251 (d) 232

Q.20. If $y = f(x) = \frac{x+2}{x-1}$, then it is incorrect to say

(a) $x = \frac{y+2}{y-1}$ (b) $f(0) = -2$

(c) $f(1) = 0$ (d) $f(-2) = 0$

Direction for questions 21 and 22: Read the passage below and solve the questions based on it.

A function $f(x)$ is defined as $f(x) = \log(g(x))$ where $g(x)$ is any function of x. Then

Q.21. For which of the values of $g(x)$, can $f(x)$ be expressed as $f1(x) + f2(x)$, where $f1(x)$ and $f2(x)$ are any two functions of x?
(a) $g(x) = e^x$ (b) $g(x) = x^2$
(c) $g(x) = \log(x)$ (d) None of these

Q.22. For what value of $g(x)$, $g(x) = g(f(x))$?
(a) e (b) $\log x$ (c) e^x (d) None of these

Q.23. For what value of $g(x)$ is the function $f(x) = g(x)$?
(a) e^{-1} (b) $\log x^2$
(c) e^{2x} (d) None of these

Q.24. If $f(t) = \frac{t^3+2t}{t-1}$, then $\frac{f(t) + f(-t)}{f(t) - f(-t)}$ is equal to

(a) t^3 (b) $\frac{1}{t}$ (c) t (d) t^2

Q.25. If $f(x) = 2x^2 + 6x - 1$ and $g(x) = |x + 5|$ then the value of $f[g\{f(g(-6))\}]$ is
(a) 225 (b) 249 (c) 341 (d) 359

Q.26. If $f(x) = x^3 + 1$ and the value of the expression $f(2x) - 2f(x) = 383$, the value of x is
(a) 1 (b) 0 (c) (d) (d) 6

Direction for questions 27 and 28: Read the passage below and solve the questions based on it.

$F(x) = 0$ when $x = 1$
$= 1$ when $x = 0$
$= 0.5$ when $x = 0.5$
$g(x) = 1$ when $x = 1$
$= 0.5$ when $x = 0$
$= 0$ when $x = 0.5$

Q.27. If $f[g[f[g(t)]]] = 0$, the value of t is
(a) 1 (b) 0
(c) 0.5 (d) None of these

Q.28. If $f[g(t) + f(t)] + g[g(t)] = 1$, the value of t is
(a) 1 (b) 0
(c) 0.5 (d) None of these

Q.29. If $f(x) = x^2$ and $h(x) = \sqrt{x}$, then
i. h of (−2) = 4
ii. h of (4) = 4
iii. h of (3) = 6
(a) i is correct but other two are incorrect
(b) ii is correct but other two are incorrect
(c) i and iii are correct but ii is incorrect
(d) None of the above is true

Q.30. Let $h(x) = \frac{1-x}{1+x}$, find the domain of $h^{-1}(x)$.

i. $(-\infty, +\infty)$ ii. $(-\infty, -1)$
iii. $(1, +\infty)$ iv. $[-1, +1]$
(a) i alone is correct
(b) ii ∪ iii are correct, where ∪ represents union of the sets.
(c) ii ∪ iii ∪ iv are correct, where ∪ represents union of the sets.
(d) None of these

Q.31. If $f(n) = 2n^2 - n$ for $n > 5$, $= n^2 + n$ for n 5, then $f(1) + f(2) + \ldots + f(10)$ equals
(a) 705 (b) 440 (c) 690 (d) 715

Q.32. If $y = f(x) = (ax + b)/(cx - a)$, then which of the following is equal to $f(y)$?
(a) x (b) $2x$ (c) $x/2$ (d) x^2

Q.33. If $y = f(x) = (x + 2)/(x - 1)$, then which of the following is equal to $f(y)$?
(a) x (b) $2x$ (c) $x/2$ (d) x^2

Q.34. If $f(x) = \frac{x}{x-1}$, then find the value of $\frac{f\left(\frac{a}{b}\right)}{f\left(\frac{b}{a}\right)}$.

(a) a/b (b) $-b/a$ (c) $-a/b$ (d) b/a

Q.35. Find the domain of the function $y = f(x)$, which is defined as $f(x)\frac{1}{\sqrt{|x|+x}}$.
(a) $(-\infty, +\infty)$ (b) $(0, +\infty)$
(c) $[0, +\infty)$ (d) $[1, +\infty)$

Q.36. Find the domain of the function $y = f(x)$, which is defined as $f(x) = \frac{1}{\sqrt{\{x-[x]\}}}$ $[x]$ is the greatest integer function.
(a) x is any real number other than integers
(b) Any real value of x
(c) All the natural numbers
(d) None of these

Q.37. Are these two functions identical?
$f(x) = \log(x-2) + \log(x-3)$
and $g(x) = \log(x-2)(x-3)$

(a) Yes
(b) No
(c) Cannot be determined
(d) None of these

Q.38. Are these two functions identical?
$f(x) = 2 \log x$ and $g(x) = \log x^2$.
(a) Yes
(b) No
(c) Cannot be determined
(d) None of these

Q.39. The domain of the function $f(x) = \log_{10}\log_{10}(1+x^3)$ is
(a) $(-1,+\infty)$ (b) $(0,+\infty)$
(c) $(0,+\infty)$ (d) $(-1, 0)$

Q.40. The set of real values of x satisfying $|x - 1| \le 3$ and $|x - 1| \ge 1$ is
(a) $[2, 4]$
(b) $(-\infty, 2) \cup [4, +\infty]$
(c) $[-2, 0] \cup [2, 4]$
(d) None of these

MODERATE

Q.1. If $b = f(a)$ and $f(a) = \dfrac{a-1}{a+1}$ which of the following is true?
(a) $f(2a) = f(a) + 1$
(b) $f\left(\dfrac{1}{a}\right) = -f(a)$
(c) $a = f(b) + f\left(\dfrac{1}{a}\right)$
(d) $a = f(b)$

Q.2. Let $f(x)$ be defined on $[-1, 1]$ in such a way that the area of the equilateral D with two of its vertices at $(0, 0)$ and $\{x, f(x)\}$ is $\dfrac{\sqrt{3}}{4}$ sq. units. Which of the following can be $f(x)$?
1. $\sqrt{1-x^2}$ **2.** $-\sqrt{1-x^2}$
(a) 1 is correct but 2 is incorrect
(b) 2 is correct but 1 is incorrect
(c) Both 1 and 2 are correct
(d) Neither 1 nor 2 is correct

Direction for questions 3 and 4: Read the passage below and solve the questions based on it.

$[x]$ = Greatest integer less than or equal to x.
$\{x\}$ = Smallest integer more than or equal to x.

Q.3. Given that 'x' is not an integer, then what is the value of $[x] - \{x\}$?
(a) -1
(b) 1
(c) 0
(d) Depends upon the value of x

Q.4. Given that 'x' is not an integer, then $[x] + \{x\}$ is
(a) An even integer (b) An odd integer
(c) Positive number (d) Non positive integer

Q.5. Let $f(n) = \left[\dfrac{1}{2} + \dfrac{n}{100}\right]$ where $[x]$ denotes the integral part of x. Then the value of $\sum_{n=1}^{100} f(n)$ is
(a) 50 (b) 51
(c) 1 (d) None of these

Q.6. If $0 < x < 1000$ and $\left[\dfrac{x}{2}\right] + \left[\dfrac{x}{3}\right] + \left[\dfrac{x}{5}\right] = \dfrac{31}{30}x$, where $[x]$ is the greatest integer less than or equal to x, the number of possible values of x is
(a) 34 (b) 32
(c) 33 (d) None of these

Q.7. If $f(x) = 1/x$, $x \ne 0$; $f^{n+1}(x) = f^{n}(f(x))$, find the value of the product $f^{13}(1) \times f^{15}(1) \times f^{19}(1)\, f^{21}(1)$.
(a) 34 (b) 16 (c) 1 (d) 17

Q.8. The set of real values of x satisfying $||x-1|-1| \le 1$ is:
(a) $[-1, 3]$ (b) $[0, 2]$
(c) $[-1, 1]$ (d) None of these

Q.9. A function is defined as $f^{n}(x) = f\{f^{n-k}(x)\}$
If n is odd, $k = 1$
If n is even, $k = 2$ and $f^{0}(x) = 1$.
Find $f^{7}(x)$ at $x = 1$, if $f(x) = x^2 + 1$.
(a) 676 (b) 677 (c) 577 (d) 1076

Q.10. $(x^2 - 1)$ is a factor of $f(x) = (x^5 + ax^4 + bx^3 + cx^2 + x + d)$. The graph of $f(x)$ intersects Y axis at $(0, -3)$. Find the value of $(a + c)$.
(a) 0 (b) 3 (c) -3 (d) -1

Q.11. Let $f(x) = ax^2 + bx + c$, where a, b and c are rational, and $f: z \to z$, where z is the set of integers. Which of the following best describes the value of $a + b$?
(a) A negative integer
(b) An integer
(c) Non integral rational number
(d) None of these

Q.12. If $f(x+1)+f(x-1)=2f(x)$ and $f(0)=0$, then $f(n)$, $n \in \mathrm{N}$ is

(a) $nf(1)$ (b) $\{f(1)\}^n$

(c) 0 (d) None of these

Q.13. Let $f(x) = 1 + |x|$, $x < -1$

$[x]$, $x \geq -1$, where [.] denotes the greatest integer function. Then

$f\{f(-2.3)\}$ is equal to

(a) 4 (b) 2 (c) –3 (d) 3

Q.14. The largest set of real values of x for which $f(x)$ $= \sqrt{(x+2)(5-x)} - \frac{1}{\sqrt{x^2-4}}$ is a real function is

(a) $[1, 2] \cup [2, 5]$ (b) $[2, 5]$

(c) $[3, 4]$ (d) None of these

Q.15. Let a, b and c be fixed positive real numbers. Let $f(x)$ $= \frac{ax}{b+cx}$ for $x \geq 1$. Then as x increases,

(a) $f(x)$ increases

(b) $f(x)$ decreases

(c) $f(x)$ increases first and then decreases

(d) None of these

Q.16. For any real number x, let (x) denote the largest integer less than or equal to x and $<x> = x - [x]$, that is, the fractional part of x. For arbitrary real numbers x, y and z, only one of the following statements is correct. Which one is it?

(a) $[x+y+z] = [x]+[y]+[z]$

(b) $[x+y+z] = [x+y]+[z] = [x]+[y+z]+[y]$

(c) $<x+y+z> = y+z-[z+z]+<x>$

(d) $[x+y+z] = [x+y]+[z+<y+x>]$

Q.17. The domain of the function $f(x) = {}^{16-x}\mathrm{C}_{2x-1} + {}^{20-3x}\mathrm{P}_{4x-5,}$ where the symbols have their usual meanings, is the set

(a) $[1, 2, 3, 4, 5]$ (b) $\{2, 3, 4\}$

(c) $\{2, 3\}$ (d) None of these

Q.18. The domain of the real valued function $f(x) = \log_e |\log_e x|$ is

(a) $(1, +\infty)$ (b) $(0, +\infty)$

(c) $(e, +\infty)$ (d) None of these

Q.19. The domain of the function $f(x) = \log_e(x-[x])$, where [.] denotes the greatest integer function, is

(a) R (b) R–Z

(c) $(0,+\infty)$ (d) None of these

Q.20. Let $f(x) = \log_{x^2} 25$ and $g(x) = \log_x 5$, then $f(x) = g(x)$ holds for x belonging to

(a) All the real values

(b) $(0, 1) \cup (2, +\infty)$

(c) ϕ

(d) None of these

Q.21. Let $f(x) = |x-2|+|x-3|+|x-4|$ and $g(x) = f(x+1)$. Then

(a) $g(x)$ is an even function

(b) $g(x)$ is an odd function

(c) $g(x)$ is neither even nor odd

(d) None of these

Q.22. If $f(x) = x^n$, $n \in \mathrm{N}$ and g of $(x) = n\,g(x)$, then $g(x)$ can be

(a) $n|x|$ (b) $3.\sqrt[3]{x}$ (c) e^x (d) $\log|x|$

Q.23. The inverse function of the function $f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$ is

(a) $\frac{1}{2}\log\frac{1+x}{1-x}$ (b) $\frac{1}{2}\log\frac{2+x}{2-x}$

(c) $\frac{1}{2}\log\frac{1-x}{1+x}$ (d) None of these

Q.24. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$. Then

(a) $f(x+2) = f(x-2)$

(b) $f(2+x) = f(2-x)$

(c) $f(x) = f(-x)$

(d) None of these

Q.25. If $f(x) = \log\frac{(1+x)}{1-x}$ and $g(x) = \frac{3x+x^3}{1+3x^2}$, then $f[g(x)] =$

(a) $[f(x)]^3$ (b) $-f(x)$

(c) $[f^3(x)]$ (d) $3[f(x)]$

Q.26. If $f(x) = 2x^2 + 3x + 4$ and $g(x) = 5 - x^2$, the min $[f(x)]$ – max $[g(x)]$ is

(a) 0 (b) 5 (c) $\frac{17}{8}$ (d) $-\frac{17}{8}$

Q.27. If $f(x) = x^3 - x^2 + x - f(x)$, where x is a whole number less than or equal to 15. For how many values of x, $f(x)$ will not be a whole number?

(a) 7 (b) 8

(c) 5 (d) None of these

Q.28. For any real number x, define Mag (x) = Maximum $(x, -x)$ and Nag (x)=Minimum $(x, -x)$. What is the value of Mag (8, (Nag (4, (Nag (7,–7))))) – (Mag (6,(Nag (7,–7)))?

(a) 7 (b) 0 (c) 2 (d) – 14

Q.29. If $f(x) = \frac{4^x}{4^x+2}$; find the value of

$f\left(\frac{1}{1999}\right) + f\left(\frac{2}{1999}\right) + \ldots + f\left(\frac{1998}{1999}\right)$.

(a) 1998 (b) 1999 (c) 998 (d) 999

Q.30. If $f(x) = 2x^2+6x-1$, then the value of

$$\frac{f\left(\frac{3}{4}\right)+1}{f\left(\frac{3}{4}\right)-1} \text{ is}$$

(a) 11/13 (b) 35/3
(c) 45/29 (d) None of these

Q.31. Let $\{x\}$ and $[x]$ denote the fractional and integral part of a real number x respectively. Solve $4\{x\} = x + [x]$.

(a) 2/3 (b) 4/7 (c) 8/3 (d) 5/3

Direction for questions 32 and 33: Read the passage below and solve the questions based on it.

Consider a function $f(n)$ defined for non-negative integer values of n.

$f(n) = (-1)^{S_{n+1}} + (-1)^{S_n} + f(n-1)$, where $f(0) = 1$ and $si = 1 + 2 + 3 + 4 + \ldots\ldots\ldots + i$, for all $i \in N$.

Q.32. Find $f(111)$.

(a) 1 (b) –1 (c) 2 (d) 0

Q.33. Evaluate $f(1) + f(2) + f(3) + \ldots f(25)$.

(a) –26 (b) –24
(c) –22 (d) None of these

Q.34. Let f be the greatest integer function and g be the modulus function then what is the value of $(gof)\left(-\frac{5}{3}\right)+(fog)\left(-\frac{5}{3}\right)$?

(a) 3/5 (b) 11/3 (c) 2/7 (d) 3

Q.35. If $f(x) = x^2 + x + 7$ then find $\frac{f(\alpha)+f(\beta)}{\alpha-\beta}$, $\alpha \neq \beta$.

(a) $\alpha^2 + \beta^2 - 2$ (b) $\alpha + \beta + 1$
(c) $\alpha + \beta + \beta^2$ (d) $\alpha + \beta - 1$

Q.36. Let $y = f(x) = \log_a x$ and $a > 1$. Then only one of the following statements is false. Which one is it?

(a) If $x = 1$, then $y = 0$
(b) If $x < 1$, then $y < 0$
(c) If $x = ½$, then $y = ½$
(d) If $x = 1$, then $y = 1$

Q.37. If $(x) = \log \frac{1+x}{1-x}$, then $f(x) + f(y)$ is

(a) $f(x+y)$ (b) $f\left(\frac{x+y}{1+xy}\right)$
(c) $(x+y)f\left(\frac{1}{1+xy}\right)$ (d) None of these

Q.38. If $f(x, y) = |x+y|$ and $g(x, y) = |x-y|$, how many ordered pairs of the form (x, y) would satisfy $f(x, y) = g(x, y)$?

(a) 1 (b) 2
(c) 4 (d) Infinitely many

Q.39. $f(x) = y + \frac{1}{y}$, where $y > 0$. If y increases in value, then $f(x)$

(a) Fluctuates up and down in value
(b) Decreases in values
(c) Increases in value but at a much higher rate than y
(d) Increases in value and tends towards y

Q.40. A function H is defined for all the positive integers that satisfy the following condition:

$H(1) + H(2) + H(3) + \ldots + H(x) = x^2 H(x)$

If $H(1) = 2006$, then find the value of $H(2005)$.

(a) $\frac{2}{2005}$ (b) $\frac{2}{2006}$
(c) $\frac{2}{2006!}$ (d) $\frac{2}{2005!}$

ADVANCED

Q.1. If $a<b<c<d<e$ and $f(x) = (x-a)^2 (x-b) (x-c) (x-d) (x-e)$ which of the following is true?

(a) $f(x)>0$, for $(x<a)$ as well as for $(d<x<e)$
(b) $f(x)<0$, for $(a<x<b)$ as well as for $(d<x<e)$
(c) $f(x)<0$, for $(b<x<c)$ as well as for $(d<x<e)$
(d) None of these

Q.2. A function $f(x)$ is defined on the interval $0 \leq x \leq 2$ as $f(x)$ = integer closest to x, if $x \neq 0.5$ or 1.5. $f(0.5) = 1$ $f(1.5) = 2$

Find the area under the graph of this function above the x–axis and the lines $x = 0$ and $x = 2$.

(a) 2.5 sq. units (b) 1.5 sq. units
(c) 4 sq. units (d) None of these

Q.3. $[x]$ is defined as the greatest integer less than or equal to x and $\{x\}$ is defined as the least integer greater than or equal to x, for all real values of x. Consider the four statements.

1. $[x] + [-x] = -1$, if x is not an integer.
2. $\{x\} + [-x] = 0$
3. $\{[x]\} - [\{x\}] = -1$
4. $[2x] + \{3x\} \le 5x$

How many of the above statements are always true for all the real values of x?

(a) 1 (b) 2 (c) 3 (d) 4

Q.4. If $f(x) = \dfrac{x-1}{x+1}$ then $f(ax)$ in terms of $f(x)$ is equal to

(a) $\dfrac{f(x)+a}{1+af(x)}$ (b) $\dfrac{(a-1)f(x)+a+1}{(a+1)f(x)+a-1}$

(c) $\dfrac{(a+1)f(x)+a-1}{(a-1)f(x)+a+1}$ (d) None of these

Q.5. Let $f(1) = 1$ and $f(n) = \sum_{r=1}^{n-1} f(r)$. Then $\sum_{n=1}^{m} f(n)$ is equal to

(a) 3^{m-1} (b) 3 m

(c) 3^{m-1} (d) None of these

Direction for questions 6 and 7: Read the passage below and solve the question based on it.

A formula is given by $a(n)=a(a(n-1))+a(n-a(n-1))$, where n is any integer greater than 2.

Q.6. If it is known that $a(1) = 1$ and $a(2) = 1$, find the value of $a(4)$.

(a) 1 (b) 2 (c) 3 (d) 4

Q.7. If it is known that $a(2) = 1$ and $a(3) = 2$, find the value of $a(1)$.

(a) −1 (b) 0

(c) 1 (d) Cannot be determined

Q.8. Which of the following functions satisfies the condition $f\left(\dfrac{x+y}{x-y}\right) = \dfrac{f(x)+f(y)}{f(x)-f(y)}$?

(a) $f(x) = x$ (b) $f(x) = ax + b$

(c) $f(x) = 2^x$ (d) $f(x) = x^2$

Q.9. [N] denotes the largest integer less than or equal to N. V is a positive integer and $\left[\dfrac{V}{5}\right] - \left[\dfrac{V}{7}\right] = 1$. If the minimum value of V is r and the maximum value of V is s, then the value of r + s is equal to

(a) 40 (b) 33

(c) 35 (d) 34

Q.10. $f(x) = |x| + |y|$

$g(x) = \max(x + y, x - y)$

$h(x) = \min(x + y, x - y)$

i. $g(x) \ge f(x)$

ii. $g(x) + h(x) \ge f(x)$

iii. $g(x) > f(x)$

Which of the following are not necessarily true?

(a) i and ii (b) i and ii

(c) ii and iii (d) i, ii and iii

Direction for questions 11 and 12: Read the instructions given below and solve the questions based on it.

Following are the certain functions:

H(a, b, c) = Greatest common divisor of a, b and c

L(a, b, c) = Least common multiple of a, b and c

A(a, b, c) = Average of a, b, and c

Min(a, b, c) = Smallest value among a, b and c

Max(a, b, c) = Largest among a, b and c.

Q.11. Which of the following is true? (a, b and c are distinctly different positive numbers)

(a) H(a, b, c)xL(a, b, c) = abc

(b) H(a, b, c)>A(a, b, c)

(c) H(a, b, c)>Min(a, b, c)

(d) H(a, b, c)<A(a, b, c)<L(a, b, c)

Q.12. If Max (a, b, c) = Min(a, b, c), then

(a) A (a, b, c) = H(a, b, c)

(b) A(a, b, c) = L(a, b, c)

(c) A(a, b, c) = Min(a, b, c)

(d) All of these

Q.13. If the area of a circle increases at a uniform rate then the rate of increase of the perimeter is

(a) Directly proportional to the radius

(b) Inversely proportional to the radius

(c) Directly proportional to the square of the radius

(d) Inversely proportional to the square of the radius

Q.14. If $f(x) = 3^x + 4^x + 5^x - 6^x$, then $f(x) < f(3)$ for

(a) Only one value of x

(b) No value of x

(c) Only two values of x

(d) Infinitely many values of x

Direction for questions 15 and 16: Read the passage below and solve the questions based on it.

For a natural number x, if x has 2 or more than 2 digits, then $f(x)$ = the sum of all digits of x and $f(x) = x$, when x has only one digit. Also given that $f^n(x) = f\{f^{n-1}(x)\}$, where $n \ge 2$.

Q.15. If $x = 123456^{654321}$, then $f^{\infty}(x)$ is equal to

(a) 9 (b) 3 (c) 1 (d) 7

Q.16. If $x = 7^{183}$, then $f^{\propto}(x)$ is equal to
(a) 7 (b) 4 (c) 1 (d) 9

Direction for questions 17 and 18: Read the passage below and solve the questions based on it.

Following is a standard example of iterative function.

```
STEP 1    X = 0
STEP 2    Y = 2
STEP 3    If X < Y, X = X * Y + 1
          Else if X > Y, Y = Y * Y
          Else stop
STEP 4    Z = X * Y + X
STEP 5    If Z = 15 stop
          Else Go To STEP 3
```

Q.17. How many times (iteration) will the process run?
(a) Once (b) Twice
(c) Thrice (d) None of these

Q.18. What will happen to the process if we make the following change in STEP 2?
Y = 1.
(a) Process will run ten times
(b) Process will run thrice
(c) Process will run infinite times
(d) Process will run once

Direction for questions 19 and 20: Read the passage below and solve the questions based on it.

A function f is defined for $x, y \in$ such that
$f(0, y) = y^2 - 1$
$f(x + 1, y) = f(x, y + 4)$

Q.19. Find the value of $f(7, 3)$.
(a) 728 (b) 840
(c) 960 (d) Cannot be determined

Q.20. Which of the following is true?
(a) $f(8, 4) = f(0, 32)$ (b) $f(0, 0) = f(-5, 20)$
(c) $f(-2, 9) = f(3, 29)$ (d) All of these

Direction for questions 21 and 22: Read the passage below and solve the questions based on it.

V(x) denotes the absolute value of x, $mn(x, y)$ denotes the minimum of x and y, $mx(x,y)$ denotes the maximum of x and y.

Q.21. Find mn [V (mx (x, y)), mx (z, V (mn(x, z)))]; for $x = -5$, $y = 3$, $z = -2$.
(a) 2 (b) 3 (c) 1 (d) 4

Q.22. When would $mn[x, mx(x, z)]$ be equal to $mx[z, mn(x, z)]$?
(a) $x = 3z$ (b) $x = \frac{1}{2}z$
(c) $x = -z$ (d) $x = z$

Q.23. The following computer programme is a very standard example of Iterative function:
A computer performs a certain algorithm to compute an output S for an input N, with both S and N being real values. The algorithm is as follows.

```
Step 1 :  S = 0.
Step 2 :  If (N ≤ –1 or 1 ≤ N), then S = S + 1 else go
          to step 5.
Step 3 :  N = N ÷ 10
Step 4 :  Go to step 2.
Step 5 :  Print S.
```

Which of the following best describes the algorithm?
(a) It calculates the highest power of 10 that can perfectly divide N
(b) It calculates the number of digits in N
(c) It calculates the number of zeros on the extreme right of N
(d) It calculates the number of digits to the left of the decimal point in N

Q.24. If A = $\{x \mid f(x) = 0\}$ and B = $\{x \mid g(x) = 0\}$, then A ∩ B will be the set of roots of the equation
(a) $[f(x)]^2 + [g(x)]^2 = 0$
(b) $[f(x)] + [g(x)] = 0$
(c) $[f(x)]^2 - [g(x)]^2 = 0$
(d) $[f(x)] - [g(x)] = 0$

Q.25. If $f(x, y) = 2x^2 + 4xy - y^2$, then what is the value of $f(x + 1, y - 1)$?
(a) $2y^2 + 4xy - x^2 - 6y + 3$
(b) $2x^2 - 4xy + y^2 + 6y - 3$
(c) $2y^2 + 4xy - x^2 + 6y - 3$
(d) $2x^2 - 2xy + 2y^2 - 6y + 3$

Direction for questions 26 to 29: Read the passage below and solve the questions based on it.

If P (x, y, z) = max (x, y) + min (y, z)
Q (x, y, z) = max (y, z) + min (x, y)
R (x, y, z) = max (x, y, z)
S (x, y, z) = min (x, y, z)
T (x, y, z) = max (x, z) – min (y, z)
U (x, y, z) = max (y, z) – min (x, y)

Q.26. For, $x = 1, y = 2, z = 3$ which of the following is equal to 1?

(a) $\dfrac{p(x,y,z)+s(x,y,z)}{2\times U(x,y,z)}$

(b) $\dfrac{Q(x,y,z)+S(x,y,z)}{2\times T(x,y,z)}$

(c) $\dfrac{Q(x,y,z)+U(x,y,z)}{2\times R(x,y,z)}$

(d) $\dfrac{R(x,y,z)+S(x,y,z)}{2\times P(x,y,z)}$

Q.27. For $x=1, y=2, z=3$, which of the following is greater than 1?

(a) $\dfrac{P(x,y,z)}{2\times S(x,y,z)}$ (b) $\dfrac{U(x,y,z)}{Q(x,y,z)}$

(c) $\dfrac{S(x,y,z)}{R(x,y,z)}$ (d) $\dfrac{T(x,y,z)}{R(x,y,z)}$

Q.28. For $x = 1, y = 2, z = 3$, which of the following is equal to zero?

(a) $P(x, y, z) + Q(x, y, z)$

(b) $Q(x, y, z) - R(x, y, z) - S(x, y, z)$

(c) $R(x, y, z) - S(x, y, z) - T(x, y, z)$

(d) $T(x, y, z) - U(x, y, z) - S(x, y, z)$

Q.29. For $x = 1, y = 2, z = 3$, which of the following is less than 1?

(a) $\dfrac{Q(x,y,z)}{R(x,y,z)+S(x,y,z)}$

(b) $\dfrac{Q(x,y,z)-U(x,y,z)}{2S(x,y,z)}$

(c) $\dfrac{Q(x,y,z)+U(x,y,z)}{R(x,y,z)+T(x,y,z)}$

(d) $\dfrac{U(x,y,z)}{R(x,y,z)+S(x,y,z)}$

Q.30. $f(x) = x^3 + ax^2 + bx + c$ and $g(x) = x^2 + 1$, where a, b and c belongs to $\{1, 2, 3, \ldots, 10\}$. What are the number of polynomials which are divisible by $g(x)$?

(a) 0 (b) 1 (c) 10 (d) 100

Direction for questions 31 to 34: Read the passage below and solve the questions based on it.

The pages of a book are numbered 0, 1, 2, ... P, P>0. There are four categories of instructions that direct a person in positioning the pages in the book. The instruction types with their meanings are given below:

i. Open Position of the book at page number 1.
ii. Close Position of the book at page number 0.
iii. Forward, n From the current page, move forward by n pages. If, in this process, page number P is reached, stop at P.
iv. Backward, n From the current page, move backward by n pages. If, in this process, page number 0 is reached, stop at page number 0.

In each of the following questions, you will find a sequence of instructions. Let p1 be the page number before the instructions are given and p2 be the page number after executing all the instructions given.

Q.31. Forward, 30; Backward, 12. Which of the following statements is true?

(a) p1 = p2, if P = 12 and p1 = 0

(b) P = 25, provided p1 > 0

(c) p1 > 35, provided P = 100

(d) p1 = 40, provided P = 30

Q.32. Backward, 7; Forward, 7. Which of the following statements is true?

(a) p1 = p2, if p1 ≥ 7

(b) p1 = p2, if p1 > 0

(c) p2 =7, provided P>0

(d) p1>p2, provided P>0

Q.33. Forward, 15; Forward, 15. Which of the following statements is true?

(a) p2 – p1 = 30, only if p1= 0

(b) p2 – p1 = 30, only if p1= 1 and P > 30

(c) p2 – p1 = 15, only if p1= 0 and P = 31

(d) p2 > p1, if P > 0

Q.34. Forward, 5; Backward,4. Which of the following statements is true?

(a) p2 = p1 + 4 provided 1 < p1<7

(b) p2 = p1, if P<6

(c) p2 = p1 + 1, if P – p1 ≥5

(d) p2 – p1 < 0, if P>0

Q.35. If $f(x) = a^{1+x}$, then $f(x).\,f(y).\,f(z) =$

(a) $f(x + y + z)$ (b) $f(x + y + z + 1)$

(c) $f(x + y + z + 2)$ (d) $f(x + y + z + 3)$

Q.36. A polynomial function $f(x)$ satisfies $f(x)\, f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)$. If $f(10) = 1001$, then what is the value of $f(20)$?

(a) 2002 (b) 8004

(c) 8001 (d) None of these

Q.37. The function $f(x) = \max\{(1-x), (1+x), 2\}$, where x can take any real value, is equivalent to

(a) $f(x) = \begin{array}{ll} 1-x & x\le -1 \\ 2 & -1\langle x\langle 1 \\ 1+x & x\ge 1 \end{array}$

(b) $f(x) = \begin{array}{ll} 1+x & x\le -1 \\ 2 & -1\langle x\langle 1 \\ 1-x & x\ge 1 \end{array}$

(c) $f(x) = \begin{array}{ll} 1-x & x\le -1 \\ 1 & -1\langle x\langle 1 \\ 1+x & x\ge 1 \end{array}$

(d) None of these

Q.38. The equivalent definition of $f(x) = ||x|-1|$

(a) $f(x) = \begin{cases} x+1 & x \geq 0 \\ x+1 & x \leq 0 \end{cases}$

(b) $f(x) = \begin{cases} -x-1 & x \leq -1 \\ x+1 & -1 \langle x \leq 0 \\ x-1 & 0 \leq x \leq 1 \\ x+1 & x \geq 1 \end{cases}$

(c) $f(x) = \begin{cases} -x-1 & x \leq -1 \\ x+1 & -1 \langle x \leq 0 \\ 1-x & 0 \leq x \leq 1 \\ x-1 & x \geq 1 \end{cases}$

(d) None of these

Q.39. If $f(x) = ||x|-1|$, f of (x) is

(a) $f(x) = \begin{cases} |x|-2, & |x| \geq 2 \\ 2-|x| & 1 \langle |x| \langle 2 \\ |x| & |x| \leq 1 \end{cases}$

(b) $f(x) = \begin{cases} |x|+2, & |x| \geq 2 \\ |x|-2 & 1 \leq |x| \leq 2 \\ |x| & |x| \leq 2 \end{cases}$

(c) $f(x) = \begin{cases} |x|-2, & |x| \geq 2 \\ |x|+2 & 1 \leq |x| \leq 2 \\ |x| & |x| \leq 1 \end{cases}$

(d) None of these

Q.40. If $f(x)$ is the greatest integer less than or equal to $x/2$, how many of the following statements are true?

1. $f(x)$ is half the least even integer greater than or equal to x
2. . $f(x)$ is the least integer greater than or equal to $\frac{x}{2}$ -1.
3. $f(x)$ is one less than half the least even integer greater than x.

(a) 0 (b) 1 (c) 2 (d) 3

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(b)	(a)	(d)	(b)	(c)	(b)	(b)	(c)	(d)	(d)	(b)	(a)	(b)	(a)	(b)	(d)	(b)	(a)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(d)	(b)	(d)	(d)	(b)	(a)	(c)	(b)	(d)	(d)	(d)	(d)	(c)	(b)	(c)	(c)	(b)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(f)	(c)	(d)	(c)	(c)	(a)	(b)	(b)	(c)	(c)	(a)	(c)	(b)	(a)	(b)	(a)	(b)	(c)

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(a)	(b)	(a)	(c)	(c)	(a)	(b)	(b)	(b)	(a)	(d)	(b)	(a)	(d)	(c)	(d)	(b)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(d)	(a)	(b)	(d)	(d)	(b)	(c)	(d)	(d)	(d)	(a)	(d)	(d)	(b)	(c)	(b)	(d)	(d)	(a)

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(d)	(c)	(b)	(b)	(c)	(a)	(d)	(d)	(d)	(d)	(a)	(d)	(a)	(c)	(c)	(d)	(c)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(d)	(a)	(a)	(c)	(c)	(a)	(b)	(d)	(c)	(a)	(a)	(b)	(c)	(c)	(c)	(a)	(c)	(a)	(d)

HINTS AND SOLUTIONS

Moderate

1. Go thorough the options.

[redacted]

$[x]$ = Greatest integer less than or equal to x
$\{x\}$ = Smallest integer more than or equal to x
Take x = P such that P = I + F, where I is the integral and F(F > 0) is the fractional part of the number P(P cannot be an integer).
$[x] = [I + F] = I$ and similarly $\{x\} = \{I + F\}$
$= I + 1$

3. Hence, $[x] - \{x\} = -1$

4. $[x] + \{x\} = I + I + 1 = 2I + 1$ = Odd integer
Alternatively, we can do both the problems by assuming any non-integral value of x (like 3.5 or 4.2 or -2.6 etc.)

5. For all the values of $n < 50$, $f(n) = 0$
And for all the $n \geq 50$, $f(n) = 1$. Hence, 51 such values are there.

6. $x/2 + x/3 + x/5 = 31x/30$
It means $[x/2] = x/2$, $[x/3] = x/3$ and $[x/5] = x/5$
Now $[N] = N$ is possible only if N is an integer.
Hence, $x/2$, $x/3$ and $x/5$ are integers. So, x is divisible by 2, 3 and 5. Or, x is divisible by 30.
Total 33 values are possible.

7. We have to find out some powers of $f(1)$, which is in all the cases equal to 1

10. $x = 1$ and $x = -1$ are two roots of $f(x)$. Similarly, if we put $x = 0$, then $f(x) = -3$. Now solve the equation to get the value of $a + c$.

13. $f(x) = 1 + |x|, x < -1$ and $[x], x \geq -1$,
So, $f(-2.3) = 1 + |-2.3| = 1 + 2.3 = 3.3$
$f\{f(-2.3)\} = f(3.3) = [3.3] = 3$

14. Expression under square root should be non-negative and the denominator cannot be equal to zero. Now find the common values.
Alternatively, we can do this problem by using options.

15. For any increase in the value of x, increase in the numerator will be more than the increase in the denominator. This can be verified through taking a few values of x. Alternatively, this can be verified through plotting the graphs. The slope of the graph of the numerator will be more than the slope of the graph in the denominator.
Hence, $f(x)$ will increase.

16. Assume some values of x, y and z and then go through options.

17. For ${}^{16-x}C_{2x-1}$ to be defined, $(16 - x) \geq (2x - 1)$ and similarly, for ${}^{20-3x}P_{4x-5}$ to be defined, $(20 - 3x) \geq (4x - 5)$.
Now solve.

20. In this case, $f(x) = g(x)$ is possible only if the domain of the respective functions are same. Now, either solve it or go through the options.

21. $g(x) = f(x + 1) = |x - 2 + 1| + |x - 3 + 1| + |x - 4 + 1| = |x - 1| + |x - 2| + |x - 3|$
Obviously, this is neither odd nor even.
Alternatively, we know the graph of this function will neither be symmetrical to axis or origin. [See the topic Graphs and Maxima Minima]

28. Let us find out (Mag (6, (Nag (7, −7)))
Nag (7, −7) = −7, so, (Mag (6, (Nag (7, −7))) = 6
Mag (8, (Nag (4, (Nag (7, −7))))) = 8, so the answer = b

30. Applying componendo and dividendo in $\dfrac{f\left(\dfrac{3}{4}\right)+1}{f\left(\dfrac{3}{4}\right)-1}$ =

$[2.f(3/4)]/2 = f(3/4)$

$f(3/4) = 2(3/4)^2 + 6(3/4) - 1$

$f(x) = 2x^2 + 6x - 1$, then the value of $\dfrac{f\left(\dfrac{3}{4}\right)+1}{f\left(\dfrac{3}{4}\right)-1}$ is

So, the answer is option (d).

36. Go through the options.

38. $f(x, y) = g(x, y)$ is possible for all the sets of $(x, y) = (x, 0)$.

39. Draw the graph of y or see it by assuming different values of y.

40. Using H(1) + H(2) + H(3) + … + H(x) = n^2 H(x)
For $n = 1$: H(1) = 1 . H(1)
For $n = 2$:H(1) + H(2) = 4 × H(2), or, H(2) = [H(1)]/3
For $n = 3$:H(1) + H(2) + H(3) = 9 × H(3), or, H(3) = [H(1)]/2.3
Now go through the same method to find H(2005)

Advanced

1. Use the following:
If $a < x$, then $(x - a) > 0$, $b < x$, then $(x - b) > 0$, $c < x$, then $(x - c) > 0$ and $d < x$, then $(x - d) > 0$, $e < x$, then $(x - e) > 0$. And $a > x$, then $(x - a) < 0$, similarly other also can be calculated. Now use the options. Hence, the answer is option (d) None of these.

3. Assume different values of x in an attempt to refute the statements given. After a bit of hit and trial, we can observe that none of the statements can be refuted. So, the answer is option (d).

8. Using the options, if $f(x) = x$, then $f[(x + y)/(x - y)] = (x + y)/(x - y)$ and similarly it can be seen that RHS will be same as LHS.

9. Minimum value of V = 5 and maximum value of V = 29. So, sum = 34

15. This question requires finding out the digital sum of x given.
Digital sum of $x = 123456^{654321}$ is same as digital sum of $x = (1 + 2 + 3 + 4 + 5 + 6)^{654321} = f^{\infty}(x) = 21^{654321} = 3^{654321}$
Now digital sum of any power of 3 greater than 1 is 9. So the digital sum of $3^{654321} = 9$

16. Digital sum of $7^1 = 7$
Digital sum of $7^2 = 49 = 4 + 9 = 1 + 3 = 4$
Digital sum of $7^3 = 343 = 1$
Digital sum of $7^4 = 2401 = 7$
Digital sum of $7^5 = 16807 = 4$
So, digital sum of $7^{183} = 1$

[illegible]

V(x) denotes the absolute value of x, $mn(x, y)$ denotes the minimum of x and y, $mx(x, y)$ denotes the maximum of x and y.

21. Find mn [V (mx (−5, 3)), mx (−2, V (mn (−5, −2)))];
Now solve it according to the conditions given.

25. Put $x + 1$ at the place of x and $y - 1$ at the place of y and check the options.

[illegible]

31. Checking the options:
Option (a). If $p1 = 0$, then after forward 30, page number 12 will be obtained. After this, if we apply Backward 12, we will again get back to $0 = p1 = p2$.

32. If $p1$ is greater than or equal to 7 pages, then $p1 = p2$.

33. Use options.

35. $f(x).f(y).f(z) = a^{1+x} \times a^{1+y} \times a^{1+z} = a^{3+x+y+z}$
This is equal to $f(x + y + z + 2)$.

36. Let $f(x) = x^n + 1$, so $f(1/x) = (1/x^n) + 1$
Checking it for $f(x)$ $f\left(\dfrac{1}{x}\right) = f(x) + f\left(\dfrac{1}{x}\right)$ satisfies the value of $f(x)$ assumed.
$f(x) = x^n + 1 = 1001$, so $x^n = 1000$
Hence, $n = 3$.
$f(20) = 20^3 + 1 = 8001$

37. Go through the options.

38. Go through the options.

39. Go through the options.

40. Assume the values of x to check the options. All the three statements are true.

13

CHAPTER

Graphs and Maxima Minima

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Maxima and minima, and how they are calculated
- Some standard graphs and their shifting
- Definitions and properties
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

In case of maxima, minima and graphs, the biggest mistake any student can make is to see this chapter as a purely mathematical concept. However, though there is some mathematics involved, as far as the questions asked in the CAT are concerned, with good logic and common sense, they can be handled without much effort. This chapter will help the students understand the basic concepts on which the problem-solving techniques are built.

MAXIMUM AND MINIMUM VALUE

MAXIMUM VALUE

Let $f(x)$ be a function with the domain $D \subset R$. Then $f(x)$ is said to have attained the maximum value at a point 'a' if $f(a) \geq f(x)$ for all the values of x. In this case, 'a' is called the point of maximum and $f(a)$ is known as the maximum value or the greatest value or the absolute maximum value of $f(x)$.

For example, in case of $y = f(x) = |x|$, we will have the maximum value of y as $+\infty$.

MINIMUM VALUE

Let $f(x)$ be a function with domain $D \subset R$. Then $f(x)$ is said to attain the minimum value at a point 'a' if $f(x) \geq f(a)$. In such a case, the point 'a' is called the point of minimum and $f(a)$ is known as the minimum value or the least value or the absolute minimum value of $f(x)$.

Let us first see the maximum and minimum values of some very basic functions

Example 1 Find the maximum and the minimum values of $y = f(x) = -|x - 1| + 4$.

	$y = f(x)$	$y = f(x)$	$y = f(x)$	$y = f(x)$	$y = f(x)$
	$= x$	$= x^2$	$= \frac{1}{x}$	$= \log_e x$	$= \lvert x-2 \rvert$
Maximum	$+\infty$	$+\infty$	$+\infty$	$+\infty$	$+\infty$
Minimum	$-\infty$	0	$-\infty$	$-\infty$	0

Solution We have $f(x) = -|x - 1| + 4$ for all the real values of x.

$|x - 1| \geq 0$ for all real values of x

Hence, $-|x - 1| \leq 0$

$|x - 1| + 4 \leq 4$

So, the maximum value of $y = 4$

Now, $f(x) = 4 \Rightarrow -|x - 1| + 4 = 4 \Rightarrow |x - 1| = 0 \Rightarrow x = 1$.

Hence, $f(x)$ attains the maximum value 4 at $x = 1$.

To find out the minimum value, we can make $-|x-1|$ as small as possible by taking different values of x. So, the minimum value of y will be $-\infty$.

Example 2 What is the maximum and minimum value of $y = f(x) = |x + 3|$?

Solution We have $y = f(x) = |x + 3|$

$|x + 3| \geq 0$

$\Rightarrow f(x) \geq 0$ for all $x \varepsilon R$

So the minimum value of $f(x)$ is 0, which is attained at $x = -3$.

To find out the maximum value, it can be seen that $y = f(x) = |x + 3|$ can be made as big as possible. So, the maximum value of y is $+\infty$.

Finding Maxima and Minima in Some Typical Cases

1. Maxima and minima of quadratic equation

The standard methods of finding the maxima and the minima of all the quadratic equations, are as follows:

Let us discuss these methods with respect to a particular equation $y = x^2 - 5x + 6$.

(a) Graphical method

As we have seen in the concepts of quadratic equation, graph of $y = ax^2 + bx + c = 0$ will have its minimum value at $x = \frac{-b}{2a}$ and $y_{\min} = \frac{-D}{4a}$.

So, the minimum value of $y = x^2 - 5x + 6$ will be $y_{\min} = \frac{-D}{4a} = \frac{-1}{4}$

(b) Quadratic equation method

Suppose we have find the minimum value of $y = x^2 - 5x + 6$

We can find the minimum value by breaking this equation into the sum of a whole square and whatever is left i.e., $P^2 + Q$.

$$x^2 - 5x + 6 = [x^2 - 2.x.5/2 + (5/2)^2] - ¼$$

$$= \left(x - \frac{5}{2}\right)^2 - \frac{1}{4}$$

We know that the minimum value of $\left(x - \frac{5}{2}\right)^2$ is equal to zero, so the minimum value of $y = -\frac{1}{4}$

Example 3 Find the minimum value of $y = (x - a)(x - b)$.

(a) ab (b) $\frac{(a-b)^2}{4}$

(c) 0 (d) $-\frac{(a-b)^2}{4}$

Solution $x^2 - x(a + b) + ab$

$$= \left[x^2 - 2\frac{(a+b)}{2}x + \left(\frac{a+b}{2}\right)^2\right] - \frac{(a-b)^2}{4}$$

$$= \left[x - \left(\frac{a+b}{2}\right)\right]^2 - \frac{(a-b)^2}{4}$$

Now we know that the minimum value of any perfect square is 0.

So, the minimum value of $y = -\frac{(a-b)^2}{4}$

2. Maxima and minima of any modulus

We have seen above how to find the maxima and the minima of any modulus. Here we will discuss some more situations involving modulus. Let us see this with the help of an example:

Finding the maximum and the minimum value of $y = f(x) = |x + 3| + |x - 5| + |x - 7|$

Maximum value Obviously, the maximum value can be extended upto $+\infty$.

Minimum value In this case, all the three parts of $f(x)$ i.e., $|x + 3|$, $|x - 5|$ and $|x - 7|$ can not be simultaneously equal to zero. So, we are required to find the critical points here at first, and then these points in $f(x)$ to see that which one gives us the minimum value.

To obtain critical points, put all the three components $|x + 3|$, $|x - 5|$ and $x - 7|$ one by one equal to zero.

$|x + 3| = 0$, so, $x = -3$

$|x - 5| = 0$, so, $x = 5$

and $|x - 7| = 0$, so, $x = 7$

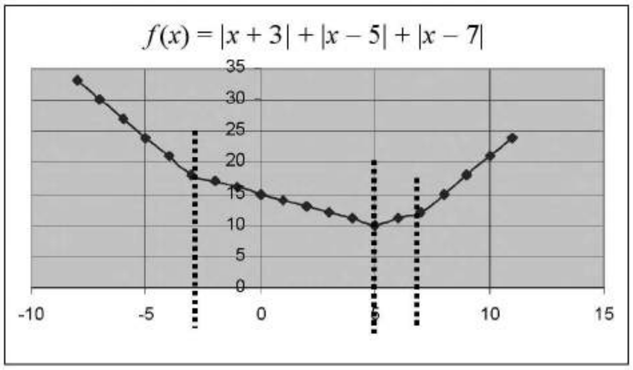
Now putting these values in $f(x)$ gives us the following result:

At $x = -3$, $f(x) = 18$

At $x = 5$, $f(x) = 10$

At $x = 7$, $f(x) = 12$

So, the minimum value of $f(x) = 10$



Alternatively, drawing the graph gives us the clear picture regarding the movement of the graph which gives us the minimum value of $f(x)$.

The dotted lines in the above graph show the movement of the graph. Obviously, at $x = 5$, graph is at its lowest point which is the minimum value of the graph.

The inspection of the graph gives us the maximum value of $f(x)$ which is $+\infty$.

3. Finding the maximum or the minimum value of the product/sum of two or more than two variables when the sum/product of these variables is given

1. If sum of two or more than two variables is given, then there product will be the maximum when the value of all the variables are equal.

Example 4 Find the maximum and the minimum value of xy subject to $x + y = 8$.

(a) 8 (b) 16 (c) 20 (d) 24

Solution The maximum value of xy will occur when $x = y = 4$

So, the maximum value of $xy = 4 \times 4 = 16$

To obtain the minimum value, we can take either x or y to be negative, and then the product of xy will be negative. And this process continues till $-\infty$. So, the minimum value of $xy = -\infty$

2. If the product of two or more than two positive variables is given, then there sum will be the minimum when the value of all the variables are equal.

Example 5 What is the minimum value of $f(x)$ $= \dfrac{x^3 + x + 2}{x}, x > 0$?

Solution Since $\dfrac{x^3 + x + 2}{x} = x^2 + 1 + \dfrac{1}{x} + \dfrac{1}{x}$

Now, the product of all the terms x^2, 1, $1/x$ and $1/x$ is 1, the sum of these terms will have the minimum value for $x^2 = 1 = 1/x$ i.e., for $x = 1$. Hence, the minimum value of $f(x)$ is 4.

Example 6 If $ab^2c^3 = 27 \times 2^8$, then find the minimum value of $a + b + c$ (given that $a, b, c > 0$).

Solution To find the minimum value of $a + b + c$, we need to know the product of a, b and c.

Assuming $a = x$, $b = 2y$ and $c = 3z$, we have $x \times 4y^2 \times 27z^3 = 27 \times 2^8$. So now we have to calculate the minimum value of $x + 2y + 3z$.

Thus, $ab^2c^3 = 27 \times 2^8 \Rightarrow x \times y^2 \times z^3 = 2^6$.

Now, $x + 2y + 3z = x + y + y + z + z + z$ and we know that $x \times y \times y \times z \times z \times z = 2^6$. So, the least value will occur when all are equal to 2. Thus, the least sum will be 12.

Corollary For any positive value of x, the minimum value of the sum of x and its reciprocal will be 2.

$$\left(x + \frac{1}{x}\right)_{\text{min}} = 2$$

Example 7 If $a > 1$, $b > 1$ then the minimum value of $\log_b a + \log_a b$ is

(a) 0 (b) 1

(c) 2 (d) None of these

Solution Assume $\log_a b = N$, so $\log_b a = 1/N$

So, the minimum value of $(\log_b a + \log_a b)$ = minimum value of $(N + 1/N) = 2$

4. Using arithmetic mean, geometric mean and harmonic mean to find the maxima and the minima

i. AM ≥ GM ≥ HM

ii. $(\text{GM})^2 = \text{AM} \times \text{HM}$ (for two numbers only)

Example 8 If a, b, c and d are positive real numbers such that $a + b + c + d = 2$, then which of the following is true regarding the values of $N = (a + b)(c + d)$?

(a) $0 \leq N \leq 1$ (b) $1 \leq N \leq 2$

(c) $2 \leq N \leq 3$ (d) $0 \leq N \leq 1$

Solution Using AM ≥ GM

$$\frac{1}{2}[(a+b)+(c+d)] \ge [(a+b)]^{1/2} = \frac{1}{2}(2) \ge (N)^{1/2}$$

Also, $(a+b)(c+d) \ge 0$

So, $0 \le N \le 1$

GRAPHS

Sketching a Graph

To sketch any graph, we make a table of points on the X-axis and corresponding points on the Y-axis.

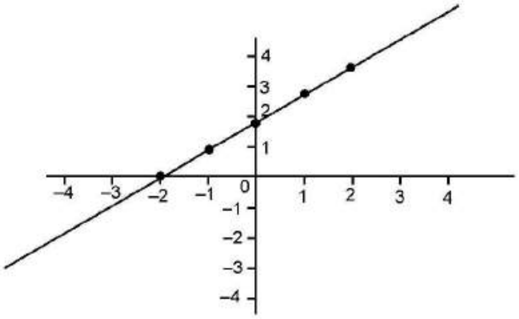
1. $y = x + 2$

Making a table of points for x and y

y	x
2	0
3	1
4	2
1	–1
0	–2

And hence the graph:

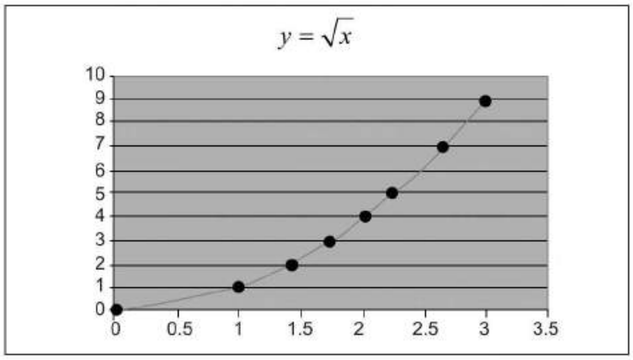
$y = x + 2$



2. $y = \sqrt{x}$

y	x
0	0
1	1
1.414214	2
1.732051	3
2	4

And hence the graph:



3. $y = |x - 1| + |x - 2|$

To draw the graph of any modulus function, we need to work with the help of the critical points, which help us in determining the intervals in which the modulus will be +ve or –ve.

Arranging the critical points on the number line:

$$-\infty \qquad 1 \qquad 2 \qquad +\infty$$

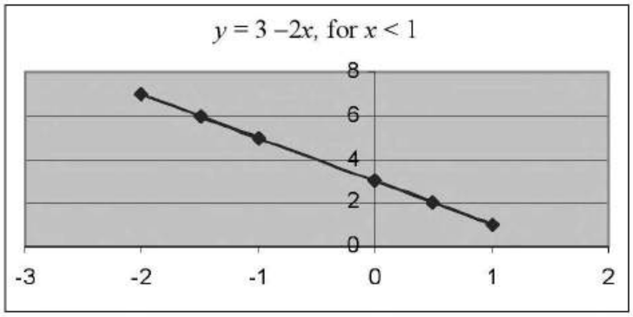
For all the values $-\infty < x < 1$

$|x - 1| + |x - 2| = (1 - x) + (2 - x) = 3 - 2x$

So, the table for values in between this interval is:

$y = 3 - 2x$	x
3	0
2	0.5
6	–1.5
5	–1
7	–2

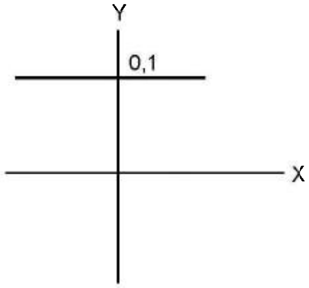
So, the graph for this interval will look like:



For all the values $1 < x < 2$

$|x - 1| + |x - 2| = (x - 1) + (2 - x) = 1$

So, the graph for this interval will look like:

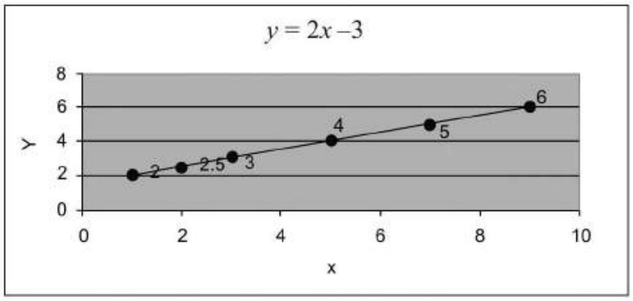


For all the values $x > 2$

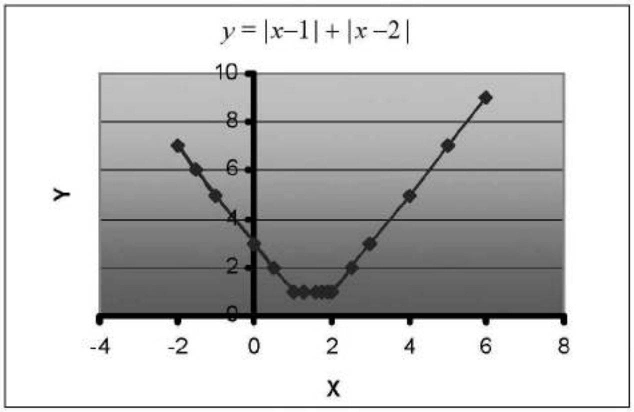
$|x - 1| + |x - 2| = (x - 1) + (x - 2) = 2x - 3$

$y = 2x - 3$	x
2	2.5
1	2
3	3
5	4
7	5
9	6

And its graph will look like:

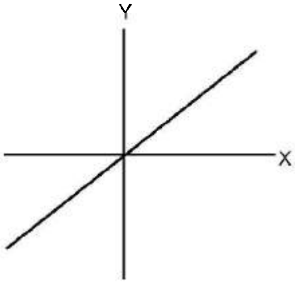


Combining all these graphs, we finally get,

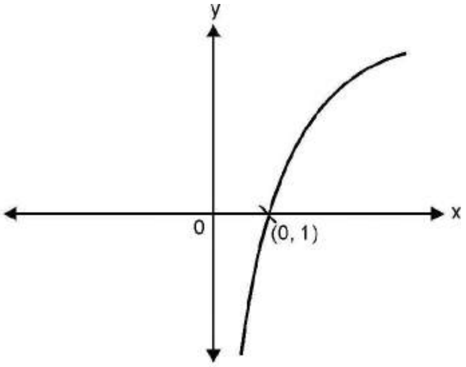


Some Standard Graphs

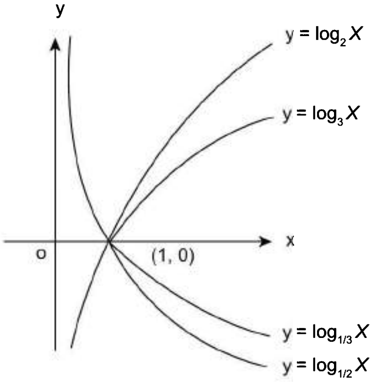
1. $y = x$



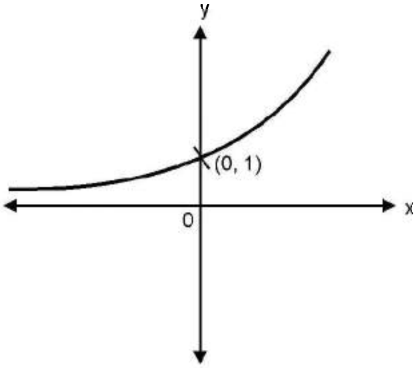
2. $y = \log_e x$



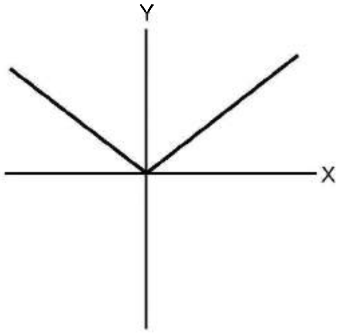
3. $y = \log_a x$



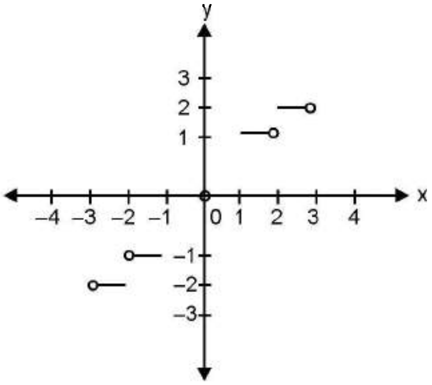
4. $y = e^x$



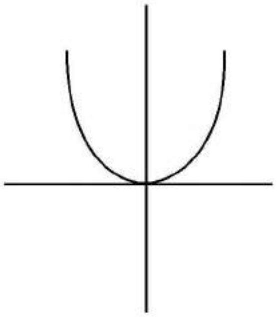
5. $y = |x|$



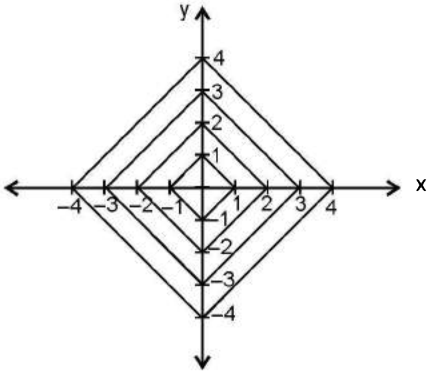
6. $y = [x]$, where $[x]$ represents the greatest integer less than or equal to x.



7. $y = x^2$



8. $|x| + |y| = K$



Shifting of a Graph

To know how a graph would shift, we need to know two things:

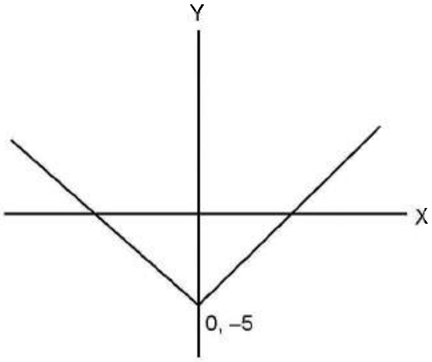
i. The shifting is either on the X-axis or on the Y-axis or on both of them,

ii. The shifting is in the +ve direction or in the –ve direction.

Let us understand this phenomena with the help of an example $y = f(x) = |x|$

Case 1

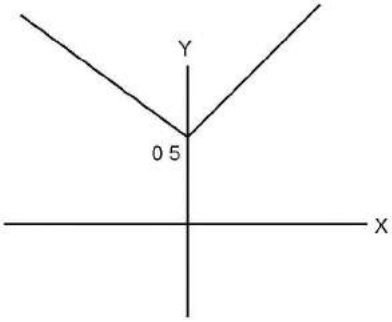
If $y = f(x)$ is changed to	Impact on the graph of $y = f(x)$
$Y = f(x) - 5$	It will shift 5 units downwards, i.e., 5 units on the –ve Y axis



$y = |x| - 5$

Case 2

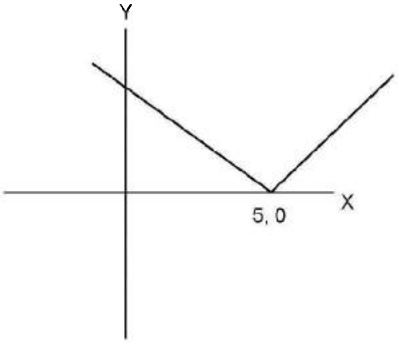
If $y = f(x)$ is changed to	Impact on the graph of $y = f(x)$
$Y = f(x) + 5$	It will shift 5 units upwards, i.e., 5 units on the +ve Y axis



$y = |x| + 5$

Case 3

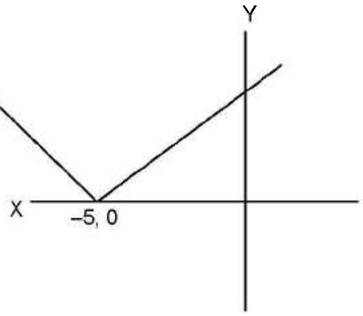
If $y = f(x)$ is changed to	Impact on the graph of $y = f(x)$
$Y = f(x) - 5$	It will shift 5 units rightwards, i.e., 5 units on the +ve X axis



$y = |x - 5|$

Case 4

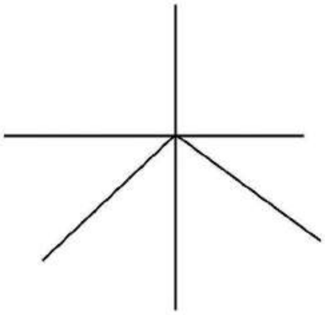
If $y = f(x)$ is changed to	Impact on the graph of $y = f(x)$
$Y = f(x) - 5$	It will shift 5 units leftwards, i.e., 5 units on the +ve X axis



$y = |x + 5|$

Case 5

If $y = f(x)$ is changed to	Impact on the graph of $y = f(x)$
$Y = -f(x)$	It will be reflected along the X-axis as if X-axis is a mirror



$y = -|x|$

Case 6

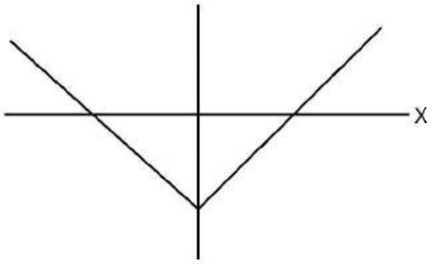
If $y = f(x)$ is changed to	Impact on the graph of $y = f(x)$
$Y = \|f(x)\|$	Graph below the X-axis will be reflected along the X-axis on its above as if X-axis is a mirror

We can summarize the whole phenomena of the shifting of a graph as:

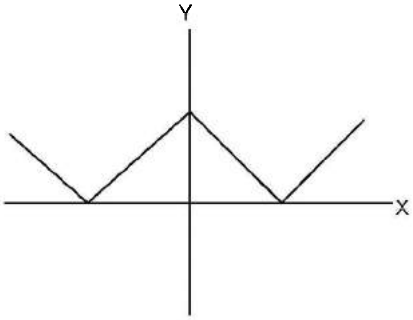
1. If there is a change in the value of X-axis, then it will move on the X-axis and similarly for the Y-axis. The graph of $y = f(x-5)$ has a change in the values of x where as graph of $y = f(x) - 5$, which can be written as $y + 5 = f(x)$ has a change in the values of y.
2. If there is an addition, then the graph will move on the −ve side, either on the X-axis or on the Y-axis and if there is a subtraction, then graph will move on the +ve side.

Example 9 The graph of $y = |x|$ is given. Draw the graph of $y = ||x|-2|$.

Solution Let us first sketch $|x|-2$.

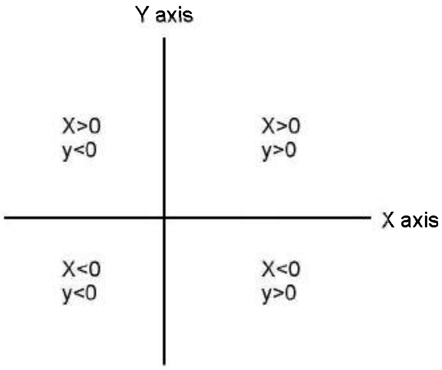


Now $||x|-2|$ will be having all the negative part of $|x| - 2$ on the positive side of Y axis, or above X-axis.



Area of a Graphs

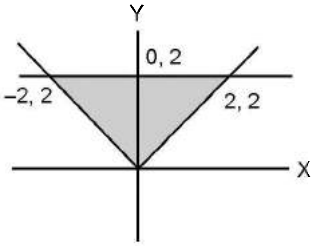
Before we proceed ahead with calculating the area of the combination of graphs, we should be clear with the quadrants and the signs of X and Y in the same.



To find the area of graphs, we first need to sketch the graphs of the equations and then by using geometry/ coordinate geometry we can find the area of the enclosed figure.

Example 10 Find out the area of the region enclosed by $y = |x|$ and $y = 2$.

Solution Following is the area enclosed by the equations given above:



The area of the enclosed figures will be,
Area = $\frac{1}{2} \times 4 \times 2 = 4$ sq. units

Example 11 In the X – Y plane, the area of the region bounded by the graph $|x + y| + |x - y| = 4$ is

(a) 8 (b) 12 (c) 16 (d) 20

Solution Let $x \geq 0, y \geq 0$ and $x \geq y$
Then $|x + y| + |x - y| = 4$
$\Rightarrow x + y + x - y = 4 \Rightarrow x = 2$
Similarly, $x \geq 0, y \geq 0, x \leq y$
$x + y + y - x = 4 \Rightarrow y = 2$
The area in the first quadrant is 4.
By using symmetry, the total area in all the four quadrants $= 4 \times 4 = 16$ sq. units

Example 12 If p, q, and r are any real numbers, then

(a) max $(p, q) <$ max (p, q, r)
(b) min $(p, q) = \frac{1}{2}(p + q - |p - q|)$
(c) min $(p, q) <$ min (p, q, r)
(d) None of these

Solution If we take $r < p, q$, then (1) and (3) cannot hold. For (2), if $p \geq q$, then $|p - q| = p - q$.

$\frac{1}{2}(p + q - |p - q|) = \frac{1}{2}(p + q - p + q|) = q =$ min (p, q)

Similarly, If $p < q$ then $|p - q| = q - p$.

$(p + q - |p - q|) = \frac{1}{2}(p + q - q + p) = p =$ min (p, q)

So, option (b) is the answer.

Example 13 If $a\,b\,c\,d = 1, a > 0, b > 0, c > 0, d > 0$, then what is the minimum value of $(a + 1)(b + 1)(c + 1)(d + 1)$?

(a) 1 (b) 8
(c) 16 (d) None of these

Solution The minimum value will occur when $a = b = c = d = 1$

So, the minimum value of $(a + 1)(b + 1)(c + 1)(d + 1) = 16$

Example 14 Consider a triangle drawn on the X-Y plane with its three vertices (41, 0), (0, 41) and (0, 0), each vertex being represented by its (X,Y) coordinates. What is the number of points with integer coordinates inside the triangle (excluding all the points on the boundary)?

(a) 780 (b) 800 (c) 820 (d) 741

Solution The equation formed from the data is $x + y < 41$

The values which will satisfy this equation are
(1, 39), (1, 38) ...(1, 1)
(2, 38), (2, 37), ...(2,1)
(39, 1)

So the total number of cases are 39 + 38 + 37 ... + 1

$= \frac{39 \times 40}{2} = 780$

Example 15 A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs 250 and Rs 300 per day respectively. In addition, a male operator gets Rs 15 per call he answers and female operator gets Rs 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

(a) 15 (b) 14 (c) 12 (d) 10

Solution First let us form both the equations:
40 m + 50 f = 1000
250 m + 300 f + 40 × 15 m + 50 × 10 × f = A
850 m + 8000 f = A
Where m and f are the number of males and females and A is the amount paid by the service provider.

Then the possible values for f are 8, 9, 10, 11, 12
If f = 8, then m = 15
If f = 9, 10 and 11 then m will not be an integer while f = 12 then m will be 10.

By putting f = 8 m = 15 and A = 18800. When f = 12 and m = 10 then A = 18100.

Hence, the number of males will be 10.

Example 16 If a, b and c are the sides of a triangle, then what is the maximum value of the expression $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b}$?

(a) 1 (b) 3/2 (c) 2 (d) 5/2

Solution Assume $2s = a + b + c$. We know that $b + c > a$, so we get $2(b + c) > a + b + c = 2s$

Hence, $b + c > s$

Similarly, $c + a > s$, $a + b > s$

Hence, $\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} < \frac{a+b+c}{s} = \frac{2s}{s} = 2$

Example 17 At how many distinct points the graphs of $y = x^{-1}$ and $y = \log_e x$ intersect? (CAT 2003)

Solution We can see that the graphs of $y = x^{-1}$ and $y = \log_e x$ intersect just once.

Example 18 How many integral solution is/are possible for the equation $|y-18| + |y - 9| + |y + 9| + | y + 18| = 54$?

Solution Here, $|y-N|$ should be seen as nothing but the distance of the point y from the point N on the number line, a person standing at a point N. Hence $|y - 18| + |y - 9| + |y + 9| + |y + 18|$ is the sum of the distances of the point y from 18, 9, –9 and –18. Now, for any point y, where $p \le y \le q$, the sum of the distances from p and q is $q - p$. So, for point y, where $-9 \le y \le 9$, the sum of the distances of y from –18, –9, 9 and 18 is $[18-(-18)] + [9-(-9)] = 54$

For points outside this limitation, this expression will have different values. Hence, the required numbers are –9, –8, … , 8, 9. So, there will be a total of 19 values.

PRACTICE EXERCISES

■■■

WARM UP

Q.1. What is the minimum value of the expression $x^2 + 8x + 10$?

(a) 0 (b) +∝

(c) –6 (d) +6

Q.2. What is the maximum value of the expression $x - 6x - x^2$?

(a) 0 (b) 14

(c) –∝ (d) None of these

Q.3. What is the maximum value of the function $y = \frac{2x^2 + 3x + 4}{x^2 + x + 3}$?

(a) 11/23 (b) 23/11

(c) 4/7 (d) 7/4

Q.4. In the above question, what is the minimum value of y?

(a) 0 (b) 1

(c) –1 (d) Cannot be determined

Q.5. If $f(x) = \max(4x + 3, 3x + 6)$ for $x \in$? [–6, 10], find the maximum value of $f(x)$?

(a) +∝ (b) 36

(c) 43 (d) None of these

Q.6 What is the maximum value of $f(x) = \min(4 - 5x, x - 3)$ for every $x \in (0, 4)$?

(a) –1 (b) 2

(c) 4 (d) None of these

Direction for questions 7 to 11: Go through the following definitions and solve the questions based on it.

$A(x, y, z) = \min(x + y, y + z, z + x)$
$B(x, y, z) = \min(x - y, y - z, z - x)$
$C(x, y, z) = \max(A(x, y, z)\ B(x, y, z))$
$D(x, y, z) = \min(A(x, y, z), B(x, y, z))$

Q.7. What is the value of C (1, 2, 3)?

(a) 1 (b) 2

(c) 3 (d) 4

Q.8. Find the value of D (0, 1, 2).

(a) –1 (b) 1

(c) 0 (d) None of these

Q.9. Find the value of A (2, 3, 4) + B (2, 3, 4).

(a) 13 (b) 4

(c) 7 (d) None of these

Q.10. Find the value of C (0, 1, D (1, 2, 3)).

(a) 1 (b) 0

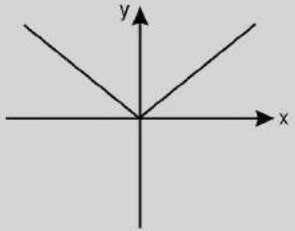
(c) –2 (d) None of these

Q.11. What is the value of D (1, 2, C (0, 1, 2))?

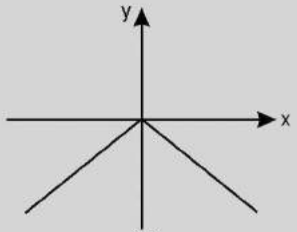
(a) 1 (b) 2

(c) 3 (d) 0

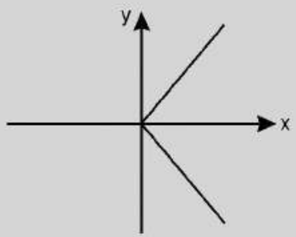
Direction for questions 12 to 16: Following graphs represent various functions. Match the figure with the appropriate function.



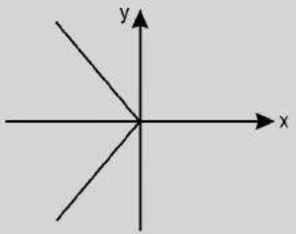
(A)



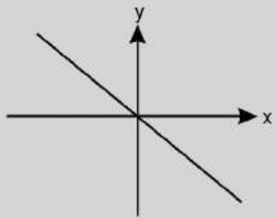
(B)



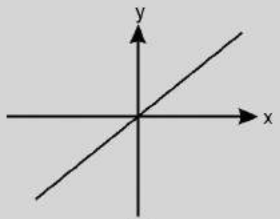
(C)



(D)



(E)



(F)

Q.12. $f(x) = -x$
(a) B (b) C (c) E (d) D

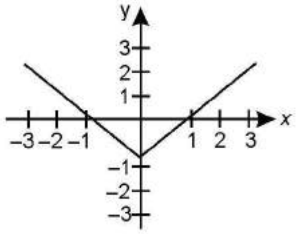
Q.13. $f(x) = |x|$
(a) A (b) B (c) C (d) D

Q.14. $f(x) = -|x|$
(a) E (b) B (c) A (d) F

Q.15. $f(x) = x$
(a) E (b) F
(c) A (d) B

Q.16. $f(x) = |x| - x$
(a) A (b) B
(c) C (d) None of these

Q.17. Which of the following represents the following graph?



(a) $|x - 1|$ (b) $|x + 1|$
(c) $|x| - 1$ (d) $|x| + 1$

Q.18. The graph of $y = (x+2)^2-3$ is the graph of $y = x^2$ –
(a) Shift 2 units up and 3 units rightwards towards +ve X axis.
(b) Shift 2 units rightwards towards +ve X axis and 3 units downwards towards –ve Y axis.
(c) Shift 2 units leftwards towards –ve X axis and 3 units downwards towards –ve Y axis.
(d) Shift 2 units up and 3 units rightwards towards +X axis.

Q.19. The graph of the function $y = f(x)$ is symmetrical about the line $x = 2$, then
(a) $f(x + 2) = f(x-2)$ (b) $f(2 + x) = f(2 - x)$
(c) $f(x) = f(-x)$ (d) None of these

Q.20. If $x + y + z = 24$, then the maximum value of xyz is
(a) 215 (b) 512
(c) 125 (d) 576

FOUNDATION

Q.1. The minimum value of $4^x + 4^{1-x}$, $x \in R$, is
(a) 2 (b) 4
(c) 1 (d) None of these

Q.2. a, b and c are three positive numbers and abc^2 has the greatest value $\frac{1}{64}$. Then
(a) $1 = b = \frac{1}{2}, c = \frac{1}{4}$ (b) $a = b = \frac{1}{4}, c = \frac{1}{2}$
(c) $a = b = c = \frac{1}{3}$ (d) None of these

Q.3. The sum of the products of the ten numbers ±1, ±2, ±3, ±4, ±5 taking two at a time is
(a) 165
(b) –55
(c) 55
(d) None of these

Q.4. If $ab = 2a + 3b$, $a > 0$, $b > 0$ then the minimum value of ab is
(a) 12 (b) 24
(c) $\frac{1}{4}$ (d) None of these

Q.5. If $a > 1$, $b > 1$ then the minimum value of $\log_b a + \log_a b$ is
(a) 0 (b) 1
(c) 2 (d) None of these

Q.6. What is the maximum value of $\frac{x}{y}$ for $(x - 2)^2 = 4$ and $(y - 3)^2 = 25$?
(a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $-\frac{1}{8}$ (d) $-\frac{5}{2}$

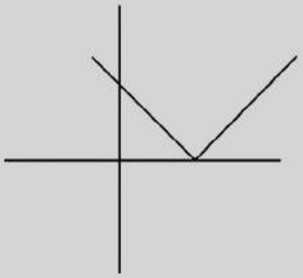
Q.7. If a and b are both positive real values, then the minimum value of $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)$ is
(a) 0 (b) 2
(c) −2 (d) 4

Q.8. If $|X + 2| > 3$, then the solution set for X is
(a) $X > -1$ (b) $X > 1$
(c) $X < -5$ (d) $X > 1$ or $X < -5$

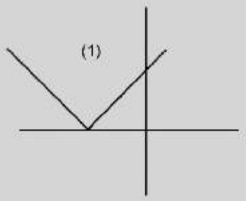
Q.9. If a and b are both positive real values, then the maximum value of $(a+b)\left(\frac{1}{a}+\frac{1}{b}\right)$ is
(a) 0 (b) 2 (c) 4 (d) $\propto$

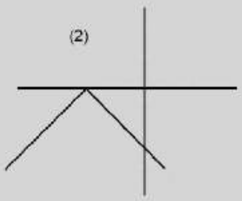
Direction for questions 10 to 13: Read the passage below and solve the questions based on it.

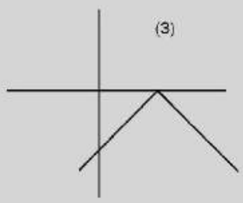
The graph of $y = f(x)$ is given as

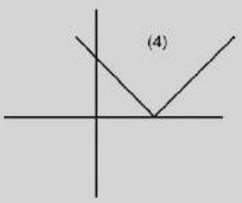


Following are the four graphs:


(1)


(2)


(3)


(4)

Q.10. Which of the following defines graph (1)?
(a) $f(x) - 1$ (b) $f(x - 1)$
(c) $f(x) + 1$ (d) $f(x + 1)$

Q.11. Which of the following defines graph (2)?
(a) $f(x) - 1$ (b) $f(x-1)$
(c) $f(x) + 1$ (d) None of these

Q.12. Which of the following defines graph (3)?
(a) $-f(x)$ (b) $f(x-1)$
(c) $f(x) + 1$ (d) $f(x+1)$

Q.13. Which of the following defines graph (4)?
(a) $f(x) - 1$ (b) $f(x)$
(c) $f(x) + 1$ (d) $f(x+1)$

Q.14. What is the maximum value of $y = |x - 5| + |x - 7|$?
(a) −2 (b) 2 (c) 0 (d) $\propto$

Q.15. If a lies between 2 and 3, both included, and b lies between 4 and 6, both included, then what is the ratio of the minimum and the maximum limits of $a^2 - b^2$?
(a) −4 (b) 4 (c) $\frac{32}{7}$ (d) $-\frac{28}{6}$

Q.16. If $y = |x| - 5$, then the minimum possible value of y is
(a) 5 (b) −5
(c) 0 (d) Both a and b

Q.17. Given that $(x - 2)^2 = 9$ and $(y - 3)^2 = 25$. What is the minimum value of $\frac{x}{y}$?
(a) $\frac{1}{2}$ (b) $\frac{5}{8}$ (c) $-\frac{1}{8}$ (d) $-\frac{5}{2}$

Q.18. When $x^2 + 4xy + 4y^2$ takes a minimum value, then
(a) $x = -2y$ (b) $x = 2y$
(c) $2x = y$ (d) $-2x = y$

Q.19. A man finds that it costs Rs (300 + 4N) per day to make N articles. If the selling price of each article is Rs 7, then what is the minimum number of articles to be produced per day if he has to make a profit?
(a) 300 (b) 150
(c) 100 (d) None of these

Q.20. If a, b and c are all the real numbers, then the minimum value of $ab + bc + ca$ is
(a) 1 (b) $-\frac{1}{2}$
(c) $-\frac{3}{2}$ (d) None of these

Q.21. $|x + y| = 10$, where x and y are integers, then what is the minimum value of $x^2 + y^2$?
(a) 75 (b) 100
(c) 50 (d) None of these

Q.22. If $|x| + |y| = 7$, then what is the sum of the minimum and the maximum values of $x + y$?
(a) $\frac{3}{2}$ (b) −7 (c) 7 (d) 0

Q.23. If $|x| - |y| = 13$, then which of the following cannot be the value of $x - y$?

(a) -18 (b) -9
(c) -17 (d) None of these

Q.24. What is the minimum value of

$$\left(\frac{a+1}{a}\right)^2 + \left(\frac{b+1}{b}\right)^2 + \left(\frac{c+1}{c}\right)^2 ?$$

(a) 1 (b) 4 (c) 8 (d) 12

Q.25. Let $y = \max (x + 3), (7 - 2x)$ what is the minimum value of y for $2 \geq \times \geq 1$?

(a) 4.50 (b) 4.00 (c) 4.67 (d) 4.33

Q.26. The minimum value of $3x + 3y + z$ subjected to the condition $xyz = 24$, where x, y and z are all positive real numbers, is

(a) $14 \times 3^{1/3}$ (b) 18 (c) 216 (d) 12

Q.27. Find the maximum and the minimum values of the function $\dfrac{x^2 - x + 1}{x^2 + x + 1}$ for real values of x.

(a) 3 and -3 (b) $\frac{1}{3}$ and $-\frac{1}{3}$
(c) 3 and $\frac{1}{3}$ (d) None of these

Q.28. A group of children walked to the Ramnarayan's shop and bought pastries for a total of Rs 168 and ice candies for a total of Rs 126. Each child has one pastry and one ice candy. How many children could be there in the group?

(a) 28 (b) 21 (c) 42 (d) 36

Q.29. Let x, y, z be three positive numbers such that $x + y + z = 1$. The minimum value of

$$\left(x + \frac{1}{x}\right)^2 + \left(y + \frac{1}{y}\right)^2 + \left(z + \frac{1}{z}\right)^2 \text{ will be}$$

(a) 16 (b) 24
(c) 33 (d) None of these

Q.30. If a, b and c are the positive real numbers, find the greatest value of $a^2\ b^3\ c^4$ subject to the condition $a + b + c = 18$.

(a) $+\propto$ (b) $4^2.6^3.8^4$
(c) 16686 (d) None of these

Q.31. What is the least value of $(x-2)\ (x-4)^2\ (x-6) + 6$, for real values of x?

(a) 6 (b) 4/3
(c) 4 (d) None of these

Q.32. Consider the following functions:

$f(x) = x^2 + 3x$

$g(x) = 3x + 4$

For what value of x will $f[g(x)]$ have its minimum value?

(a) 0 (b) $\frac{-11}{2}$
(c) $\frac{-11}{6}$ (d) None of these

Q.33. If a, b and c are all positive then what is the minimum value of $\dfrac{a}{b+c} + \dfrac{b}{c+a} + \dfrac{c}{a+b}$?

(a) 1 (b) 3 (c) 9 (d) 27

Q.34. If a lies between 2 and 3, both included, and b lies between 4 and 6, both included, then what is the ratio of the minimum and the maximum limits of $a^2 - b^2$?

(a) -4 (b) 4
(c) $\frac{-28}{5}$ (d) None of these

Q.35. The least value of $\dfrac{1}{x} + \dfrac{1}{y} + \dfrac{1}{z}$ for positive x, y and z, satisfying the condition $x + y + z = 9$ is

(a) $\frac{15}{7}$ (b) $\frac{1}{9}$ (c) 3 (d) 1

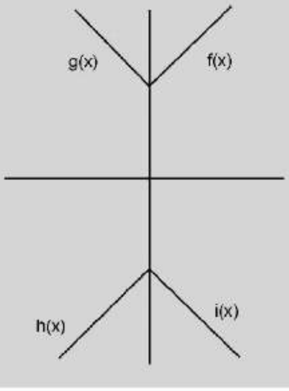
Q.36. The minimum value of $3x + 4y$, subject to the conditions $x^2 y^3 = 6$ and x and y are positive, is

(a) 10 (b) 14 (c) 13 (d) 13

Q.37. The minimum value of $f(x) = x^8 + x^6 - x^4\ 2x^3 - x^2 - 2x + 9$ is

(a) 5 (b) 1 (c) 0 (d) 9

Direction for questions 38 to 40: Go through the graph given below and solve the questions based on it (Neglect fine errors).



Q.38. Which of the following is true?

(a) $f(x) = g(-x)$ (b) $f(x) = -g(x)$
(c) $f(x) = -g(-x)$ (d) None of these

Q.39. Which of the following is true?

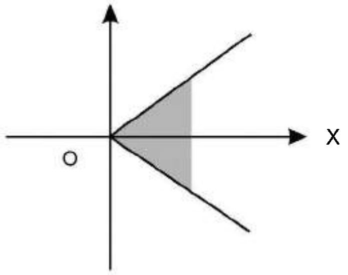
(a) $i(x) = g(-x)$ (b) $i(x) = -g(x)$
(c) $i(x) = -g(-x)$ (d) None of these

Q.40. How many of the following statements are true?

(a) $g(x) = h(-x)$ (b) $f(x) = -h(-x)$
(c) $h(x) = -i(x)$ (d) $f(x) = -i(x)$

(a) 1 (b) 2
(c) 3 (d) 4

Q.41. The shaded region in the diagram represents the relation



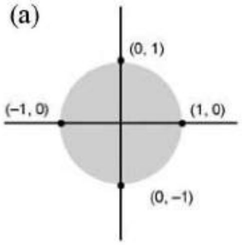
(a) $y \le x$ (b) $|y| < |x|$
(c) $y \le |x|$ (d) $|y| \le x$

Q.42. If P, Q and R are three vertices with coordinates (1,4), (4, 2) and (m, 2m−1) respectively, then the value of m for which PR + RQ is minimum is

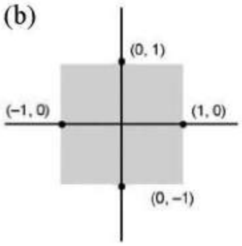
(a) $\frac{17}{8}$ (b) $\frac{5}{2}$ (c) $\frac{7}{4}$ (d) $\frac{11}{8}$

Q.43. The set $\{(x, y): |x| + |y| \le 1\}$ is represented by the shaded region in one of the four figures. Which one is it?

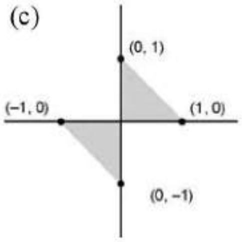
(a)



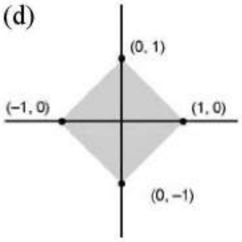
(b)



(c)



(d)



Q.44. If x, y and z are three integers satisfying $x + y + z = 13$ and $xy + xz = 42$, what is the minimum possible value of $y^2 + z^2$?

(a) 18 (b) 22.5
(c) 16 (d) 24.5

Q.45. What will be the maximum value of $\left(\frac{1}{x}\right)^x$?

(a) e (b) e^{-1}
(c) $e^{1/e}$ (d) $e^{-1/e}$

Direction for questions 46 to 48: Read the passage below and solve the questions based on it.

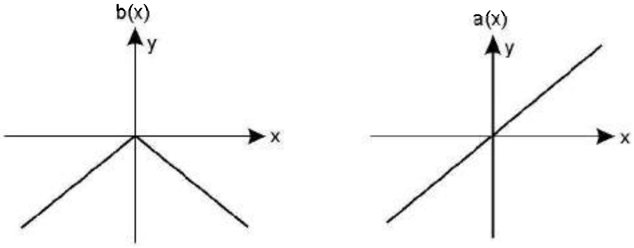
Mark option (a) as the answer if $a(x) = b(-x)$ for $x < 0$, and $a(x) = -b(-x)$ for $x > 0$

Mark option (b) as the answer if $a(x) = -b(x)$ for $x < 0$, and $a(x) = b(-x)$ for $x > 0$

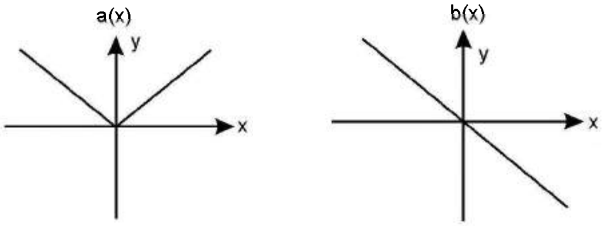
Mark option (c) as the answer if $a(x) = -b(x)$ for all the values of x.

Mark option (d) as the answer if none of the above is true.

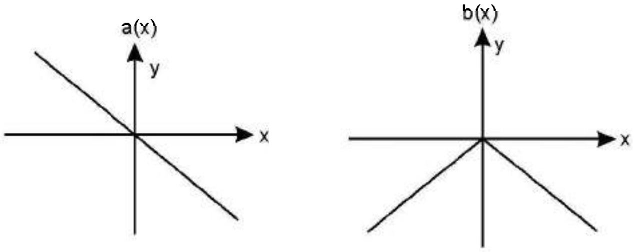
Q.46.



Q.47.



Q.48.



Q.49. Let a, b, c and d be the real numbers. If G = max{min(a, b), min(c, d)} and H = min {max (a, c), max(b, d)}. Then

(a) G ≤ H, for all values of a, b, c and d
(b) G = H, for all values of a, b, c and d
(c) G ≥ H, for all values of a, b, c and d
(d) Any one of the above three options can be true for some particular values of a, b, c and d.

Q.50. If $x = \frac{5}{a+b}$ and $y = 5(a + b)$, and if both x and y are greater than zero, then $(x + y)$ is:

(a) < 5 (b) ≥ 10
(c) Between 5 and 10 (d) None of these

MODERATE

Q.1. We have $|a + b| = |a| + |b|$, $|c + d| < |c| + |d|$, $|e + f| < |e| + |f|$. If $a, b, c, d, e, f \in$ R, then the product $abcdef$ is
(a) Equal to zero
(b) Necessarily non-negative
(c) Necessarily non-positive
(d) Either positive or negative

Q.2. For how many values of x, is the equality $|||x - 2| - 3| - 1| = 0$, true?
(a) 0 (b) 2
(c) 3 (d) None of these

Q.3. Let $f(x) = |x - a| + |x - 2a| + |x - 3a|$, where $a > 0$. Let the minimum value of $f(x)$ be denoted by m and the corresponding value of x be denoted by b. Which of the following is true?
(a) $m = a; b = a$ (b) $m = a; b = 2a$
(c) $m = 2a; b = a$ (d) $m = 2a; b = 2a$

Q.4. The minimum of $f(x) = |x - 1| + |x - 4| + |x - 5|$ is attained at $x =$
(a) 1 (b) 3 (c) 4 (d) 5

Q.5. Let $y = k^2(x^2+1) + 2k(3x + 4) + 27$. If k and x are real numbers, what is the minimum value of y?
(a) 1 (b) 3
(c) 4 (d) None of these

Q.6. If $x < -3$ and $y > 2$, then which of the following is true for all real values of x and y?
(a) $x + 2y < -1$ (b) $3y + 2x > 0$
(c) $2x - xy > 0$ (d) None of these

Q.7. If $a_1, a_2, \ldots a_n$ are n integers such that $a_1a_2\ldots a_n = 1$, then which of the following must be true?
(a) The minimum value of $(a_1 + a_2 + \ldots + a_n)$ is $-(n - 1)$, if n is odd.
(b) The minimum value of $(a_1 + a_2 + \ldots + a_n)$ is $(n - 2)$, if n is even.
(c) The minimum value of $(a_1 + a_2 + \ldots + a_n)$ is $(2 - n)$, if n is even.
(d) None of these

Q.8. If a and b are positive real values, and $ab + (a+b) = 21$, what is the minimum value of $(a + b)$?
(a) 6 (b) 8
(c) 12 (d) Cannot be determined

Q.9. The least integral value of k for which $(k - 2)x^2 + 8x + k + 4 > 0$ for all $x \in$ R, is:
(a) 5 (b) 4
(c) 3 (d) None of these

Q.10. If $f(x) = 2x^2 + 3x + 4$ and $g(x) = 5 - x^2$, the min $[f(x)] - \max[g(x)]$ is
(a) 0 (b) 5 (c) $\frac{17}{8}$ (d) $-\frac{17}{8}$

Q.11. If $a + b + c = 20$ and $\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right) = 30,$ then the value of $\frac{a}{b} + \frac{b}{c} + \frac{a}{c} + \frac{c}{a} + \frac{b}{c} + \frac{c}{b}$ is
(a) 597 (b) 8 (c) 350 (d) 441

Q.12. The minimum value of the expression $\frac{1+V^2}{1+V}$, for positive values of V, is
(a) $\sqrt{2}$ (b) $\frac{2}{\sqrt{2}+1}$
(c) 1 (d) None of these

Q.13. If p, q, r, s are real numbers such that $p^2 + q^2 = 1$, $r^2 + s^2 = 1$, then the maximum value of $pr + qs$ would be
(a) −1 (b) 2 (c) −2 (d) 1

Q.14. If p, q, r denote the sides of a triangle, then $\frac{p}{q+r} + \frac{q}{r+p} + \frac{r}{p+q}$ always lie between (both end values inclusive)
(a) 3 and 4 (b) 2 and 3
(c) 3.5 and 5 (d) 1.5 and 2

Q.15. p, q, r, s are any four positive real numbers, the minimum value of $\frac{p}{q} + \frac{q}{r} + \frac{r}{s} + \frac{s}{p}$ is
(a) 1 (b) 2 (c) $2\sqrt{2}$ (d) 4

Q.16. $p, q,$ and r are all positive numbers. If $p + q + r = 1$, then the least value of $\left(\frac{1}{p} - 1\right)\left(\frac{1}{q} - 1\right)\left(\frac{1}{r} - 1\right)$ is
(a) 10 (b) 25
(c) 24 (d) None of these

Q.17. Find the maximum value of $y = 5 - |x + 1| - |x - 3|$.
(a) 1 (b) 5 (c) 4 (d) ∞

Q.18. If three positive real numbers x, y and z are in an AP such that $xyz = 4$, then what will be the minimum value of y?
(a) $2^{1/3}$ (b) $2^{2/3}$
(c) $2^{1/4}$ (d) $2^{3/4}$

Q.19. Find the least value of the expression $2\log_{10} x - \log_x (.01)$ for $x>1$.

(a) 4 (b) 2

(c) 1 (d) None of these

Q.20. When the curve $y = x^2 + 4x - 5$ and $y = \frac{1}{(1+x^2)}$ are drawn in the $x - y$ plane, how many times do they intersect for values of $x > 0$?

(a) Never (b) Once

(c) Twice (d) More than twice

Q.21. If p, q, and r are three real number such that $p + q + r = 4$ and $p^2 + q^2 + r^2 = 6$, then what can be the maximum value of q?

(a) 1 (b) 2/3

(c) 4 (d) None of these

Q.22. In the above question, what will be the value of p when q is minimum?

(a) 2 (b) 1/3

(c) 2/3 (d) None of these

Q.23. Given $y = \min (x - 5, 7 - x)$. What is the minimum value of y?

(a) 0 (b) $-\infty$ (c) $+\infty$ (d) 1

Q.24. Given $y = \min (x - 5, 7 - x)$. What is the maximum value of y?

(a) 0 (b) $-\infty$ (c) $+\infty$ (d) 1

Q.25. Given $y = \max (x - 5, 7 - x)$. What is the minimum value of y?

(a) 0 (b) $-\infty$ (c) $+\infty$ (d) 1

Q.26. Given $y = \max (x - 5, 7 - x)$. What is the maximum value of y?

(a) 0 (b) $-\infty$ (c) $+\infty$ (d) 1

Q.27. Let y be the maximum of $\{(x + 5), (3 - x)$. What is the minimum value of y, if $-2 \le x \le 0$?

(a) 3 (b) 2

(c) 1 (d) None of these

Q.28. The maximum possible value of $x^2 + 3y^2 + 9z^2$, subject to $x + 2y + 3z = 12$, where x, y and z are real numbers, is

(a) 48 (b) 216

(c) 240 (d) None of these

Q.29. At how many different points, $y = x^{-1}$ and $y = \log_e x$ intersect?

(a) 0 (b) 1 (c) 2 (d) Infinite

Q.30. A rectangle of the largest area is inscribed in a semi-circle of radius r. What is the area of the rectangle?

(a) $1.5 \times r^2$ (b) $r^2 \times \sqrt{2}$

(c) r^2 (d) $r^2\sqrt{3}$

Q.31. The circumference of a circle, the perimeter of a pentagon and an octagon are the same. Their areas are denoted by C, P and O respectively. Which of the following is true for their areas?

(a) C > P > O (b) O>P>C

(c) O>C>P (d) C>O>P

Q.32. At what value of x, $\left(\frac{1}{x}\right)^x$ will attain the maximum value?

(a) e (b) e^{-1}

(c) e^{-2} (d) e^2

Q.33. The altitude of a right circular cone of the minimum volume circumscribed about a sphere of the radius r is

(a) 2r (b) 3r

(c) 4r (d) None of these

Q.34. The minimum value of $(x - a)(x - b)$ is

(a) ab (b) 0

(c) $\frac{(a-b)^2}{4}$ (d) $-\frac{(a-b)^2}{4}$

Q.35. If x, y and z are arbitrary positive real numbers satisfying the equation $4xy + 6xz + 8zx = 9$, find the maximum possible value of the product xyz.

(a) $\frac{1}{2\sqrt{2}}$ (b) $\frac{\sqrt{3}}{4}$

(c) $\frac{3}{8}$ (d) None of these

Q.36. The minimum value of the quantity

$$\frac{(a^2+3a+1)(b^2+3b+1)(c^2+3c+1)}{abc},$$

where a, b and c are positive real numbers, is

(a) $\frac{11^3}{2^3}$ (b) 125 (c) 25 (d) 27

Q.37. A right circular cylindrical container closed on both the sides contains a fixed volume of motor oil. Suppose its base has the diameter d and its height is h. The overall surface area of the container is minimum when

(a) $h = \frac{4}{3}\pi d$ (b) $h = 2d$

(c) $h = d$ (d) None of these

Q.38. A rectangular box of volume 48 cu ft is to be constructed, so that its length is twice its width. The material to be used for the top and the four sides is three times costlier per ft² than that used for the bottom. Then, the height (in ft) of the box that minimizes the cost is equal to.

(a) $\frac{8}{27}$ (b) $\frac{8\sqrt[3]{4}}{3}$ (c) $\frac{4}{27}$ (d) $\frac{8}{3}$

Q.39. A truck is to be driven for 300 km on a highway at a constant speed of x km/h. The speed rules of the highway requires $30 \le x \le 60$. The fuel costs Rs 10 per litre and is consumed at the rate of $2+\frac{x^2}{600}$ litres per hour. The wages of the driver are Rs 200 per hour. The most economical speed to drive the truck, in km/h, is

(a) 30 (b) 60

(c) $30\sqrt{3.3}$ (d) $20\sqrt{33}$

Q.40. A square tin sheet of side 12 inches is converted into a box with an open top in the following steps–The sheet is placed horizontally. Then, equal sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what is the value of x that maximizes the volume of the box?

(a) 3 (b) 4 (c) 1 (d) 2

Q.41. How many integral points are there within the graph of $|x| + |y| \le 4$?

(a) 32 (b) 41

(c) 9 (d) None of these

Q.42. The sides of a rectangle of the greatest area which can be inscribed into an ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1= 1$ is:

(a) 5, 3 (b) $5\sqrt{2}$, 3

(c) 5, $3\sqrt{2}$ (d) $5\sqrt{2}$, $3\sqrt{2}$

(See geometry chapter for ellipse)

Q.43. The sets

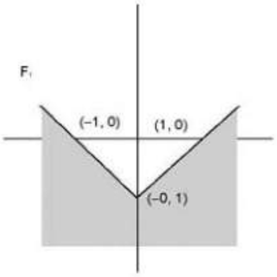
$\{[x, y] : |y-1| - x \ge 1\}$

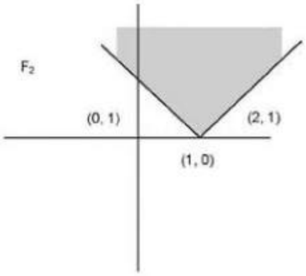
$\{[x, y] : |x| - y \ge 1\}$

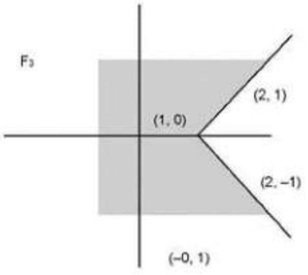
$\{[x, y] : |x-1| - y \le 1\}$

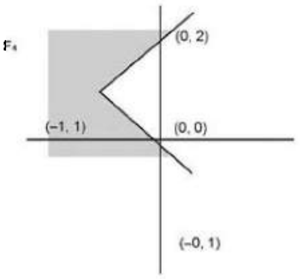
$\{[x, y] : |y-1|x - 1| \ge 0\}$

are represented by the shaded regions in the figures given below in some order.









Then the correct order of the figures is

(a) F_4, F_1, F_2, F_3 (b) F_4, F_2, F_3, F_1

(c) F_1, F_4, F_3, F_2 (d) F_4, F_1, F_3, F_2

Direction for questions 44 to 46: Read the passage below and solve the questions based on it.

In each question, choose the function which best describes the graph.

$f(x) = (|x|-1)(|x|-2)$

$g(x) = ||x-1|-2|-2$

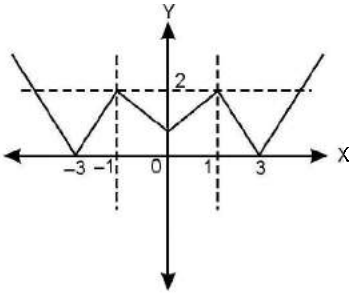
$h(x) = ||\,|x| - 1| - 2|$

$j(x) = (|x|+1)(|x|-2)$

$k(x) = |x-1|\,|x-3|$

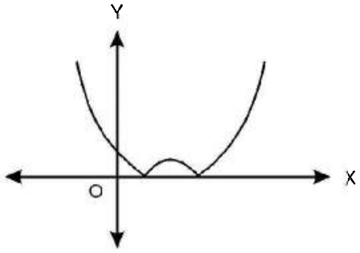
$l(x) = ||\,|x| - 2| - 1|$

Q.44.



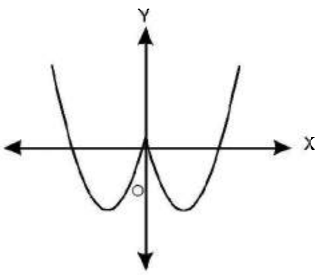
(a) $g(x)$ (b) $l(x)$ (c) $h(x)$ (d) $k(x)$

Q.45.



(a) $f(x)$ (b) $l(x)$ (c) $j(x)$ (d) $k(x)$

Q.46.



(a) $f(x)$ (b) $k(x)$ (c) $j(x)$ (d) $h(x)$

Q.47. From a square tin sheet of side 12 feet a box with its top open is made by cutting equal squares at the four corners and then bending the tin sheet so as to form the sides of the box. The side of the removed square for which the box has the maximum possible volume in feet is.

(a) 3 (b) 1

(c) 2 (d) None of these

Q.48. From a circular sheet of paper of radius a, a sector with a central angle is cut out and folded into the shape of a conical funnel. The volume of this funnel is maximum when θ equals

(a) $\frac{2\pi}{\sqrt{2}}$ (b) $2\pi\sqrt{\frac{2}{3}}$

(c) $\frac{\pi}{2}$ (d) π

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(b)	(b)	(c)	(d)	(c)	(b)	(c)	(d)	(a)	(c)	(a)	(b)	(a)	(d)	(c)	(c)	(b)	(b)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(b)	(c)	(a)	(d)	(d)	(d)	(d)	(d)	(a)	(b)	(d)	(c)	(c)	(b)	(a)	(d)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(d)	(b)	(d)	(d)	(b)	(c)	(c)	(d)	(b)	(d)	(c)	(c)	(d)	(d)	(a)	(a)	(a)	(c)	(b)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(d)	(a)	(d)	(a)	(c)	(d)	(d)	(b)	(a)	(b)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(d)	(d)	(c)	(d)	(b)	(d)	(a)	(a)	(d)	(a)	(b)	(d)	(d)	(d)	(d)	(a)	(b)	(a)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(b)	(d)	(d)	(c)	(d)	(d)	(b)	(c)	(d)	(b)	(d)	(d)	(c)	(b)	(c)	(d)	(b)	(d)
Q. No.	41	42	43	44	45	46	47	48												
Answer	(b)	(d)	(d)	(b)	(d)	(c)	(c)	(b)												

HINTS AND SOLUTIONS

Moderate

1. $|a + b| = |a| + |b|$ is possible only if both a and b are having the same signs or $a = b = 0$.
$|c + d| < |c| + |d|$ is possible only if both c and d are having opposite signs, and the same is true a bout $|e + f| < |e| + |f|$.
sign of ab = +ve or $ab = 0$
Sign of cd = –ve and sign of ef = –ve
Hence, the sign of $abcdef$ = Non negative.

2. $||x - 2| - 3| - 1 = 0$, or, $||x - 2| - 3| = 1$, or, $|x - 2| - 3 = 1$, or, $|x - 2| = 4$,
If $x - 2 > 0$, then $x = 6$, and if $x - 2 < 0$, then $-(x - 2) = 4$, so, $x = -2$.
Similarly,
If $x - 2 > 0$, then $|x - 2| = (x - 2)$
Or, $|x - 2| - 3 = -1$, or, $(x - 2) = 2$, or, $x = 4$.
If $x - 2 < 0$, $|x - 2| = -(x - 2)$
So, $|x - 2| - 3 = -1$, or, $-(x - 2) = 2$, or, $2 - x = 2$, so $x = 0$.
So, four values of x are possible.

3. Obviously, the minimum value will be attained at the median of the critical points which is $x = 2a$ and the minimum value will be $f(x) = 2a$

4. Minimum value of $f(x)$ will be attained at the median of the critical points of $f(x)$ ie., $x = 4$.

6. Going through the options.
Checking option 2. $3y > 6$ and $2x < -6$, so $3y + 2x > 0$.

8. $ab + 2(a + b) = 21 \rightarrow ab + 2(a + b) + 4 = 21 + 4$.
So, $(a + 2)(b + 2) = 25$.
For minimum value of $(a + b)$, a and b should be equal.
Hence, $(a + 2) = (b + 2) = 5$, (we will not take -5 because it will give negative values of a abd b).
So, $a = b = 3$
Alternatively, the quadratic equation can also be used to find out the minimum value of $(a + b)$.
Minimum value will occur if $a = b$
So, $ab + 2(a + b) = 21$ will be equal to $a^2 + 2(2a) - 21 = 0$
Or, $(a + 7)(a - 3) = 0$, hence, $a = 3 = b$

9. $D = 8^2 - 4(k - 2)(k + 4) > 0$
Or, $(k + 6)(k - 4) < 0$
Or, $4 < k < 6$. So, $k = 5$

10. $\min[f(x)] = \min(2x^2 + 3x + 4) = 46/16$
$\max[g(x)] = 5$

11. Use AM ≥ GM ≥ HM for all the four fractions.

15. Use AM ≥ GM ≥ HM for all the four fractions.

17. Find the maximum value of $y = 5 - [|x + 1| + |x - 3|]$
Assume $[|x + 1| + |x - 3|] = z$
So, $y = 5 - z$, y will attain the maximum value at minimum value of z.
Now, Min of z will occur at $x = -1$ [Using critical points]
So, minimum value of Z = 4
Maximum value of $y = 1$

18. $2y = x + z$
Now, AM of x, y and z will be more than their GM.
$Y \geq (xyz)^{1/3}$
Hence, $y_{\min} = 4^{1/3} = 2^{2/3}$

19. $2\log_{10} x - \log_x(.01) = \log_{10}x^2 - \log_x 10^{-2} = 2[\log_{10}x + 1/\log_{10}x]$
Minimum value of $[\log_{10}x + 1/\log_{10}x] = 2$

23. Drawing the graphs of $x - 5$ and $7 - x$ tells us that both the graphs are heading towards $+\infty$ and $-\infty$, depending upon the values. So, minimum of y will be $-\infty$.

24. Now the direction of movement of both the graphs are opposite (one is going towards $+\infty$ and other one is going towards $-\infty$ for a particular set of values and vice-versa.)
For $x < 6$, $x - 5$ will provide us the minimum value and for $x > 6$, $7 - x$ will provide us with the minimum value and at $x = 1$, we will get the values equal. And this $x = 6$ is the point for which $y_{\max} = \min(x - 5, 7 - x)$ is obtained.

28. Any of x, y or z can be taken in such a way that $x^2 + 3y^2 + 9z^2$ can go to $+\infty$. Actually it was a problem of common sense, rather than maxima and minima (refer to introduction of this topic.)

29. Drawing the curves tells us that it will intersect at one and only one point.

31. We know that for a fixed perimeter of polygons, polygon with the more number of sides will have more area with respect to the polygon with the less number of sides. And similarly, if circumference of a circle and perimeter of polygon are same, then the area of circle will be more than area of any polygon. [Circle can be treated to be a polygon of n sides, n tending to infinity].

32. Go through the options.

34. Make a quadratic equation in the format of $z^2 + y$. Minimum value of $z^2 = 0$, so, mininum value = y.

36. $\dfrac{(a^2+3a+a)(b^2+3b+1)(c^2+3c+1)}{abc}$

= [a + (1/a) + 3] [b + (1/b) + 3] [c + (1/c) + 3]

Minimum value of [a + (1/a)] = [b + (1/b)]
= [c + (1/c)] = 2

39. Go through the options.

44. At $x = 0, y = 1$ is the key. Otherwise also, you don't have to draw the graph. Just check the options using the points given on the graph.

45. Since no part of the graph is negative, function has to involve an overall modulus. So, it cannot be $f(x)$ or $j(x)$. Now, this graph is having two points on the +ve side of X-axis.
Checking option (b):
$l(x) = 0 = ||\,|x| - 2| - 1|$
Or, $||x| - 2| = 1$
Or, $|x| - 2 = \pm 1$, or, $|x| - 2 = +1$, or, $|x| = 3$, or, $x = \pm 3$
Or, $|x| - 2 = -1$, or, $|x| = 1$, or, $x = \pm 1$
We get four points of x for $l(x) = 0$. So, not possible.
Checking option (d):
$K(x) = |x - 1|\,|x - 3| = 0$, Hence, $x = 1$ or $x = 3$
So, we get two points on +ve X-axis for $k(x) = 0$
Hence, option (d) is the answer.
Otherwise also, since we have eliminated the first three options, the fourth option has to be the answer.

46. Here, we get two values of x, one +ve and other one –ve, for $y = 0$.
Checking options:
Option 1 – $f(x) = (|x| - 1)(|x| - 2)$
We are getting four points on X-axis for $f(x) = 0$. So, not possible.
Option 2 – $k(x)$ is given in 45. please go through.
Option 3 – $j(x) = (|x| + 1)(|x| - 2) = 0$
So, either $(|x| + 1) = 0$ or $(|x| - 2) = 0$
If $(|x| + 1) = 0$, then $|x| = -1$, which is not possible.
If $(|x| - 2) = 0$, then $x = \pm 2$.
Hence, option (c) is the answer.

14

CHAPTER

Logarithm

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Meaning of logarithm and its bases
- Definitions and properties
- Value of the log pertaining to negative and positive values
- Methods of solving questions

Introduction to the topic

Considering logarithm in terms of the number of questions being asked in the CAT, this chapter is quite important. The CAT paper usually has 1–2 questions from this topic. The concept of logarithm is simple and a simple formula is sufficient to solve the questions without getting into lengthy derivations.

LOGARITHM

The logarithmic function is defined as the inverse of an exponential function.

If any number N is expressed in the form a^x then the index 'X' is called the logarithm of the number N to the base 'a'.

Thus, if $N = a^x$

Then $x = \log_a N$.

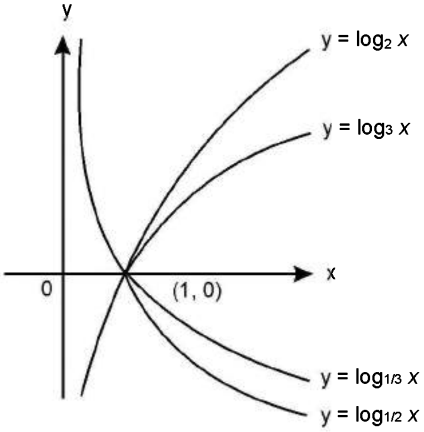
Generally, logarithm of any number is calculated to the base 10. When run base is not mentioned, it should be taken as 10.

Restrictions with logarithm of any number For logarithm of any number to be defined, the number should be greater than zero and base should be positive and not equal to 1.

For $\log_a x$ to be defined, $x > 0$ and $a > 0$ as well $a \neq 1$.

It can also be seen with the help of the graph of log x (given alongside).

Following observations can be made from this graph:



- Value of y can be negative for some value of x.
- Value of x cannot be negative in any case.
- For constant x, if base is lying in between 0 and 1, then log x becomes a decreasing function. Otherwise it is an increasing function.

Some Important Properties

In case of all the following properties the standard restrictions on logarithm will be used.

1. $\log_a(XY) = \log_a X + \log_a Y$
   If the restrictions given regarding log of any number are not used, then a good number of contradictions about the numbers can be seen. One of the examples of a similar nature is as follows:
   $\log(12) = \log(-4 \times -3) = \log(-4) + \log(-3)$
   Now on the left hand side, we have a defined value, but on the right hand side the value is not defined. It is due to the fact that $\log_a(XY) = \log_a X + \log_a Y$ is possible only if $X > 0$ and $Y > 0$.
2. $\log_a(X/Y) = \log_a X - \log_a Y$
3. $\log_a(X^k) = k \log_a X$, $\log_a \sqrt[k]{X} = 1/k \times \log_a X$
4. $\log_a 1 = 0$ (As $a^0 + 1$)
5. $\log_X X = 1$
6. $\log_a X = 1/\log_x a$
7. Base change rule
   $\log_a X = \log_b X/\log_b a$
   $\Rightarrow \log_a X \div \log_b a = \log_b X$
8. $a^{(\log_a x)} = x$
9. If $a > 1$ and $X > a$, $\log_a X > 0$
10. If $0 < x < y$ then $\log x < \log y$ (i.e., $\log x$ is an increasing function). In particular if $x > 1$ then $\log x > 0$ and if $0 < x < 1$ then $\log x < 0$.
11. If $a > 1$ and $0 < x < y$ then $\log_a x < \log_a y$ and $a^x < a^y$. If $0 < a < 1$ and $0 < x < y$ then $\log_a x > \log_a y$ and $a^x > a^y$.
12. $\log_a 1 = 0$
13. Log of 0 and negative numbers is not defined.

CHARACTERISTICS AND MANTISSA

The integral part of logarithm is called *Characteristic* and its decimal part is called *Mantissa*. Logarithms to the base 10 are called *Common logarithms*. The characteristic of common logarithm can be found out by a visual inspection. The characteristics of the logarithm (base 10) of a number greater than 1 is less by one than the number of digits in the integral part and is a positive. However, if a decimal fraction number is less than 1 but positive, its characteristic will be greater by unity than the number of consecutives zeros immediately after the decimal point and is a negative.

Example 1 If $5 \log_{27}(y) + 2 \log_9(81y) = 20$, then y is equal to

(a) 1/7 (b) 81 (c) 729 (d) 243

Solution $5 \log_{27}(y) + 2 \log_9(81y) = \frac{5}{3} \log_3(y) + \frac{2}{2} \log_3(81y)$

$= \log_3(81) + \log_3\left(y^{\frac{8}{3}}\right)$

$= 4 + \frac{8}{3} \log_3(y) = 20$

Hence, $\log_3 y = 6$

So, $y = 3^6 = 729$

Alternatively, this question can be solved by using options too.

Example 2 If $x \geq y$ and $y > 1$, then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be

(a) -1 (b) -0.5

(c) 0 (d) 1

Solution $\mathrm{P} = \log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$

$= \log_x X - \log_x y + \log_y y - \log_y y$

$= 2 - \log_x y - \log_y x$

Assume now that $\log_x y = t$

$$p = 2 - \frac{1}{t} - t = -\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2$$

This can never be equal to 1.

PRACTICE EXERCISES

FOUNDATION

Q.1. If $3^{x+1} = 6^{\log_2 3}$ then x is

(a) 2 (b) 3

(c) $\log_3 2$ (d) $\log_2 3$

Q.2. If $\log_4 5 = a$ and $\log_5 6 = b$ then what is the value of $\log_3 2$?

(a) $\frac{1}{2a+1}$ (b) $\frac{1}{2b+1}$

(c) 2ab + 1 (d) $\frac{1}{2ab-1}$

Q.3. $\log_5 \mathrm{P} \times \log_\mathrm{P} x = 2$. What is the value of x?

(a) 125 (b) P^2 (c) 25 (d) $5^{1/2}$

Q.4. Find the value of $3\log\frac{81}{80} + 5\log\frac{25}{24} + 7\log\frac{16}{15}$.

(a) log 2 (b) log 3

(c) 1 (d) None of these

Q.5. If [N] = the greatest integer less than or equal to N, then $[\log_{10} 6730.4]$ is equal to

(a) 6 (b) 4

(c) 5 (d) None of these

Q.6. What is the value of x in the following expression?

$$1 - \log_{10} 5 = \frac{1}{3}\left(\log_{10}\frac{1}{2} + \log_{10} x + \frac{1}{3}\ \log_{10} 5\right)$$

(a) 1 (b) $16 \times 5^{-1/3}$

(c) $16 \times 5^{1/3}$ (d) None of these

Q.7. What is the number of zeros coming immediately after the decimal point in the value of $(0.2)^{25}$? ($\log_{10}2 = 0.30103$)

(a) 15 (b) 16

(c) 17 (d) None of these

Q.8. What is the value of $\log_{32} 27 \times \log_{243} 8$?

(a) $\frac{\log 9}{\log 4}$ (b) $\frac{\log 3}{\log 2}$

(c) log 27 (d) None of these

Q.9. Find the value of $\log_{64}(1/16)$.

(a) –2/3 (b) 2/3

(c) 3/2 (d) –3/2

Q.10. Find the value of $\frac{1}{\log_2 X} + \frac{1}{\log_3 X} + \frac{1}{\log_4 X} + \frac{1}{\log_5 X} + \frac{1}{\log_7 X} + \frac{1}{\log_6 X}$.

(a) 1 (b) $\log_{27} X$

(c) $\log_{5040} X$ (d) $\log_x 5040$

Q.11. If $\log_y \mathrm{N} = 5$ and $\log_{2y} 8\mathrm{N} = 4$, then find the value of N.

(a) 16 (b) 32

(c) 5 (d) 25

Q.12 If $\log_y^3 \mathrm{N} = \log_N^3 y$, then what is the value of $\log_N^3 y$?

(a) 1/3 (b) – 1/3

(c) Either 1 or 2 (d) None of these

Q.13. If $\frac{\log X}{\log 4} = \frac{\log 343}{\log 49} = \frac{\log y}{\log 64}$, then what is the value of $x + y$?

(a) 520 (b) 740

(c) 880 (d) Cannot be determined

Q.14. Arrange the following in an ascending order

$\mathrm{A} = \log_7 2401, \mathrm{B} = \log_7 \sqrt{7} \times 343\ \mathrm{C} = \log_{\sqrt{6}} 216, \mathrm{D} = \log_2 32.$

(a) ABCD (b) BDCA

(c) BDAD (d) BADC

Q.15. If $\log_{10}\left[1 - \left\{1 - \left(1 - x^2\right)^{-1}\right\}^{-1}\right]^{-1/2} = 1$, then which of the following is the value of x.

(a) 1 (b) 2

(c) 10 (d) None of these

Q.16. What is the value of N in the following expression: $4^{\log}2^8 + 27^{\log}\,27^{81} = 144 + \log_{10} \mathrm{N}$?

(a) 1 (b) 10

(c) 12 (d) 100

Q.17. What is the value of x if $\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \frac{25}{4}$?
(a) 9 (b) 27
(c) 81 (d) None of these

Q.18. If $\log_y x = 8$ and $\log_{10y} 16x = 4$, then find the value of y.
(a) 1 (b) 2
(c) 3 (d) 5

Q.19. What is the value of x in the following expression:
$\log_7 \log_5 \left[\sqrt{(x+5)} + \sqrt{x}\right] = 0$?
(a) 1 (b) 2
(c) 3 (d) 4

Q.20. What is the value of x in the following expression: $\log_2 (3-x) + \log_2 (1-x) = 3$?
(a) 1 (b) 0
(c) -1 (d) Not possible

Q.21. If $\log N^2 - \log 2N = 3 \log 3 - \log 6$, then what is the value of N?
(a) 1 (b) 2
(c) 9 (d) More than one value

Q.22. What is the value of $\log_{3\sqrt{3}} 27$?
(a) 2 (b) 3 (c) 4 (d) 5

Q.23. Evaluate $\log\left(36\sqrt{6}\right)$ to the base 6.
(a) 1/2 (b) 5/2
(c) 3/2 (d) 7/2

Q.24. What is the value of P if $\log_e 2.\ \log_p 625 = \log_{10} 16.\log_e 10$?
(a) 2 (b) 4
(c) 5 (d) 7

Q.25. If $3\log_{(3x)} 27 - 2\log_{(3x)} 9 = 0$, then what is the value of x?
(a) 1/243 (b) 1/7
(c) 1/49 (d) None of these

MODERATE

Q.1. $N = n\,!$, where $n>2$. Find the value of $(\log_2 N)^{-1} + (\log_3 N)^{-1} + (\log_4 N)^{-1} + \ldots (\log_n N)^{-1}$.
(a) 0 (b) 1
(c) 10 (d) N

Q.2. What is the value of x in the following expression?
$x + \log_{10} (1 + 2^x) = x \log_{10} 5 + \log_{10} 6$.
(a) 1 (b) 0
(c) -1 (c) 3

Q.3. If $a_1, a_2, a_3, \ldots$ are positive numbers in GP then $\log a_n$, $\log a_{n+1}$, $\log a_{n+2}$ are in
(a) AP (b) GP
(c) HP (d) None of these

Q.4. If $x = \log_a (bc)$, $y = \log_b (ca)$ and $z = \log_c (ab)$ when which of the following is equal to 1?
(a) $x + y + z$
(b) $(1 + x)^{-1} + (1 + y)^{-1} + (1 + z)^{-1}$
(c) xyz
(d) None of these

Q.5. If $\log x : \log y : \log z = (y - z):(z - x):(x - y)$ then
(a) $X^y.Y^z.Z^x = 1$ (b) $X^x Y^y Z^z = 1$
(c) $x\sqrt{x}, y\sqrt{y}, z\sqrt{z} = 1$ (d) None of these

Q.6. If $\log_{0.04} (x - 1) \geq \log_{0.2} (x - 1)$ then x belongs to the interval
(a) $(1, 2]$ (b) $(-\infty, 2]$
(c) $[2, +\infty)$ (d) None of these

Q.7. If $\log_k x.\ \log_5 k = 1 k \neq 1, k > 0$, then x is equal to
(a) k (b) $\frac{1}{5}$
(c) 5 (d) None of these

Q.8. If $\frac{\log a}{b-c} = \frac{\log b}{c-a} = \frac{\log c}{a-b}$ then $a^a b^b c^c$ is
(a) 0 (b) 1 (c) 0.5 (d) 2

Q.9. If $x > 1, y > 1, z > 1$ are three numbers in GP then $\frac{1}{1+\log x}, \frac{1}{1+\log y}, \frac{1}{1+\log z}$ are in
(a) AP (b) HP
(c) GP (d) None of these

Q.10. If $\log_k N = 6$, and $\log_{25k}(8N) = 3$, then k is
(a) 12.5 (b) $(12.5)^2$
(c) $\frac{5^{\frac{2}{3}}}{2}$ (d) $(12.5)^{\frac{2}{3}}$

Q.11. If a, b and c are distinct positive numbers ($\neq 1$) such that

$(\log_b a \log_c a - \log_a a) + (\log_a b \log_c b - \log_b b) + (\log_a c \log_b c - \log_c c) = 0$. What is the value of abc?
(a) 1 (b) 0
(c) -1 (d) None of these

Q.12. If $4^{\log_9 3}+9^{\log_2 4}=10^{\log_x 83}, x \varepsilon R$, how many values can x take?

(a) 0 (b) 3

(c) 2 (d) None of these

Q.13. How many values of x ($x > 1$) satisfy the following equation:

$\log_2 \times.\log_4 \times.\log_6 x = \log_2 x.\log_4 x + \log_2 x.\ \log_6 x + \log_4 x.\log_6 x$?

(a) 0 (b) 1

(c) 2 (d) More than 2

Q.14. If $a^{\log b} = k\, b^{\log a}$, find the value of $\log_{ab} k$, where $a, b > 1$.

(a) 1 (b) 0

(c) 2 (d) Cannot be determined

Q.15. What is the value of x in the following expression $\log_{3/4} \log_2 (x^2+7) \log_{1/4}(x^2+7)^{-1} = -2$?

(a) +3 (b) −3

(c) ±3 (d) None of these

Q.16. If $2[\log(x+y) - \log 5] = \log x + \log y$, then what is the value of $x^2 + y^2$?

(a) $20-xy$ (b) $23xy$

(c) $25-xy$ (d) $28xy$

Q.17. The logarithm of a number to a certain base is 9. The logarithm of 64 times the number to a base which is 11 times the original base is 6. Find the actual base.

(a) 11/6

(b) 2.33/7

(c) 121/4

(d) Cannot be determined

Q.18. If $\log_{30}3 = x$ and $\log_{30}5 = y$, then $\log_8 30$ is equal to

(a) $3(1-x-y)$ (b) $\frac{1}{3(1-x-y)}$

(c) $\frac{3}{(1-x-y)}$ (d) $\frac{1-x-y}{3}$

Q.19. What is the value of $\frac{1}{\log_2 n}+\frac{1}{\log_3 n}+\ldots+\frac{1}{\log_{40} n}$?

(a) $\frac{1}{\log(40)!n}$ (b) $\log(40!)n$

(c) 1 (d) None of these

Q.20. The number of solutions of $\log_2(x+5) = 6 - x$ is

(a) 2 (b) 0

(c) 3 (d) None of these

Q.21. x, y and z are three angles of a triangle. Which of the following set of values of x, y and z satisfies $\log(x \times y \times z) = 3 \log X + 4 \log 2$, given that x, y and z are integers?

(a) 30°, 70°, 80° (b) 30°, 60°, 90°

(c) 20°, 80°, 80° (d) 30°, 50°, 100°

Q.22. What is the value of $\log_3 2.\ \log_4 3.\ \log_5 4 \ldots \log_{16} 15$?

(a) 1/2 (b) 1/3 (c) 2/3 (d) 1/4

Q.23. If $\log_{12} 18 = a$, $\log_{24} 54 = b$, find the value of $ab + 5(a-b)$.

(a) −1 (b) 1 (c) 0 (d) 2

Q.24. What is the value of x in the following expression?

$x \log_{10}(10/3) + \log_{10} 3 = \log_{10}(2+3^x) + x$

(a) 1 (b) −2 (c) 0 (d) 2

ANSWERS

■■■

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(c)	(a)	(d)	(b)	(c)	(d)	(a)	(d)	(b)	(c)	(a)	(d)	(c)	(b)	(d)	(b)	(d)	(c)
Q. No.	21	22	23	24	25															
Answer	(c)	(a)	(b)	(c)	(a)															

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(a)	(a)	(b)	(b)	(c)	(c)	(b)	(b)	(a)	(a)	(d)	(b)	(b)	(c)	(c)	(c)	(b)	(a)	(d)
Q. No.	21	22	23	24	25															
Answer	(c)	(d)	(b)	(c)	(a)															

HINTS AND SOLUTIONS

Foundation

1. $3^{x+1} = (3\times 2)^{\log_2^3}$

$\Rightarrow\ 3^{x+1} = 3^{\log_2^3} \times 2^{\log_2^3}$

$\Rightarrow\ 3^{x+1} = 3^{\log_2^3} \times 3$

$\Rightarrow\ 3^x = 3^{\log_2^3} \quad \Rightarrow x = \log_2^3$

2. $\log_4^5 = a$ and $\log_5^6 = b$

$\Rightarrow \log_4^5 \times \log_5^6 = ab$

$\Rightarrow \log_4^6 = ab \Rightarrow \frac{1}{2}\log_2^6 = ab$

$\Rightarrow (1 + \log_2^3) = 2ab$

$\therefore\ \log_2^3 = 2ab - 1$

$\therefore\ \log_3^2 = \frac{1}{2ab-1}$

3. $\log_5 P \times \log_P x = 2$

$\Rightarrow \log_5 x = 2$

$\therefore\ x = 5^2 = 25$

4. $\log\left(\frac{81}{80}\right)^3 + \log\left(\frac{25}{24}\right)^5 + \log\left(\frac{16}{15}\right)^7$

$= \log\left[\frac{3^{12}\times 5^{10}\times 2^{28}}{2^{12}\times 5^3\times 2^{15}\times 3^5\times 3^7\times 5^7}\right]$

$= \log 2$

5. $[\log_{10} 6730.4] = [\log_{10} 6.7304 \times 10^3]$

$= [3 + \log_{10} 6.7304]$

= 3 [Value of $\log_{10} n$ will lie in between 0 and 1 when $1<n<10$]

6. $1 - \log_{10}^5 = \frac{1}{3}\left[\log_{10}\left(\frac{x.5^{1/3}}{2}\right)\right]$

$= 3[\log_{10} 10 - \log_{10}^5] = \log_{10}\left(\frac{x.5^{1/3}}{2}\right)$

$\Rightarrow \log_{10} 2^3 = \log_{10}\left(\frac{x.5^{1/3}}{2}\right)$

$\Rightarrow\ 2^3 = \frac{x.5^{1/3}}{2}$

$\therefore\ x = 16 \times 5^{-1/3}$

7. $\log (0.2)^{25} = 25 \log\left(\frac{2}{10}\right)$

$= 25(\log 2 - \log 10)$

$= -25(1 - 0.30103)$

$= -25 \times 0.69897 = -17.47425$

$\therefore$ Number of zeros after the decimal point is 17.

8. $\log_{32}^{27} \times \log_{243}^{8} = \log_{2^5}^{3^3} \times \log_{3^5}^{2^3}$

$= \frac{3}{5}\log_2^3 \times \frac{3}{5}\log_3^2$

$= \left(\frac{3}{5}\right)^2 \log_2^3 \times \log_3^2 = \frac{9}{25}$

9. $\log_{64}(1/16) = \log_{4^3} 4^{-2} = \frac{-2}{3}$

10. Expression is equal to $\log_x 2 \times 3 \times 4 \times 5 \times 6 \times 7 = \log_x 5040$

11. Go through options.

14. $A = \log_7 2401 = 4$

$B = \log_7(\sqrt{7}.343)$

$\Rightarrow B = \log_7(7^{1/2} \times 7^3) = 3\frac{1}{2}$

$C = \log_{\sqrt{6}} 216 = 2.\log_6 6^3 = 3$

$D = \log_2 32 = 5$

$\therefore$ Arranging in ascending order: *BADC*

15. $\log_{10}\left[1-\left\{1-(1-x^2)^{-1}\right\}^{-1}\right]^{-\frac{1}{2}}=1$

$\Rightarrow \log_{10}\left[1-\left\{1-\dfrac{1}{1-x^2}\right\}^{-1}\right]^{-\frac{1}{2}}=1$

$\Rightarrow \log_{10}\left[1-\left\{\dfrac{-x^2}{1-x^2}\right\}^{-1}\right]^{-\frac{1}{2}}=1$

$\Rightarrow \left[\dfrac{1}{x^2}\right]^{-\frac{1}{2}}=10 \Rightarrow x = 10$

17. $\log_3 x + \log_9 x + \log_{27} x + \log_{81} x = \dfrac{25}{4}$

$\Rightarrow \log_3 x + \dfrac{1}{2}\log_3 x + \dfrac{1}{3}\log_3 x + \dfrac{1}{4}\log_3 x = \dfrac{25}{4}$

$\Rightarrow \log_3 x\ [1 + 1/2 + 1/3 + 1/4] = \dfrac{25}{4}$

$\Rightarrow \dfrac{25}{12} \times \log_3 x = \dfrac{25}{4}$

$\Rightarrow \log_3 x = 3 \Rightarrow x = 27$

18. $\log_y x = 8 \Rightarrow y^8 = x$ .... (1)
$\log_{10y} 16x = 4 \Rightarrow 10^4.y^4 = 16x$ .... (2)
Dividing (2) by (1) $10^4.y^{-4} = 16 \Rightarrow y = 5$

19. Go through options. (Use $\log 1 = 0$)

20. Going through options.

Option (a) is not possible.

Checking option (c): For $x = -1$, expression $= \log_2 4 + \log_2 2 = \log_2 8 = 3\log_2 2 = 3$

21. $\log N^2 - \log 2N = 3\log 3 - \log 6$

OR, $\log\left(\dfrac{N^2}{2N}\right) = \log\dfrac{3^3}{6}$ or, $\dfrac{N}{2} = \dfrac{9}{2}$

$\therefore N = 9$

22. $\log_{3\sqrt{3}} 27 = \log_{3\sqrt{3}} (3\sqrt{3})^2 = 2$

23. $\log_6 36\sqrt{6} = \log_6 36 + \log_6 \sqrt{6}$

$= 2 + 0.5 = 2.5$

24. $\log_e 2.\ \log_p 625 = \log_{10} 16.\ \log_e 10$

$\Rightarrow \log_p 625 = \log_{10} 2^4 \cdot \dfrac{\log_e 10}{\log_e 2}$

$\Rightarrow \log_p 5^4 = 4 \log_{10} 2.\ \log_2 10$

or, $4 \log_p 5 = 4$

$\therefore \log_p 5 = 1 \Rightarrow P = 5$

25. $3\log_{3x^2} 27 - 2 \log_{3x} 9 = 0 \Rightarrow 3\log_{3x^2} 3^3 - 2 \log_{3x} 3^2$
$= 0$

$\Rightarrow 9\log_{3x^2} 3 = 4 \log_{3x} 3$

$\Rightarrow \dfrac{9}{\log_{3x} 3} = \dfrac{4}{\log_{3x^2} 3}$

$\Rightarrow 9 \log_3 3x = 4.\ \log_3 3x^2 \Rightarrow \log_3(3x)9 = \log_3(3x^2)^4$
$\Rightarrow 3^9 \times x^9 = 3^4 \times x^8 \Rightarrow x = 3^{-5}$

$\therefore\ x = \dfrac{1}{243}$

Moderate

1. $N = n!$ (given)

$\dfrac{1}{\log_2 N} + \dfrac{1}{\log_3 N} + \dfrac{1}{\log_4 N} + \ldots\ldots\ldots \dfrac{1}{\log_n N}$
$= \log_N 2 + \log_N 3 + \log_N 4 + \ldots\ldots. + \log_N n$
$= \log_N n! = \log_N N = 1$

2. $x + \log_{10}(1 + 2^x) = x \log_{10} 5 + \log_{10} 6$
$\Rightarrow \log_{10} 10^x + \log_{10}(1 + 2^x) = \log_{10} 5^x.6$
$\Rightarrow 10^x (1 + 2^x) = 5^x .6 \Rightarrow 2^x(1 + 2^x) = 6$
$\therefore\ 2^x = 2$, so $x = 1$.

3. $a_1 = a_1$; $a_2 = a_1 r$; $a_3 = a_1 r^2$; $a_4 = a_1 r^3$ are in G.P. with common difference 'r'
$\therefore \log a_1 = \log a_1$
$\log a_2 = \log a_1 + \log r$
$\log a_3 = \log a_1 + 2\log r$
$\log a_4 = \log a_1 + 3\log r$
$\log a_{n+1} = \log a_1 + n \log r$
$\log a_{n+2} = \log a_1 + (n + 1) \log r$
$\therefore \log a_1, \log a_2, \log a_3, \log a_4, \ldots\ldots\ldots \log a_{n+2}$ are in A.P. with common difference '$\log r$'.
Alternatively,
Assume $a_1 = 10$, $a_2 = 100$ and $a_3 = 1000$.
To find out about $\log a_n$, $\log a_{n+1}$, $\log a_{n+2}$, take $n = 1$.
Hence we have to find that log 10, log 100, log 1000 are in which series.
log 10 = 1, log 100 = 2 and log 1000 = 3
So, log 10, log 100, log 1000 are in AP.

4. $x = \log_a bc \quad \therefore 1 + x = \log_a abc$
$y = \log_b ca \quad \therefore 1 + y = \log_b abc$
$z = \log_c ab \quad \therefore 1 + z = \log_c abc$

$\therefore\ \dfrac{1}{1+x} + \dfrac{1}{1+y} + \dfrac{1}{1+z} = \log_{abc} a + \log_{abc} b + \log_{abc} c$
$= \log_{abc} abc = 1$

5. $\log x : \log y : \log z = (y - z) : (z - x) : (x - y)$

$\therefore \dfrac{\log x}{y-z} = \dfrac{\log y}{z-x} = \dfrac{\log z}{x-y} = k$ (say)

$\therefore \log x = k(y - z)$

$\Rightarrow x \log x = kx(y - z)$

Similarly $y \log y = ky(z - x)$

$z \log z = kz(x - y)$

Adding all 3 equations

$x \log x + y \log y + z \log z = 0 \Rightarrow \log X^x.Y^y.Z^z = 0$

$\therefore X^x.Y^y.Z^z = 1$

6. $\log_{0.04}(x - 1) \geq \log_{0.2}(x - 1)$

$\Rightarrow \dfrac{\log_e(x-1)}{\log_e 0.04} \geq \dfrac{\log_e(x-1)}{\log_e 0.2}$

$\Rightarrow \dfrac{\log_e(x-1)}{\log_e(0.2)^2} \geq \dfrac{\log_e(x-1)}{\log_e 0.2}$

$\Rightarrow \dfrac{\log_e(x-1)}{2\log_e(0.2)} \geq \dfrac{\log_e(x-1)}{\log_e(0.2)}$

$\because \log_e 0.2 < 0$

$\therefore \log_e(x - 1) \leq 2\log_e(x - 1)$

$\therefore x - 1 \geq 1$

$\therefore x \geq 2 \Rightarrow x \in [2, \infty]$

7. $\log_k x.\ \log_5 K = 1$

$\Rightarrow \log_5 x. = 1 \Rightarrow x = 5$

9. Solution 1:

x, y, z are in G.P.

$\therefore \log x, \log y, \log z$ are in A.P. as shown in solution of question No. 3 in moderate section

$\therefore 1 + \log x,\ 1 + \log y,\ 1 + \log z$ are in A.P.

$\therefore \dfrac{1}{1+\log x}, \dfrac{1}{1+\log y}, \dfrac{1}{1+\log z}$ are in H.P.

Solution 2:

Since the numbers are in GP, let us assume the numbers x, y and z to be equal to 10, 100 and 1000.

Then, $\dfrac{1}{1+\log x} = \dfrac{1}{2}$, $\dfrac{1}{1+\log y} = \dfrac{1}{3}$ and $\dfrac{1}{1+\log z} = \dfrac{1}{4}$

These are in HP.

10. $\log_K N = 6 \qquad \Rightarrow K^6 = N$

$\log_{25K} 8N = 3 \quad \Rightarrow (25K)^3 = 8N$

$\therefore \dfrac{25^3 K^3}{K^6} = 8 \Rightarrow 25^3.\ K^{-3} = 2^3$

$\therefore K = \dfrac{25}{2} = 12.5$

11. $\log_b a \log_c a + \log_a b \log_c b + \log_a c \log_b c = 3$

$\Rightarrow \dfrac{(\log a)^2}{\log b.\log c} + \dfrac{(\log b)^2}{\log a \log c} + \dfrac{(\log c)^2}{\log a \log b} = 3$

$\Rightarrow \dfrac{(\log a)^3 + (\log b)^3 + (\log c)^3}{\log a.\log b.\log c} = 3$

$\Rightarrow (\log a)^3 + (\log b)^3 + (\log c)^3 = 3 \log a.\ \log b.\ \log c$

We know, $x^3 + y^3 = z^3 = 3xyz$ when $x + y + z = 0$

$\log a + \log b + \log c = 0 \Rightarrow \log abc = 0 \Rightarrow abc = 1.$

13. $\log_2 x.\log_4 x.\log_6 x = \log_2 x.\log_4 x + \log_2 x.\log_6 x + \log_4 x.\log_6 x$

Dividing LHS and RHS by $\log_2 x.\log_4 x.\log_6 x$

$\Rightarrow 1 = \dfrac{1}{\log_6 x} + \dfrac{1}{\log_4 x} + \dfrac{1}{\log_2 x}$

$\Rightarrow 1 = \log_x 6 + \log_x 4 + \log_x 2$

$\Rightarrow \log_x 6 \times 4 \times 2 = 1$

$\Rightarrow x = 48.$ $\therefore$ only 1 value of x.

14. $a^{\log b} = kb^{\log a}$

$\Rightarrow \log b.\ \log a = \log k + \log a.\ \log b$ {taking log on both sides}

$\Rightarrow \log k = O$

$\therefore K = 1$

$\therefore \log_{ab} k = O$

15. Go through the options.

16. $2[\log(x + y) - \log 5] = \log x + \log y$

$\Rightarrow \log \dfrac{(x+y)^2}{5^2} = \log xy$

$\Rightarrow \dfrac{(x+y)^2}{25} = xy \Rightarrow x^2 + y^2 + 2xy = 25xy$

$\therefore x^2 + y^2 = 23xy$

17. Let the base be 'b'.

$\therefore \log_b N = 9$ and $\log_{11b} 64N = 6$

$\Rightarrow N = b^9$ and $64N = (11b)^6$

$\therefore 64 = \dfrac{11^6 b^6}{b^9} \qquad \therefore b^3 = \dfrac{11^6}{64}$

$\therefore b = \dfrac{11^2}{2^2} = \dfrac{121}{4}$

18. $\log_{30} 3 = x,\ \log_{30} 5 = y$

$\therefore x + y = \log_{30} 15$

$\Rightarrow x + y = \log_{30}(30/2)$

$\Rightarrow (x + y) = 1 - \log_{30} 2 \Rightarrow \log_{30} 2 = 1 - x - y$

$\Rightarrow 3\log_{30} 2 = 3(1 - x - y) \Rightarrow \log_{30} 8 = 3(1 - x - y)$

$\therefore \log_8 30 = \dfrac{1}{3(1-x-y)}$

19. $\frac{1}{\log_2 n} + \frac{1}{\log_3 n} + \frac{1}{\log_4 n} + \ldots \frac{1}{\log_{40} n}$

$= \log_n 2 + \log_n 3 + \log_n 4 + \ldots + \log_n 40$

$= \log_n (40!)$

$= \frac{1}{\log_{(40!)} n}$

20. $\log_2(x + 5) = 6 - x$

Draw the graph of $\log_2(x + 5)$ and $(6 - x)$:

$\therefore$ The two graphs will intersect at only one point.

21. It can be simply seen that in the expression $\log (x \times y \times z) = 3 \log X + 4 \log 2$, when we have replaced y and z on the LHS, answer is in constant with a term of x only. It means y and z has to be replaceable with each other. Only option 3 satisfies this relationship.

22. $\log_3 2.\log_4 3.\log_5 4.\ldots.\log_{16} 15.$

$$= \frac{\log 2}{\log 3}.\frac{\log 3}{\log 4}.\frac{\log 4}{\log 5}\ldots\ldots\frac{\log 15}{\log 16} = \frac{\log 2}{\log 16} = \frac{\log 2}{\log 2^4}$$

$$= \frac{\log 2}{4 \log 2} = \frac{1}{4}$$

23. Put the values of a and b in $ab + 5(a - b)$ after converting them to same base.

24. Use options to solve this question.

25. Use options to solve this question.

15

CHAPTER

Permutation and Combination

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Difference between permutation and combination
- Different cases of permutation such as word formation
- Different cases of combination such as selection of a team
- Application of P and C to derive more formulae
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

Permutations and combinations has lately emerged as an important topic for management entrance examinations. This is primarily because questions from this topic require analytical skills and a logical bent of mind. Even students who do not have math as a subject can handle them if they have a fairly good understanding of the concepts and their application. However, a point to note is that questions from permutations and combinations asked in the CAT are not based on mathematical concepts. Hence anyone who is well-versed in different methods of counting and basic calculations will be able to solve these problems easily.

MEANING OF PERMUTATION AND COMBINATION

If we go by the dictionary meaning of the words permutation and combination, then permutation is the number of ways in which a set or a number of things can be put in an order or arranged and; combination refers to the number of ways in which a group of things can be chosen from a larger group without regard to their arrangement.

Let us go through an example. Suppose there are four different batsmen A, B, C and D and we have to select a group of three batsmen out of these four. Now, we can select any combination of three batsmen so that no set of batsmen has all the same three batsmen. These set of batsmen will be — ABC, BCD, ABD, ACD. This is a case of combination as for every set of selection of three batsmen, order of selection does not play any role (i.e., we can select anybody—first or second or third—and it does not create any difference in the final selection as well as in the total number of selections).

Now, if we try to define their batting order also, i.e., who bats first and second and so on, then corresponding to every selection of a set of three batsmen, we will have six different arrangements of their batting order. It can be seen below that corresponding to the selection of ABC as a team, following is the list of different batting orders:

ABC, ACB, BAC, BCA, CAB, CBA.

This is a case of permutation since the order of occurrence has become important. Since there are four different ways of selecting a group of three batsmen and every selection can be arranged in 6 different ways, so the total number of ways of arranging 3 batsmen (or, distinct things) out of 4 batsmen (or, distinct things) = 4 × 6 = 24 ways.

Permutation and combination can be better understood through the examples of hand-shake and gifts exchange also. Assume that there are 20 persons in a party and everybody shakes hand with each other and also presents a gift. Now if we take a case of two persons A and B, then the event of shaking hand between them is a case of combination because when A shakes hand with B or B shakes hand with A, the number of hand shake is just one. So, there is no order as such and hence it is a case of combination.

Similarly, the event of presenting the gift is a case of permutation because the gift given to B by A and the gift given to A by B are two different gifts. So, the order of case plays a role here and hence it is a case of permutation.

$n!$ = Product of all the natural numbers from n to $1 = n(n-1)(n-2)(n-3)\ldots \times 3 \times 2 \times 1$.

$0! = 1$

Factorials are defined only for whole numbers, and not for negative numbers or fractions (≠ whole numbers).

FUNDAMENTAL PRINCIPLES OF COUNTING : TWO BASIC THEOREMS

1. Multiplication theorem

If there are two jobs in such a way that one of them can be done in m ways and when it is completed in any of the m ways, the second job can be completed in n ways, then the whole job can be done in $m \times n$.

2. Addition theorem

If there are two jobs in such a way that one of them can be done in m ways and the second one can be done in n ways independently, then either of the jobs can be done in $(m + n)$ ways.

Basically, there is one point where these two theorems differ—in multiplication the job does not get completed while in addition it gets completed. In a layman's language, we multiply the number of ways when the job has not been completed and we add the number of ways when the job has been completed.

Example 1 There are 10 girls and 15 boys in a class. In how many ways can

i. a class representative be selected?
ii. a team of two students be chosen with one girl and one boy?

Solution

i. A class representative can be a girl or a boy. Now, one girl can be selected from 10 girls in 10 ways (any of the girls can be selected) and one boy can be selected from 15 boys in 15 ways (any of the boys can be selected). So the ways of selecting a class representative includes either selecting a boy or a girl. So, the total number of ways of selecting a class representative =10 + 15 = 25
In this case, the moment a girl gets selected, the job is completed. There are some more ways of doing this by selecting a boy. So, it is a case of addition.
ii. One girl can be chosen from 10 girls in 10 ways. Now corresponding to every selected girl, any one of the 15 boys can be selected in 15 ways.
It can be seen in the following presentation:
Girl selected (Assume name of the girls are $G_1, G_2, G_3, \ldots G_9, G_{10}$) – G_1

Boy selected (Assume the names of the boys are $B_1, B_2, B_3, \ldots B_{14}, B_{15}$)– B_1, Or B_2, or B_3, or B_{15}.

So, corresponding to G_1, the total number of selection of a boy = 15

Corresponding to G_2, the total number of selection of a boy = 15

Corresponding to G_3, the total number of selection of a boy = 15

… … … … …

Corresponding to G_{15}, the total number of selection of a boy = 15

So, the total number of ways of selecting a team of one boy and a girl = the total number of ways of selecting a girl × the total number of ways of selecting a boy = 10 × 15 = 150

In this case, just by selecting a girl or a boy, work has not been completed. So, it is a case of multiplication.

Another example of multiplication theorem If there are three cities A, B and C located in such a way that there are 3 roads joining A and B, and 4 roads joining B and C, then the number of ways one can travel from A to C is 3 × 4, i.e., 12.

PERMUTATIONS

As we have seen, the arrangements made by taking some or all elements out of a number of things is called a permutation. Permutation implies "arrangement" where "order of the things" is important.

The permutations of three things, *a b* and *c*, taken two at a time are *ab, ba, ac, ca, cb* and *bc*. Since the order in which the things are taken is important, *ab* and *ba* are counted as two different arrangements.

The number of permutations of n things taking r at a time is denoted by ^nP_r.

COMBINATIONS

As we have seen, the groups or selections made by taking some or all elements out of a number of things is called a combination. In combination, the order in which the things are taken is not important.

The combination of three things, *a*, *b* and *c*, taken two at a time are *ab*, *bc* and *ca*. Here, *ab* and *ba* are same because the order in which *a* and *b* are taken is not important. What is required is only a combination including *a* and *b*. The words "combination" and "selection" can be used without any differentiation.

The number of combinations of n things taking r at a time is denoted by ^nC_r.

Approaching a Problem

Mostly the questions asked in the CAT are self-explanatory, i.e., they clearly mention what process is to be used—permutation or combination.

In case, the question does not specify this, you should try to find out whether it is a case of permutation or a case of combination. Sometimes the problem very clearly states whether it is the number of permutations (or arrangements) or the number of combinations (or selections) that has to be found out. The questions can be as follows:

For permutations:

"What is the number of permutations that can be done … " or "What is the number of arrangements that can be made… " or "Find the different numbers of ways in which something can be arranged… etc.

For combinations:

"What is the number of combinations that can be done … " or "What is the number of selections that can be made … " or "Find the different numbers of ways in which things can be selected etc.

Some other standard examples of permutation and combination are:

Permutation Word formation, number formation and circular permutation etc.

Combination Selection of a team, forming geometrical figures, distribution of things (except some particular cases).

Still, sometimes the questions may not explicitly state what you have to find—permutation or combination. In that case, the nature of what is to be found out will decide whether it is the number of permutations or the number of combinations.

See the example given below

I have to invite two of my eight friends to my anniversary party. In how many different ways can I do this?

Assume my eight friends are A, B, C, D, E, F, G and H. Whether the two friends that I call for the party are A and B or B and A, does not make any difference. As we have discussed before, that what matters most in case of permutation is the order of occurrence of things. As order does not play any role here, it is clearly the case of combination.

Meaning and Derivation of ^nP_r and ^nC_r

Number of permutations of n different things taking r at a time = ^nP_r

In this statement, we take the following two assumptions:

- All the n things are distinct (or no two things are of the same type)
- Each thing is used at most once (i.e., nothing is repeated in any arrangement)

Let us assume that there are r boxes and each of them can hold one thing. When all the r boxes are filled, what we have is an arrangement of r things taken from the given n things. So, each time we fill up the r boxes with things taken from the given n things, we have an arrangement of r things taken from the given n things without repetition. Hence, the number of ways in which we can fill up the r boxes by taking things from the given n things is equal to the number of permutations of n things taking r at a time.

The first box can be filled in n ways (because this box can be filled by any one of the n things given). After filling the first box, we now have only $(n-1)$ things to fill the second box; any one of these things can be used to fill the second box and hence the second box can be filled in $(n-1)$ ways; Similarly, the third box can be filled in (n–2) ways and so on the rth box can be filled in (n–(r–1) ways, i.e., [n–r+1] ways. Hence, from the fundamental rules of counting, all the r boxes together can be filled up in → $n. (n-1). (n-2) \ldots (n-r+1)$ ways

Hence, ${}^nP_r = n. (n-1). (n-2) \ldots (n-r+1)$

This can be simplified by multiplying and dividing the right hand side be $(n-r)\ (n-\overline{r-1}) \ldots 3.2.1.$

$$ {}^nP_r = n\,(n-1)(n-2) \ldots [n-\overline{r-1}] $$
$$ = \frac{n!}{(n-r)!} $$

The number of arrangements of n distinct things taken r things at a time is

$$ {}^nP_r = \frac{n!}{(n-r)!} $$

If we take n things at a time, then we get ${}^nP_n$. From the discussion similar to that we had for filling the r boxes above we can find that ${}^nP_n$ is equal to $n!$ The first box can be filled in n ways, the second one in $(n-1)$ ways, the third one in $(n-2)$ ways and so on, then the *nth* box in 1 way; hence, all the n boxes can be filled in

$$ {}^nP_n = n! $$

But if we substitute $r = n$ in the formula for ${}^nP_n$ then we get ${}^nP_n = \frac{n!}{0!}$; since we already found that ${}^nP_n = n!$

We can conclude that $0! = 1$

The number of combinations of n distinct things taking r at a time = ${}^nC_r$

Let the number of combinations ${}^nC_r$ be S. Consider one of these S combinations. Since this is a combination, the order of the r things is not important. If we now impose the condition that order is required for these r things, we can get $r!$ arrangement from this one combination. So each combination can give rise to $r!$ permutations. S combinations will thus give rise to S × r! permutations. But since these are all permutations of n things taking r at a time, this must be equal to ${}^nP_r$. So,

$$ S \times r! = {}^nP_r = \frac{n!}{(n-r)!} $$

$$ \text{So, } S = {}^nC_r = \frac{n!}{(n-r)!} \times \frac{1}{r!} $$

It can also be deduced from here that the number of selections of n distinct things taken all at a time will be equal to 1 (since there is only one way in which all the articles can be selected).

$$ \text{Alternatively } {}^nC_n = \frac{n!}{0! \times n!} = 1 $$

Out of n things kept in a bag, if we select r things and remove them from the bag, we are left with $(n-1)$ things inside the bag i.e., whenever r things are selected out of n things, we automatically have another selection of $(n-1)$ things. Hence, the number of ways of making combinations taking r out of n things is the same as selecting (n–r) things out of n given things, i.e.,

$$ {}^nC_r = {}^nC_{n-r} $$

Before we move ahead, let us once again make it clear that whenever we are using ${}^nC_r$ and ${}^nP_r$, our assumption is that all the things are distinct, i.e., no two of them are same.

Example 2 Munchun has 10 children. She takes 3 of them to the zoo at a time, as often as she can, but she does not take the same three children to the 0 more than once. How many times Munchun will be required to go to the zoo?

(a) 120 (b) 45 (c) 90 (d) 180

Solution Number of times (read ways) 3 children (read distinct things) can be selected from 10 children (read distinct things) = ${}^{10}C_3$.

So, she will be required to go to the zoo ${}^{10}C_3$ times.

So, option (a) is the answer.

Example 3 In the above question, how many times a particular child will go?

(a) 72 (b) 45 (c) 90 (d) 36

Solution Consider the case for any particular child C_1 Since C_1 has already been selected, so out of the rest 9 children Munchun will be required to select 2 more children. This can be done on ${}^9C_2$ ways.

So, option (d) is the answer.

Example 4 In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is

(a) 200 (b) 216 (c) 235 (d) 256

Solution Let there be m boys and n girls

$$ {}^nC_2 = 45 = \frac{n(n-1)}{2} \Rightarrow n(n-1) = 90 \Rightarrow n = 10 $$

$$ {}^mC_2 = 190 = \frac{m(m-1)}{2} = 190 \Rightarrow m\ (m-1) = 380 \Rightarrow m = 20 $$

Number of games between one boy and one girl

$= {}^{10}C_1 \times {}^{20}C_1 = 10 \times 20 = 200$

Hence, option (a) is the answer.

Example 5 In how many ways can three persons be seated on five chairs?

Solution This question is a very fundamental problem of arrangements without repetition. The first person can sit in 5 ways (into any of the five chairs), the second person can take place in 4 ways (into any of the remaining 4 chairs) and the third person can sit in 3 ways.

So, the total number of ways in which these 3 persons can arrange themselves on 5 chairs is 5 × 4 × 3 = 60.

Some Important Derivations

While deriving an expression for ${}^nP_r$, we imposed two constraints, viz. distinct things and repetition being not allowed over it and learned how to find the number of permutations. Let us now see what will happen if we do not impose these two restrictions on ${}^nP_r$.

Number of arrangements of *n* things of which *p* are of one type, *q* are of a second type and the rest are distinct When all the things are not distinct, then we cannot use the general formula for ${}^nP_r$ for any value of *r*. If we want to find out ${}^nP_r$ for a specific value of *r* in that given problem, we will be required to use it on the basis of the given situation.

The number of ways in which *n* things may be arranged taking all of them at a time, when *p* of the things are exactly alike of one kind, *q* of them exactly alike of another kind, *r* of them exactly alike of a third kind, and the rest all is distinct is

$$\frac{n!}{p!.q!.r!}$$

Number of permutations of *n* distinct things where each one of them can be used for any number of times (i.e., repetition allowed) Derivation for this is based upon common sense. If I have 5 friends and 3 servants and I have to send the invitation letters to all my friends through any of my servants, I obviously have 3 options for the invitation card to be sent to friend1, the same 3 options for the invitation card to be sent to friend2, and similarly 3 options for the invitation card to be sent to each of the friends. So the total number of ways of sending the invitation letters = 3^5 And it will not be 5^3, as friends are not going to the servants to get the letter.

In general, the number of perambulations of *n* things, taking *r* at a time when each of the thing may be repeated once, twice,... up to *r* times in any arrangement is n^r.

Total Number of Combinations

Out of *n* things, the number of ways of selecting one or more things:

where we can select 1 or 2 or 3 ... and so on *n* things at time; hence, the number of ways is ${}^nC_1 + {}^nC_2 + {}^nC_3 + \dots {}^nC_n$. $= 2^n - 1$, where *n* is the number of things.

Above derivation can also be understood in the following manner:

Let there be *n* bags.

The first bag can be dealt in two ways—it is either included or not included. Similarly, the second bag can be dealt in two ways, the third one in two ways and so on, the *nth* bag in two ways. Using multiplication theorem of counting, the number of ways of dealing with all the bags together is $2 \times 2 \times 2 \times \dots n$ times $= 2^n$ ways. But out of these, there is one combination where we do not include any of the bags. This is not allowed because we have to select at least one thing.

Hence, the number of ways of selecting one or more things from *n* given things is $2^n - 1$.

Distributing the given things (*m* + *n*) into two groups where one group is having m things and other one *n* things If we select *m* things (which can be done in ${}^{m+n}C_m$ ways), then we will be left with *n* things, i.e., we have two groups of *m* and *n* things respectively. So, the number of ways of dividing (*m* + *n*) things into two groups of *m* and *n* things respectively is equal to ${}^{m+n}C_m$.

$${}^{m+n}C_m = \frac{(m+n)!}{n!.m!}$$

If we take *m* = *n*, then the above expression will denote "Distributing 2m things equally between two distinct groups $= {}^{2m}C_m = \dfrac{(2m)!}{m!.m!}$

However, when the groups are identical, then we will be required to divide the above result by 2!.

Hence, in that case it becomes $\dfrac{(2m)!}{2!m!.m!}$

(Refer to word formation examples)

All the above derivations with their different applications can be seen below in a summarized form.

1. Fundamental Principle of counting:
   (a) *Multiplication rule* If a work is done only when all the number of works are done, then the number of ways of doing that work is equal to the product of the number of ways of doing separate works.
   (b) *Addition rule* If a work is done only when any one of the number of works is done, then the number of ways of doing that work is equal to the sum of the number of ways of doing separate works.
   Thus, if a work is done when exactly one of the number of works is done, then the number of ways of doing this work = sum of the number of ways of doing all the works.
2. If ${}^nC_x = {}^nC_y$ then either $x = y$ or $x + y = n$.
3. $\underline{|n} = 1.2.3 \dots n$; $\underline{|0} = 1$
4. (a) The number of permutations of *n* different articles taking *r* at a time is denoted by ${}^nP_r$ and ${}^nP_r = \dfrac{\underline{|n}}{\underline{|n-r}}$
   (b) The number of permutations of n different articles taking all at a time = $\underline{|n}$.
   (c) The number of permutations of *n* articles, out of which *p* are alike and are of one type, *q* are alike and are of second type and rest are all different $= \dfrac{\underline{|n}}{\underline{|p} = \underline{|q}}$.

5. The number of permutations (arrangements) of n different articles taking r at a time when articles can be repeated any number of times = $n \times n \times \ldots r$ times = n^r.
6. Circular permutations:
   i. The number of circular permutations (arrangements) of n different articles = $\underline{|n-1}$.
   ii. The number of circular arrangements of n different articles when clockwise and anticlockwise arrangements are not different i.e., when the observation can be made from both the sides $= \dfrac{\underline{|n-1}}{2}$.
7. The number or combinations of n different articles taking r at a time is denoted by ${}^nC_r$ and ${}^nC_r = \dfrac{\underline{|n}}{\underline{|r}\,\underline{|n-r}}$.
8. The number of selections of r articles ($r \le n$) out of n identical articles is 1.
9. Total number of selections of zero or more articles from n distinct articles = ${}^nC_0 + {}^nC_1 + {}^nC_2 + \ldots {}^nC_n = 2^n$.
10. Total number of selections of zero or more articles from n identical articles = $1 + 1 + 1 + \ldots$ to $(n + 1)$ terms = $n + 1$.
11. The number of ways of distributing n identical articles among r persons when each person may get any number of articles = ${}^{n+r-1}C_{r-1}$.
12. The number of ways of dividing $m + n$ different articles in two groups containing m and n articles respectively ($m \ne n$)

$$= {}^{m+n}C_n \times {}^mC_m = \frac{\underline{|m+n}}{\underline{|m}\,\underline{|n}}.$$

13. The number of ways of dividing $2m$ different articles each containing m articles $= \dfrac{\underline{|2m}}{\underline{|m}\,\underline{|m}\,\underline{|2}}$.
14. The number of ways of dividing $3m$ different articles among three persons and each is getting m articles = $\dfrac{\underline{|3m}}{\underline{|m}\,\underline{|m}\,\underline{|m}\,\underline{|3}}\underline{|3}$.
15. The number of ways of selecting n distinct articles taken r at a time when p particular articles are always included = ${}^{n-p}C_{r-p}$.
16. ${}^nC_{r-1} + {}^nC_r = {}^{n+1}C_r$
17. ${}^np_r = r.{}^{n-1}p_{r-1} + {}^{n-1}P_r$

SOME STANDARD FORMATS OF QUESTIONS

Word Formation

As we know that order of occurrence of letters decide the formation of words, so word formation is one standard example of permutation.

Let us understand with the help of some examples:

Example 6 How many words can be formed with the word 'LUCKNOW' which have

i. no restriction
ii. L as the 1st letter of the word
iii. L and W as the terminal letters
iv. all the vowels together
v. L always occuring before U
vi. L always occuring before U and U always occuring before W

Solution

i. The total number of distinct letters = 7 (L, U, C, K, N, O, W)
   So, the total number of words that can be formed = 7!
ii. Now we can arrange only 6 letters (as place of L is restricted),
   So, the total number of words that can be formed = 6!
iii. Now we can arrange only 5 letters (as place of L and W are restricted),
   So, the number of arrangements = 5!
   But the place of L and W can be interchanged between themselves.
   So, the total number of words that can be formed = 5! × 2!
iv. U and O should be together, so we will assume these two letters to be tied up with each other.
   Now we have 6 distinct things to be arranged– (L, U, O, C, K, N, W)
   So, the number of arrangements = 6!
   But place of U and O can be interchanged between themselves.
   So, the total number of words that can be formed = 6! × 2!
v. There is an equal likelihood occurrence of all the letters in the word, so in half of the cases L will occur before U and in the remaining half, U will occur before O.
   So, the total number of words that can be formed = 7!/2
vi. There are six possible arrangements (3!) corresponding to L, U and W. However, only one out of these six will be in the prescribed order: L always occurs before U and U always occurs before W.
   So, corresponding to 7! arrangements, the number of ways in which the condition will be satisfied = 7!/3! ways.

Example 7 How many new words can be formed with the word 'PATNA'?

Solution Total number of letters— P, T, N occur once while A occurs twice.

So, the total number of words that can be formed = 5!/2! = 60

Total number of new words = 60 – 1 = 59

Example 8 How many words can be formed with the word 'ALLAHABAD'?

Solution Letters are— A – Four times

L – Twice

H, B and D occur once.

So, the total number of words = $\frac{9!}{4!2!}$

Example 9 How many 4-lettered distinct words can be formed from the letters of the word 'EXAMINATION'?

Solution Letters are: A – Twice

I – Twice

N – Twice

E, X, M, T, O – Once

Words will be of three types:

i. All distinct, ii. Two same, two distinct, iii. Two same and of one kind: two same and of other kind.

i. All distinct = ${}^8P_4$ (Distinct letters are – A, I, N, E, X, M, T and O)

ii. Two same, two distinct

Selection of one pair out of the three pairs (A, I, N) can be chosen in ${}^3C_1$ ways. Now rest of the two distinct letters can be chosen in ${}^7C_2$ ways.

Total number of words = ${}^3C_1 \times {}^7C_2 \times \frac{4!}{2!}$

iii. Two same and are of one kind, two same and are of other kind = Out of the three pairs of letters (A, I, N), we can select two pairs in ${}^3C_2$ ways.

Total number of words = ${}^3C_2 \times \frac{4!}{2! \times 2!}$

Number Formation

Number formation is another standard example of permutation. Here we will discuss the box diagram method of solving the questions.

If a three-digit number is to be constructed, then we will use

Hundred's place	Ten's place	Unit's place

If a four-digit number is to be constructed, then we will use

Thousand's place	Hundred's place	Ten's place	Unit's place

and so on.

While solving the questions related to number formation, we should know two things very clearly:

- While using the box diagram, we should start with the digit which has restriction, i.e., some condition is imposed on that digit.
- When nothing about the repetition of digits is mentioned in the question, we will assume that the repetition is allowed.

Example 10 How many different 3-digit numbers can be formed using the digits 1, 2, 3, 4 and 5?

i. When repetition is not allowed

ii. When repetition is allowed

Solution The box given below represents the respective positioning of digits in a three-digit number.

Hundred's place	Ten's place	Unit's place

i. Since repetition of the digits is not allowed, we can fill the unit's place in 5 ways, ten's place in 4 ways and hundred's place in 3 ways.

3	4	5

Using Multiplication Theorem, the total number of numbers which can be formed = 5 × 4 × 3 = 60

Alternatively, 3 digits can be selected out of 5 digits in ${}^5P_3 = 60$

ii. Since repetition of the digits is allowed here, we can fill each of the hundred's, ten's and unit's place in 5 ways.

5	5	5

Using Multiplication Theorem, the total number of numbers which can be formed = 5 × 5 × 5 = 125

Example 11 How many 4-lettered numbers divisible by 4 can be formed from the digits 0, 1, 2, 3, 4, 5?

Solution Any number divisible by 4 will have the number formed by its last two digits divisible by 4.

In this case, last two digits of the number can be 00, 04, 12, 20, 24, 32, 40, 44, 52.

Corresponding to any one of 00, 04, 12, 20, 24, 32, 40, 44, 52, we can have the following digits at its hundred's and thousand's place:

5	6	1	1

Thousand's place cannot be filled by 0, so it can be filled in 5 ways.

Hundred's place can be filled by any of the 0, 1, 2, 3, 4, 5; hence 6 ways.

So, corresponding to any one of 00, 04, 12, 20, 24, 32, 40, 44, 52, the total number of ways = 5 × 6 = 30

So, the total number of numbers which can be formed = 30 × 9 = 270

Example 12 In the above question, how many numbers can be formed if repetition of the digits is not allowed?

Solution Last two digits of this number can be—04, 12, 20, 24, 32, 40, 52.

At this point, we will have to divide the process of solving this question—one part will have those numbers which contain '0' as one of its last two digits viz., 04, 20, 40 and other part will have the remaining numbers viz., 12, 24, 32, 52.

1st part – Last two digits are 04, 20, 40.

4	3	1	1

$= 4 \times 3 = 12$ ways

Hence, the total number of numbers which can be formed $= 12 \times 3 = 36$

2nd part — Last two digits are 12, 24, 32, 52.

'0' cannot occur at thousand's place.

3	3	1	1

$= 3 \times 3 = 9$ ways

Hence, the total number of numbers which can be formed $= 9 \times 4 = 36$

Total numbers = 36 + 36 = 72

Example 13 How many odd integers from 1000 to 8000 have none of its digits repeated?

Solution There are two restrictions operating in this questions:

i. For a number to be odd, unit digit should be either 1 or 3 or 5 or 7 or 9.

ii. Thousand's place cannot be filled with 8 or 9.

For unit's digit — When it is filled with 9, thousand's place can be filled in 7 ways namely any digit from 1 to 7, and the remaining two places can be filled in $8 \times 7 = 56$ ways.

So, the total number of numbers formed in this way = $56 \times 7 = 392$

Now, if the unit's place is filled with any of the four digits 1, 3, 5 or 7, the thousand's place can be filled in 6 ways (0 will be excluded), and the remaining two places can be filled in $8 \times 7 = 56$ ways.

So, the total number of numbers formed in this way = $56 \times 6 \times 4 = 1344$

So, the total number of numbers = 392 + 1344 = 1736

Example 14 How many integers from 6000 to 6999 have atleast one of its digits repeated?

Solution Total number of numbers = None of its digits repeated numbers + atleast one of its digits repeated number (i.e., either the digits will be repeated or not repeated).

Total numbers with none of its digits repeated $= 1 \times 9 \times 8 \times 7 = 504$

So, the numbers having atleast one of its digits repeated = 1000 – 504 = 496

Example 15 How many natural numbers less than a million can be formed using the digits 0, 7 and 8?

Solution The numbers formed would be of a single digit, two digits, three digits, four digits, five digits and six digits.

Single-digit numbers = 7 and 8

For two-digit numbers,

2	3

$= 2 \times 3 = 6$ numbers

For three-digit numbers,

2	3	4

$= 2 \times 3 \times 3 = 18$ numbers

For four-digit numbers,an

2	3	3	3

$= 2 \times 3 \times 3 \times 3 = 54$ numbers

For five-digit numbers,

2	3	3	3	3

$= 2 \times 3 \times 3 \times 3 \times 3 = 162$ numbers

For six-digit numbers,

2	3	3	3	3	3

$= 2 \times 3 \times 3 \times 3 \times 3 \times 3 = 486$ numbers

So, the total number of numbers = 728

CIRCULAR PERMUTATION

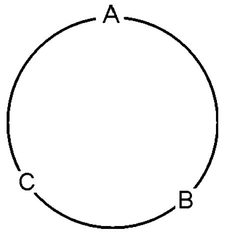
When n distinct things are to be arranged in a straight line, we can do this in $n!$ ways. However, if these n things are arranged in a circular manner, then the number of arrangements will not be $n!$.

Let us understand this:

The number of ways A, B and C can be arranged in a straight line = 3! = 6

The possible arrangements are — ABC, ACB, BAC, BCA, CAB, CBA

Now arrange these three people A, B and C in a circle



What we observe here is that the arrangements ABC, BCA and CAB are the same. And similarly the arrangements ACB, CBA and BAC are the same.

So, there are only two permutations in this case of circular permutation.

To derive the formula for circular permutation, we first fix the position of one thing then the remaining $(n - 1)$ things can be arranged in $(n - 1)!$ ways.

Hence, the number of ways in which n distinct things can be arranged in a circular arrangement is (n – 1)!

It can be seen in the following way also:

If n things are arranged along a circle, then corresponding to each circular arrangement the number of linear arrangement = n

So, the number of linear arrangements of n different things = n × (number of circular arrangements of n different things)

Hence, the number of circular arrangements of n different things = $(1/n)$ × number of linear arrangements of n different things = $(1/n) \times n! = (n-1)!$

Clockwise and Anti-clockwise Circular Arrangements

If we take the case of four distinct things A, B, C and D sitting around a circular table, then the two arrangements ABCD (in clockwise direction) and ADCB (the same order but in anti-clockwise direction) will be different and distinct. Hence, we can conclude that the clockwise and anti-clockwise arrangements are different. However, if we consider the circular arrangement of a necklace made of four precious stones A, B, C and D, the two arrangements discussed as above will be the same because we take one arrangement and turn the necklace around (front to back), then we get the other arrangement. Hence, the two arrangements will be considered as one arrangement because the order of stones is not changing with the change in the side of observation. So, in this case there is no difference between the clockwise and the anti-clockwise arrangements.

Summarizing the above discussion, the number of circular arrangements of n distinct things is $(n-1)!$ if there is a difference between the clockwise and anti-clockwise arrangements and $(n-1)!/2$ if there is no difference between the clockwise and anti-clockwise arrangements.

Example 16 In how many ways 5 Indians and 4 Americans can be seated at a round table if

i. There is no restriction
ii. All the four Americans sit together
iii. No two Americans sit together
iv. All the four Americans do not sit together

Solution

i. Total number of persons = 9. These 9 persons can be arranged around a circular table in 8! ways.

ii. Assuming all the Americans to be one group, we have 6 things (5 Indians + 1 group) to be arranged around a circular table which can be arranged in 5! ways. However, these 4 Americans can be arranged in 4! ways among themselves.
So, the total number of arrangements = 5! × 4!

iii. Since there is no restriction on Indians. The 5 Indians can be seated around a table in 4! ways. The Americans will now be seated between two Indians, i.e., 5 places. 4 Indians can be seated on these 5 places in ${}^5P_4$ ways.

iv. The total number of arrangements when there is no restriction = 8! and the number of arrangements when all the four Americans sit together = 5! × 4!
So, the total number of arrangements when all the four Americans do not sit together = 8! – 5! × 4!.

PERMUTATION AND COMBINATION IN GEOMETRY

It is quite difficult to quantify the importance of P and C in geometry, a good number of P and C questions which use the concepts of geometry (and vice-versa) have been asked in the CAT and other premier B-school exams.

Example 17 How many diagonals will be there in an n-sided regular polygon?

Solution An n-sided regular polygon will have n vertices. And when we join any of these two vertices (${}^nC_2$) we get a straight line, which will be either a side or a diagonal.

So, ${}^nC_2$ = Number of sides + number of diagonals = n + number of diagonals

Hence, the number of diagonals = ${}^nC_2 - n = \frac{n(n-3)}{2}$

Above written result can be used as a formula also.

Example 18 Ten points are marked on a straight line and 11 points are marked on another parallel straight line. How many triangles can be constructed with vertices among the above points?

Solution Triangles will be constructed by taking one point from the 1st straight line and two more points from the 2nd straight line, and vice versa.

So, the total number of Δ formed = ${}^{10}C_2 \times {}^{11}C_1 + {}^{11}C_2 \times {}^{10}C_1 = 1045$

Example 19 There is an n-sided polygon (n>5). Triangles are formed by joining the vertices of the polygon. How many triangles can be constructed which will have no side common with the polygon?

Solution An n-sided polygon will have n vertices. Triangles constructed out of these n vertices will be of three types:

i. Having two sides common with the polygon,
ii. Having one side common with the polygon,
iii. Having no side common with the polygon. And total number of triangles formed will be ${}^nC_3$.

i. Having two sides common with the polygon — Out of total n vertices, any combination of three consecutive vertices will give us the triangle which is having two sides common with polygon = n

ii. Having one side common with the polygon — Number of selection of three vertices out of which two are consecutive (If we select A_5 and A_6 as the two vertices, then A_7 or A_4 should not be the third vertex because it will constitute the two sides of the common triangle).
$= n \times {}^{(n-4)}C_1$

iii. So, the total number of triangles having no side common with polygon = ${}^{n}C_3 - n \times {}^{(n-4)}C_1 - n$

Some More Important Results:

1. Maximum No.of points of Intersection among n Straight Lines = ${}^{n}C_2$
2. Maximum No. of points of Intersection among n Circles = ${}^{n}P_2$

FINDING THE RANK OF A WORD

To find the rank of a word out of all the possibilities using all the letters given in the word is nothing but the extension of the concept of alphabetically arranging the words in a dictionary. However, unlike the case of the dictionary, we can have 'meaningless' words also in the case of finding the rank.

Example 20 All the letters of the word 'LUCKNOW' are arranged in all possible ways. What will be the rank of the word LUCKNOW?

Solution Alphabetical order of occurrence of letters— C, K, L, N, O, U, W.

Number of words starting with C = 6!
Number of words starting with K = 6!
All the words starting with LC – 5!
All the words starting with LK – 5!
All the words starting with LN – 5!
All the words starting with LO – 5!
Next word will start with LU – C – K – N – O – W.
So, rank of LUCKNOW – 2 × 6! + 4 × 5! + 1 = 1921.

Example 21 In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is

(a) 200 (b) 216 (c) 235 (d) 256

Solution Let there be m boys and n girls

$${}^{n}C_2 = 45 = \frac{n(n-1)}{2} \Rightarrow n(n-1) = 90 \Rightarrow n = 10$$

$${}^{m}C_2 = 190 = \frac{m(m-1)}{2} = 190 \Rightarrow m(m-1) = 380 \Rightarrow m = 20$$

Number of games between one boy and one girl
$= {}^{10}C_1 \times {}^{20}C_1 = 10 \times 20 = 200$
Hence, option (a).

Example 22 If each permutation of the digits 1, 2, 3, 4, 5, 6 is listed in the increasing order of the magnitude, the 289th term will be

(a) 326541 (b) 341256
(c) 356241 (d) 314256

Solution $289 = (2 \times 5!) + (2\times 4!) + 1$
So, the number will be 341256.

Example 23 There are 12 intermediate stations between two places A and B. In how many ways can a train be made to stop at 4 of these 12 intermediate stations provided no two of them are consecutive?

Solution

1st Method Let $S_1, S_2, \ldots, S_8$ denote the stations where the train does not stop. The four stations where the train stops should be at any four of the nine places indicated by cross.

$$\therefore \text{ Required number} = {}^{9}C_4 = \frac{9.8.7.6}{\underline{|4}} = 126$$

2nd Method Let S_1, S_2, S_3, S_4 be the four intermediate stations where the train stops.

$$\boxed{a}\ S_1\ \boxed{b}\ S_2\ \boxed{c}\ S_3\ \boxed{d}\ S_4\ \boxed{e}$$

Let a, b, c, d, e be the number of stations between A and S_1, S_1 and S_2, S_2 and S_3, S_3 and S_4 and B respectively.

Then, $a + b + c + d + e = 8$ ...(1)
Where $a \geq 0, b \geq 1, c \geq 1, d \geq 1, e \geq 0$
Let $x = a, y = b - 1, z = c - 1, t = d - 1, w = e$
Now $x + y + z + t + w = a + b + c + d + e - 3$
$= 8 - 3 = 5$
Or $x + y + z + t + w = 5$, where $x, y, z, t, w \geq 0$ ...(2)

Required number = number of non negative integral solutions

$= {}^{n+r-1}C_r = {}^{5+5-1}C_5 = {}^{9}C_5 = 126$

Example 24 Find the number of integral solutions of equation $x + y + z + t = 25$, $x > 0, y > 1, z > 2$ and $t \geq 0$.

Solution Given, $x + y + z + t = 25$, ...(1)
Where $x \geq 1, y \geq 2, z \geq 3, t \geq 0$
Let $p = x - 1, q = y - 2, r = z - 3, s = t$
Then $p + q + r + s = x + y + z + t - 6 = 25 - 6 = 19$, where, $p, q, r, s \geq 0$

$\therefore p + q + r + s = 19, p, q, r, s \geq 0$ ...(2)

$\therefore$ Required number = number of ways in which 19 identical things can be distributed among 4 persons when each person can get any number of things

$= {}^{n+r-1}C_{r-1} = {}^{22}C_3$

Example 25 There are 4 oranges, 5 apricots and 6 alphonso in a fruit basket. In how many ways can a person make a selection of fruits from among the fruits in the basket?

Solution Whenever we are talking about fruits, we assume them to be identical. However when we are talking about men, we treat them to be distinct.

Zero or more oranges can be selected out of 4 identical oranges in 4 + 1 = 5 ways.

Zero or more apricot can be selected out of 5 identical apricots in 5 +1 = 6 ways.

Zero or more can be selected out of 6 identical alphanso in 6 + 1 = 7 ways.

∴ The total number of selections when all the three types of fruits are selected (the number of any type of fruit may also be zero)

$= 5 \times 6 \times 7 = 210.$

But in one of these selections the number of each type of fruit is zero and hence this selection must be excluded.

∴ Required number = 210 – 1 = 209.

Example 26 Twelve different letters of alphabet are given. Words with six letters are formed from these given letters. Find the number of words which have at least one letter repeated.

Solution The total number of letters = 12. Words of six letters are to be framed.

The total number of words of 6 letters when any letter may be repeated any number of times (This also includes the number of words formed when no letter is repeated.)

$= 12 \times 12 \times 12 \times 12 \times 12 \times 12 = 12^6$

Number of words of 6 letters when no letter is repeated $= {}^{12}P_6$.

So, Number of words of 6 letters which have at least one letter repeated $= 12^6 - {}^{12}P_6$

PRACTICE EXERCISES

■■■

WARM UP

Q.1. How many numbers between 100 and 1000 can be formed using the digits 0, 2, 4, 6, 8, 5, if repetition of digits in a number is not allowed?

(a) 99 (b) 82 (c) 100 (d) 120

Q.2. How many numbers between 100 and 1000 can be formed using the digits 0, 2, 4, 6, 8, 5, if repetition of digits in a number is allowed?

(a) 164 (b) 180 (c) 192 (d) 100

Q.3. Two out of six paper sets for an examination are of mathematics. What is the number of ways in which the papers can be set so that the two mathematics papers are not together?

(a) 480 (b) 520 (c) 492 (d) 512

Q.4. In a letter lock, each of the three rings is marked with 15 letters. What is the maximum number of unsuccessful attempts that one has to make before the lock is opened?

(a) 3374 (b) 5284 (c) 8457 (d) 8101

Q.5. A certain code consists of 5 variables, with each variable having 4 different constant values possible. What is the total number of coded messages that can be sent with 5 constants one from each variable?

(a) 1024 × 5! (b) 1024 × 4!

(c) 1024 × 3! (d) None of these

Q.6. From 3 different soft drinks, 4 Chinese dishes and 2 ice-creams, how many different meals are possible if at least one of each of the three items is to be included, depending upon the number of people likely to turn up?

(a) 315 (b 282

(c) 864 (d) None of these

Q.7. The governing council of an institute has fifteen members and wants to hold its annual meeting. In how many ways can the council be seated around a round table if the chairman and the vice-chariman of the council are always seated together?

(a) 10 × 12! (b) 14 × 10!

(c) 13! (d) None of these

Q.8. Akshay is planning to give a birthday party at his place. In how many ways can he invite one or more of five friends and seat them at a circular table?

(a) 84 (b) 89 (c) 78 (d) 81

Q.9. In the above question, In how many ways can he invite one or more of his five friends and seat them at a circular table with him?

(a) 325 (b) 205 (c) 265 (d) 320

Q.10. A cricket team of eleven is to be chosen from 8 batsmen, 6 bowlers and 2 wicket-keepers. In how many ways can the team be chosen if there must be at least four batsmen, at least four bowlers and exactly one wicket-keeper?

(a) 1681 (b) 5304

(c) 1652 (d) None of these

Q.11. How many words can be formed using the letters of the word "CORRESPONDENCE"?

(a) $[(14!)/(2!)^4]$ (b) 14!

(c) $[(14!)/(2!)^4(3!)]$ (d) $[(14!)/(2!)^3(3!)]$

Q.12. How many words can be formed using the letters of the word "CORRESPONDENCE" if the consonants are always written together?

(a) 182 (b) 184

(c) $216 \times 9!$ (d) None of these

Q.13. If six persons are selected out of ten, in how many ways will a particular person be found among those six?

(a) 124 (b) 126 (c) 144 (d) 84

Q.14. A committee of five is to be chosen from among six men and four ladies. In how many ways can this be done in order to include at least one lady?

(a) 252 (b) 246 (c) 244 (d) 152

Q.15. How many ways are there to pick two different cards from a deck of 52 cards such that the first card is an ace and the second is not a queen

(a) 188 (b) 198

(c) 164 (d) None of these

Q.16. In the above question, what is the number of ways such that the first card is a spade and the second is not a queen?

(a) 612 (b) 612

(c) 164 (d) None of these

Q.17. How many different license-plates involving three letters and three digits are there if the three letters appear together, either at the beginning or at the end of the license?

(a) $2 \times 26^3 \times 10^3$ (b) 54102

(c) $4 \times 25^2 \times 10^4$ (d) None of these

Q.18. A joint student teacher committee of 5 members is to be formed with 4 teachers, 3 male students and 5 female students. How many different committees can be formed if the committee must consist of at least 2 teachers, 1 male student and 2 female students?

(a) 170 (b) 152 (c) 180 (d) 104

Q.19. There are 8 different locks, with exactly one key for each lock. All the keys have been mixed up. What is the maximum number of trials required in order to determine which key belongs to which lock?

(a) 44 (b) 28 (c) 24 (d) 32

Q.20. Hoppers' Stop stocks four styles of trousers. For each style, there are ten different possible waist sizes, six different trousers lengths and four colour choices. How many different types of trousers could the store have?

(a) 1024 (b) 960

(c) 921 (d) 924

FOUNDATION

Q.1. In a group of boys, the number of arrangements of 4 boys is 12 times the number of arrangements of 2 boys. The number of boys in the group is

(a) 10 (b) 8

(c) 6 (d) None of these

Q.2. From a group of persons the number of ways of selecting 5 persons is equal to that of 8 persons. The number of persons in the group is

(a) 13 (b) 40 (c) 18 (d) 21

Q.3. For Board of Cricket Control of India (BCCI), a selection committee is to be chosen consisting of 5 ex-cricketers. Now there are 10 representatives from four zones. It has further been decided that if Kapil Dev is selected, Sunil Gavaskar will not be selected and vice versa. In how many ways can this be done?

(a) 140 (b) 112 (c) 196 (d) 56

Q.4. At a get-together, it was found that a total of 66 handshakes took place at the conclusion of the party. If each guest shook hand only once with all the others, how many people were present.

(a) 33 (b) 22 (c) 12 (d) 13

Q.5. A committee is to be formed comprising of 7 members such that there is a majority of men and at least 1 woman in every committee. The shortlist consists of 9 men and 6 women. In how many ways can this be done?

(a) 3724 (b) 3630

(c) 3526 (d) 4914

Q.6. A number of 6-digit numbers that can be made with the digits 0, 1, 2, 3, 4 and 5, so that even digits occupy odd places, is

(a) 24 (b) 36

(c) 48 (d) None of these

Q.7. The Total number of ways in which six '+' and four '–' signs can be arranged in a line such that no two '–' signs occur together, is

(a) $\frac{7!}{3!}$ (b) $6!\times\frac{7!}{3!}$

(c) 35 (d) None of these

Q.8. Let A be the set of 4-digit numbers $a_1a_2a_3a_4$ where $a_1>a_2>a_3>a_4$ then how many values of A are possible?

(a) 126 (b) 84
(c) 210 (d) None of these

Q.9. A number of 5-digit numbers in which no two consecutive digits are identical, is:

(a) $9^2 \times 8^3$ (b) 9×8^4
(c) 9^5 (d) None of these

Q.10. The total number of selections of at most n things from $(2n + 1)$ different things is 63. Then the value of n is

(a) 3 (b) 2
(c) 4 (d) None of these

Q.11. From 4 gentlemen and 6 ladies, a committee of five is to be formed. The number of ways in which the committee can be formed so that the gentlemen are in majority, is

(a) 66 (b) 156
(c) 60 (d) None of these

Q.12. The number of all four digit numbers, which are divisible by 4, that can be formed from the digits 1,2, 3, 4 and 5 is

(a) 125 (b) 30
(c) 95 (d) None of these

Q.13. The total number of integral solutions for (x, y, z) such that $xyz = 24$, is

(a) 36 (b) 90
(c) 120 (d) None of these

Q.14. The number of ways to give 16 different things to three persons A, B, C so that B gets 1 more than A and C gets 2 more than B, is

(a) $\frac{16!}{4!5!7!}$ (b) 4!5!7!

(c) $\frac{16!}{3!5!8!}$ (d) None of these

Q.15. The number of positive integral solutions of $x + y + z = n, n \in \mathrm{N}, n \geq 3$, is

(a) ${}^{n-1}C_2$ (b) ${}^{n-1}P_2$
(c) $n(n-1)$ (d) None of these

Q.16. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in an AP then n is

(a) 8 (b) 9
(c) 14 (d) 10

Q.17. There are 20 questions in a question paper. If no two students solve the same combination of questions but solve equal number of questions, then the maximum number of students who appeared in the examination is

(a) ${}^{20}C_9$ (b) ${}^{20}C_{11}$
(c) ${}^{20}C_{10}$ (d) None of these

Q.18. The number of ways, to distribute 32 different things equally among 4 persons, is

(a) $\frac{32!}{(8!)^3}$ (b) $\frac{32!}{(8!)^4}$

(c) $\frac{1}{4}(32!)$ (d) None of these

Q.19. In a dinner party, there are 10 Indians, 5 Americans and 5 Englishmen. In how many ways can they be arranged in a row so that all persons of the same nationality sit together?

(a) $10!\times5!\times5!$ (b) 20!
(c) $3!\times10!\times5!\times5!$ (d) 20! 3!

Q.20. In how many ways can 15 I.Sc. and 13 B.Sc. candidates be arranged in a line so that no two B.Sc. candidates may occupy consecutive positions?

(a) $15!\times13!$ (b) $15!\times16p_{12}$
(c) $13!\times{}^{16}P_{12}$ (d) $2!\times15!\times13!$

Q.21. How many new words can be formed from the letters of the word 'circle' taken together?

(a) 720 (b) 719 (c) 360 (d) 359

Q.22. How many different words can be formed with the letters of the word 'Vice-Chancellor' so that the vowels are together?

(a) $10 \times 5!$ (b) $10! \times 5!$
(c) $5 \times 10!/2$ (d) $5 \times 10!$

Q.23. In how many ways can the letters of the word 'Lucknow' be arranged so that the two vowels do not come together?

(a) 720 (b) 1440
(c) 3600 (d) None of these

Q.24. How many words can be formed out of the letters of the word 'Article' so that the vowels occupy the even places?

(a) 72 (b) 144 (c) 288 (d) 36

Q.25. How many numbers greater than four millions (4000000) can be formed with the digits 2, 3, 0, 3, 4, 2, 5,?

(a) 280 (b) 380
(c) 360 (d) None of these

Q.26. How many signals can be made by hoisting 2 blue, 2 red and 5 black flags on a pole at the same time?

(a) 378 (b) 1512
(c) 756 (d) None of these

Q.27. Find the number of arrangements of the letters of the word 'Maruti' if u always comes before a.

(a) 60 (b) 180 (c) 420 (d) 360

Q.28. In a class of students there are 6 boys and 4 girls. In how many ways can they be seated, around a table so that all the 4 girls sit together?

(a) $6! \times 5!$ (b) $2 \times 6! \times 4!$
(c) $5! \times 4!$ (d) $6! \times 4!$

Q.29. A round table conference is to be held among 20 delegates of 20 countries. In how many ways can they and the host be seated if two particular delegates are always to sit on either side of the host?
(a) $19! \times 2!$ (b) $19!$
(c) $18! \times 2!$ (d) $20! \times 2!$

Q.30. Four gentlemen and four ladies are invited to a certain party. Find the number of ways of seating them around a table so that only ladies are seated on the two sides of each gentleman.
(a) 72 (b) 144 (c) 36 (d) 56

Q.31. Five men, 6 boys and 7 women be seated in a row so that the men, women and boys may not be separated in
(a) $5!\times6!\times7!$ ways (b) $6!6!7!$ ways
(c) $2!5!6!7!$ ways (d) $3!5!6!7!$ ways

Q.32. In how many ways 12 different books can be distributed equally among 4 persons?
(a) 195 (b) 154 (c) 210 (d) 256

Q.33. Eight chairs are numbered 1 to 8. Two women and three men wish to occupy one chair each. First two women choose the chairs from amongst the chairs marked 1 to 4; and then the men selected the chairs from amongst the remaining. Find the number of possible arrangement.
(a) 720 (b) 1440
(c) 2110 (d) None of these

Q.34. How many numbers between 100 and 1000 can be formed with the digits, 1, 2, 3, 4, 5, 6, 7; with no digit being repeated?
(a) 210 (b) 420 (c) 105 (d) 320

Q.35. How many different numbers greater that 5000 can be formed with the digits 0, 1, 5, 9, with no digit being repeated?
(a) 18 (b) 12 (c) 24 (d) 6

Q.36. How many numbers less than 1000 and divisible by 5 can be formed in which no digit occurs more than once in the same number?
(a) 154 (b) 162 (c) 176 (d) 132

Q.37. Find the number of numbers of 4 digits formed with the digits 1, 2, 3, 4, 5 in which 3 occurs in the thousands place and 5 occurs in the units place.
(a) 4 (b) 8
(c) 12 (d) 6

Q.38. Find the sum of all the four digit numbers that can be formed with the digits 3, 2, 3, 4.
(a) 40982 (b) 39996
(c) 41682 (d) None of these

Q.39. In how many ways can three prizes be given away to 5 students when each student is eligible for any of the prizes?
(a) 243 (b) ${}^5C_3$ (c) ${}^5P_3$ (d) 125

Q.40. A round table conference is to be held between 20 delegates. If two particular delegates are always to sit together, they can be seated in
(a) $2 \times 19!$ ways (b) $19!$ ways
(c) $20!$ ways (d) $2 \times 18!$ ways

Q.41. How many quadrilaterals can be formed joining the vertices of a polygon of n sides?
(a) ${}^nC_2 \times 2$ (b) ${}^nC_4$
(c) ${}^nC_4$ (d) None of these

Q.42. A delegation of 6 members is to be sent abroad out of the 12 members. In how many ways can the selection be made so that a particular member is included?
(a) 376 (b) 424 (c) 448 (d) 462

Q.43. The sides AB, BC, CA of a triangle ABC have 3, 4 and 5 interior points respectively on them. Find the number of triangles that can be constructed using given interior points as vertices.
(a) 210 (b) 195 (c) 205 (d) ${}^{12}C_3$

Q.44. Let S be the set of five-digit numbers formed by digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd position are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S?
(a) 228 (b) 216 (c) 294 (d) 192

Q.45. Triplet x, y and z are chosen from the set {1, 2, 3, ... 24, 25} such that $x \le y < z$. How many such triplets are possible?
(a) 25_{C_2} (b) 600
(c) $25_{C_2} + 25_{C_3}$ (d) 1200

Q.46. From 8 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady?
(a) 736 (b) 728 (c) 280 (d) 792

Q.47. Out of 7 men and 4 ladies a committee of 5 is to be formed. In how many ways can this be done so as to include at least 3 ladies?
(a) 84 (b) 91
(c) 90 (d) 102

Q.48. A person has 12 friends of whom 8 are relatives. In how many ways can he invite 7 friends such that at least 5 of them may be relatives?
(a) 284 (b) 384
(c) 456 (d) 512

Q.49. A person wishes to throw as many different parties as he can out of his 20 friends such that each party

consists of the same number of persons. The number of friends he should invite at a time is
(a) 5 (b) 10
(c) 8 (d) None of these

Q.50. If all permutations of the letters of the word *AGAIN* are arranged as in dictionary then fiftieth word is
(a) NAAGI (b) NAGAI
(c) NAAIG (d) NAIAG

MODERATE

Q.1. The number of distinct rational numbers x such that $0 < x < 1$ and $x = \frac{p}{q}$, where $p, q \in (1, 2, 3, 4, 5, 6)$, is
(a) 15 (b) 13 (c) 12 (d) 11

Q.2. The total number of 9-digit numbers of different digits is
(a) 10 (9!) (b) 8 (9!)
(c) 9 (9!) (d) None of these

Q.3. How many three-digit numbers with distinct digits can be formed such that the product of the digits is the cube of a positive integer?
(a) 21 (b) 24 (c) 36 (d) 30

Q.4. Seven different lecturers are to deliver lectures in seven periods of a class on a particular day. A, B and C are three of the lecturers. The number of ways in which a routine for the day can be made such that A delivers his lecture before B, and B before C, is
(a) 420 (b) 120
(c) 210 (d) None of these

Q.5. Let A = $\{x \mid x$ is a prime number and $x < 30\}$. The number of different rational numbers whose numerator and denominator belong to A is
(a) 90 (b) 180
(c) 91 (d) None of these

Q.6. Find the number of non-negative integral solutions to the system of equations $x + y + z + u + t = 20$ and $x + y + z = 5$.
(a) 228 (b) 336 (c) 448 (d) 528

Q.7. Find the number of integral solution to $|x|+|y|+|z| = 15$.
(a) 902 (b) 728 (c) 734 (d) 904

Direction for questions 8 and 9: Read the passage below and solve the questions based on it.

There are 6 boys and 4 girls sitting for a photo session. They were posing for the photograph standing in 2 rows one behind the other. There were 5 people sitting in the front row and 5 standing in the back row.

Q.8. If the boys were divided equally among the front and back rows, in how many ways can the photo session be arranged?
(a) 5! × 5! (b) ${}^6C_3 \times {}^4C_2 \times 5! \times 5!$
(c) ${}^6C_3 \times {}^4C_2 \times 10!$ (d) ${}^6C_3 \times {}^4C_2 \times 5! \times 5! \times 2!$

Q.9. In how many ways can the photos be taken such that no two boys and no two girls are standing or sitting together
(a) ${}^6C_3 \times {}^4C_2 \times 10!$ (b) ${}^6C_3 \times {}^4C_2 \times (3!)2(2!)2$
(c) ${}^6C_3 \times {}^4C_2$ (d) None of these

Q.10. A teacher takes 3 children from her class to the zoo at a time as often as she can, but she does not take the same three children to the zoo more than once. She finds that she goes to the zoo 84 times more than a particular child goes to the zoo. The number of children in her class is
(a) 12 (b) 10
(c) 60 (d) None of these

Q.11. ABCD is a convex quadrilateral. 3, 4, 5 and 6 points are marked on the sides AB, BC, CD and DA respectively. The number of triangles with vertices on different sides is
(a) 270 (b) 220 (c) 282 (d) 342

Q.12. The number of triangles that can be formed with 10 points as vertices, n of them being collinear, is 110. Then n is
(a) 3 (b) 4 (c) 5 (d) 6

Q.13. If x, y, and z are integers and $x \geq 0$, $y \geq 1$, $z \geq 2$, $x + y + z = 15$ then the number of values of the ordered triplet (x, y, z) is
(a) 91 (b) 455
(c) ${}^{17}C_{15}$ (d) None of these

Q.14. If a, b, c are positive integers such that $a + b + c \leq 8$ then the number of possible values of the ordered triplet (a, b, c) is
(a) 84 (b) 56
(c) 83 (d) None of these

Q.15. The product of r consecutive integers is necessarily divisible by
(a) r (b) $\sum_{k=1}^{r-1} k$
(c) (r + 1)! (d) None of these

Q.16. A candidate is required to answer six out of ten questions which are divided into groups, each containing five questions and he is not permitted to attempt more than 4 from any group. In how many ways can he make up his choice?

(a) 184 (b) 192
(c) 200 (d) None of these

Q.17. In an examination, the question paper contains three different sections A, B and C containing 4, 5 and 6 questions respectively. In how many ways, a candidate can make a selection of 7 questions, selecting at least two questions from each section?

(a) 890 (b) 1420
(c) 2700 (d) 3140

Q.18. A tea party is arranged for 16 people along the two sides of a long table with 8 chairs on each side. Four men wish to sit on one particular side and two on the other side. In how many ways can they be seated?

(a) ${}^{10}C_4 \times 8!$ (b) ${}^{10}P_4 \times (8!)^2$
(c) 10C4 × (8!)2 (d) $4! \times 2! \times (8!)^2$

Q.19. A candidate is required to answer 7 questions out of 12 questions, which are divided into two groups, each containing 6 questions. He is not permitted to attempt more than 5 from either of the group. In how many different ways can he choose the seven questions?

(a) 640 (b) 780 (c) 920 (d) 1020

Q.20. The number of parallelograms that can be formed from a set of four parallel lines intersecting another set of three parallel lines, is

(a) 6 (b) 18 (c) 12 (d) 9

Q.21. We are required to form different words with the help of the word INTEGER. Let m_1 be the number of words in which I and N are never together and m_2 be the number of words which begin with I and end with R, then m_1/m_2 is equal to

(a) 42 (b) 30 (c) 6 (d) 50

Q.22. In an election there are five candidates and three members are to be elected, and an elector can vote for any number of candidates not greater than the number to be elected. Then the number of ways in which an elector may vote is

(a) 25 (b) 30
(c) 32 (d) None of these

Q.23. The greatest possible number of points of intersection of 8 straight lines and 4 circles is

(a) 32 (b) 64
(c) 76 (d) 104

Q.24. A lady gives a dinner party to 5 guests to be selected from nine friends. The number of ways of forming the party of 5, given that two particular friends A and B will not attend the party together is

(a) 56 (b) 126
(c) 91 (d) None of these

Q.25. In a plane there are 37 straight lines, of which 13 pass through the point A and 11 pass through the point B. Besides, no three lines pass through one point, no line passes through both points A and B, no two are parallel. Then the number of intersection points the lines have is equal to

(a) 535 (b) 601
(c) 728 (d) None of these

Q.26. If a denotes the number of permutations of $x + 2$ things taking all at a time, b the number of permutations of x things taking 11 at a time and c the number of permutations of $x - 11$ things taking all at a time such that $a = 182bc$, then the value of x is

(a) 15 (b) 12 (c) 10 (d) 18

Q.27. There are 4 letters and 4 directed envelopes. The number of ways in which all the letters can be put in the wrong envelope is

(a) 8 (b) 9
(c) 16 (d) None of these

Q.28. The total number of natural numbers of six digits that can be made with digits 1, 2, 3, 4, if all digits have to appear in the same number at least once, is

(a) 1560 (b) 840 (c) 1080 (d) 480

Q.29. In a club election the number of contestants is one more than the number of maximum candidates for which a voter can vote for. If the total number of ways in which a voter can vote is 62, then what is the number of candidates?

(a) 7 (b) 5
(c) 6 (d) None of these

Q.30. Seven women and seven men are to sit round a circular table such that there is a man on either side of every women then the number of seating arrangement is

(a) $(7!)^2$ (b) $(6!)^2$
(c) 6! x 7! (d) 7!

Q.31. How many 10 digits numbers can be written by using the digits 1 and 2

(a) ${}^{10}C_1 + {}^9C_2$ (b) 210
(c) ${}^{10}C_2$ (d) 10!

Q.32. All possible two factors products are formed from the numbers 1, 2, 3, 4 ..., 200. The number of factors obtained out of the total which are multiples of 5 is

(a) 5040 (b) 7180
(c) 8150 (d) None of these

Q.33 How many different flags can be made with 5 different stripes using them any number at a time?

(a) 3^5 (b) 5^3
(c) 385 (d) 625

Q.34. Three apples and two bananas have to be distributed among three boys—Anuj, Bairag and Chandan, and two girls—Mugdha and Vaidehi, such that each person gets one fruit. In how many different ways can this be done if at least one girl gets an apple?

(a) 6 (b) 5
(c) 9 (d) None of these

Q.35. Find the distinct number of 7 digit numbers the sum of whose digits is even.

(a) 90×10^6 (b) 45×105
(c) 640000 (d) None of these

Q.36. Find the number of non-congruent rectangles that can be found on a chessboard normal 8×8 chessboard.

(a) 24 (b) 36
(c) 48 (d) None of these

Q.37 A dinner menu is to be designed out of 5 different starters, 6 identical main courses and 4 distinct desserts. In how many ways menu be designed such that there is atleast one of each of the starters, main courses and desserts?

(a) $31 \times 6 \times 15$ (b) $32 \times 6 \times 16$
(c) $31 \times 7 \times 15$ (d) $5 \times 6 \times 4$

Q.38. A man has 7 relatives. 4 of them are ladies and 3 gentlemen; his wife also has 7 relatives. 3 of them are ladies and 4 gentlemen. In how many ways can they invite 3 ladies and 3 gentlemen for a dinner party so that there are 3 of the man's relatives and 3 of the wife's relatives?

(a) 395 (b) 465
(c) 485 (d) 515

Q.39. Find the number of integral solutions of equation $x + y + z + t = 29$, $x > 0$, $y > 1$, $z > 2$ and $t \geq 0$.

(a) ${}^{27}C_3$ (b) ${}^{28}C_3$
(c) 2600 (d) ${}^{29}C_4$

Q.40. Find the number of numbers between 2×10^4 and 6×10^4 having sum of the digits even.

(a) 20,000 (b) 19999
(c) 24000 (d) 25000

Q.41. Eight straight lines in a set are parallel to each other and the distance between any two adjacent lines in 1 cm. Another set of 6 straight lines are parallel to each other and the distance between any two adjacent lines is 1 cm. These 6 straight lines of second set intersect with the first set of 8 straight lines to form parallelograms. How many of such formed parallelograms will not be rhombuses?

(a) 285 (b) 365
(c) 335 (d) None of these

Q.42. Ten guests of which three are ladies are to be seated in a row. The ladies insist on sitting together while two of the gentlemen refuse to take consecutive seats. In how many ways can the guests be seated?

(a) 256 (b) 512
(c) $8!.3! - 7!.2!$ (d) $8!.3! - 7!.3!.2!$

Q.43. Find the number of non-negative integral solutions of $2x + 2y + z = 10$

(a) ${}^{12}C_2$ (b) ${}^{14}C_4$
(c) ${}^{11}C_2$ (d) None of these

Q.44. How many committees of 11 persons can be made out of 50 persons if three particular persons A, B and C are not to be included together in the committee?

(a) ${}^{50}C_{11} - {}^{47}C_8$ (b) ${}^{50}C_{11} - {}^{47}C_9$
(c) ${}^{50}C_{11} \times {}^{47}C_8$ (d) None of these

Q.45. If each of the m points on the one straight line be joined to each of the n points on the other straight line, then excluding the points on the given two lines, number of points of intersection of these lines is

(a) $\frac{1}{4}mn(m-1)(n-1)$ (b) ${}^{m+n}C_2$
(c) ${}^{mn}C_2$ (d) None of these

ADVANCED

Q.1. There is a regular decagon. Triangles are formed by joining the vertices of the polygon. What is the number of triangles which have no side common with any of the sides of the polygon?

(a) 50 (b) 300 (c) 44 (d) 294

Q.2. There are 12 intermediate stations between two places A and B. In how many ways can a train be made to stop at 4 of these 12 intermediate stations that no two stations are consecutive?

(a) ${}^{15}C_3$ (b) ${}^{11}C_3$
(c) ${}^{9}C_4$ (d) ${}^{9}C_3$

Q.3. A conference attended by 200 delegates is held in a hall. The hall has 7 doors, marked A, B, ... G. At each door, an entry book is kept and the delegates entering through that door sign it in the order in which they enter. If each delegate is free to enter any time and through any door he likes, how many different sets of seven lists would arise in all? (Assume that every person signs only at his first entry.)

(a) ${}^{206}C_6$ (b) ${}^{199}P_5$
(c) ${}^{199}C_5$ (d) ${}^{206}P_6$

Q.4. In an election for the post of president of students union, every student can vote in 56 ways. Every student votes for atleast one candidate and a student can vote for more than one candidate. If the total number of candidates exceeds the maximum number that can be elected by 2, then what is the number of candidates?

(a) 5 (b) 7
(c) 6 (d) None of these

Q.5. The number of positive integral solution of $abc = 30$ is:

(a) 27 (b) 81
(c) 243 (d) None of these

Q.6. A train going from Cambridge to London stops at nine intermediate stations. Six persons enter the train during the journey with six different tickets of the same class. How many different sets of tickets would they have?

(a) ${}^{11}C_5$ (b) ${}^{35}C_6$
(c) ${}^{45}C_6$ (d) None of these

Q.7. Five balls of different colours are to be placed in three boxes of different sizes. Each box can hold all five balls. In how many different ways can we place the balls so that no box remains empty?

(a) 119 (b) 150
(c) 180 (d) 210

Q.8. An eight-oared boat is to be manned by a crew chosen from 11 men of whom 3 can steer but cannot row and the rest cannot steer. In how many ways can the crew be arranged if two of the men can only row on bow side?

(a) ${}^{3}C_1.{}^{4}C_2.6!$ (b) ${}^{3}P_1.{}^{4}P_2$
(c) 3P1.4P2.6! (d) ${}^{3}C_1.{}^{4}C_2$

Q.9. Find the number of n-digit number formed with the digits 1, 2, 3 ..., 9 in which no two consecutive digits are the same.

(a) 9^{n-1} (b) 9.8^{n-1}
(c) 9.8^{n} (d) 9^{n}

Q.10. If the number of ways in which n different things can be distributed among n persons so that at least one person does not get any thing is 232 then what is the value of n?

(a) 3 (b) 4
(c) 5 (d) None of these

Q.11. If m be the greatest value of k for which 100! is divisible by 3^k, then the number of ways of distributing k different things among 5 persons so that two of them get 10 things each, two get 12 things each and the fifth one gets four things is

(a) $\dfrac{48!}{(101)^2(121)^2 41}$ (b) $\dfrac{48!}{(10!)^2(12!)^2}$
(c) $\dfrac{48!}{(10!)^2(12!)^2 4}$ (d) $\dfrac{48!}{(10!)^2(12!)^2}.\dfrac{5}{4}$

Q.12. A bag contains n white and n red balls. Pairs of balls are drawn without replacement until the bag is empty. If the number of ways in which each pair consists of one red and one white ball is 14400, then what is the value of n?

(a) 5 (b) 10 (c) 15 (d) 20

Direction for questions 13 to 15: Read passage below and solve the questions based on it.

In a horticultural show, a participant arranged a total of 1000 fruits of four types in a long straight line. He first placed one apple, then two mangoes, three oranges, four custard apples, five apples, six mangoes, seven oranges, eight custard apples and so on (with apples followed by mangoes, oranges, custard apples in that order) upto the 1000th fruit.

Q.13. What was the 1000th fruit?

(a) Apple (b) Mango
(c) Orange (d) Custard apple

Q.14. What was the position of the 100th Mango?

(a) 387 (b) 437 (c) 417 (d) 363

Q.15. Among the last 700 fruits, which type of fruit occurred the greatest number of times?

(a) Apple (b) Mango
(c) Orange (d) Custard apple

Q.16. There is a framework of a cuboid of length 6, breadth 5 and height 7. The cuboid is only composed of a skeleton of 210 cubes of side 1. An insect is on one corner of the cube and it wants to travel to the opposite end of the longest diagonal. It can only travel along the sides of the small cubes and it always takes the shortest possible route.

Find the number of choices the insect has.

(a) $\dfrac{18!}{7!6!5!}$ (b) $\dfrac{15!}{6!5!4!}$
(c) $\dfrac{17!}{7!6!5!}$ (d) $\dfrac{16!}{6!5!4!}$

Q.17. In the above question, if the insect suddenly realizes that one of the faces on the opposite side of the cuboid having maximum area has been sprayed with pesticides due to which it can not reach the original destination and if the insect still wants to reach the opposite end of longest diagonal now possible, then in how many ways can it do so?

(a) $\dfrac{16!}{7!6!5!}$ (b) $\dfrac{18!}{6!5!4!}$

(c) $\dfrac{15!}{6!6!5!}$ (d) $\dfrac{16!}{6!5!5!}$

Direction for questions 18 and 19: Read the passage below and solve the questions based on it.

James Bond has to crack a password of 3 symbols. The symbols can be +, – or ×. Every time, he makes an attempt to crack the password, a feedback is given in the form of 'R' and 'W'. 'R' indicates that one of the symbols is in the right place. 'W' indicates that one of the digits is in the wrong position. All the codes are used.

This is his first two attempts with their respective feedbacks:

Attempt 1	Feedback
+ + ×	R R W
Attempt 1	
– + ×	R W W

Q.18. What is the minimum number of additional attempts required to crack the code?
(a) 0 (b) 1 (c) 2 (d) 3

Q.19. What is the right code?
(a) × – + (b) + – ×
(c) + + – (d) – + ×

Q.20. At the beginning of a party, each present person shook hands with all other people present, and in all there were 28 handshakes. In the midst of the party, 2 persons left due to an emergency. Now, the number of males and females present in the party was equal. At the end, each female shook hands only with every female present and each male shook hands only with every male present. What is the total number of handshakes that took place at the party?
(a) 52 (b) 40
(c) 34 (d) 36

Q.21. Consider S = (1, 2, 3,… 10). In how many ways two numbers from S can be selected so that the sum of the numbers selected is a double digit number?
(a) 36 (b) 16
(c) 29 (d) ${}^9C_2 - {}^5C_2$

Q.22. In a chess tournament every person played one game with every other person in the group. The total number of games that men played between themselves exceeded those played by men with women by 18. If there were 4 women in the tournament, in all how many games were played in the tournament?
(a) 114 (b) 66
(c) 78 (d) 120

Q.23. The number of ways of painting the faces of a cube with six different colour is
(a) 1 (b) 6
(c) 6! (d) None of these

Q.24. Find the number of non-negative integer solutions to the system of equations $a + b + c + d + e = 20$ and $a + b + c = 5$ is
(a) 240 (b) 336 (c) 672 (d) 1008

Q.25. If the number of ways of selecting k coupons out of an unlimited number of coupons bearing the letters A,T,C so that they can not be used to spell the word CAT is 93, then what is the value of k?
(a) 2 (b) 5
(c) 8 (d) None of these

Q.26. In a test of 10 multiple choice questions of one correct answer, each having 4 alternative answers, then the number of ways to put ticks at random for the answers to all the questions is
(a) 410 (b) 10^4
(c) $4^{10}-4$ (d) 10^4-10

Q.27. The streets of a city are arranged like the lines of a chessboard. There are m streets running north-south and n streets running in east-west direction. What is the number of ways in which a man can travel on these streets from the north-west to the south-east corner, going by the shortest possible distance.
(a) $(m + n – 2)!$
(b) $(m + n – 1)!/ [(m – 1)!]$
(c) $(m + n – 1)!/[(n – 1)!]$
(d) $(m + n – 2)!/[(m – 1!)(n – 1)!]$

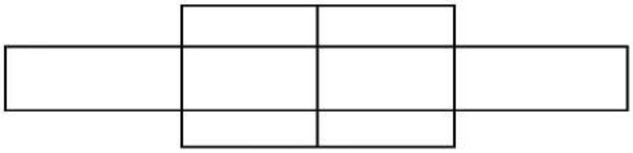
Q.28. Let $(1 + x + x^2)^9 = \alpha_0 + \alpha_1 x + ,,,,, \alpha_{18}x^{18}$, then which of the following is true?
(a) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18} = \alpha_1 + \alpha_3 + \ldots + \alpha_{17}$;
(b) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18}$ is even;
(c) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18}$ is divisible by 9;
(d) $\alpha_0 + \alpha_2 + \ldots + \alpha_{18}$ is divisible by 3 but not by 9.

Q.29. If N be an element of the set A = {1, 2, 3, 5, 6, 10,15, 30} and P, Q and R are integers such that PQR = N, then the number of positive integral solutions of PQR = N, is
(a) 32 (b) 64
(c) 96 (d) 128

Q.30. In how many ways can one arrange letters in the word 'INSTITUTION' such that no two same letters comes together along with the following conditions:
(A) There is no 'T' which is immediately preceded as well as followed by 'N'.
(B) There is no 'T' which is immediately preceded as well as followed by 'T'.
(a) 32256 (b) 40320
(c) 60480 (d) None of these

Q.31. Consider S = (1, 2, 3,… 10). In how many ways the two numbers from S can be selected so that the sum of the numbers selected is a double digit number?
(a) 36 (b) 16
(c) 29 (d) $^9C_2 - {}^5C_2$

Q.32. Six X's have to be placed in the squares of the figure such that each row contains at least one X.



In how many different ways can this be done?
(a) 28 (b) 27
(c) 26 (d) None of these

Q.33. Number of ways in which *n* distinct things can be distributed among *n* persons so that atleast one person does not get anything is 232. Find *n*.
(a) 3 (b) 4 (c) 5 (d) 6

Q.34. Six persons A, B, C, D, E and F are to be seated at a circular table. In how many ways can this be done if A must always have either B or C on his immediate right and B must always have either C or D on his immediate right?
(a) 16 (b) 24 (c) 30 (d) 18

Q.35. The number of employees in a nationalised bank in a small town is 10, out of which 4 are female and the rest male. A committee of 5 is to be formed. If m be the number of ways to form such a committee in which there is atleast one female employee and n be the number of ways to form such a committee which includes at least two male employees, then find the ratio m:n.
(a) 3:2 (b) 5:1
(c) 1:1 (d) 0.8:0.9

Q.36. A question paper is split into two parts — Part A and Part B. Part A contains 5 questions and part B has 4 questions. Each question in part A has an alternative. A student has to attempt at least one question from each part. Find the number of ways in which the student can attempt the question paper?
(a) 1,830 (b) 2,440
(c) 3,630 (d) 4,260

Q.37. How many integers between 1 and 100000 have the sum of their digits equal to 18?
(a) 89,93 (b) 25,927
(c) 36,592 (d) 24,987

Q.38. A person invites his 10 friends on a party and places them 5 at one round table, 5 on the other round table. Find the number of ways in each case in which he can arrange the guests.
(a) $^{10}P_5/25$ (b) $^{10}P_5 \times {}^5P_5/25$
(c) 10!/25 (d) None of these

Q.39. In the above question, find the number of ways in which he can arrange 4 at one round table and 6 at the other round table.
(a) $^9P_5/24$ (b) $^9P_5 \times {}^5P_5/24$
(c) 10!/24 (d) None of these

Q.40. In how many ways 4 men and 4 women can be seated in a row so that men and women are alternate?
(a) 28 (b) 36
(c) 4!.4! (d) 2.4!.4!

ANSWERS

■■■

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(a)	(a)	(a)	(a)	(d)	(b)	(a)	(c)	(c)	(d)	(b)	(b)	(a)	(a)	(a)	(c)	(b)	(b)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(a)	(c)	(c)	(d)	(a)	(c)	(c)	(c)	(a)	(a)	(a)	(c)	(a)	(a)	(c)	(c)	(b)	(c)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(d)	(c)	(b)	(c)	(c)	(d)	(d)	(c)	(b)	(d)	(a)	(b)	(a)	(b)	(a)	(d)	(b)	(d)	(d)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(b)	(d)	(c)	(b)	(b)	(a)	(b)	(c)	(b)	(c)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(c)	(d)	(d)	(c)	(b)	(a)	(b)	(b)	(b)	(d)	(c)	(a)	(b)	(b)	(c)	(c)	(c)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(c)	(c)	(c)	(a)	(b)	(b)	(a)	(c)	(c)	(b)	(b)	(c)	(c)	(b)	(b)	(a)	(c)	(c)	(a)
Q. No.	41	42	43	44	45															
Answer	(c)	(d)	(a)	(a)	(a)															

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(d)	(c)	(c)	(c)	(b)	(c)	(b)	(b)	(d)	(a)	(a)	(b)	(d)	(a)	(d)	(a)	(b)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(d)	(a)	(b)	(b)	(a)	(d)	(b)	(b)	(c)	(c)	(c)	(b)	(d)	(c)	(c)	(b)	(c)	(c)	(d)

HINTS AND SOLUTIONS

Moderate

1. Total sets possible = ${}^6C_2 = 15$
   But ½ = 2/4 = 3/6
   1/3 = 2/6
   2/3 = 4/6
   So, distinct values of $x = 11$

2. It will be a number of 9 digits.
   $9 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 = 9 \times 9!$

3. Without any restriction, 7 can be arranged in 7! Ways.
   Now, keeping 4 of them fixed, A, B and C can be arranged in 3! Ways, i.e., out of every 3! Arrangement, we have one choice for which A-B-C order is followed.
   So, [7!]/[3!]
   It can be seen that – For AB = [7!]/[2!]
   – For A-B-C (or any three) = [7!]/[3!]
   – For A-B-C-D (or any four) = [7!]/[4!]

– For A-B-C-D-E (or any five) = [7!]/[5!]
– For A-B-C-D-E-F (or any six) = [7!]/[6!]
– For A-B-C-D-E-E-F = [7!]/[7!]

6. *n* identical things can be distributed among r persons in ${}^{n+r-1}C_{r-1}$

7. Same as 6.

10. ${}^{n}C_3 - {}^{n-1}C_2 = 84$, where n is the number of students. Now use options.

13. Here, Since $x \geq 0, y \geq 1, z \geq 2$
Now, x can get any number of things, y should get a minimum of 1 thing and z should get a minimum of 2 things. So, let us give these to y and z.
Now, total things left = 12
Now use the formula n identical things can be distributed among r persons in ${}^{n+r-1}C_{r-1}$
Where n = 12, r = 3

17. Various ways of doing this is as follows:

A	B	C
2	2	3
2	3	2
3	2	2

Now do the selections.

18. Four persons have chosen to sit on one particular side (assume side A) and two persons on the other side (assume side B). So, we are supposed to select four persons for side A from the remaining 10 persons and remaining six persons will be sitting on side B.
Number of ways 4 persons can be selected from 10 persons = ${}^{10}C_4$
Number of ways 6 persons can be selected from the remaining 6 persons = ${}^{6}C_6$
Number of ways 8 persons can be arranged on side A = 8!
Number of ways 8 persons can be arranged on side B = 8!
Total number of ways = ${}^{10}C_4 \times {}^{6}C_6 \times 8! \times 8!$

20. This is nothing but ${}^{4}C_2 \times {}^{3}C_2$.

24. This can be done in three ways:
   i. A is selected but B is not selected – A has already been selected, rest 4 is to be selected from 7 persons = ${}^{7}C_4 = 35$
   ii. B is selected but A is not selected - ${}^{7}C_4 = 35$
   iii. Neither A nor B is selected - ${}^{7}C_5 = 21$
   Total = 91

26. $(x + 2)! = a$, ${}^{x}P_{11} = b$ and $(x - 11)! = c$
$(x + 2)! = 182 \times {}^{x}P_{11} \times (x - 11)!$
Now, go through the options.

31. $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$.

33. Case I. When a boy sits at the first place:
Possible arrangement will be of the form

B	G	B	G	B	G	B	G

Now there are four places namely 1st, 3rd, 5th and 7th for four boys, therefore four boys can be seated in $\lfloor 4$ ways. Again there are four places namely 2nd, 4th, 6th and 8th for four girls.
Four girls can be seated in $\lfloor 4$ ways.
Number of ways in this case = $\lfloor 4 \, \lfloor 4$.
And similarly G B G B G B G.B.
So, total = $2 \times 4! \times 4!$

36. Find 1×2 rectangles, 2×3 rectangles, 3×4 rectangles, 4×5 rectangles, 5×6 rectangles, 6×7 rectangles, 7×8 rectangles.

37. Given $x + y + z + u + t = 20$
$x + y + z = 5$
Given, system of equations is equivalent to $x + y + z = 5$ ... (1)
and $u + t = 15$ ... (2)
Number non-negative integral solutions of equation (1)
$= {}^{n+r-1}C_r = {}^{3+1-1}C_5 = {}^{7}C_5$,
Number of non-negative integral solution of equation (2)
$= {}^{n+r-1}C_r = {}^{2+15-1}C_{15} = {}^{16}C_{15}$
$\therefore$ Required number = ${}^{7}C_5 \, {}^{16}C_{15} = 336$

39. This situation is similar to distributing 29 identical things among 4 persons x, y, z and t in such a way that x is getting a minimum 1 thing, y is getting minimum 2 things, z is getting minimum 3 things and t is getting minimum 0 thing.
So, let us distribute 1 thing to x, 2 things to y, 3 things to z and 0 thing to t. After doing this distribution, we are free to distribute the remaining things among x, y, z and t in any ways.
Now the equation is: $x + y + z + t = 23$
The total number of ways of distributing 23 identical things among 4 persons in such a way that anybody can get any number of things = ${}^{26}C_3$

Advanced

1. Total number of triangles = Triangles having no sides common + Triangles having one side common + Triangles having two sides common + Triangles having three sides common. Now take help of Worked out Example 19.

2. Let $S_1, S_2, \ldots, S_8$ denote the stations where the train does not stop. The four stations where the train stops should be at any four of the nine places indicated by cross.

$$\therefore \text{ Required number} = {}^{9}C_4 \frac{9.8.7.6}{\lfloor 4} = 126$$

5. If $a = 1 = b$, then $c = 30$. Similarly, we can have $a = c = 1$ and $b = c = 1$. These are three sets.
Now, take $a = 1$, then $bc = 30 = 2^1 \times 3^1 \times 5^1$
Possible sets of b, c for the product $bc = 30$ is 13.
So, total number of sets for any one of these to be equal to zero = 39.
Now take $a = 2$, then $bc = 15 = 3^1 \times 5^1$. Now again find the sets of (b, c) and similarly check all the possibilities.

10. Go through the options using formula.

Questions 13 to 15

Fruits are ordered like

1. Apple	2. Mangoes
3. Oranges	4. Custard apple

Fruits are being put up first one, then two, then three, then four and so on.

13. $1 + 2 + 3 + \ldots + 44 = 990$
So, the 45th fruit will be the 100th fruit. Since we are having a set of four different types of fruits, 45th will be Apple.

14. Mangoes will come like 2, 6, 10, 14 etc.
100th mango will come $2 + 6 + 10 + 14 + 18 + 22 + 26 + 2 = 100$
100th mango will come in its 8th turn.
Before that apple must have got its 8th turn, orange and custard apple must have got their 7th turn.
Total apples displayed till now
$= 1 + 5 + 9 + \ldots + 29 = 120$
Total oranges displayed till now
$= 3 + 7 + 11 + \ldots + 27 = 105$
Total custard apples displayed till now
$= 4 + 8 + 12 + \ldots + 28 = 112$
So, total fruits displayed till now (other than mangoes) = 337
So, the position of the 100th mango = 437

Questions 18 and 19

Use the options of 19 to solve both the questions simultaneously.

21. If the sum is a two-digit number, then both the values simultaneously cannot be less than 5. Now start making the sets by taking different values greater than or equal to atleast one of the numbers.

22. ${}^{M}C_2 - {}^{M}C_1\,{}^{W}C_1 = 18$, where M and W are the number of men and women respectively.

24. Given $a + b + c + d + e = 20$ ...(1)
$a + b + c = 5$ ...(2)
Given, system of equations is equivalent to $a + b + c = 5$ ...(3)
and $d + e = 15$

Number non-negative integral solutions of equation (3)
$= {}^{n+r-1}C_r = {}^{3+1-1}C_5 = 7\,C_5$,
Number of non-negative integral solution of equation (4)
$= {}^{n+r-1}C_r = {}^{2+15-1}C_{15} = {}^{16}C_{15}$
∴ Required number $= {}^{7}C_5\,{}^{16}C_{15} = 336$

26. Each of the questions can be answered in 4 ways. So, 10 questions can be answered in 4^{10} ways.

28. Eliminate the options by assuming several values of $x = 1$ or 2 etc.

29. This is nothing but the application of the concept of number of factors (see Number System).

33. $n^n - n! = 232$. Use options now.

34. If B is sitting on the immediate right of A, then C or D can sit on the immediate right of B and the rest 3 places can be filled in 3! ways. So, the total number of ways when B is sitting to the immediate right of A $= 2 \times 3! = 12$.
Similarly, when C is sitting to the immediate right of A, then D has to sit on the immediate right of B, so B-D becomes a pair. Now for rest of 2 can be seated at three different positions in two ways each (This is dependent of the positions of B - C). Number of arrangements possible here = 6
So, total arrangements = 18

35.

Males	Females
6	4

Now take different cases to fulfill the conditions given.

40. Case I. When a boy sits at the first place:
Possible arrangement will be of the form

B	G	B	G	B	G	B	G

Now there are four places namely 1st, 3rd, 5th and 7th for four boys, therefore four boys can be seated in $\underline{|4}$ ways. Again there are four places namely 2nd, 4th, 6th and 8th for four girls.

∴ Four girls can be seated in $\underline{|4}$ ways.

∴ Number of ways in this case $= \underline{|4}\ \underline{|4}$.

Similarly, another possible arrangement will be like:

B	G	B	G	B	G	B	G

Number of ways of doing this $= \underline{|4}\ \underline{|4}$.

So, total ways $= 2\ \underline{|4}\ \underline{|4}$.

16

CHAPTER

Probability

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Probability
- Meaning and definitions of the different terms
- Different theorems
- Methods of solving questions

Introduction to the topic

Probability is an expectation of the happening of some event. In our day-to-day life, we often predict the outcome of several events, and often hear phrases such as probably it will be a hot day today or it will probably rain tomorrow. These statements involve an element of uncertainty and hence questions such as how to measure the uncertainty of the happening of something arises. Probability is just an attempt to quantify this method of predicting the future events with the help of some concepts. The theory of probability deals with those situations in which a particular result or outcome is not certain, but it can be any one of the several possible outcomes given to us for consideration. As a decision maker, we will be required to use this tool of probability to obtain called "satisfycing output", keeping all the variables in mind.

PROBABLITY AND ITS APPLICATION

Probability is the measure of uncertainty. We turn our attention to one of the problems that is widely held responsible for the development of the theory of probability, namely, that of throwing a dice. As all of us know, a dice is a well-balanced cube with six faces marked with numbers from 1 to 6, one number on one face.

Let's throw a dice once. What are the possible outcomes? Clearly, a dice can fall with any of its faces uppermost. The number on each of the faces is, therefore a possible outcome and all the outcomes are equally probable. Hence, it is as likely to show up a number, say 2, as any other number 1, 3, 4, 5, or 6.

Since there are six equally likely outcomes in a single throw of a dice and there is only one way of getting a particular outcome (say 2). Therefore, the chance of the number 2 coming up on the dice is 1 by 6. In other words, the same phenomenon is known as probability of getting 2 in a single throw of dice is 1/6.

We write this as $P(2) = 1/6$

Similarly, when an ordinary coin is tossed, it may show up head (H) or tail (T). Hence, the probability of getting a head in a single toss of a coin is given by

$$P(H) = \tfrac{1}{2}$$

Before we define the process to finding out the probability, it is essential to understand the various terms associated with probability.

Trial and Elementary Events

When we repeat a random experiment under identical conditions, then the experiment is known as a trial and the possible outcomes of the experiment are known as elementary events.

For example,

- tossing of a coin is a trial and getting head or tail is an elementary event.
- throwing of a dice is a trial and getting 5 on its upper face is an elementary event.

Compound Events

Events obtained by combining two or more elementary events are known as the compound events. A compound event is said to occur if one of the elementary events associated with it occurs.

Exhaustive Number of Cases

The total number of possible outcomes of a random experiment in a trial is known as the exhaustive number of cases.

For example, in throwing of a dice the exhaustive number of cases is 6, since any one of the six face marked with 1, 2, 3, 4, 5, 6 may appear on its upper face.

Mutually Exclusive Events

Events are said to be mutually exclusive if the occurrence of any one of them prevents the occurrence of all the others, i.e., if no two or more than two events can occur simultaneously in the same trial.

Equally Likely Events

Events are equally likely if there is no reason for an event to occur in preference to any other event. For example, while throwing a dice, chances of occurring of Head or Tail are an equally likely event.

Favourable Number of Cases

The number of cases favourable to an event in a trial is the number of elementary events such that if any one of them occurs, we say that the event happens.

In other words, the number of cases favourable to an event in a trial is the total number of elementary events such that the occurrence of any one of them ensures the happening of the event.

For example, in throwing of a dice, the number of cases favourable to the appearance of a prime number is 3 viz., 2, 3 and 5.

Independent Events

Events are said to be independent if the happening (or non-happening) of one event is not affected by the happening (or non-happening) of others.

For example, if two cards are drawn from a well-shuffled pack of 52 cards one after the other with replacement, then getting an ace in the first draw is independent of getting a jack in the second draw. But, if the first card drawn in the first draw is not replaced, then the second draw is dependent on the first draw.

Sample Space

This is the most important factor in probability and is defined as the set of all possible outcomes of a random experiment associated with it.

These examples suggest the following definition of probability (assuming that outcomes are equally likely).

Now, Probability of an event E, written as P(E), is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to E}}{\text{Total number of possible outcomes (sample space)}}$$

Points to Remember

1. $0 \leq P(E) \leq 1$
2. $P(E) + P'(E) = 1$

Example 1 In a single throw of two die, what is the probability that the sum on the top face of both the die will be more than 9?

Solution When two die are thrown, sum of the numbers appearing on the faces can be anything from 2 to 12. To find the number of favourable cases we will be required to find the cases in which the sum is more than 9.

Following are the cases—(5, 5), (6, 4), (4, 6), (6, 5), (5, 6), (6, 6)

So, the total number of favourable cases = 6

The total number of possible outcomes = $6 \times 6 = 36$

Hence probability = 6/36 = 1/6

Example 2 Six dice are thrown simultaneously. Find the probability that all of them show the same face.

Solution Sample space of throwing six dice = $6 \times 6 \times 6 \times 6 \times 6 \times 6 = 6^6$.

All dice are showing the same face implies that we are getting same number on the entire six dice. The number of ways for which is ${}^6C_1$.

Hence, the required probability = $\frac{{}^6C_1}{6^6} = \frac{1}{6^5}$

Example 3 In the above question, find the probability that all of them show a different face.

Solution The total number of ways in which all the dice show different numbers on their top-faces is the same as the number of arrangement of 6 numbers 1, 2, 3, 4, 5, 6 by taking all at a time.

So, the number of favourable cases = 6!

Hence, the required probability = $\frac{6!}{6^6}$

Example 4 Five persons enter a lift on the ground floor of an 8-floor apartment. Assuming that each one of them independently and with equal probability can leave the lift at any floor beginning with the first. What is the probability that all the five persons are leaving the lift at different floors?

Solution Apart from the ground floor, there are 7 floors.

A person can leave the lift at any of the seven floors. Hence the total number of ways in which each of the five persons can leave the lift at any of the 7 floors = 7^5.

So, the sample space = 7^5

Five persons can leave the lift at five different floor = ${}^7P_5$ ways.

So, the favourable number of ways = ${}^7P_5$.

Hence, the required probability = $\frac{{}^7P_5}{7^5}$

Example 5 If you have 3 tickets of a lottery for which 10 tickets were sold and 5 prizes are to be given, the probability that you will win at least one prize is: (JMET 2005)

(a) $\frac{7}{12}$ (b) $\frac{9}{12}$ (c) $\frac{1}{12}$ (d) $\frac{11}{12}$

Solution Probability that you will win at least one prize = 1 – probability that you will not win any prize.

$$= 1 - \frac{5C_3}{10C_3} = \frac{11}{12}$$

Odds in Favour and Odds Against

$$\text{Odds in Favour} = \frac{\text{Number of favourable cases}}{\text{Number of unfavourable cases}}$$

$$\text{Odds in Against} = \frac{\text{Number of unfavourable cases}}{\text{Number of favourable cases}}$$

Understanding And/Or To understand the role played by And/Or in our calculation, let us take the example of throwing an unbiased dice. Let A and B be the two events associated with it such that

A = getting an even number, B = getting a multiple of 3.

Then A = {2, 4, 6}, and B = {3, 6}.

We now define a new event "A or B" which occurs if A or B or both occur i.e., at least one of A or B occurs. Clearly the event "A or B" occurs if the outcome is any one of the outcomes {2,3,4,6}. Thus, the event "A or B" is represented by the subset A U B.

Similarly, "A and B" means occurrence of both A and B which is possible if the outcome is {6}.

Hence, it is represented by the subset $A \cap B$.

ADDITION THEOREM

If A and B are two events associated with a random experiment, then

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Corollary: If the events are mutually exclusive, then $P(A \cup B) = P(A) + P(B)$

Addition theorem can be extended for any number of events.

Example 6 A basket contains 20 apples and 10 oranges out of which 5 apples and 3 oranges are defective. If a person takes out 2 at random what is the probability that either both are apples or both are good?

Solution Out of 30 items, two can be selected in ${}^{30}C_2$ ways. So, exhausted number of cases = ${}^{30}C_2$.

Consider the following events:

A = getting two apples; B = getting two good items

Required probability = $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

There are 20 apples, out of which two can be drawn in ${}^{20}C_2$ ways.

$$\therefore P(A) = \frac{20C_2}{{}^{30}C_2}$$

There are 8 defective pieces and the remaining 22 are good. Out of 22 good pieces, two can be selected in $^{22}C_2$ ways.

$$\therefore\ P(B) = \frac{22C_2}{^{30}C_2}$$

Since there are 15 pieces which are good apples of which 2 can be selected in $^{15}C_2$ ways, therefore

P (A∩B) = Probability of getting 2 pieces which are good apples = $\frac{15C_2}{^{30}C_2}$

From (i)

Required probability = P(A) + P(B) – P(A∩B)

$$= \frac{^{20}C_2}{^{30}C_2} + \frac{^{22}C_2}{^{30}C_2} - \frac{^{15}C_2}{^{30}C_2} = \frac{316}{435}$$

CONDITIONAL PROBABILITY

Let A and B be the two events associated with a random experiment. Then, the probability of occurrence of A under the condition that B has already occurred and P (B) ≠ 0, is called the conditional probability and it is denoted by P(A/B).

Thus, P(A/B) = Probability of occurrence of A given that B has already happened.

Similarly, P (B/A) = Probability of occurrence of B given that A has already happened.

Sometimes, P (A/B) is also used to denote the probability occurrence of A when B occurs. Similarly, P(B/A) is used to denote the probability of occurrence of B when A occurs.

Following examples illustrate the various meanings of these notations:

Example 7 A bag contains 5 white and 4 red balls. Two balls are drawn form the bag one after the other without replacement. Consider the following events.

A = drawing a white ball in the first draw, B = drawing a red ball in the second draw.

Solution Now, P(B/A) = Probability of drawing a red ball in the second draw given that a white ball has already been drawn in the first draw.

Since 8 balls are left after drawing a white ball in the first draw and out of these 8 balls 4 balls are red, therefore

$$P\ (B/A) = \frac{4}{8} = \frac{1}{2}$$

Note that P (A/B) is not meaningful in this experiment because A cannot occur after the occurrence of B.

Example 8 One card is drawn from a pack of 52 cards, each of the 52 cards being equally likely to be drawn. Find the probability that the card drawn is

i. a king
ii. either a red or a king
iii. a red and a king

Solution Out of 52 cards, one card can be drawn in $^{52}C_1$ ways. Therefore, exhaustive number of cases = $^{52}C_1$ = 52

i. There are 4 kings in a pack of cards, out of which one can be drawn in $^4C_1$ ways. Therefore, the favourable number of cases = $^4C_1$ = 4, so the required probability $= \frac{4}{52} = \frac{1}{13}$
ii. There are 28 cards in a pack of cards which are either a red or a king. Therefore, one can be drawn in $^{28}C_1$ ways. Therefore, the favourable number of cases = $^{28}C_1$=28. So the required probability = $\frac{28}{52} = \frac{7}{13}$
iii. There are 2 cards which are red and king, i.e., red kings. Therefore, the favourable number of cases = $^2C_1$ = 2. so the required probability = $\frac{2}{52} = \frac{1}{26}$

Example 9 An urn contains 9 blue, 7 white and 4 black balls. If 2 balls are drawn at random, find the probability that

(i) Both the balls are blue
(ii) One ball is white

Solution There are 20 balls in the bag out of which 2 balls can be drawn in $^{20}C_2$ ways. So the total number of cases(sample space) = $^{20}C_2$ = 190.

i. There are 9 blue balls out of which 2 balls can be drawn in $^9C_2$ ways. Therefore, the favourable number of cases = $^9C_2$ = 36. So the required probability = $\frac{36}{190} = \frac{18}{95}$
ii. There are 7 white balls out of which one white can be drawn in $^7C_1$ ways. One ball from the remaining 13 balls can be drawn in $^{13}C_1$ ways. Therefore, one white and one other colour ball can be drawn in $^7C_1 \times {}^{13}C_1$ ways. So the favourable number of cases = $^7C_1 \times {}^{13}C_1$ = 91.

So, the required probability = $\frac{91}{190}$

Example 10 Three persons A, B and C are to speak at a function along with five others. If they all speak in random order, the probability that A speaks before B and B speaks before C is

(a) 3/8
(b) 1/6
(c) 3/5
(d) None of these

Solution The total number of ways in which 8 persons can speak is $^8P_8$= 8!. The number of ways in which A, B and C can be arranged in the specified speaking order is $^8C_3$. There are 5! ways in which the other five can speak. So, the favourable number of ways is $^8C_3$ = 5!.

$$\text{Hence, the required probability} = \frac{^8C_3 \times 5!}{8!} = \frac{1}{6}$$

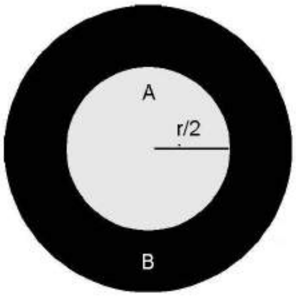
Example 11 There is a point inside a circle. What is the probability that this point is closer to the circumference than to the centre?

Solution

Assume that the radius of the bigger circle is r, and the radius of the inner circle is r/2. Point will be closer to the circumference than to the centre if the point is lying in the segment B.

Area of segment B = ¾ πr^2

So, the probability of point being closer to the circumference = ¾ $\pi r^2/\pi r^2$ = 3/4



PRACTICE EXERCISES

WARM UP

Q.1. What is the chance of drawing an ace from a deck of cards?
(a) 8/52 (b) 16/52
(c) 2/52 (d) 4/52

Q.2. A three digit number is to be formed by using the digits 1, 2, 3,… 9. What is the probability that the number formed is greater than 500, if repetition is not allowed?
(a) 280/504 (b) 54/280
(c) 58/204 (d) 24/504

Q.3. Mayank and Amit are throwing an unbiased dice. If Mayank throws 1, what is the Amit's chance of throwing a higher number?
(a) 1/5 (b) 5/6
(c) 2/12 (d) None of these

Q.4. A bag contains 4 red and 7 green balls. If three balls are drawn from the bag, replaced and once again three balls are drawn from the bag, what is the probability of obtaining 3 red balls on the first drawing and 3 green balls on the second drawing?
(a) 14/5445 (b) 14/27225
(c) 28/5445 (d) None of these

Q.5. One number is selected at random from the first 25 natural numbers. What is the probability that it is a multiple of either 5 or 7?
(a) 2/12 (b) 8/25
(c) 4/25 (d) None of these

Q.6. A bag contains 5 green apples and 7 red apples. If two apples are drawn from the bag, what is the probability that one is red and the other is green?
(a) 12/66 (b) 35/66
(c) 2/12 (d) 2/35

Q.7. The probability of a problem being solved by A is 1/4 the and B solving it is 1/5 the. If they try independently, then what is the probability that the problem is not solved?
(a) 2/5 (b) 4/9 (c) 3/5 (d) 2/9

Q.8. Ten horses are running in a race, the chances that A will win are 30%, that B will win are 20% and C will win are 10%. What is the probability that one of them will win?
(a) 0.689 (b) 0.598
(c) 0.498 (d) 0.398

Q.9. A's chance of winning a single game against B is 3/4 th. Find the chance that in a series of five games with B, A wins exactly 3 games?
(a) 135/1024 (b) 135/512
(c) 235/1024 (d) None of these

Q.10. It is known that at noon the sun is hidden by clouds on an average of two days out of every three. Find the probability that at noon on at least four out of five days the sun will be shining.
(a) 9/11 (b) 11/243
(c) 11/81 (d) None of these

Q.11. If on an average 1 vessel in every 10 is wrecked, find the chance that out of 5 vessels expected, at least 4 will arrive safely?
(a) 61246/50000
(b) 4592/5000
(c) 35927/50000
(d) None of these

Q.12. Two fair dice are thrown. What is the probability that the number of dots on the first dice exceeds 3 and that on the second exceeds 4?

(a) 2/6 (b) 3/6 (c) 1/6 (d) 5/6

Q.13. What is the probability that a card drawn at random from a pack of 52 cards is either a King or a Spade?

(a) 2/52 (b) 4/13
(c) 4/26 (d) None of these

Q.14. Nazneen was dialing a telephone number of six digits. She forgot the last three digits of the six digit telephone number but remembered that the number formed by the last three digits in the same order was a perfect square. What is the probability that the she dialed a right number?

(a) 1/32 (b) 2/32
(c) 2/16 (d) 1/23

Q.15. What is the probability that there are 53 Sundays and 53 Tuesdays in a leap year?

(a) 0 (b) 1
(c) 0.5 (d) None of these

Q.16. What is the probability that there are 52 Sundays in a leap year?

(a) 0 (b) 1
(c) 0.5 (d) None of these

Q.17. What is the probability that there are 53 Sundays and 53 Saturdays in a leap year?

(a) 1/7 (b) 2/7
(c) 0 (d) None of these

Q.18. What is the probability that there are 53 Sundays in a leap year?

(a) 2/7 (b) 1/7
(c) 0 (d) None of these

FOUNDATION

Q.1. Five boys and three girls are seated at random in a row. The probability that no boy sits between two girls is

(a) $\frac{1}{56}$ (b) $\frac{1}{8}$
(c) 3/28 (d) None of these

Q.2. In a convex hexagon two diagonals are drawn at random. The probability that the diagonals intersect at an interior point of the hexagon is

(a) $\frac{5}{12}$ (b) $\frac{7}{12}$
(c) $\frac{2}{5}$ (d) None of these

Q.3. Two cards are drawn at random from a pack of 52 cards. The probability of getting at least a spade and an ace is

(a) $\frac{1}{34}$ (b) $\frac{8}{221}$ (c) $\frac{1}{26}$ (d) $\frac{2}{51}$

Q.4. If the letters of the word ATTEMPT are written down at random, the chance that all the Ts are consecutive is

(a) $\frac{1}{42}$ (b) $\frac{6}{71}$
(c) $\frac{1}{7}$ (d) None of these

Q.5. Seven white balls and 3 black balls are placed in a row at random. The probability that no two black balls are adjacent is

(a) $\frac{1}{3}$ (b) $\frac{7}{15}$ (c) $\frac{2}{15}$ (d) $\frac{1}{3}$

Q.6. Four gentlemen and 4 ladies take seats at random round on a table. The probability that they are sitting alternately is

(a) $\frac{4}{35}$ (b) $\frac{1}{70}$ (c) $\frac{2}{35}$ (d) $\frac{1}{35}$

Q.7. Let $x = 33^n$. The index n is given a positive integral value at random. The probability that the value of x will have 3 in the units place is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$
(c) $\frac{1}{3}$ (d) None of these

Q.8. Three dice are thrown simultaneously. The probability of getting a sum of 15 is

(a) $\frac{1}{72}$ (b) $\frac{5}{36}$
(c) $\frac{5}{72}$ (d) None of these

Q.9. Three dice are thrown. The probability of getting a sum which is a perfect square is

(a) $\frac{2}{5}$ (b) $\frac{9}{20}$
(c) $\frac{1}{4}$ (d) None of these

Q.10. A box contains 6 red balls, 7 green balls and 5 blue balls. Each ball is of a different size. The probability that the red ball being selected is the smallest red ball is

(a) $\frac{1}{18}$ (b) $\frac{1}{3}$ (c) $\frac{1}{6}$ (d) $\frac{2}{3}$

Q.11. Two distinct numbers are selected at random from the first twelve natural numbers. The probability that the sum will be divisible by 3 is

(a) $\frac{1}{3}$ (b) $\frac{23}{66}$

(c) $\frac{1}{2}$ (d) None of these

Q.12. Three different numbers are selected at random from the set A={1, 2, 3,… 10}. The probability that the product of two of the numbers is equal to the third is

(a) $\frac{3}{4}$ (b) $\frac{1}{40}$

(c) $\frac{1}{8}$ (d) None of these

Q.13. There are 7 seats in a row. Three persons take seats at random. The probability that the middle seat is always occupied and no two persons are consecutive is

(a) $\frac{9}{70}$ (b) $\frac{9}{35}$

(c) $\frac{4}{35}$ (d) None of these

Q.14. From a group of 10 persons consisting of 5 lawyers, 3 doctors and 2 engineers, four persons are selected at random. The probability that the selection contains at least one of each category is

(a) $\frac{1}{2}$ (b) $\frac{1}{3}$

(c) $\frac{2}{3}$ (d) None of these

Direction for questions 15 and 16: Read the passage below and solve the questions based on it.

A and B are two possible events of an experiment such that $P(A \cup B) = 0.7$ and $P(A) = 0.4$, then find P(B) given that

Q.15. A and B are mutually exclusive events

(a) 0.6 (b) 0.3 (c) 0.2 (d) 0.5

Q.16. A and B are independent events.

(a) 0.5 (b) 0.3 (c) 0.6 (d) 0.2

Q.17. The odds against an event are 3 to 4 and the odds in favour of another independent event are 2 to 5. The probability that exactly one of them occurs is

(a) $\frac{23}{49}$ (b) $\frac{26}{49}$ (c) $\frac{6}{49}$ (d) $\frac{8}{49}$

Q.18. In the previous question, what is the probability that atleast one of them occurs is

(a) $\frac{34}{49}$ (b) $\frac{15}{49}$ (c) $\frac{23}{49}$ (d) $\frac{26}{49}$

Q.19. Chunmun picks a letter from the set of English alphabet and finds it to be a vowel. What is the probability that the letter is E?

(a) 1/26 (b) 5/6

(c) 1/5 (d) None of these

Q.20. There are 3 red, 3 blue and 3 green balls. What is the probability that out of 3 selected balls 2 are red?

(a) 1/4 (b) 2/5

(c) 3/7 (d) None of these

Q.21. In the horse race, the horse Exaclibur's chances of winning are $\frac{1}{6}$, the horse Noorjehan's chances are $\frac{1}{10}$ and those of the horse Bombay are $\frac{1}{8}$. If the race starts with twelve horses and only one horse wins, then what are the chances that one of these three horses will win?

(a) $\frac{57}{120}$ (b) $\frac{47}{120}$ (c) $\frac{37}{120}$ (d) $\frac{27}{120}$

Q.22. If a year has 360 days and all the months with 30 days. What is the probability that your birthday falls on a Monday and that is an even day of an even month, given that January 1 is a Monday?

(a) $\frac{1}{10}$ (b) $\frac{13}{360}$

(c) $\frac{1}{28}$ (d) $\frac{11}{360}$

Q.23. If 4 whole numbers are taken at random, and multiplied together, then the probabaility that the last digit in the product is 1, 3, 7 or 9 will be

(a) 13/976 (b) 17/529

(c) 16/625 (d) 13/625

Q.24. There are 500 students in an examination. 150 students passed the first paper, 350 students passed the second paper and 50 students passed both the papers. Find the probability that a student selected at random has failed in both the papers.

(a) 1/5 (b) 1/10

(c) 3/10 (d) 3/5

Q.25. A person is asked to randomly pick two balls from a bag which has 15 yellow & 5 red balls. The probability that the person picks two balls of the same colour is

(a) 33/38 (b) 23/38

(c) 38/43 (d) 15/38

MODERATE

Q.1. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 is

(a) $\frac{1}{5}$ (b) $\frac{1}{7}$ (c) $\frac{1}{4}$ (d) $\frac{1}{49}$

Q.2. If the probability of A failing in an examination is $\frac{1}{5}$ and that of B failing the same examination is $\frac{3}{10}$, then the probability that either A or B fails is

(a) $\frac{1}{2}$ (b) $\frac{11}{25}$

(c) $\frac{19}{50}$ (d) None of these

Q.3. India plays two matches each with West Indies and New Zealand. In any match the probabilities of India getting 0,1 and 2 points are 0.45, 0.05 and 0.50 respectively. Assuming that the outcomes are independent, the probability of India getting at least 7 points is

(a) 0.0875 (b) $\frac{1}{16}$

(c) 0.1125 (d) None of these

Q.4. Let A and B be two independent events such that P(A) = $\frac{1}{5}$, $P(A\cup B) = \frac{7}{10}$ Then $P(\bar{B})$ is equal to

(a) $\frac{3}{8}$ (b) $\frac{2}{7}$

(c) $\frac{7}{9}$ (d) None of these

Q.5. Let A and B be two independent events in such a way that their probabilities are $\frac{3}{10}$ and $\frac{2}{5}$ respectively. The probability of exactly one of the events happening is

(a) $\frac{25}{30}$ (b) $\frac{1}{2}$

(c) $\frac{31}{50}$ (d) None of these

Q.6. A, B, C are three events for which P(A) = 0.6, P(B) = 0.4, P(C) = 0.5, P(A∪B) = 0.8, P(A∩B) = 0.3 and P(A∩B∩C) = 0.2. If $\Pi(A\cup B\cup C) \geq 0.85$, then the interval of values of P(B∩C) is

(a) [0.2,0.35] (b) [0.55, 0.7]

(c) [0.2,0.55] (d) None of these

Q.7. A dice is thrown $2n+1$ times, $n\in N$. The probability that faces with even numbers shown odd number of times is

(a) $\frac{2n+1}{4n+3}$ (b) Less than $\frac{1}{2}$

(c) Greater than $\frac{1}{2}$ (d) None of these

Q.8. A shooter firing at a target has 10% chance of hitting the target in one shot. The number of times he must fire at the target to have above 50% chance of hitting the target is

(a) 11 (b) 9 (c) 7 (d) 5

Q.9. Let A = {2, 3, 4, … , 20, 21}. A number is chosen at random from the set A and it is found to be a prime number. The probability that it is more than 10 is

(a) $\frac{9}{10}$ (b) $\frac{1}{10}$

(c) $\frac{1}{5}$ (d) None of these

Q.10. The probabilities that a student passes in mathematics, physics and chemistry are *m*, *p* and *c* respectively. Of these subjects, a student has a 75% chance of passing in at least one, a 50% chance of passing in at least two, and a 40% chance of passing in exactly two subjects. Which of the following relations are true?

(a) $p+m+c = \frac{19}{20}$ (b) $p+m+c = \frac{27}{20}$

(c) $pmc = \frac{1}{20}$ (d) $pmc = \frac{1}{4}$

Q.11. If A and B are two events such that $P(A\cup B) \geq \frac{3}{4}$ and $\frac{1}{8} \leq P(A\cap B) \leq \frac{3}{8}$ then

(a) $P(A) + P(B) \leq \frac{11}{8}$

(b) $P(A).P(B) \leq \frac{3}{8}$

(c) $P(A) + P(B) \geq \frac{7}{8}$

(d) None of these

Q.12. Three natural numbers are taken at random from the set $A=\{x|x, 1\leq x\leq 100, x\in N\}$. The probability that the AM of the numbers taken is 25 is

(a) $\frac{{}^{77}C_2}{{}^{100}C_3}$ (b) $\frac{{}^{25}C_2}{{}^{100}C_3}$

(c) $\frac{{}^{74}C_{72}}{{}^{100}C_3}$ (d) None of these

Q.13. The probability of a number n showing in a throw of a dice marked 1 to 6 is proportional to *n*. then the probability of the number 3 showing in throw is

(a) $\frac{1}{2}$ (b) $\frac{1}{6}$ (c) $\frac{1}{7}$ (d) $\frac{1}{21}$

Direction for questions 14 to 16: Read the passage below and solve the questions based on it.

Three mountaineers Amit, Vinit and Nishit are climbing up a mountain with their respective probabilities of reaching the summit being $\frac{1}{3}, \frac{1}{5}$ and $\frac{1}{4}$ respectively. What is the probability that

Q.14. All of them reach the summit?

(a) $\frac{3}{60}$ (b) $\frac{1}{60}$ (c) 1 (d) 0

Q.15. At least one of them reaches the summit?

(a) $\frac{59}{60}$ (b) $\frac{36}{60}$ (c) $\frac{17}{60}$ (d) $\frac{1}{60}$

Q.16. Exactly one of them reaches the Summit?

(a) $\frac{13}{30}$ (b) $\frac{17}{30}$ (c) $\frac{19}{30}$ (d) $\frac{11}{30}$

Q.17. The coach of the football team says that the probability that player A plays and team wins is $\frac{1}{4}$, player B plays and team wins is $\frac{1}{2}$ and both play and team wins is $\frac{1}{4}$. He also added that the team has $\frac{3}{4}$ chances to win. What is the probability that when either player A or player B played, the team won?

(a) $\frac{2}{3}$

(b) $\frac{1}{2}$

(c) Cannot be determined

(d) None of these

Direction for questions18 and 19: Read the passage below and solve the questions based on it.

Two teams Patel Nagar and Boring Road are playing a cricket tournament. If team Patel Nagar wins the toss, probability of it winning the match is 0.55. If Boring Road wins the toss, probability of its winning the match is 0.65. Probability of winning the toss is same for both the teams Patel Nagar and Boring Road.

Q.18. In a five-matche tournament, what is the probability of Patel Nagar winning all the matches, if it wins at most two tosses?

(a) 0.45 (b) 0.41 (c) 0.39 (d) 0.51

Q.19. What is the probability of Boring Road winning all the matches, if Patel Nagar wins 3 tosses in a five match tournament?

(a) 0.16

(b) 0.14

(c) 0.21

(d) None of these

Q.20. Rakesh went to take an exam. After the exam, while explaining the structure of the question paper, the teacher said that 40% of the total questions were difficult and 20% of those were on algebra and 30% of those were on geometry. Also, 10% and 20% of easy questions were on algebra and geometry respectively. All the questions that Rakesh attempted were either on algebra or on geometry. What is the probability that Rakesh found all the questions difficult (Questions are either easy or difficult)?

(a) $\frac{14}{100}$ (b) $\frac{14}{27}$

(c) $\frac{20}{27}$ (d) None of these

Direction for questions 21 and 22: Read the passage below and solve the questions based on it.

In a game, one can score as many points as he gets in every throw till he gets a score (in one throw) which is not a multiple of points scored in the previous throw.

Q.21. If a person scores 5 points in total, what is the probability that the dice was thrown 2 times?

(a) $\frac{3}{4}$ (b) $\frac{3}{5}$

(c) $\frac{1}{2}$ (d) None of these

Q.22. If a person throws the dice 3 times, what is the probability that the total score is 11?

(a) 5/43

(b) 5/41

(c) 5/37

(d) None of these

Q.23. Triangles are formed by joining vertices of an octagon. Any one of those triangle is selected at random. What is the probability that the selected triangle has no side common with the octagon?

(a) 3/7 (b) 2/7 (c) 5/7 (d) 1/7

Q.24. What is the probability that a 4 digit number of the type abcd (a, b, c, d are all different) is a multiple of 11?

(a) 0.142 (b) 0.048

(c) 0.072 (d) 0.091

Q.25. A month is randomly selected from the months in a non leap year and it is found that it has five Sundays. What is the probability that it has five Mondays?

(a) 1/7 (b) 2/7 (c) 2/3 (d) 20/33

ADVANCED

Q.1. Manoj throws a fair dice. He is promised an amount thrice the value of the number showing up if the number showing up is odd and an amount twice the value of the number showing up if it is even. What is the maximum amount Manoj is willing to pay each time to throw the dice, if in the long run he wants to make an average profit of Rs 5 per throw?

(a) Rs 3.50 (b) Rs 8.5

(c) Rs 5 (d) None of these

Direction for questions 2 to 4: Read the passage below and solve the questions based on it.

A shopkeeper inspected some items in a box containing a, b and c number of items of type P, Q and R respectively. He picked up an item randomly, and put it back into the box if he found that the item selected was regularly sold, he also added the equal number of items of the same type as those previously in the box otherwise he removed all the items of that type.

Q.2. If exactly one of the three items is regularly sold then what is the probability that the item selected in the second draw is similar to that in the first draw? (Given that the item selected in the second draw is a regularly sold item.)

(a) $2\left(\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}\right)$

(b) $\left(\frac{2a^2}{2a+b+c}+\frac{2b^2}{a+2b+c}+\frac{2c^2}{a+b+2c}\right)\frac{1}{a+b+c}$

(c) $\left(\frac{a}{2a+b+c}+\frac{b}{1+2b+c}+\frac{c}{a+c+2c}\right)\frac{1}{a+b+c}$

(d) None of these

Q.3. In the above problem, what is the probability that the item selected in the second draw is not a regularly sold item?

(a) $\frac{a}{a+b+c}+\frac{b}{a+b+c}+\frac{c}{a+b+c}$

(b) $\frac{b+c}{2a+b+c}+\frac{a+c}{a+2b+c}+\frac{a+b}{a+b+2c}$

(c) $\frac{a}{b+c}+\frac{b}{a+c}+\frac{c}{a+b}$

(d) None of these

Q.4. What is the probability that the items selected in the second draw is different from that in the first draw, given that the item selected in the first draw is a regularly sold item? (Assume exactly two of the three items are not regularly sold items.)

(a) $\left(\frac{a(a+c)}{2a+b+c}+\frac{b(a+c)}{a+2b+c}+\frac{c(a+b)}{a+b+2c}\right)\frac{1}{a+b+c}$

(b) $\left(\frac{a(b+c)}{2a+b+c}+\frac{b(a+c)}{a+2b+c}+\frac{c(a+b)}{a+b+2c}\right)\frac{1}{a+b+c}$

(c) $\left(\frac{a}{2a+b+c}+\frac{b}{a+2b+c}+\frac{c}{a+b+2c}\right)\frac{1}{a+b+c}$

(d) 0

Q.5. There is a frame of a cuboid of length 6, breadth 5 and height 7. The cuboid is only composed of skeleton of 210 cubes of side 1. An insect is on one corner of the cube and it wants to travel to the opposite end of the longest diagonal. It can only travel along the sides of the a small cubes and it always takes the shortest possible route. Find the probability that it passes through atleast one of the corners.

(a) $\frac{1}{6}$ (b) $\frac{7}{12}$

(c) $\frac{5}{18}$ (d) None of these

Q.6. A coin is so biased that the heads occurs four times as frequently as tails. Another coin is biased such that the heads occurs 65% of the times. When the two coins are tossed simultaneously, what is the probability of at least one tail turning up?

(a) 35% (b) 87% (c) 48% (d) 73%

Q.7. Only one bomb is needed to destroy a bridge and there are three bombers engaged in an attempt to destroy the bridge. The probabilities of the bombers hitting

the bridge are respectively 0.3, 0.4 and 0.7. What is the probability of the bridge being destroyed?
(a) 0.874 (b) 0.126 (c) 0.916 (d) 0.084

Q.8. A pack of playing cards was found to contain only 51 cards. If the first 13 cards, which are examined, are all red, what is the probability that the missing card is black?
(a) 3/7 (b) 5/7 (c) 2/3 (d) 3/8

Q.9. A speaks truth in 75% of the cases and B in 80% of the cases. In what percentage of cases are they likely to contradict each other in stating the same fact?
(a) 15% (b) 20% (c) 5% (d) 35%

Direction for questions 10 to 12: Read the passage below and solve the questions based on it.

In a manufacturing unit machines A, B, C manufacture 15%, 25%, 60% of the total production of bolts respectively. Of all the bolts manufactured by the machine 4% by A, 2% by B and 3% by C are defective respectively. A bolt is drawn at random and is found to be defective.

Q.10. What is the probability that it was produced by machine A?
(a) 6/29 (b) 5/29 (c) 18/29 (d) 4/29

Q.11. What is the probability that it was produced by machine B?
(a) 6/29 (b) 5/29 (c) 18/29 (d) 4/29

Q.12. What is the probability that it was produced by machine C?
(a) 6/29 (b) 5/29 (c) 18/29 (d) 4/29

Q.13. There is a 30% chance that it rains on any particular day. What is the probability that there is at least one rainy day within a period of 3 days?
(a) $(0.7)^3$ (b) $1-(0.7)^3$
(c) 0.3^3 (d) None of these

Q.14. The numbers are selected at random, one at a time, from the two-digit numbers 00, 01, 02, … 99 with replacement. An event E occurs only if the product of the two digits of a selected numbers is 18. If four numbers are selected, find the probability that the event E occurs at least 3 times.
(a) 0.0025 (b) 0.00025
(c) 0.025 (d) 0.25

Q.15. A and B are two independent events. The probability that both A and B occur is 1/6 the and the probability that neither of them occurs in 1/3rd. Find the probability of the occurrence of A.
(a) 1/6
(b) 1/4
(c) 1/3
(d) Cannot be determined

Q.16. A biased dice is tossed and the respective probabilities for various faces to turn up are

Face	1	2	3	4	5	6
Probability	0.1	0.24	0.19	0.18	0.15	0.14

If an even face has turned up, then the probability that it is face 2 or face 4 is
(a) 0.25 (b) 1.42 (c) 0.75 (d) 0.9

Q.17. A natural number x is chosen at random from the first one hundred natural numbers. What is the probability that $x+\frac{100}{x}>50$?
(a) 13/20 (b) 3/5 (c) 9/20 (d) 11/20

Q.18. A dice is loaded in such a way that the probability of getting i on the dice is proportional to i. This dice is rolled three times. What is the probability of getting a product of 15?
(a) 10/1029 (b) 14/1029
(c) 16/1029 (d) 18/1029

Q.19. A target may be either at point I with probability $\frac{8}{9}$ or at point II with probability $\frac{1}{9}$. We have 21 shells, each of which can be fired either at point I or at point II. Each shell may hit the target, independently of the other shells, with probability $\frac{1}{2}$ How many shells must be fired at point I to hit the target with maximum probability?
(a) 10 (b) 11
(c) 9 (d) 12

Q.20. Let N cells be numbered 1, 2, 3, … N. We randomly throw balls into them. The process is continued until any one of the cells receives two balls. What is the probability that more than n throws will be necessary?
(a) ${}^{N}P_{N}/N^{n}$
(b) ${}^{N}/N^{n}$
(c) ${}^{N}C_{n}/N^{n}$
(d) None of these

Q.21. The probability that a man will live 10 more years is $\frac{1}{4}$ and the probability that his wife will live 10 more years is $\frac{1}{3}$.Then the probability that neither will be alive in 10 years is
(a) $\frac{5}{12}$ (b) $\frac{7}{12}$
(c) $\frac{1}{2}$ (d) $\frac{11}{12}$

Q.22. A locker can be opened by dialing a fixed three-digit code between 000 and 999. A stranger, who does not know the code, tries to open the locker by dialing three digits at random. Find the probability that the stranger succeeds at the sth trial. (Assume that the stranger does not repeat unsuccessful combinations.)

(a) 2s/1000 (b) 2s/999
(c) s/1000 (d) s/999

Q.23. The probability of a bomb hitting a bridge is $\frac{1}{2}$ and two direct hits are needed to destroy it. The least number of bombs required so that the probability of the bridge being destroyed is greater than 0.9 is

(a) 8 (b) 9
(c) 10 (d) 11

Q.24. From a bag which is known to contain 4 balls each of which is just as likely to be black as white, a ball is drawn and is found to be white. What is the probability that the bag contained 3 white and 1 black balls?

(a) 3/7 (b) 5/8
(c) 3/8 (d) 4/7

Q.25. On a loaded dice, any even number occurs thrice as frequently as an odd number. If the dice is thrown twice, what is the probability that the sum is 10 or more?

(a) 55/72 (b) 17/72
(c) 39/72 (d) 23/72

ANSWERS

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Answer	(d)	(a)	(b)	(c)	(b)	(b)	(c)	(d)	(b)	(b)	(d)	(c)	(b)	(a)	(a)	(b)	(a)	(a)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(a)	(c)	(c)	(b)	(d)	(a)	(d)	(d)	(c)	(a)	(b)	(c)	(a)	(b)	(a)	(b)	(a)	(c)	(d)
Q. No.	21	22	23	24	25															
Answer	(b)	(b)	(c)	(b)	(b)															

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(a)	(a)	(a)	(d)	(d)	(c)	(d)	(c)	(d)	(c)	(c)	(b)	(b)	(a)	(a)	(b)	(d)	(b)
Q. No.	21	22	23	24	25															
Answer	(b)	(a)	(b)	(a)	(d)															

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(d)	(a)	(d)	(c)	(c)	(c)	(d)	(a)	(b)	(c)	(b)	(b)	(d)	(d)	(d)	(a)	(d)	(c)
Q. No.	21	22	23	24	25															
Answer	(c)	(c)	(b)	(c)	(b)															

HINTS AND SOLUTIONS

Moderate

1. For the sum to be divisible by 5, unit digit of $7^m + 7^n$ should be either 0 or 5.
   Different unit digit possible for 7 = 7, 9, 3 and 1.
   So, if 7^m is giving unit digit 7, 7^n should give 3, or vice-versa.
   Similarly, if 7^m is giving unit digit 9, 7^n should give 1, or vice-versa.

2. Use P(AUB) = 1/5 + 3/10 – (1/5) × (3/10)

5. Do as given in 2.

9. Total number of primes = 8 and number of numbers more than 10 = 11

14. All of them reach the summit = 1/3 × 1/5 × 1/4 = 1/60

15. To find out "At least one of them reaches the summit", we will find the probability of nobody reaching the summit = 2/3 × 4/5 × 3/4 = 2/5. Hence, the probability of at least one of them reaching the summit = 1 – 2/5 = 3/5

16. This is same as: Amit reaches the summit, but Vinit and Nishit do not reach the summit. Or, Vinit reaches the summit, but Amit and Nishit do not reach the summit. Or, Nishit reaches the summit, but Vinit and Amit do not reach the summit.
    Amit reaches the summit, but Vinit and Nishit do not reach the summit = 1/3 × 4/5 × 3/4 = 1/5 and similarly we will find all the other possibilities.

23. Total number of triangles formed = ${}^8C_3$ = 56
    Triangles having two sides common = 8
    Triangles having one side common = ${}^8C_1 \times {}^4C_1$ = 32
    Triangles having three sides common = 0
    Triangles having no side common = 56 – 40 = 16
    So, probability = 16/56 = 2/7.

Advanced

7. It is nothing but [1 – probability of bridge not getting destroyed] = 1 – 0.084

9. Contradiction can occur if A speaks truth and B lies, B speaks truth and A lies.
   So, required probability = 0.75 × 0.2 + 0.8 × 0.25 = 0.35

13. Probability that it rains = 0.3, so probability that it does not rain = 0.7.

15. 1 – Neither of them occurs = At least one of them occurs
    So, 2/3 = P(A) + P(B) – P(A $\cap$ B), So, it cannot be determined.

17. $x^2 - 50x + 100 > 0$. Find the range of the values of x and solve the question.

21. The probability that a man will not live 10 more years = 3/4 and the probability that his wife will not live 10 more years =2/3. Then the probability that neither will be alive in 10 years = 3/4 × 2/3.

BENCHMARKING TEST 3

This test paper contains 30 questions of 4 marks each. One-third of the mark allotted to a particular question will be deducted in case of wrong answer.

Time Given: 70 Minutes **Total Marks: 120** **Qualifying Marks: 55 Marks**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
88+	99+
83	98.5+
78	98
55	92
40	85
30	80

Q.1. The first term, a term of an AP = the common ratio of a GP and the first term of the GP = common difference of the AP. If the sum of the first two terms of the GP is equal to the sum of the first 2 terms of the AP, then the ratio of first term of GP to the first term of AP is
(a) > 1
(b) Independent of first term of AP
(c) Independent of first term of GP
(d) < 1

Q.2. In a college of 300 students, every student reads 5 newspapers and every newspaper is read by 60 students. The number of newspapers is
(a) At least 30 (b) At most 20
(c) Exactly 25 (d) None of these

Q.3. The condition that both the roots of a quadratic equation $ax^2 + bx + c = 0$ are positive, is
(a) a and c have opposite sign that of b
(b) b and c have opposite sign that of a
(c) a and b have opposite sign that of c
(d) None of these

Q.4. The 4th degree polynomial equation $p(x) = x^4 - 2x^3 + 3x^2 - 4x + 5 = 0$ has
(a) At least one real root
(b) No real root
(c) Two real roots
(d) All real roots

Q.5. Let $g(x)$ be a function such that $g(x + 1) + g(x - 1) = g(x)$ for every real x.Then for what value of p is the relation $g(x + p) = -g(x)$ necessarily true for every real x?
(a) 5 (b) 3 (c) 2 (d) 6

Q.6. If f is real valued function and $f(x + y) = f(y)$ $\forall$ $x \in$ R. Given $f(1) = 3$ and $\sum_{r=1}^{r=n} f(r) = 1092$. Find the value of n.
(a) $n = 5$ (b) $n = 6$
(c) $n = 7$ (d) None of these

Q.7. Let $f(x) = 0$ be the polynomial equation of the least possible degree with rational coefficients. One of the roots of this equation is $\sqrt{7} + \sqrt[3]{49}$. What is the product of all the roots of $f(x) = 0$?
(a) 21 (b) 35 (c) 42 (d) 56

Q.8. If $f(x)$ is a real valued function and $f\{x + f(y)\} = x + f(y)$. Then $f(100)$ is

(a) 100 (b) 1000

(c) 10000 (d) 100000

Q.9. The quadratic equation $px^2 + qx + r = 0$ satisfies the following conditions:

1. p, q and r are distinct
2. $p, q, r \{1, 2, 3, 4, 5, 6\}$
3. $(x + 1)$ is a factor of quadratic equation $px^2 + qx + r = 0$

How many such quadratic equations are possible?

(a) 8 (b) 12

(c) 16 (d) None of these

Q.10. Given are $f(x) = 4^x + 4^{-x}$ and $g(x) = 5/4$. At how many points graphs of both the functions will intersect?

(a) 0 (b) 1 (c) 2 (d) Infinite

Q.11. How many values of p $(p > 1)$ satisfy the following equation:

$\log_2 p \,.\log_4 p.\log_6 p = \log_2 p.\log_4 p + \log_2 p \,.\, \log_6 p + \log_4 p \,.\, \log_6 p$?

(a) 0 (b) 1

(c) 2 (d) More than 2

Q.12. If $\sum_{r=0}^{\infty} \frac{1}{(2r-1)^2} = \text{N}$, then which of the following is equal to $\sum_{r=1}^{\infty} \frac{1}{r^2}$?

(a) (3/4N) – 1 (b) (4/3N) + 1

(c) 3/4(N + 1) (d) 4/3 (N – 1)

Q.13. Let p, q, r and s be real numbers. If N = max [min (p, q), min(r, s)] and S = min[max (p, r), max (q, s)]. Then which of the following is true?

(a) $\text{N} \leq \text{S}$, for all values of p, q, r and s

(b) $\text{N} \geq \text{S}$, for all values of p, q, r and s

(c) N = S, for all values of p, q, r and s

(d) Cannot be uniquely determined

Q.14. The maximum possible value of $x^2 + 4y^2 + 9z^2$ is N. What is the value of N if $x + 2y + 3z = 12$ (x, y and z are real numbers)?

(a) 224 (b) 16

(c) 4096 (d) Infinite

Q.15. x, y, z and t are positive numbers which satisfy the following conditions:

1. If $x > y$, then $z > t$ and
2. If $x > z$, then $y < t$

Then which of the following is true?

(a) If $x < y$, then $z < t$

(b) If $x < z$, then $y >$ t

(c) If $x > y + z$, then $z > y$

(d) If $x > y + z$, then $z < y$

Q.16. If $h(x)$ is the greatest integer less than or equal to $h/2$, then how many of the following statements is/are true?

1. $h(x)$ is half the least even integer greater than or equal to h.
2. $h(x)$ is the least integer greater than or equal to $\frac{h}{2} - 1$.
3. $h(x)$ is one less than half the least even integer greater than h.

(a) 0 (b) 1 (c) 2 (d) 3

Q.17. The number of ways in which three distinct numbers in AP and GP can be selected from 1, 2, 3,... 24 is A and G respectively. Which of the following is the value of A : G?

(a) 21 (b) 12

(c) 9 (d) None of these

Q.18. $\text{N}_1 + \text{N}_2 + \text{N}_3 + \ldots + \text{N}_\text{N} = 100$, where $\text{N}_1, \text{N}_2, \text{N}_3 \ldots$ and N_N are $n(n>1)$ consecutive natural numbers such that $\text{N}_1 < \text{N}_2 < \text{N}_3 < \text{N}_4 \ldots < \text{N}_{\text{N}.}$ If $\text{N}_1 > 8$, then how many values of n is/are possible?

(a) 4 (b) 8

(c) 2 (d) Infinite

Q.19. $\text{P} = \frac{10^{999} + 9.10^{998} + 9^2.10^{997} + \ldots + 9^{999}}{10^{10^3} - 9^{10^3}}$, then find the value of $\frac{(P+1)(P+100)}{P(P-2)}$?

(a) 1

(b) 101

(c) 202

(d) None of these

Q.20. Let $f(x)$ be a function such that $f(x - 1) + (x + 1) = \sqrt{2} f(x)$. Then for what value of y is the relation $f(x + y) = f(x)$ necessarily true for every real x?

(a) 4 (b) 6 (c) 8 (d) 12

Q.21. The sum of all the real roots of the equations $|x - 2|^2 + |x - 2| - 2 = 0$ is

(a) 2 (b) –4 (c) –2 (d) 4

Q.22. Let p and q be roots of the equation $x^2 - 2x + \text{A} = 0$ and let r and s be the roots of the equations $x^2 - 18x + \text{B} = 0$. If $p < q < r < s$ are in an arithmetic progression then what is the value of A + B?

(a) –77 (b) 74 (c) –74 (d) 77

Q.23. Let 't' be any root of the equation $x^n + x^{n-1} + x^{n-2} + \ldots + 1 = 0$. Then $(t^{2n+2} + 3)(t^{3n+3} - 4)$ equals

(a) 12 (b) – 12 (c) 6 (d) – 6

Q.24. An n – digit number is a positive number with exactly n digits. Nine hundred distinct n – digit numbers are to be formed using only the three digits 2, 5 and 7. The smallest value of n for which this is possible is

(a) 6 (b) 7 (c) 8 (d) 9

Q.25. Let $a_1, a_2, \ldots, a_{10}$ be in A.P. and $h_1, h_2, \ldots, h_{10}$ be in H.P. If $a_1 = h_1 = 2$ and $a_{10} = h_{10} = 3$, then $a_4 h_7$ is

(a) 2 (b) 3 (c) 5 (d) 6

Q.26. If the integers m and n are chosen at random between 1 and 100, then the probability that a number of the form $7^m + 7^n$ is divisible by 5 equals

(a) $\frac{1}{4}$ (b) $\frac{1}{7}$

(c) $\frac{1}{8}$ (d) $\frac{1}{49}$

Q.27. Let $S_1, S_2 \ldots$ be squares such that for each $n \geq 1$, the length of a side of S_n equals the length of a diagonal of S_{n+1}. If the length of a side of S_1 is 10 cm^2, then for how many values of n is the area of S_n less than 1 cm^2?

i. 7 ii. 8 iii. 9 iv. 10

(a) 1 (b) 2

(c) 3 (d) 4

Q.28. If a, b, c, d are positive real numbers such that $a + b + c + d = 2$, then M = $(a + b)(c + d)$ satisfies the relation

(a) $0 \leq M \leq 1$ (b) $1 \leq M \leq 2$

(c) $2 \leq M \leq 3$ (d) $3 \leq M \leq 4$

Q.29. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions?

(a) 16 (b) 36

(c) 60 (d) 180

Q.30. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) 4

ANSWERS

■■■ Benchmarking Test 3

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(c)	(a)	(b)	(b)	(b)	(d)	(a)	(b)	(a)	(b)	(d)	(a)	(d)	(c)	(d)	(b)	(c)	(d)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(d)	(b)	(b)	(b)	(d)	(a)	(c)	(a)	(c)	(b)										

Module 4

Measurement

- Geometry
- Mensuration
- Co-ordinate Geometry

17

CHAPTER

Geometry

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Triangles, polygons, circles, and quadrilaterals
- Different theorems and their applications
- Similarity and conguency
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

Over the years, Geometry has become as important as the number system. In other words, it is one of those areas which can help students in getting good marks in QA on its own. In CAT, questions for close to 20% marks are asked from geometry alone. However, this is the only the secondary reason of this topic being so significant; the primary reasons being (i) creation of level-playing field for everyone as all the questions asked from this chapter pertain to close10 level maths only and (ii) most of the questions can be solved by the method of measurement of simple construction, without using any theorem of geometry.

Geometry, deals with measures and properties of points, lines, surfaces and solids. Here we will be discussing:

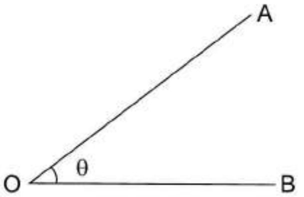
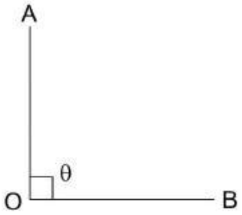
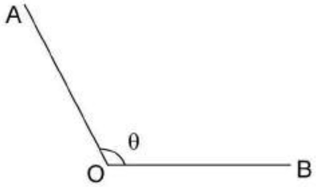
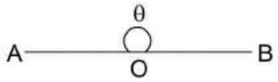
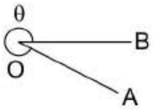
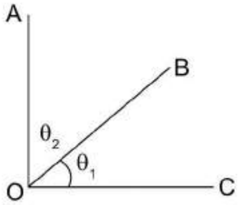
1. Lines and their properties
2. Polygons and their properties
3. Triangles and their properties
4. Quadrilaterals and their properties
5. Circles and their properties

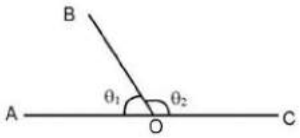
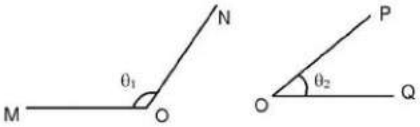
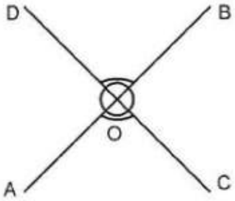
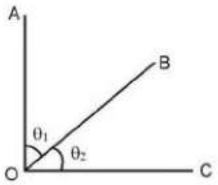
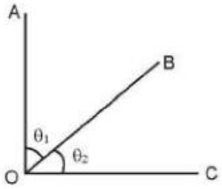
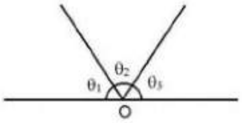
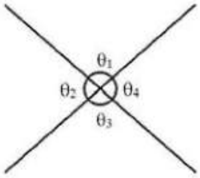
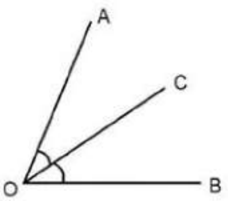
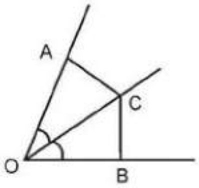
LINES AND THEIR PROPERTIES

A line is a set of points placed together that extends into infinity is both directions.

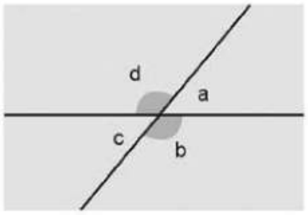
a) Different angles and pairs of angles

Measurement and nomenclature

Types of Angles	*Property*	*Diagram*
Acute Angle	$0° < \theta < 90°$ ($\angle AOB$ is an acute angle)	
Right Angle	$\theta = 90°$	
Obtuse Angle	$90° < \theta < 180°$	
Straight Line	$\theta = 180°$	
Reflex Angle	$180° < \theta < 360°$	
Complementary Angle	$\theta_1 + \theta_2 = 90°$ Two angles whose sum is 90° are complementary to each other	

Supplementary Angle	$\theta 1 + \theta 2 = 180^\circ$ Two angles, whose sum is 180°, are supplementary to each other	 B θ_1 θ_2 A O C  N P θ_1 θ_2 M O O Q
Vertically Opposite Angle	$\angle DOA = \angle BOC$ and $\angle DOB = \angle AOC$	
Adjacent Angles	$\angle AOB$ and $\angle BOC$ are adjacent angles Adjacent angles must have a common side (e.g., OB)	 A B θ_1 θ_2 O C
Linear Pair	$\angle AOB$ and $\angle BOC$ are linear pair angles. One side must be common (e.g., OB) and these two angles must be supplementary.	 A B θ_1 θ_2 O C
Angles on One Side of a Line	$\theta_1 + \theta_2 + \theta_3 = 180^\circ$	 θ_2 θ_1 θ_3 O
Angles Round the Point	$\theta_1 + \theta_2 + \theta_3 + \theta_4 = 360^\circ$	 θ_1 θ_2 θ_4 θ_3
Angle Bisector	OC is the angle bisector of $\angle AOB$. i.e., $\angle AOC = \angle BOC = \frac{1}{2} (\angle AOB)$ When a line segment divides an angle equally into two parts, then it is said to be the angle bisector (OC)	  (Angle bisector is equdistant from the two sides of the angle) i.e.,

b) Angles associated with two or more straight lines



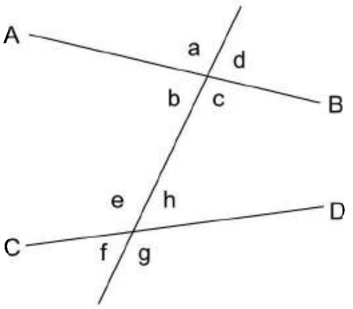
When two straight lines cross each other, $\angle d$ and $\angle b$ are the pair of vertical angles.

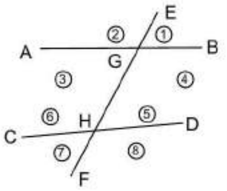
$\angle a$ and $\angle c$ are the pair of vertical angles.

Vertical angles are equal in value.

Alternate angles and corresponding angles

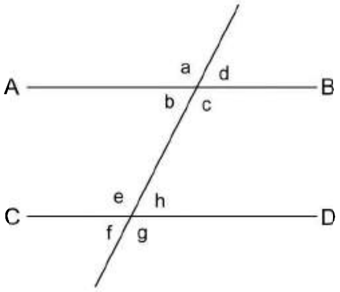
In the figure given below, corresponding angles are $\angle a$ and $\angle e$, $\angle b$ and $\angle f$, $\angle d$ and $\angle h$, $\angle c$ and $\angle g$. The alternate angles are $\angle b$ and $\angle h$, $\angle c$ and $\angle e$.



Corresponding angles	When two lines are intersected by a transversal, then they form four pairs of corresponding angles (a) $\angle AGE, \angle CHG = (\angle 2, \angle 6)$ (b) $\angle AGH, \angle CHF = (\angle 3, \angle 7)$ (c) $\angle EGB, \angle GHD = (\angle 1, \angle 5)$ (d) $\angle BGH, \angle DHF = (\angle 4, \angle 8)$	

c) Angles associated with parallel lines

A line passing through two or more lines in a plane is called a transversal. When a transversal cuts two parallel lines, then the set of all the corresponding angels will be equal and similarly, the set of all the alternate angles will be equal.



In the figure given above, corresponding $\angle a = \angle e$ and corresponding $\angle b = \angle f$

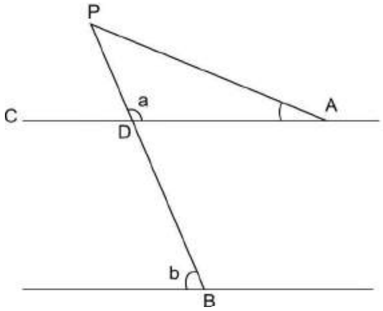
Similarly, alternate $\angle b = \angle h$ and alternate $\angle c = \angle e$.

Now, $\angle b + \angle c = 180°$, so $\angle b + \angle e = \angle h + \angle c = 180°$

So we can conclude that the sum of the angles on one side of the transversal and between the parallel lines will be equal to 180°.

Converse of the above theorem is also true. When a transversal cuts two lines, and if the corresponding angles are equal in size, or if alternate angles are equal in size, then the two lines are parallel.

Example 1 In the figure given below, find the value of $\angle b$ in terms of $\angle a$.



Solution In the given figure, $\angle b = \text{Alternate } \angle PDC = 180° - \angle PDA = 180° - a$

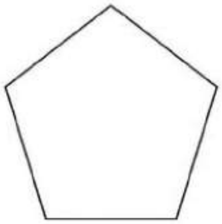
POLYGONS AND THEIR PROPERTIES

Any closed plane figure with *n* sides is known as a polygon. If all the sides and the angles of this polygon are equivalent, the polygon is called a regular polygon. Polygons can be convex or concave. The word “polygon” derives from the Greek *poly*, meaning “many,” and *gonia*, meaning “angle.” The most familiar type of polygon is the regular polygon, which is a convex polygon with equal side lengths and angles.

The generalization of a polygon into three dimensions is called a *polyhedron*, and into four dimensions is called a *polychoron*.

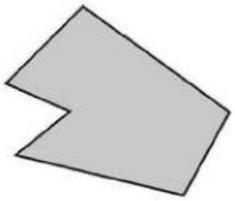
A **convex polygon** is a simple polygon that has the following features:

- Every internal angle is at most 180 degrees.
- Every line segment between the two vertices of the polygon does not go outside the polygon (i.e., it remains inside or on the boundary of the polygon). In other words, all the diagonals of the polygon remain inside its boundary.
- Every triangle is strictly a convex polygon.



Convex Polygon

If a simple polygon is not convex, it is called **concave**. At least one internal angle of a concave polygon is larger than 180°.



Concave Polygon

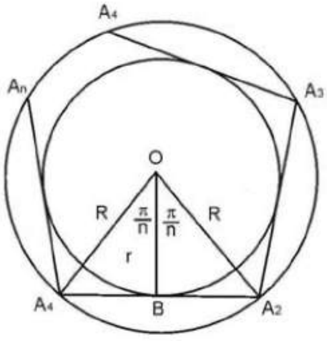
As we can see in the above figure, one of the internal angles is more than 180°.

Here onwards, all the discussion about polygon refer to regular polygons only.

Polygons are named on the basis of the number of sides they have. A list of some of the polygons are given below:

Number of sides	*Name of the polygon*
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon

Area and Perimeter of a Regular Polygon



Given, $A_1\, A_2\, A_3\, A_4 \ldots A_n$ is a with regular polygon, '*n*' sides.

$A_1A_2 = A_2A_3 = A_3A_4 = \ldots . = A_{(n-1)}A_n$ = a units

OB (in radius) = r and $OA_1 = OA_2$ (Circumradius) = R

(i) Perimeter (P) = na

(ii) Area $= \frac{na}{2} \times r = \frac{p}{2} \times r$

(iii) Area $= \frac{na}{2}\sqrt{R^2 - \left(\frac{a}{2}\right)^2} = \frac{p}{2}\sqrt{R^2 - \left(\frac{a}{2}\right)^2}$

(iv) Area $= \frac{na^2}{4} \times \cot\left(\frac{\pi}{n}\right)^{\circ}$

(v) Area $= nr^2 \times \tan\left(\frac{\pi}{n}\right)^{\circ}$

(vi) r (in radius) $= \frac{a}{2}\cot\left(\frac{\pi}{n}\right)^{\circ}$

Properties of a Polygon

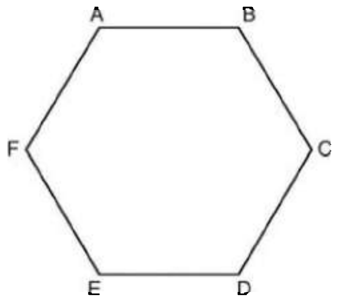
1. Interior Angle + Exterior Angle = 180°
2. The number of diagonals in an n-sided polygon = $n(n-3)/2$
3. The sum total of all the exteriors angles of any polygon = 360°
4. The measure of each exterior angle of a regular polygon $= \frac{360°}{n}$
5. The ratio of the sides of a polygon to the diagonals of a polygon is 2 : $(n-3)$
6. The ratio of the interior angle of a regular polygon to its exterior angle is $(n-2) : 2$
7. The sum total of all the interior angles of any polygon $= (2n - 4) \times 90°$

   So, each interior angle $= \frac{(2n-4)90°}{n}$

Some frequently used polygons

Apart from triangles and quadrilaterals, regular hexagon and regular octagon are also worth mentioning.

Regular hexagon In the figure given below, ABCDEF is a regular hexagon with each side measuring a unit. Point O inside the hexagon is the centre of the hexagon.



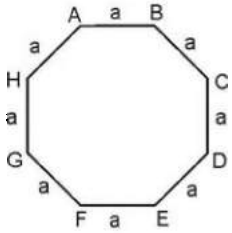
Sum of the interior angles = 720°

Each interior angle = 120°

Each exterior angle = 60°

$$\text{Area} = \frac{3\sqrt{3}}{2}a^2$$

Regular octagon In the figure given below, ABCD-EFGH is a regular octagon with each side measuring 'a' unit.



Sum of the interior angles = 1080°

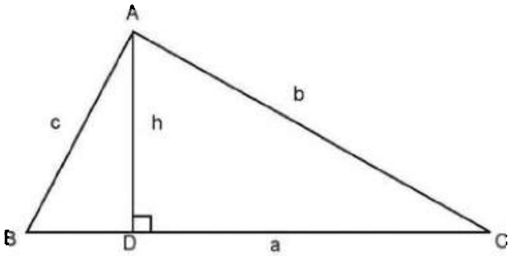
Each interior angle = 135°

Each exterior angle = 45°

$$\text{Area} = 2a^2(1+\sqrt{2})$$

TRIANGLES AND THEIR PROPERTIES

A triangle is a figure enclosed by three sides. In the figure given below ABC is a triangle with sides AB, BC and CA measuring c, a and b units respectively. Line AD represents the height of the triangle corresponding to the side BC and is denoted by h.



In any triangle ABC

$$\text{Area} = \frac{1}{2}\times \text{BC}\times \text{AD} = \frac{1}{2}\,a\times h$$

Properties of a Triangle

- The sum of all the angles of a triangle = 180°
- The sum of lengths of the two sides > Length of the third side
- The difference of any two sides of any triangle < length of the third side
- The area of any triangle can be found by several methods:

(a) Area of any triangle $= \frac{1}{2}\times$ base × perpendicular to base from the opposite vertex.

(b) Area of any triangle $= \sqrt{S(S-a)(S-b)(S-c)}$, where S is the semi-perimeter of the triangle and a, b and c are the sides of a triangle.

(c) Area of any triangle $= \left(\frac{1}{2}\right)\times$ bc Sin A

Besides, there are some formulae which we use exclusively in some particular cases.

Example 2 What is the number of distinct triangles with integral valued sides and perimeter as 14?

(a) 6 (b) 5 (c) 4 (d) 3

Solution The sum of the lengths of the two sides > the length of the third side

So, the maximum length of any particular side can be 6 units.

Now if a = 6, then b + c = 8, so the possible sets are (6, 6, 2), (6, 5, 3) and (6, 4, 4)

If a = 5, then b + c = 9, so the possible set is (5, 5, 4)

So, the number of distinct triangles = 4

CLASSIFICATION OF TRIANGLES

Based Upon Sides

1. Scalene triangle

A triangle whose all sides are of a different length is a scalene triangle.

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}, \text{ where}$$

$$\text{S (semi perimeter)} = \frac{a+b+c}{2}$$

Example 3 What is the area of the triangle with side lengths 4 units, 5 units and 10 units?

Solution This triangle is not possible. (Since the sum of lengths of the two sides > Length of the third side)

2. Isosceles triangle

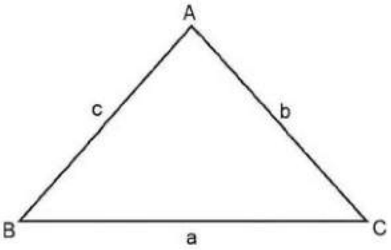
A triangle whose two sides are of an equal length is an isosceles triangle.

$$\text{Height} = \sqrt{\frac{4a^2 - b^2}{2}}$$

$$\text{Area} = \frac{b}{4}\sqrt{4a^2 - b^2}$$

3. Equilateral triangle

A triangle whose all sides are of an equal length is called an equilateral triangle.



In any equilateral triangle, all the three sides are of an equal length, so $a = b = c$

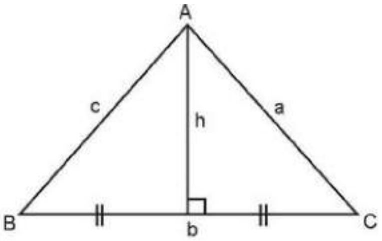
$$\text{Height} = (\text{side})\ \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{2}a$$

$$\text{Area} = \frac{\sqrt{3}}{4}(\text{side})^2 = \frac{\sqrt{3}}{4}a^2$$

Based Upon Angles

1. Right-angled triangle

A triangle whose one angle is of 90° is called a right-angled triangle. The side opposite to the right angle is called the hypotenuse.



$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{perpendicular}$$

Pythagoras Theorem

Pythagoras theorem is applicable in case of right-angled triangle. It says that, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

$a^2 + b^2 = c^2$

For more proofs of Pythagoras theorem, go to www.cut-the-knot.org/pythagoras/index.shtml

The smallest example is $a = 3$, $b = 4$ and $c = 5$. You can check that

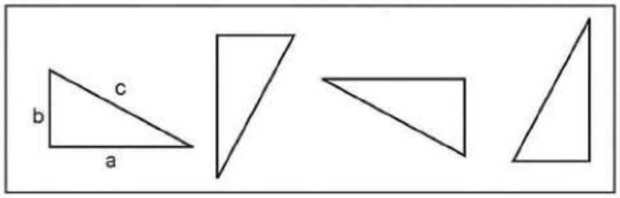
$3^2 + 4^2 = 9 + 16 = 25 = 5^2$.

Sometimes we use the notation (a, b, c) to denote such a triple.

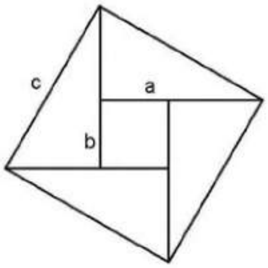
Notice that the greatest common divisor of the three numbers 3, 4 and 5 is 1. Pythagorean triples with this property are called *primitive*.

Proofs of Pythagoras theorem

Proof 1



Now we start with four copies of the same triangle. Three of these have been rotated at 90°, 180°, and 270°, respectively. Each has the area ab/2. Let's put them together without additional rotations so that they form a square with side c.

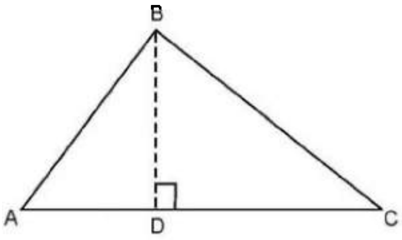


The square has a square hole with the side $(a - b)$. By summing up its area $(a - b)^2$ and 2ab, the area of the four triangles (4·ab/2), we get

$C^2 = (a-b)^2 + 2ab = a^2 + b^2$. QED

Proof 2 ABC is a right-angled triangle at B

To Prove: $AC^2 = AB^2 + BC^2$



Construction: Draw BD ⊥ AC

Proof: ΔADB ~ΔABC (Property 8.5)

$\therefore \frac{AD}{AB} = \frac{AB}{AC}$ (Sides are proportional)

Or $AB^2 = AD \times AC$ (1)

Also, $\Delta CDB \sim \Delta CBA$

$\therefore \frac{CD}{BC} = \frac{BC}{CA}$

$\therefore \frac{BC^2}{AC^2} = \frac{BD \times BA}{AB \times AD}$ (Sides are proportional)

or $BC^2 = CD \times CA$

Adding (1) and (2)

$AB^2 + BC^2 = AD \times AC + CD \times AC$

$= AC\,[AD + CD]$

$= AC \times AC = AC^2$ QED.

Pythagorean triplets

A *Pythagorean triplet* is a set of three positive whole numbers a, b and c that are the lengths of the sides of a right triangle.

$a^2 + b^2 = c^2$

It is noteworthy to see here that all of a, b and c cannot be odd simultaneously. Either of a or b has to be even and c can be odd or even.

The various possibilities for a, b and c are tabled below:

a	*b*	*c*
Odd	Odd	Even
Even	Odd	Odd
Odd	Even	Odd
Even	Even	Even

Some Pythagoras triplets are:

3	4	5	$(3^2 + 4^2 = 5^2)$
5	12	13	$(5^2 + 12^2 = 13^2)$
7	24	25	$(7^2 + 24^2 = 25^2)$
8	15	17	$(8^2 + 15^2 = 17^2)$
9	40	41	$(9^2 + 40^2 = 41^2)$
11	60	61	$(11^2 + 60^2 = 61^2)$
20	21	29	$(20^2 + 21^2 = 29^2)$

Note If each term of any pythagorean triplet is multiplied or divided by a constant (say P, P > 0) then the triplet so obtained will also be a pythagorean triplet. This is because if $a^2 + b^2 = c^2$, then $(Pa)^2 + (Pb)^2 = (Pc)^2$, where $P > 0$.

For example, 3 × 2 4 × 2 5 × 2 gives
6 8 10 $(6^2 + 8^2 = 10^2)$

Using Pythagoras theorem to determine the nature of triangle

If $c^2 = a^2 + b^2$, then the triangle is right-angled triangle.

If $c^2 > a^2 + b^2$, then the triangle is an obtuse-angled triangle.

If $c^2 < a^2 + b^2$, then the triangle is an acute-angled triangle.

Mechanism to derive a Pythagorean triplet If the length of the smallest side is odd, assume the length of the smallest side = 5

Step 1 Take the square of 5 (length of the smallest side) = 25

Step 2 Break 25 into two parts P and Q, where P – Q = 1. In this case, P = 13 and Q = 12. Now these two parts P and Q along with the smallest side constitute pythagorean triplet.

However, there is another general formula for finding out all the primitive pythagorean triplets:

$a = r^2 - s^2$,
$b = 2rs$,
$c = r^2 + s^2$,
$r > s > 0$ are whole numbers,
$r - s$ is odd, and
The greatest common divisor of r and s is 1.

Table of small primitive Pythagorean triplets Here is a table of the first few primitive Pythagorean triplets:

r	*s*	*a*	*b*	*c*
2	1	3	4	5
3	2	5	12	13
4	1	15	8	17
4	3	7	24	25
5	2	21	20	29
5	4	9	40	41
6	1	35	12	37
6	5	11	60	61
7	2	45	28	53

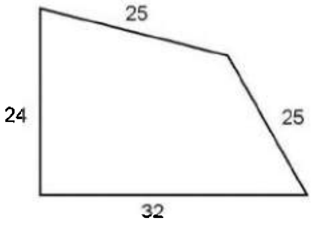
Perimeter, area, inradius and shortest side

The perimeter P and area K of a Pythagorean triple triangle are given by

$P = a + b + c = 2r(r + s)\,d.$

$K = ab/2 = rs\,(r^2 - s^2)\,d^2.$

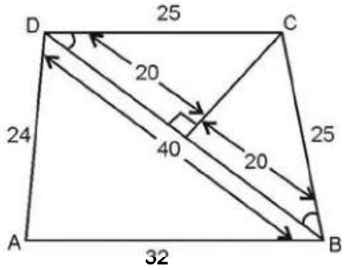
Example 4 Two sides of a plot measure 32 m and 24 m and the angle between them is a right angle. The other two sides measure 25 m each and the other three angles are not right angles.



What is the area of the plot (in m^2)?

(a) 768 (b) 534 (c) 696 (d) 684

Solution The figure given above can be seen as



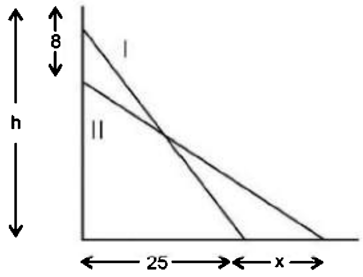
Since ABD is a right-angled triangle, so it will satisfy the Pythagoras theorem. And the triplet used here is – 3(×8), 4(×8) and 5(×8). Similarly the other part of the figure can also be bifurcated by drawing a perpendicular from C on BD.

So, the area of the plot is:

Area (ΔABD) + Area (ΔCBD) $= \frac{1}{2} \times 24 \times 32 + 2 \times (1/2 \times 20 \times 15) = 684\ m^2$

Example 5 A ladder of length 65 m is resting against a wall. If it slips 8 m down the wall, then its bottom will move away from the wall by N m. If it was initially 25 m away from it, what is the value of x?

(a) 60 m (b) 39 m (c) 14 m (d) 52 m



Solution Using Pythagorean triplets, (5, 12, 13),

$\Rightarrow$ h = 60

After it has slipped by 8 m, the new height = 52 m, and the length of the ladder = 65 m.

So $25 + x = 39$ (3, 4, 5 triplet)

$\Rightarrow x = 14$ m

2. Obtuse-angled triangle

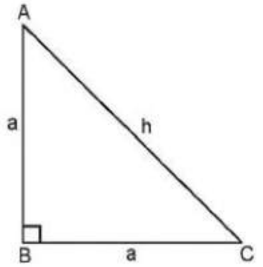
If one of the angles of the triangle is more than 90°, then the triangle is known as an Obtuse angled triangle. Obviously in this case, rest of the two angles will be less than 90°.

3. Acute-angled triangle

If all the angles of the triangle are less than 90°, then the triangle is known as acute angled triangle.

4. Isosceles right-angled triangle

A right angled triangle whose two sides containing the right angle are equal in length, is an isosceles right triangle.



In this case, Hypotenuse (h) $= a\sqrt{2}$

Perimeter = $2a + h = 2a + a\sqrt{2}$

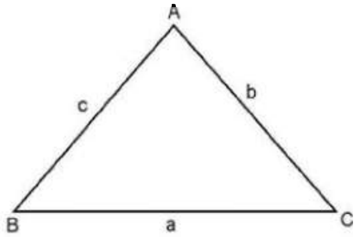
$= a\sqrt{2}(\sqrt{2} + a)$

$= h(1 + \sqrt{2})$

= Hypotenuse $(1 + \sqrt{2})$

Trigonometric formulae

In any Δ ABC,



Area of a triangle

Area of $\Delta = \frac{1}{2}$ bc sin A, where $\angle A = BAC$

Area of $\Delta = \frac{1}{2}$ ac Sin B, where $\angle B = ABC$

Area of $\Delta = \frac{1}{2}$ ab Sin C, where $\angle C = ACB$

Cosine Rule and Sine Rule

In any Δ, we have six quantities namely the three angles and the three sides. Using the following rules, we can find any of the three quantities if we are provided with the remaining three quantities.

$$\text{Cos } A = \frac{b^2 + c^2 + a^2}{2bc}$$

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\text{Cos } C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\text{Sine Rule } \frac{a}{\text{Sin } A} = \frac{b}{\text{Sin } B} = \frac{c}{\text{Sin } C}$$

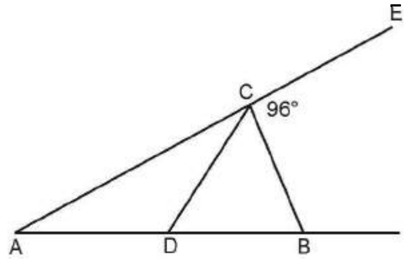
Sine and Cosine formulae are particularly more important in cases where we have one side and two angles out of the three angles of the triangle and we have to find out the value of all the sides and angles.

$$\frac{AB}{\sin \angle ACB} = \frac{BC}{\sin \angle BAC} = \frac{AC}{\sin \angle ABC} = 2R$$

Two important points

- The side opposite to the largest angle will be the largest.
- The side opposite to the smallest angle will be the smallest.

Example 6 In the figure given below, AD = CD = BC. What is the value of ∠CDB?

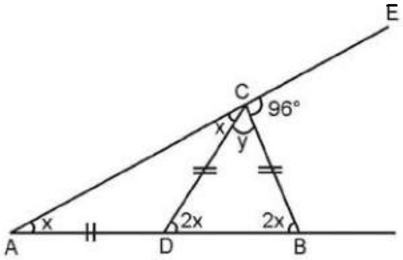


Solution $x + y = 180 - 96 = 84°$ (i)

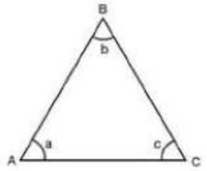
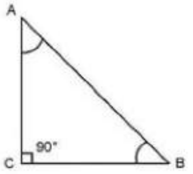
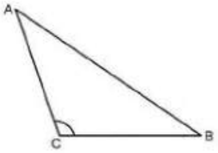
In ΔCDB, $4x + y = 180$ (ii)

Solving (i) and (ii), $x = 32°$

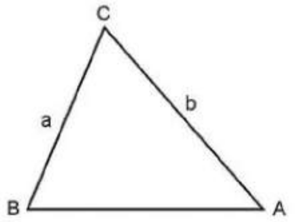
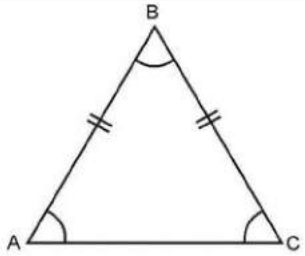
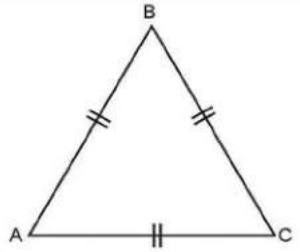
So, $2x = 64°$



Summarizing the Above Classification

Types of Triangles	*Property/Definition*	*Diagram*
Acute-angle Triangle	Each angle of a triangle is less than 90°, i.e., a < 90°, b < 90°, c < 90°	 {∠a, ∠b, ∠c} < 90°
Right-angled Triangle	If one of the angles is equal to 90°, then it is called a right-angled triangle. The rest two angles are complementary to each other.	 ∠C = 90°
Obtuse-angle Triangle	If one of the angles is obtuse (i.e., greater than 90°), then it is called an obtuse-angle triangle	 ∠C > 90°

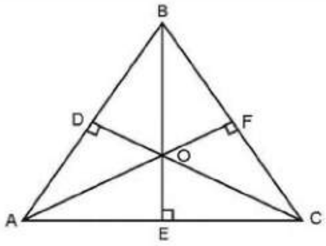
(b) According to the length of sides

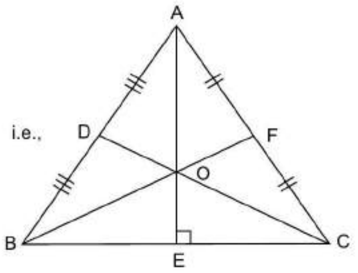
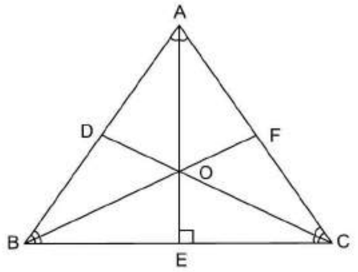
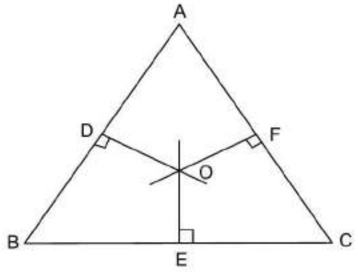
Types of Triangles	*Property/Definition*	*Diagram*
Scalene Triangle	A triangle in which none of the three sides are equal is called a scalene triangle (all the three angles are also different).	 a ≠ b ≠ c
Isosceles Triangle	A triangle in which two sides are equal is called an isosceles triangle. In this triangle, the angles opposite to the congruent sides are also equal.	 AB = AC ∠B = ∠C
Equilateral Triangle	A triangle in which all the three sides are equal is called an equilateral triangle. In this triangle each angle is congruent and equal to 60°.	 AB = BC = AC ∠A = ∠B = ∠C = 60°

Points of a Triangle

Before we move ahead to discuss different points inside a triangle, we need to be very clear about some of the basic definitions.

Basic Definitions

Nomenclature	*Property/Definition*	*Diagram*
Altitude (or height)	The perpendicular drawn from the opposite vertex of a side in a triangle is called an altitude of the triangle. There are three altitudes in a triangle.	 AF, CD and BE are the altitudes

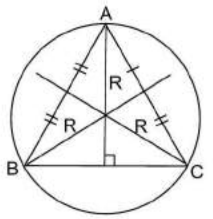
Nomenclature	*Property/Definition*	*Diagram*
Median	The line segment joining the mid-point of a side to the vertex opposite to the side is called a median. There are three medians in a triangle. A median bisects the area of the triangle. Area (ABE) = Area (AEC) $= \frac{1}{2}$ Area (ΔABC) etc.	 i.e., AE, CD and BF are the medians (BE = CE = AD = BD, AF = CF)
Angle bisector	A line segment which originates from a vertex and bisects the same angle is called an angle bisector. ($\angle$BAE = $\angle$CAE $= \frac{1}{2}$ $\angle$BAC) etc.	 AE, CD and BF are the angle bisectors
Perpendicular bisector	A line segment which bisects a side perpendicularly (i.e., at right angle) is called a perpendicular bisector of a side of triangle. All points on the perpendicular bisector of a line are equidistant from the ends of the line.	 DO, EO and FO are the perpendicular bisectors

Circumcentre

Circumcentre is the point of intersection of the three perpendicular bisectors of a triangle. The circumcentre of a triangle is equidistant from its vertices and the distance of the circumcentre from each of the three vertices is called circumradius (R) of the triangle. These perpendicular bisectors are different from altitudes, which are perpendiculars but not necessarily bisectors of the side.

The circle drawn with the circumcentre as the centre and circumradius as the radius is called the circumcircle of the triangle and it passes through all the three vertices of the triangle.

The circumcentre of a right-angled triangle is the mid-point of the hypotenuse of a right- angled triangle.



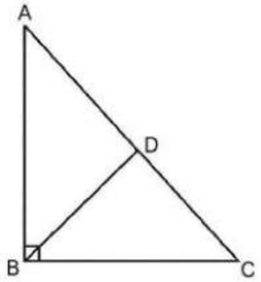
AB = c, BC = a, AC = b

The process to find the circumradius (R) For any triangle $R = \frac{abc}{4A}$, where a, b and c are the three sides and A = area of a triangle.

For equilateral triangle $R = \frac{\text{Side}}{\sqrt{3}}$

Positioning of the circumcentre

- If the triangle is acute-angled triangle, then the circumcentre will lie inside the triangle.
- If the triangle is obtuse-angled triangle, then the circumcentre will lie outside the triangle.
- If the triangle is a right-angled triangle, then the circumcentre will lie on the mid-point of the hypotenuse. This can be seen through the following diagram:

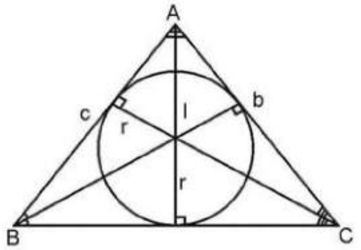


Here D is the circumcentre. So, AD = CD = BD

Incentre

Incentre is the point of intersection of the internal bisectors of the three angles of a triangle. The incentre is equidistant from the three sides of the triangle i.e., the perpendiculars drawn from the incentre to the three sides are equal in length and are called the inradius of the triangle.

The circle drawn with incentre as the centre and inradius as the radius is called the incircle of the triangle and it touches all the three sides from the inside.



AB = c, BC = a, CA = b

To find inradius (r)

For any triangle $r = \frac{A}{S}$, where

A = Area of triangle and

S = Semi-perimeter of the triangle $\frac{(a+b+c)}{2}$

For equilateral triangle $r = \frac{\text{side}}{2\sqrt{3}}$

$\angle BIC = 90° + \angle A/2$

Important derivation – In a right-angled triangle, Inradius = Semi-perimeter – Length of Hypotenuse.

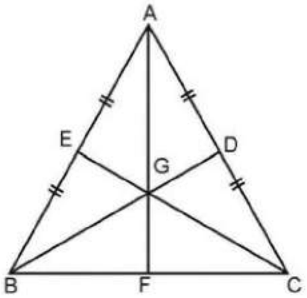
Euler's formula for inradius and circumradius of a triangle Let O and I be the Circumcentre and Incentre of a triangle with circumradius R and inradius r. Let d be the distance between O and I. Then

$d^2 = R(R - 2r)$

From this theorem, we obtain the inequality $r \geq 2r$. This is known as Euler's Inequality.

Centroid

Centroid is the point of intersection of the three medians of a triangle. The centroid divides each of the medians in the ratio 2 : 1, the part of the median towards the vertex being twice in length to the part towards the side.



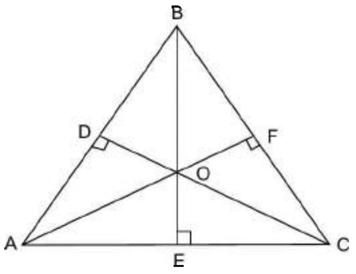
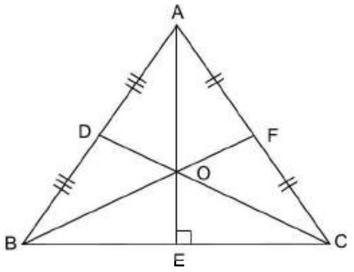
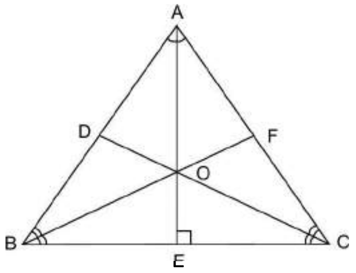
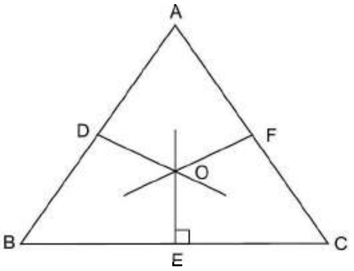
$$\frac{AG}{GF} = \frac{BG}{GD} = \frac{CD}{GE} = \frac{2}{1}$$

Median divides the triangle into two equal parts of the same area.

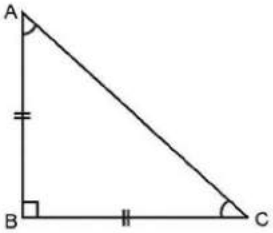
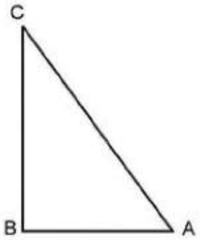
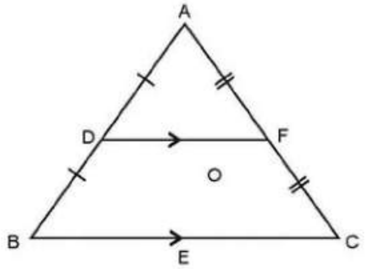
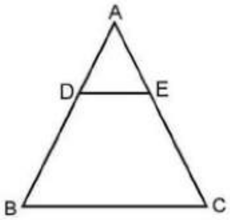
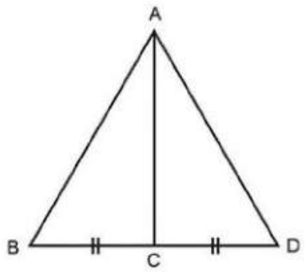
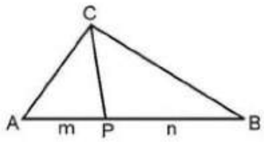
Orthocentre

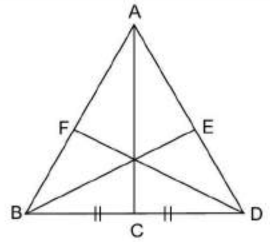
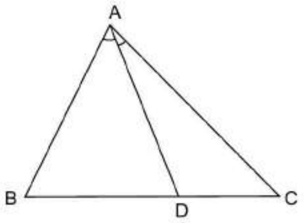
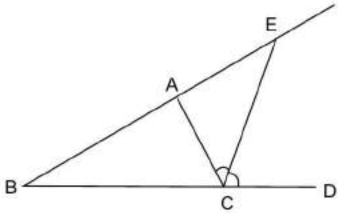
The point of concurrency of the altitudes is known as the orthocentre.

Summarizing the above discussion regarding the points of the triangle:

Orthocentre	The point of intersection of the three altitudes of the triangle is known as the orthocenter. $\angle BOC = 190 - \angle A$ $\angle COA = 190 - \angle B$ $\angle AOB = 180 - \angle B$	 'O' is the orthocenter
Centroid	The point of intersection of the three medians of a triangle is called the centroid. A centroid divides each median in the ratio 2 : 1 (vertex: base) $\frac{AO}{OE} = \frac{CO}{OD} = \frac{BO}{OF} = \frac{2}{1}$	 'O' is the centroid
Incentre	The point of intersection of the angle bisectors of a triangle is known as the incentre. Incentre O is the always equidistant from all three sides i.e., the perpendicular distance between the sides.	 'O' is the incentre
Circumcentre	The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre. OA = OB = OC = (circum radius) Circumcentre O is always equidistant from all the three vertices A, B and C Perpendicular bisectors need not be originating from the vertices.	 'O' is the incentre

Important Theorems Related to Triangle

Theorem	*Statement/Explanation*	*Diagram*
45° – 45° – 90°	If the angles of a triangle are 45°, 45° and 90°, then the hypotenuse (i.e., longest side) is $\sqrt{2}$ times of any smaller side. Excluding hypotenuse rest two sides are equal. i.e., AB = BC and AC $= \sqrt{2}$ AB $= \sqrt{2}$ BC AB : BC : AC = 1 : 1 : $\sqrt{2}$	 ∠A = 45° ∠B = 90° ∠C = 45°
30° – 60° – 90°	If the angles of a triangle are 30°, 60° and 90°, then the sides opposite to 30° angle is half of the hypotenuse and the side opposite to 60° is $\frac{\sqrt{3}}{2}$ times the hypotenuse, e.g., AB $= \frac{AC}{2}$ and $= \frac{\sqrt{3}}{2}$ AC AB : BC: AC = 1: $\sqrt{3} : 2$	 ∠C = 30°, ∠B = 90°, ∠A = 60°
Basic Proportionality Theorem (BPT) or	Any line parallel to one side of a triangle divides the other two sides proportionally. So if DE is drawn parallel to BC, it would divide sides AB and AC proportionally i.e $\frac{AD}{DB} = \frac{AF}{FC}$ or $\frac{AD}{AB} = \frac{AF}{AC}$ $\frac{AD}{DF} = \frac{AB}{BC}$.	
Mid-point Theorem	Any line joining the mid-points of two adjacent sides of a triangle are joined by a line segment, then this segment is parallel to the third side, i.e., if AD = BD and AE = CE then DE||BC.	
Apollonius' Theorem	In a triangle, the sum of the squares of any two sides of a triangle is equal to twice the sum of the square of the median to the third side and square of half the third side. i.e., $AB^2 + AD^2 = 2\,(AC^2 + BC^2)$	
Stewarts Theorem/ Generalization of Apollonius Theorem	If length of AP = m and PB = n, then $m \times CB^2 + n \times AC^2 = (m + n)\, PC^2 + mn\,(m + n)$ Also understand that *m* and *n* here are length of segments, and not their ratio.	

Theorem	*Statement/Explanation*	*Diagram*
Extension of Apollonius' Theorem	In the given ΔABC, AC, BE and DF are medians. 3 (Sum of squares of sides) = 4 (Sum of squares of medians) $3\,(AB^2 + AD^2 + DB^2) = 4\,(AC^2 + EB^2 + FD^2)$	
Interior Angle Bisector Theorem	In a triangle the angle bisector of an angle divides the opposite side to the angle in the ratio of the remaining two sides. i.e., $\frac{BD}{CD} = \frac{AB}{AC}$ and $BD \times AC - CD \times AB = AD^2$	
Exterior Angle Bisector Theorem	In a triangle the angle bisector of any exterior angle of a triangle divides the side opposite to the external angle in the ratio of the remaining two sides i.e., $\frac{BE}{AE} = \frac{BC}{AC}$	

Congruency of Triangles

Two figures are said to be congruent if, when placed one over the other, they completely overlap each other. They would have the same shape, the same area and will be identical in all respects.

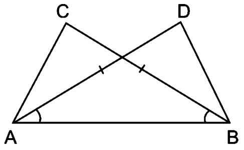
So, we can say that all congruent triangles are similar triangles, but vice-versa is not always true.

Rules for two triangles to be congruent

1. S – S – S

   If in any two triangles, each side of one triangle is equal to a side of the other triangle, the two triangles are congruent. This rule is S – S – S rule.

2. S – A – S



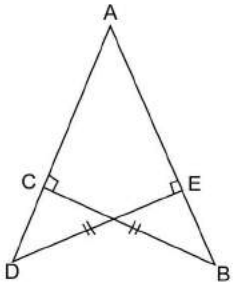
   In Δ ABC and Δ ABD,
   AB = AB (common side)
   $\angle$ABC = $\angle$ BAD (given)
   BC = AD (given)

   Thus by rule S – A – S the two triangles are congruent. This rule holds true, when the angles that are equal have to be included between the two equal sides (i.e., the angle should be formed between the two sides that are equal).

3. A – S – A

   In Δ ABC and Δ ADE,
   $\angle$ACB = $\angle$ AED (given)
   $\angle$ BAC = $\angle$ DAE (common angle)
   BC = DE (given)

   Thus by rule A – S – A the two triangles are congruent. For this rule, the side need not be the included side.



   A – S – A can be written as A – A – S or S – A – A also.

4. R – H – S

   This rule is applicable only for right-angled triangles.

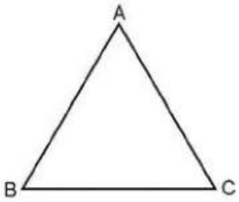
   If two right-angled triangles have their hypotenuse and one of the sides as same, then the triangles will be congruent.

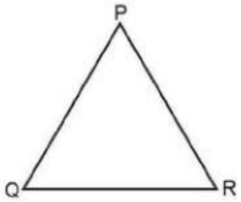
Similarity of the Triangles

If we take two maps of India of different sizes (breadths and lengths), then the map of all the 28 states of India will cover proportionally the same percentage area in both the maps.

Lets see this in geometry:

Criteria for similarity of two triangles





Two triangles are similar if (i) their corresponding angles are equal and/or (ii) their corresponding sides are in the same ratio. That is, if in two triangles, ABC and PQR, (i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, and/or (ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{CA}{RP}$, the two triangles are similar.

All regular polygons of the same number of sides such as equilateral triangles or squares, are similar. In particular, all circles are also similar.

Theorems for Similarity

1. If in two triangles, the corresponding angles are equal, then their corresponding sides will also be proportional (i.e., in the same ratio). Thus the two triangles are similar.
   This property is referred to as the AAA similarity criterion for two triangles.
   Corollary If two angles of a triangle are respectively equal to two angles of another triangle, then the two triangles are similar. This is referred to as the AA similarity criterion for the two triangles. It is true due to the fact that if two angles of one triangle are equal to the two angles of another triangle, then the third angle of both the triangles will automatically be the same.
2. If the corresponding sides of two triangles are proportional (i.e., in the same ratio), their corresponding angles will also be equal and so the triangles are similar. This property is referred to as the SSS similarity criterion for the two triangles.
3. If one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional, then the triangles are similar. This property is referred to as the SAS similarity criterion of the two triangles.
4. The ratio of the areas of the two similar triangles is equal to the ratio of the squares of their corresponding sides.
5. If a perpendicular is drawn from the vertex of the right angle of a right angled triangle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.

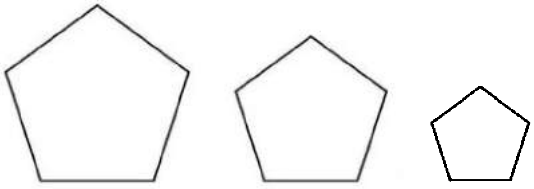
Similar Polygons

Two polygons of the same number of sides are similar, if

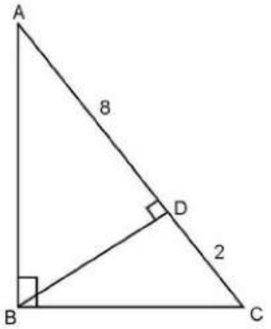
(i) Their corresponding angles are equal (i.e., they are equiangular) and

(ii) Their corresponding sides are in the same ratio (or proportional).

This can be seen in the figures given below:



Example 7 ΔABC is a right angled triangle $BD \perp AC$. If AD = 8 cm and DC = 2 cm, then BD = ?



(a) 4 cm (b) 4.5 cm
(c) 5 cm (d) Cannot be determined

Solution $\Delta ADB \sim \Delta BDC$

$$\therefore \frac{AD}{BD} = \frac{BD}{DC}$$

$$\therefore BD^2 = AD \times DC = 8 \times 2$$

$$\therefore BD^2 = 16$$

$$\therefore BD = 4 \text{ cm}$$

Important Result of this question $BD^2 = AD \times DC$ can be used as a standard result also.

Example 8 Circles with radii 3, 4 and 5 units touch each other externally. If P is the point of intersection of the tangents to these circles at their point of contact, find the distance of P from the point of contacts of the circles.

Solution Let A, B and C be the centres of the three circles. So, the point P will be the incentre of DABC and distance of P from the point of contacts of the circles will be the inradius (r).

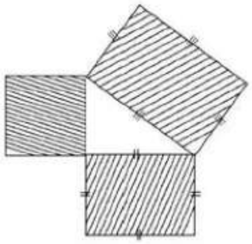
$$\text{So, } r = \frac{A}{S}$$

Sides of DABC will be 7 units, 8 units and 9 units.

So, $r = \sqrt{5}$

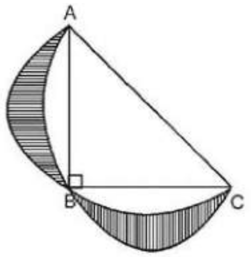
Some interesting facts

1. If we draw regular polygons on all the sides of a right angled triangle, taking the sides of the triangle as one of the sides of the figures, then the area of the shaded portion is equal to twice the area of the figure on the hypotenuse. This will also hold true for semi-circles whose diameters will be the side of the right-angled triangle.

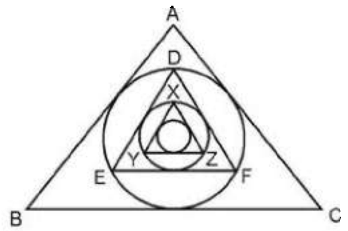


Area of the shaded region = 2 (area of figure on hypotenuse)

2. In the figure below, two semi-circles are drawn with diameters equal to the sides of the right angled triangle. The area of the shaded region (the crescents) is equal to the area of the right-angled triangle. Area of the shaded region (is equal to) the area of the right angled triangle.

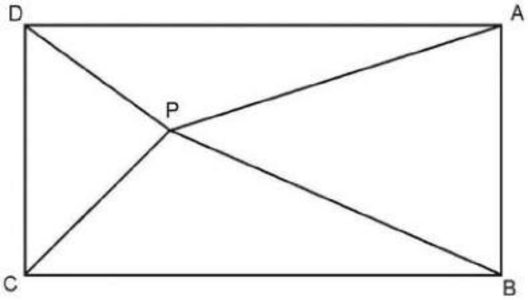


3. In the figure given below all triangles are equilateral triangles and circles are inscribed in these triangles. If the side of triangle ABC = a, then the side of triangle DEF $= \frac{a}{2}$ and the side of triangle $xyz = \frac{a}{4}$.



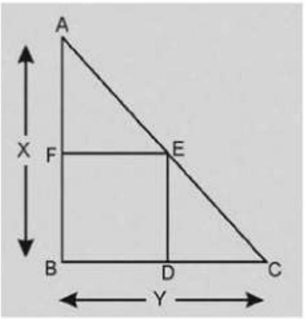
In other words, we can say in order to obtain the side of the next inner triangle divide the side of the immediate outer triangle by 2. The same algorithm holds true for the inscribed circles.

4. In the figure given below, if P is any point inside rectangle ABCD, then $PA^2 + PC^2 = PB^2 + PD^2$

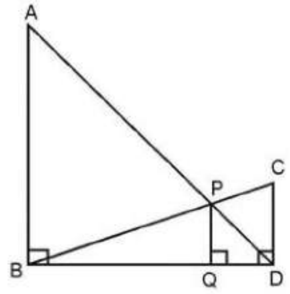


Some Important Points

1. (a) In an acute angled triangle, the circumcentre lies inside the triangle.
   (b) In a right-angled triangle, the circumcentre lies on the middle point of the hypotenuse.
   (c) In an obtuse angled triangle, the circum centre lies outside the triangle.
2. (a) In an acute angled triangle, the orthocenter lies inside the triangle.
   (b) In a right-angled triangle, the orthocenter lies on the vertex is where the right angle is formed i.e., the vertex opposite to the hypotenuse.
   (c) In an obtuse angled triangle, the orthocenter lies outside the triangle.
3. In a right angled triangle the length of the median drawn to the hypotenuse is equal to half the hypotenuse. This median is equal to the circumradius (R) of the right angled triangle.
4. In the figure given below triangle ABC is a right angle triangle, right angled at B. Side AB measures x units and BC measures y units. If a square (BDEF) the maximum area is inscribed in the triangle as shown, below then the side of the square is equal to $\frac{xy}{x+y}$.



Example 9 In the figure given below, $\angle ABD = \angle CDB = \angle PQD = 90°$. If AB:CD = 3:1, then what is the ratio of CD: PQ?

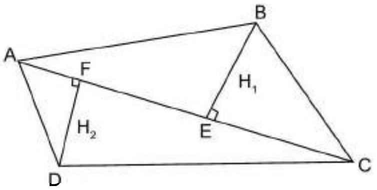


Solution Using the proportionality theorem, AB/PQ = BD/QD and PQ/CD = BQ/BD. Multiplying both these equations, we get AB/CD = BQ/QD = 3:1

Hence, CD/PQ = BD/BQ = 4 : 3.

QUADRILATERALS AND THEIR PROPERTIES

A quadrilateral is a figure bounded by four sides. In the figure given below ABCD is a quadrilateral. Line AC is the diagonal of the quadrilateral (denoted by d) and BE and DF are the heights of the triangles ABC and ADC respectively (denoted by h_1 and h_2).



AC = d, BE = h, and DE = h_2

(i) Area $= \frac{1}{2} \times$ one diagonal × (sum of perpendiculars to the diagonal from the opposite vertexes) $= \frac{1}{2}$ d $(h_1 + h_2)$

(ii) Area $= \frac{1}{2} \times$ product of diagonals × sine of the angle between them

(iii) Area of the cyclic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$ where a, b, c and d are the sides of quadrilateral and s = semi-perimeter $= \dfrac{a+b+c+d}{2}$

(iv) *Brahmagupta's formula* For any quadrilateral with sides of length a, b, c and d, the area A is given by

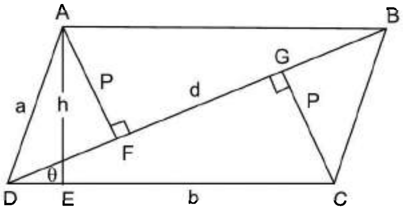
$$A = \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\frac{1}{2}(A+B)}$$

Where $s = \dfrac{a+b+c+d}{2}$ is known as the semi-perimeter, A is the angle between sides a and d, and B is the angle between the sides b and c.

Different Types of Quadrilaterals

Parallelogram

A parallelogram is a quadrilateral when its opposite sides are equal and parallel. The diagonals of a parallelogram bisect each other.



Given: AD = BC = a and AB = DC = b

BD = d

AF (height of ΔABD) = CG (height of ΔCBD) and AE = height of the parallelogram = h

$\angle ADC = \theta$

(i) Area = base × height

(ii) Area = (any diagonal) × (perpendicular distance to the diagonal from the opposite vertex)

(iii) Area = (product of adjacent sides) × (sine of the angle between them) Area = AB sin θ

(iv) Area $= 2\sqrt{s(s-a)(s-b)(s-d)}$, where a and b are the adjacent sides and d is the diagonal.

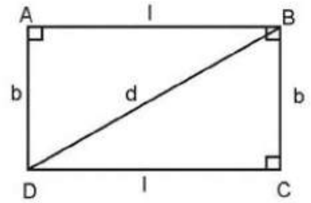
(v) $AC^2 + BD^2 = 2(AB^2 + BC^2)$

(vi) The parallelogram that is inscribed in a circle is a rectangle.

(vii) The parallelogram that is circumscribed about a circle is a rhombus.

(viii) A parallelogram is a rectangle if is diagonals are equal.

Rectangle



A rectangle is a quadrilateral when its opposite sides are equal and each internal angle equals 90°. The diagonals of a rectangle are equal and bisect each other.

Given: AD = BC = b and AB = DC = l, BD = d

(i) Area = length × breadth Area = lb

(ii) Perimeter = 2 (length + breadth) Perimeter = 2 (l + b)

(iii) Diagonal^2 = length^2 + breadth^2 (Pythagoras Theorem)

$d^2 = l^2 + b^2$ $d = \sqrt{l^2 + b^2}$

(iv) Finding area using Brahmagupta' Formula In this case, we know that $a = c$ and $b = d$, and $A + B = \pi$ So, area of Rectangle

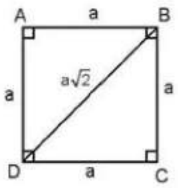
$$= \sqrt{(a+b-a)(a+b-b)(a+b-a)(a+b-b) - a.b.a.b\cos^2 90°} = ab$$

$$= ab$$

(v) The quadrilateral formed by joining the mid points of intersection of the angle bisectors of a parallelogram is a rectangle.

Square

A square is a quadrilateral when all its sides are equal and each internal angle is of 90°. The diagonals of a square bisect each other at right angles (90°)



Given: AB = BC = CD = DA = a

BD (diagonal) = $a\sqrt{2}$

(i) Area = (side)^2 $= \dfrac{(\text{diagonal})^2}{2} = \dfrac{(\text{perimeter})^2}{16}$

$$\text{Area } = a^2 = \frac{d^2}{2} = \frac{P^2}{16}$$

(ii) Using Brahmagupta's formula to find out the area of a square:

We know that $a = b = c = d$ and $A + B = \pi$

So, area of square

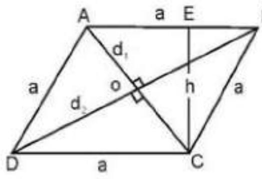
$$= \sqrt{(s-a)(s-b)(s-c)(s-d) - abcd\cos^2\frac{1}{2}(A+B)} =$$

$$\sqrt{(2a-a)(2a-a)(2a-a)(2a-a) - a.a.a.a.\cos^2 90°} = a^2$$

(ii) Perimeter = 4 (side) ⇒ Perimeter = $4a$

Rhombus

A rhombus is a quadrilateral when all sides are equal. The diagonals of a rhombus bisects each other at right angles (90°)



Given = AB = BC = CD = DA = a

∠AOB = ∠BOC = ∠COD − ∠DOA = 90°

AC = d, (AO = OC) and BD = d_2 (BO = OD) CE (height) = h

(i) Area $= \dfrac{1}{2} \times$ (product of the diagonals)

$$\text{Area } = \frac{1}{2}\, d_1 d_2$$

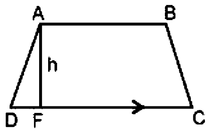
(ii) Area = base × height

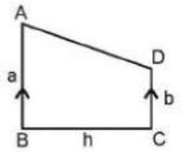
Area = a × h

(iii) A parallelogram is a rhombus if its diagonals are perpendicular to each other. Remember, the sum of the square of the diagonals is equal to four time the square of the side i.e., $d_1^2 + d_2^2 = 4a^2$

Trapezium

A trapezium is a quadrilateral in which only one pair of the opposite sides is parallel





Given: AB = a and CD = b

In Fig. 1, AF (height) = h and in Fig. 2, BC (height) = h

(i) Area $= \dfrac{1}{2} \times$ (sum of the parallel sides) × (distance between the parallel sides)

$$\text{Area } = \frac{1}{2}\,(a+b)\,h$$

(ii) The line joining the mid-points of the non-parallel sides is half the sum of the parallel sides and is known as Median.

(iii) If we make non-parallel sides equal, then the diagonals will also be equal to each other.

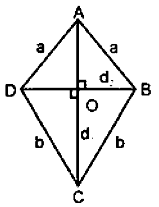
(iv) Diagonals intersect each other proportionally in the ratio of the lengths of the parallel sides.

(v) If a trapezium is inscribed inside a circle, then it is an isosceles trapezium with oblique sides being equal.

Kite

Kite is a quadrilateral when two pairs of adjacent sides are equal and the diagonals bisect each other at right angles (90°).

Given: AB = AD = a and BC = DC = b



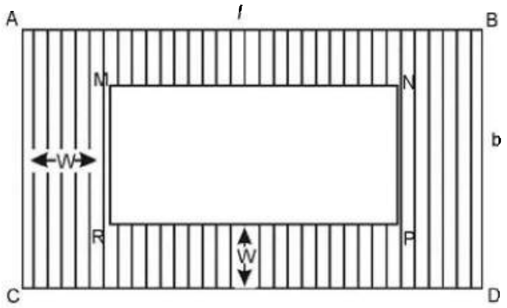
$AC = d_1$ $(AO = OC)$ and $BD = d_2$ (B0 = 0D)
$\angle AOB = \angle BOC = \angle COD = \angle DOA = 90°$

(i) $\text{Area} = \frac{1}{2} \times$ (Product of the diagonals)

$$\text{Area} = \frac{1}{2}\, d_1\, d_2$$

Area of Shaded Paths

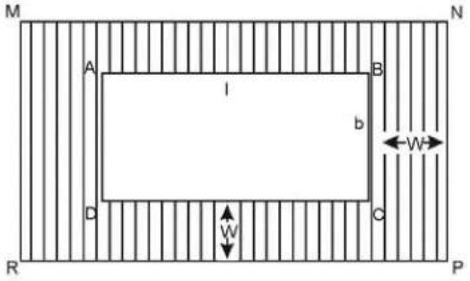
Case 1 When a pathway is made outside a rectangle having length = l and breadth = b



ABCD is a rectangle with length = l and breadth = b, the shaded region represents a pathway of uniform width = W

Area of the shaded region/pathway = 2w (l + b − 2w)

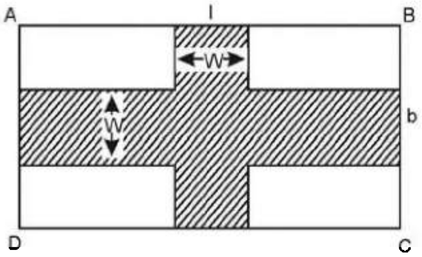
Case 2 When a pathway is made inside a rectangle having length = l and breadth = b



ABCD is a rectangle with length = l and breadth = b, the shaded region represents a pathway of uniform width = w

Area of the shaded region/pathway = 2w (l + b + 2w)

Case 3 When two pathways are drawn parallel to the length and breadth of a rectangle having length = l and breadth = b



ABCD is a rectangle with length = l and breadth = b, the shaded region represents two pathways of a uniform width = w

Area of the shaded region/pathway = W (l + b − w)

From the above figure we can observe that the area of the paths does not change on shifting their positions as long as they are perpendicular to each other.

We can conclude from here that:

1. Every rhombus is a parallelogram but the converse is not true.
2. Every rectangle is a parallelogram but the converse is not true.
3. Every square is a parallelogram but the converse is not true.
4. Every square is a rhombus but the converse is not true.
5. Every square is a rectangle but the converse is not true.

Construction of new figures by joining the mid-points

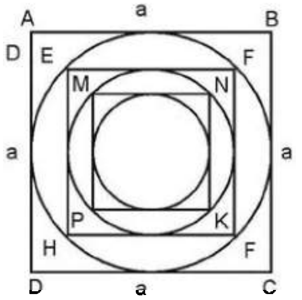
	Original Figure		Resulting Figure
Lines joining the mid-points of adjacent sides of	Quadrilateral	form	Parallelogram
	Parallelogram		Parallelogram
	Rectangle		Rhombus
	Rhombus		Rectangle
	Trapezium		Four similar Δ

Properties of diagonals

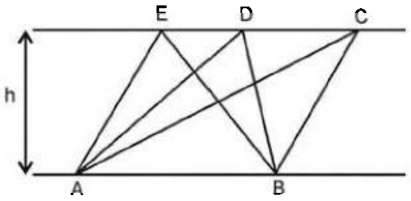
Properties		*Types of Quadrilaterals*				
Sl. No.		*Square*	*Rectangle*	*Parallelogram*	*Rhombus*	*Trapezium*
1.	Diagonals are equal	Y	N	N	Y	N
2.	Diagonals bisect each other	Y	Y	Y	Y	N
3.	Diagonals bisect vertex angles	Y	N	N	Y	N
4.	Diagonals are at right angles	Y	N	N	Y	N
5.	Diagonals make congruent triangles	Y	N	N	Y	N

Some important points

1. In the figure given below, all the side quadrilaterals are squares and circles are inscribed in these squares. If the side of the square ABCD = a, then the side of square EFGH $= \frac{a}{\sqrt{2}}$ and the side of square MNKP $= \frac{a}{2}$. In other words we can say that in order to obtain the side of the next inner square, divide the side of the immediate outer square by $\sqrt{2}$. The same procedure will be applied for the inscribed circles i.e., we divide the radius of the immediate outer circle by $\sqrt{2}$ to obtain the radius of the next inner circle.



2. Triangles on the same base and between the same parallel lines are equal in area.

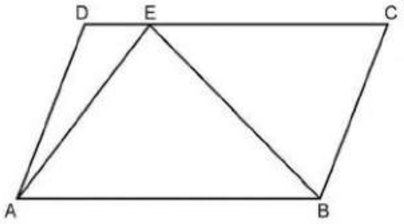


Area (∠ABC) = Area (ΔABD) = Area (ΔABE)

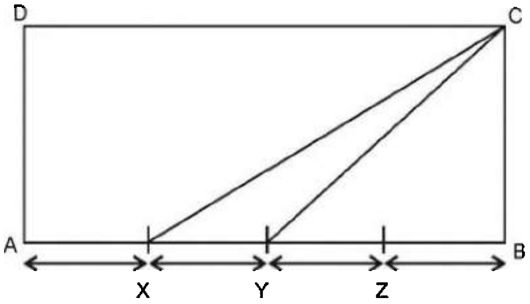
$= \frac{1}{2}$ (AB) × h

(Base AB is the same and the height 'h' is also the same.)

3. If a parallelogram and a triangle are drawn on the same base and between the same parallel lines, then the area of the parallelogram is twice the area of the triangle. Area of ABCD = 2 × (Area of Δ ABE)



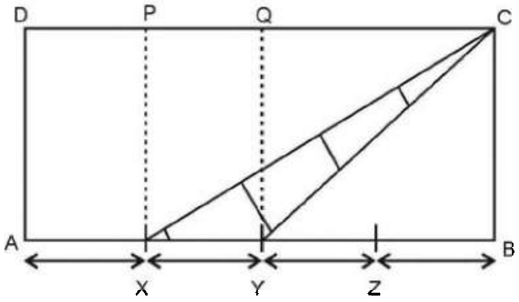
Example 10 Side AB of a rectangle ABCD is divided into four (4) equal parts as shown in the figure. Find the ratio of the area (ΔXYC) and area (€ABCD)?



Solution Let the area of the rectangle ABCD = A

Area of the rectangle XYQP = A/4

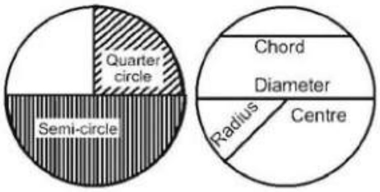
Rectangle XYQP and ΔXYC are on the same base and between the same parallel lines.



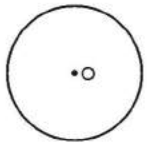
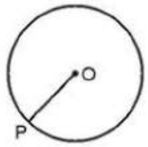
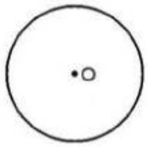
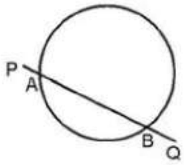
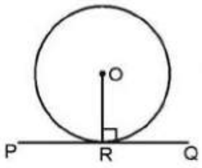
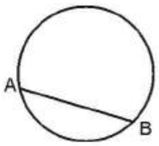
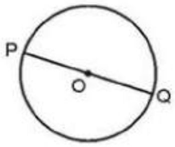
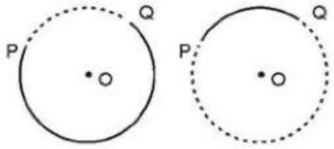
Thus area (XYC) = A/8

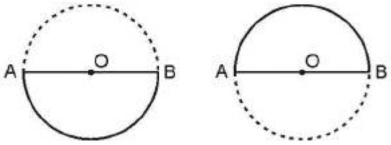
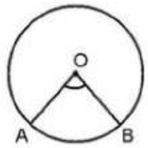
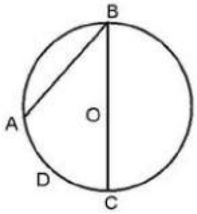
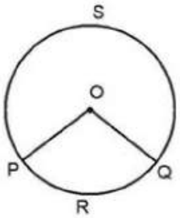
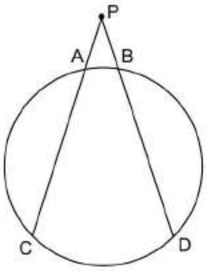
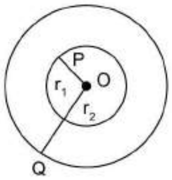
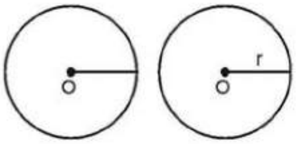
CIRCLES AND THEIR PROPERTIES

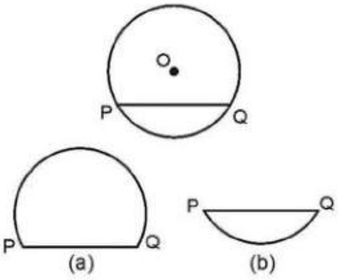
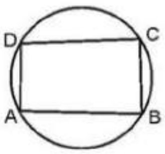
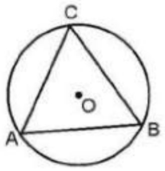
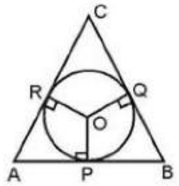
A circle is the path travelled by a point which moves in such a way that its distance from a fixed point remains constant. The fixed point is known as the centre and the fixed distance is called the radius.



Before we move ahead, let us understand the basics definitions of circle.

Nomenclature	*Definition*	*Diagram*
Centre	The fixed point is called the centre. In the given diagram ‘O’ is the centre of the circle.	
Radius	The fixed distance is called a radius. In the given diagram OP is the radius of the circle. (point P lies on the circumference)	
Circumference	The circumference of a circle is the distance around a circle, which is equal to $2\pi r$. (r →radius of the circle)	
Secant	A line segment which intersects the circle in two distinct points, is called as secant. In the given diagram secant PQ intersects circle at two points at A and B.	
Tangent	A line segment which has one common point with the circumference of a circle, i.e., it touches only at only one point is called as tangent of circle. The common point is called as point of contact. In the given diagram, PQ is a tangent which touches the circle at a point R.	 (R is the point of contact) Note: Radius is always perpendicular to tangent.
Chord	A line segment whose end points lie on the circle. In the given diagram AB is a chord.	
Diameter	A chord which passes through the centre of the circle is called the diameter of the circle. The length of the diameter is twice the length of the radius. In the given diagram PQ is the diameter of the circle. (O→is the centre of the circle)	
Arc	Any two points on the circle divides the circle into two parts the smaller part is called as minor arc and the larger part is called as major arc. It is denoted as ‘’. In the given diagram PQ is arc.	

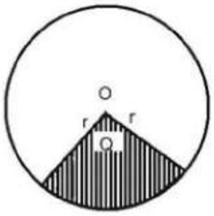
Nomenclature	*Definition*	*Diagram*
Semicircle	A diameter of the circle divides the circle into two equal parts. Each part is called a semicircle.	
Central Angle	An angle formed at the centre of the circle, is called the central angle. In the given diagram ∠AOB in the central angle.	
Inscribed Angle	When two chords have one common end point, then the angle included between these two chords at the common point is called the inscribed angle. ∠ABC is the inscribed angle by the arc ADC	
Measure of an Arc	Basically, it is the central angle formed by an arc. e.g., (a) measure of a circle = 360° (b) measure of a semicircle = 180° (c) measure of a minor arc = ∠POQ (d) measure of a major arc = 360 − ∠POQ	 m(arc PRQ = m ∠POQ m(arc PSQ) = 360° − m (arc PRQ)
Intercepted Arc	In the given diagram, AB and CD are the two intercepted arcs, intercepted by ∠CPD. The end points of the arc must touch the arms of ∠CPD, i.e., CP and DP.	
Concentric Circles	Circles having the same centre at a plane are called the concentric circles. In the given diagram, there are two circles with radii r_1 and r_2 having the common (or same) centre. These are called as concentric circl Ces.	
Congruent Circles	Circles with equal radii are called as congruent circles.	

Nomenclature	*Definition*	*Diagram*
Segment of a Circle	A chord divides a circle into two regions. These two regions are called the segments of a circle: (a) major segment (b) minor segment.	
Cyclic Quadrilateral	A quadrilateral whose all the four vertices lie on the circle.	
Circumcircle	A circle which passes through all the three vertices of a triangle. Thus the circumcentre is always equidistant from the vertices of the triangle. OA = OB = OC (circumradius)	
In Circle	A circle which touches all the three sides of a triangle i.e., all the three sides of a triangle are tangents to the circle is called an incircle. Incircle is always equidistant from the sides of a triangle.	

Now come to different formula and theorems attached to circle:

Circumference of a circle = $2\pi r$

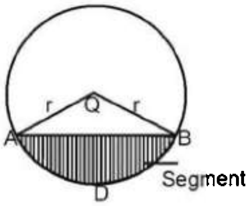
Area of a circle = πr^2, where r is the radius.



Area of a sector $= \pi r^2 \frac{\theta}{360°}$

Circumference of a sector $= 2\pi r \frac{\theta}{360} +$

Perimeter of a sector $= 2r\left(\frac{\pi\theta}{360}+1\right)$



Area of a segment = Area of a sector OADB – Area of triangle OAB

Area of a segment $= \pi r^2 \frac{\theta}{360} - \frac{1}{2}r^2 \text{Sin}\theta$

Common Tangents and Secants of Circles

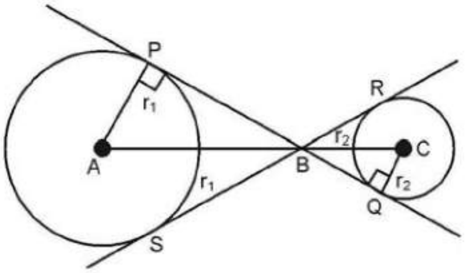
Depending upon the positioning of the circles, two or more than two circles can have a common tangent. Following is a list indicating the number of common tangents in case of two circles:

Sl. No.	*Position of two circles*	*Number of common tangents*
1.	One circle lies entirely inside the other circle	Zero
2.	Two circles touch internally	One
3.	Two circles intersect in two distinct points	Two
4.	Two circles touch externally	Three
5.	One circle lies entirely outside the other circle	Four

Direct common tangents and transverse common tangents

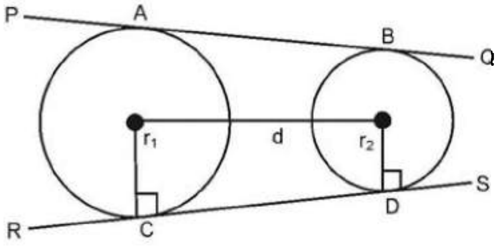
1. *Transverse common tangent* In the figure given below, PQ and RS are the transverse common tangents. Transverse common tangents intersect the line joining the centre of the two circles. They divide the line in the ratio $r_1 : r_2$.
   AB : BC = $r_1 : r_2$

   Assume AC = Distance between centres = d



   $PQ^2 = RS^2 = d^2 - (r_1 + r_2)^2$

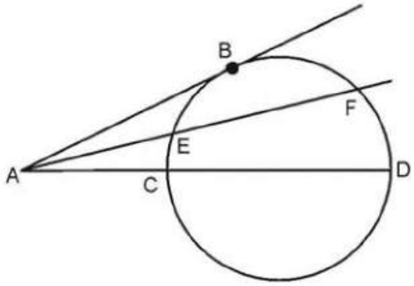
2. *Direct common tangent*



   In the figure given above, PQ and RS are direct common tangents.

   Points A and C are the point of tangency for the first circle and similarly points B and D are the point of tangency for the second circle. AB and CD are known as lengths of the direct common tangents and they will be same.
   $CD^2 = AB^2 = d^2 - (r_1 - r_2)^2$

Secants

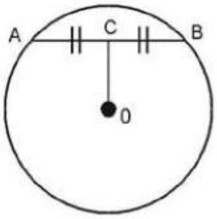


In the figure given above, AB is a tangent and ACD is a secants

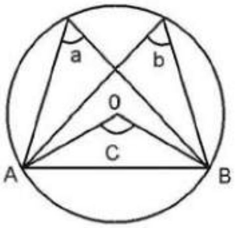
i. $AB^2 = AC \times AD$
ii. $AE \times AF = AC \times AD$

Important theorems related to circle

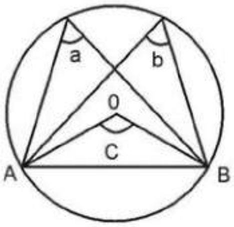
1. If C is the mid-point of AB, then OC is perpendicular to AB. And vice versa is also true.



2. Angles in the same segment will be equal.

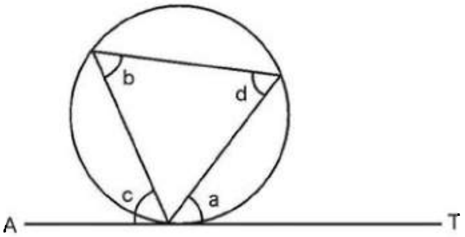


   In the figure given above, *a* = *b*.
3. Angle subtended by a chord at the centre is two times the angle subtended on the circle on the same side. In the figure given below, 2*a* = 2*b* = *c*.



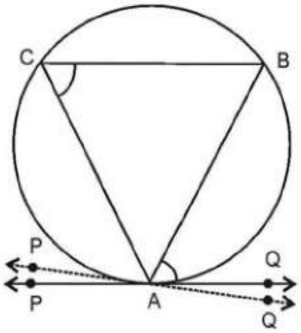
4. Angle subtended by a diameter of the circle is a right angle.

5. Alternate segment theorem



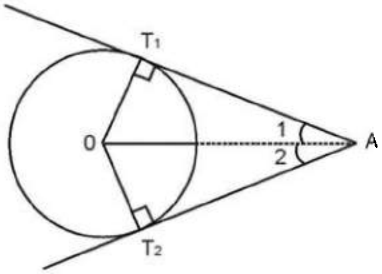
In the figure above, AT is the tangent. ∠a = Alternate segment ∠b ∠c = Alternate segment ∠d

6. Converse of alternate segment theorem If a line is drawn through an end point of a chord of a circle so that the angle formed by it with the chord is equal to the angle subtended by the chord in the alternate segment, then the line is a tangent to the circle.



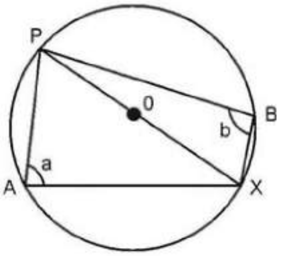
AB is a chord of a circle and a line PAQ such that ∠BAQ = ∠ACB, where C is any point in the alternate segment ACB, then PAQ is a tangent to the circle.

7. Tangent drawn to a circle from a point are same in length.
   In the figure below, tangents are drawn to the circle from point A and AT_1 and AT_2 are the tangents.
   i. $AT_1 = AT_2$ ii. $\angle 1 = \angle 2$
   iii. $AT_1^{\,2} + OT_1^{\,2} = AT_2^{\,2} + OT_2^{\,2} = AO^2$



Cyclic Quadrilateral

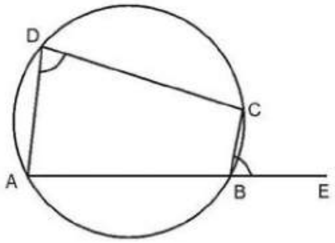
Consider the figure given below:



If we have $a + b = 180°$ and quadrilateral AXBP has all its vertices on a circle, then such a quadrilateral is called a Cyclic quadrilateral.

For a cyclic quadrilateral, the sum of the opposite angles of a quadrilateral in a circle is 180°.

It can also be seen that exterior ∠CBE = internal ∠ADC = 180° − ∠ABC.



Using Brahmagupta's formula to find out the area of a cyclic quadrilateral We know that A + B = p So, area of cyclic quadrilateral

$$= \sqrt{(s-a)(s-b)(s-c)(s-d)}$$

Where terms used are having their meaning.
[Cos 90° = 0]

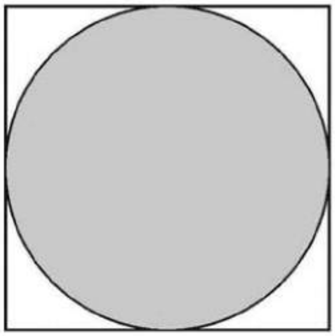
Circle Packing

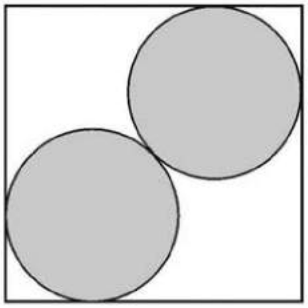
N circles have been packed inside a square of side length R unit and radius of the circle is to be calculated.

Case 1 When *n* = 1

Obviously in this case, diameter of the circle = side of the square.

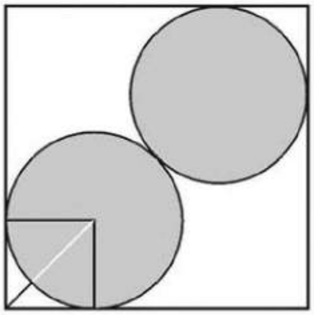
So, the radius of the circle $= \frac{1}{2}$ R unit





Case 2 When $n = 2$

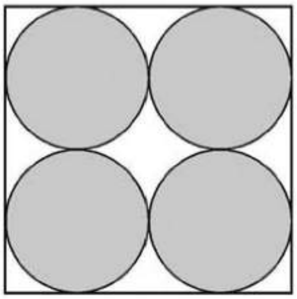
Consider this figure



This is a right-angled triangle with sides r, r and r $\sqrt{2}$. Hence the diagonal of the square = 4r + 2 r $\sqrt{2}$ = $\sqrt{2}$ R

So, $r = \dfrac{\sqrt{2}}{4 + 2\sqrt{2}} R$

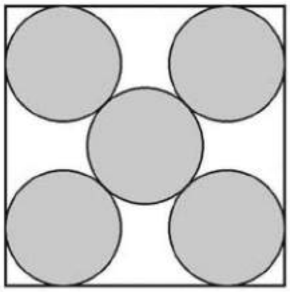
Case 3 When $n = 4$



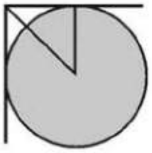
Obviously, in this case 2 × diameter of a circle = side of square.

So, the radius of a circle $= \dfrac{1}{4}$ R

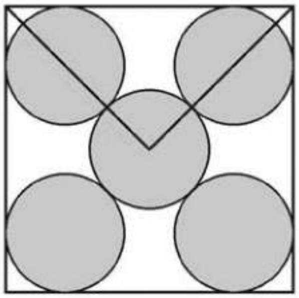
Case 4 When $n = 5$



Consider this figure:



If the radius of the circle is r, then the distance between the centre of the circle and the vertex of a square = r $\sqrt{2}$

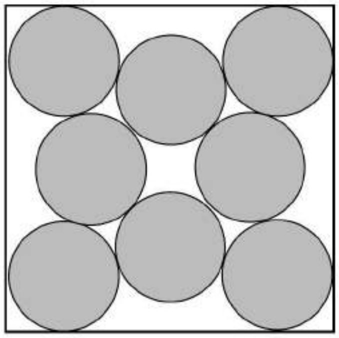


Now, consider this figure, the triangle formed here is a right-angled triangle:

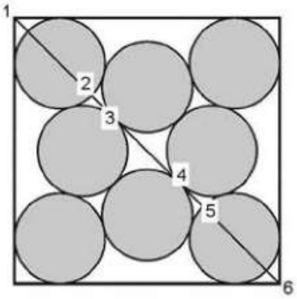
$1/\sqrt{2}$(1/2 of the diagonal of the square) = r + r + r $\sqrt{2}$

= r(2 + $\sqrt{2}$), so, $r = \dfrac{1}{\sqrt{2} \times (2 + \sqrt{2})} R$

Case 5 When $n = 8$



We will find out the value of r in 3 steps here:



Step 1 Find 1 - 2 = 5 - 6

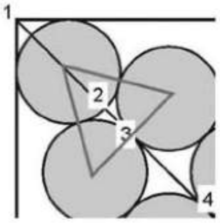
Step 2 Find 2 - 3 = 4 - 5

Step 3 Find 3 - 4

Step 1 1 - 2 = 5 - 6 = r + r √2

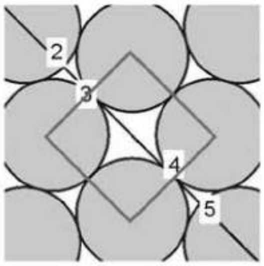
Step 2 For 2 - 3

This is an equilateral triangle with side length = 2r. Height of this triangle r √3



So, 2 - 3 = 4 - 5 = r√3 – r

Step 3

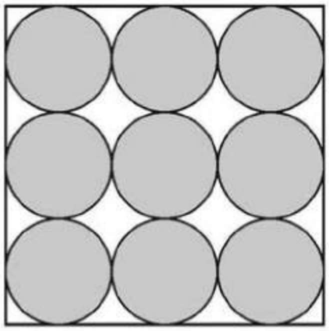


This is a square with side length 2 r and diagonal = 2r √2

So, 3 - 4 = 2r

Hence 1 – 6 = 2(r + r√2) + 2(r√3 – r) + 2r = R√2. Now r can be calculated.

Case 6 When *n* = 9



Radius of circle = r

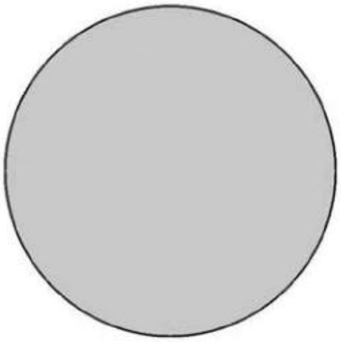
So, the side of a square = 3 × diameter of a circle = 6r

So, 6r = R unit, r = R/6 unit

Circles Inside Circles

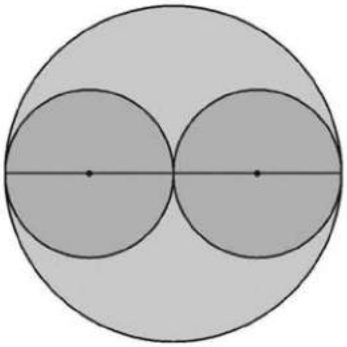
Radius of the outer circle R = 1 unit and n similar circles of ‘r’ radius have been inserted inside this outer circle.

N = 1



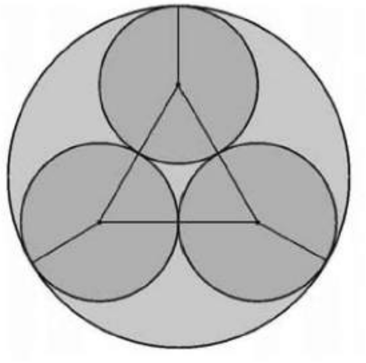
Obviously R = r

N = 2



Obviously, 4 r = 2 R Hence r = R/2

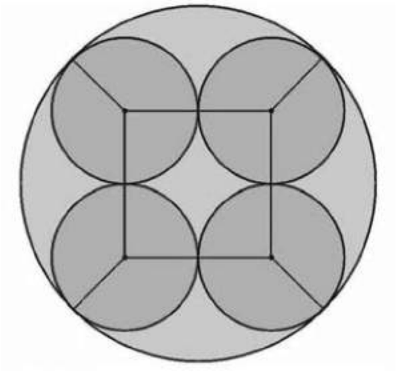
N = 3



The triangle formed inside will be an equilateral triangle of the side length 2 r. The centre of this circle will be incentre/centroid/circumcentre of this equilateral triangle. So, the distance between the centre of any smaller circle to the centre of the bigger circle = 2/3 median = *x* = 2/3 [√3/2 × a], where a = 2r.

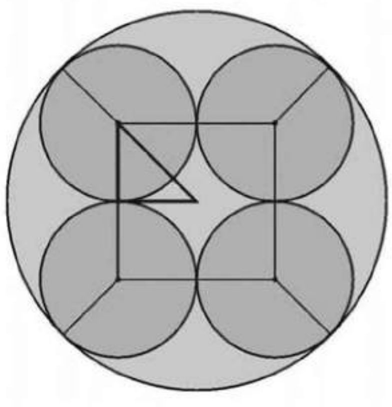
Now, r + x = R. Now put the values of x from the above condition to find the value of r.

N = 4



The square formed inside will be of the side length 2r. Now consider this figure,

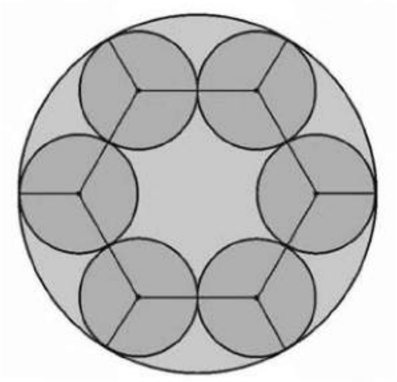
N = 5



The figure inside is a right angled triangle, with base and height being equal to r, and hypotenuse = r√2 = r + x. Hence x = r(√2 – 1)

Now 2R = 4 r + 2 x. Hence r = 2(R – x)/4[Now put the value of x from the above condition.]

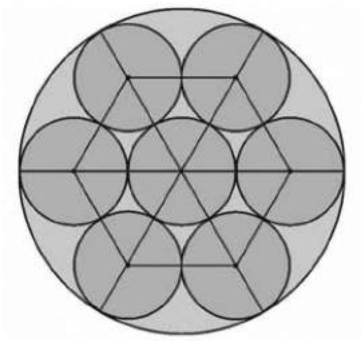
N = 6



Here we can put a similar circle inside all the six circles making it a 7 circle figure. Now, the situation is the same as that of a circle packing with N = 7

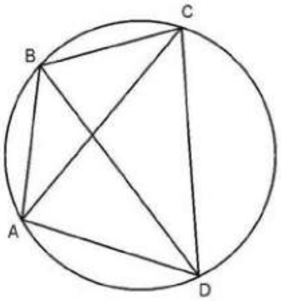
In this case, 6r = R, so, r = 1/6 R

N = 7



In this case, 6r = R, so, r = 1/6 R

Ptolemy's Theorem of Cyclic Quadrilateral



For a cyclic quadrilateral, the sum of the products of the two pair of the opposite sides equal the product of the diagonals.

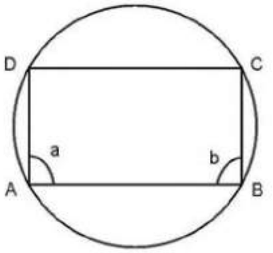
In the figure given above, AB × CD + BC × DA = AC × BD

In any cyclic quadrilateral of side lengths a, b, c and d, the length of the diagonals p and q will be equal to:

$$p = \sqrt{\frac{(ab+cd)(ac+bd)}{ad+bc}} \quad q = \sqrt{\frac{(ac+bd)(ad+bc)}{ab+dc}}$$

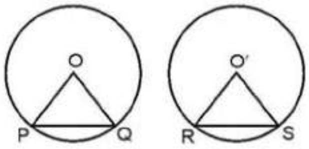
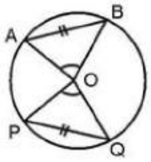
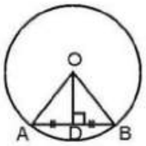
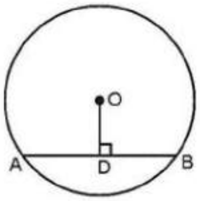
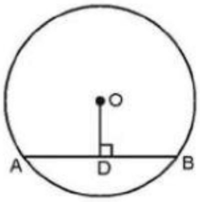
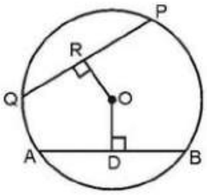
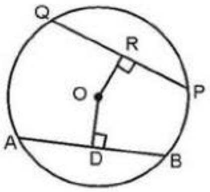
Example 11 Under some special conditions, it is given that a cyclic quadrilateral ABCD is a parallelogram. What kind of figure will ABCD be?

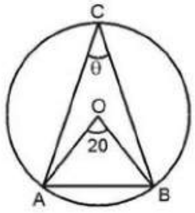
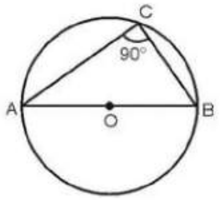
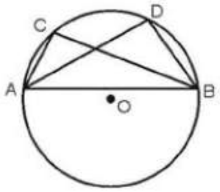
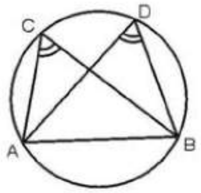
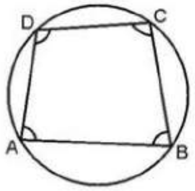
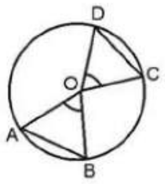
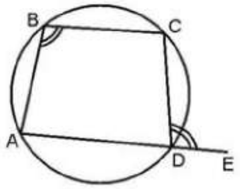
Solution ABCD is a parallelogram. Thus $a + b$ = 180°. And ∠A + ∠C = 180°

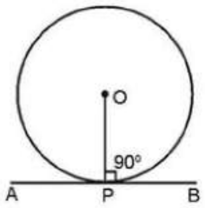
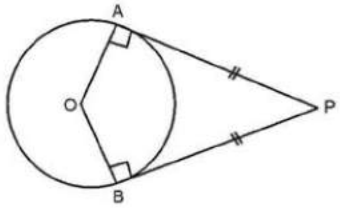
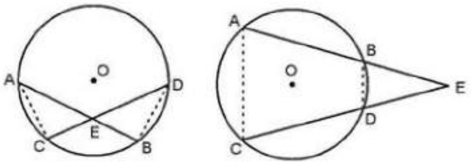
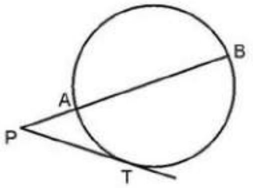
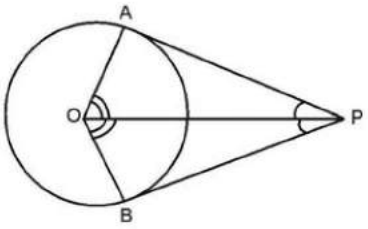
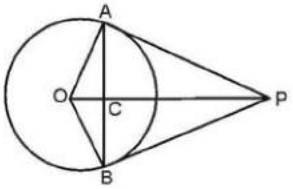
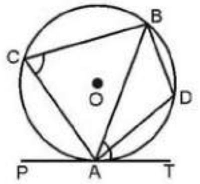


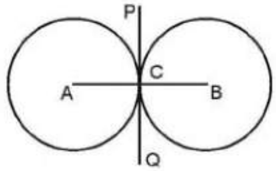
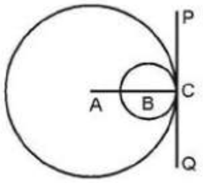
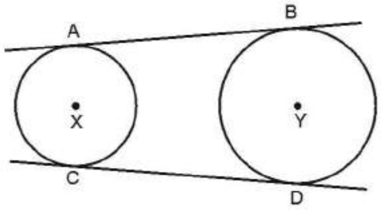
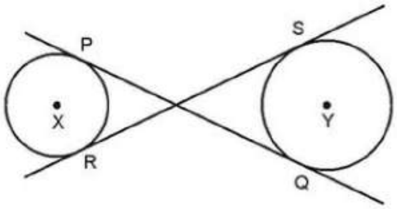
This means that a parallelogram drawn inside a circle is always a rectangle.

Summarizing the discussion regarding circle

Sl. No.	*Theorem/Property*	*Diagram*
1.	In a circle (or congruent circles) equal chords are made by equal arcs. {OP = OQ} = {O’R = O’S) PQ = RS and PQ = RS	
2.	Equal arcs (or chords) subtend equal angles at the centre PQ = AB (or PQ = AB) ∠POQ = ∠AOB	
3.	The perpendicular from the centre of a circle to a chord bisects the chord i.e., if OD ⊥ AB (OD is perpendicular to AB).	
4.	The line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord. AD = DBOD ⊥ AB	
5.	Perpendicular bisector of a chord passes through the centre i.e., OD ⊥ AB and AD = DB ∴ O is the centre of the circle	
6.	Equal chords of a circle (or of congruent circles) are equidistant from the centre ∴ AB = PQ ∴ OD = OR	
7.	Chords of a circle (or of congruent circles) are equidistant from the centre ∴ OD = OR ∴ AB = PQ	

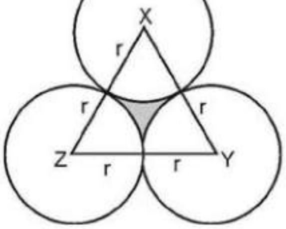
Sl. No.	*Theorem/Property*	*Diagram*
8.	The angle subtended by an arc (the degree measure of the arc) at the centre of a circle is twice the angle subtended by the arc at any point on the remaining part of the circle. m $\angle AOB = 2m \angle ACB$.	
9.	Angle in a semicircle is a right angle.	
10.	Angles in the same segment of a circle are equal i.e., $\angle ACB = \angle ADB$	
11.	If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, then the four points lie on the same circle. $\angle ACB = \angle ADB$ ∴ Points A, C, D, B are concyclic i.e., lie on the circle	
12.	The sum of pair of opposite angles of a cyclic quadrilateral is 180°. $\angle DAB + \angle BCD = 180^\circ$ and $\angle ABC + \angle CDA = 180^\circ$ (Inverse of this theorem is also true)	
13.	Equal chords (or equal arcs) of a circle (or congruent circles) subtended equal angles at the centre. AB = CD (or AB = CD) $\angle AOB = \angle COD$ (Inverse of this theorem is also true)	
14.	If a side of a cyclic quadrilateral is produced, then the exterior angle is equal to the interior opposite angle. m $\angle CDE = m \angle ABC$	

Sl. No.	*Theorem/Property*	*Diagram*
15.	A tangent at any point of a circle is perpendicular to the radius through the point of contact. (Inverse of this theorem is also true)	
16.	The lengths of two tangents drawn from an external point to a circle are equal i.e., AP = BP	
17.	If two chords AB and CD of a circle, intersect inside a circle (outside the circle when produced at a point E), then AE × BE = CE × DE	
18.	If PB be a secant which intersects the circle at A and B and PT be a tangent at T then PA × PB = $(PT)^2$	
19.	From an external point from which the tangents are drawn to the circle with centre O, then (a) they subtend equal angles at the centre (b) they are equally inclined to the line segment joining the centre of that point ∠AOP = ∠BOP and ∠APO = ∠BPO	
20.	If P is an external point from which the tangents to the circle with centre O touch it at A and B then OP is the perpendicular bisector of AB. OP ⊥ AB and AC = BC	
21.	If from the point of contact of a tangent, a chord is drawn then the angles which the chord makes with the tangent line are equal respectively to the angles formed in the corresponding alternate segments. In the adjoining diagram. ∠BAT = ∠BCA and ∠BAP = ∠BDA	

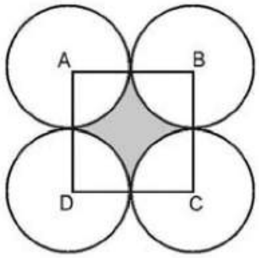
Sl. No.	*Theorem/Property*	*Diagram*
22.	The point of contact of two tangents lies on the straight line joining the two centres. (a) When two circles touch externally then the distance between their centres is equal to sum of their radii, i.e., AB = AC + BC (b) When two circles touch internally the distance between their centres is equal to the difference between their radii. i.e., AB = AC – BC	 
23.	For the two circles with centre X and Y and radii r_1 and r_2. AB and CD are two Direct Common Tangents (DCT), then the length of DCT $= \sqrt{(\text{Distance between centres})^2 - (r_1 - r_2)^2}$	
24.	For the two circles with centre X and Y and radii r_1 and r_2 PQ and RS are two transverse common tangent, then length of TCT $= \sqrt{(\text{Distance between centres})^2 - (r_1 + r_2)^2}$	

Some important points

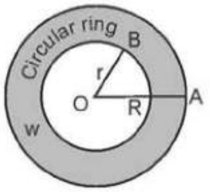
1. If three circles, each of radius r, are so kept that each circle touches the other two, then the area of the shaded region is $r^2 [\sqrt{3} - (\pi/2)]$ which is approximately equal to $(4/25)\, r^2$.



2. If four circles, each of the radius r, are so kept that each circle touches the other two, then the area of the shaded region is $r^2 (4 - \pi)$ which is equal to $(6/7)r^2$.



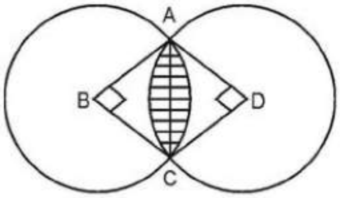
3. In the diagram given below, if the radius (OA) of the outer circle is R and the radius (OB) of the inner circle is r, then the width (w) of the ring is $(R - r)$ and the area of the shaded region is $\pi(R^2 - r^2)$ or $\pi(R - r)\,(R + r)$ or $\pi(w)\,(R + r)$



Example 12 Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. The area in sq. cm of the portion that is common to the two circles, is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}-1$ (c) $\frac{\pi}{5}$ (d) $\sqrt{2}-1$

Solution



Shaded area = 2 × (area of sector ADC – area of ΔADC)

$$= 2\times\left(\frac{\pi}{4}\times 1^2 - \frac{1}{2}\times 1\times 1\right)$$

$$= \frac{\pi}{2}-1$$

Hence, option (b)

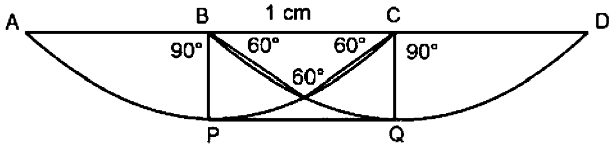
Example 13 Four points A, B, C and D lie on a straight line in the X – Y plane, such that AB = BC = CD, and the length of AB is 1 cm. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one metre of any insect repellent. The minimum distance the ant must traverse to reach the sugar particle (in m) is

(a) $3/\sqrt{2}m$ (b) $1+\pi$

(c) $\frac{4\pi}{3}$ (d) 5

Solution

In the drawn figure, ant will go along APQD since it cannot be within a distance of 1 cm from the repellents kept at B and C.



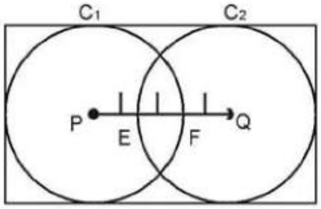
$$AP = \frac{90}{360}\times 2\pi\times 1 = \frac{\pi}{2}$$

Also AP = QD $= \frac{\pi}{2}$

So the minimum distance = AP + PQ + QD

$$= \frac{\pi}{2}+1+\frac{\pi}{2} = 1+\pi$$

Example 14 Two circles C_1 and C_2, having the same radius of 2 cm and centers at P and Q respectively, intersect each other such that the line of centers PQ intersects C_1 and C_2 at F and E respectively. EF = 1 cm. The whole assembly is enclosed in a rectangle of the minimum area. The perimeter of the rectangle is



Hence, breadth of the rectangle = 4 cm

And length = 7 cm

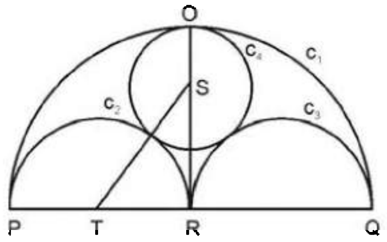
Perimeter = 2 × (7 + 4) = 22 cm

Example 15 Semi-circle C_1 is drawn with a line segment PQ as its diameter with centre at R. Semicircles C_2 and C_3 are drawn with PR and QR as diameters respectively, both C_2 and C_3 lying inside C_1. A full circle C_4 is drawn in such a way that it is tangent to all the three semicircles C_1, C_2 and C_3. C_4 lies inside C_1 and outside both C_2 and C_3. The radius of C_4 is

(a) $\frac{1}{3}PQ$ (b) $\frac{1}{6}PQ$

(c) $\frac{1}{\sqrt{2}}PQ$ (d) $\frac{1}{4}PQ$

Solution



Assume that the radius of C_4 = r and PQ = k.

Now, PR = k/2 = RQ = RO

⇒ RS = (k/2) – r

RT = k/4

ST = (k/4) + r

Applying Pythagoras theorem in triangle STR.

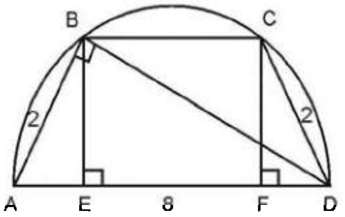
$$\left(\frac{k}{4}+r\right)^2 = \left(\frac{k}{4}\right)^2 + \left(\frac{k}{2}-r\right)^2$$

⇒ r = k/6 = PQ/6

Example 16 On a semicircle with diameter AD, chord BC is parallel to AD. Further each of the chords AB and CD has length 2 units, while AD = 8 units. What is the length of BC?

(a) 7.5 (b) 7
(c) 7.75 (d) 8

Solution



Finding area of DABD $\frac{1}{2}$ AB × BD $= \frac{1}{2}$ AD × BE

So, BE = (√15)/2

Hence AE $= \frac{1}{2}$

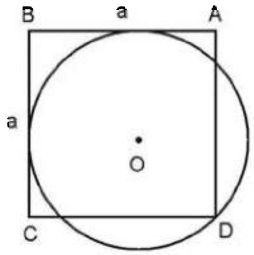
Now, BC = EF = 8 – $\left(\frac{1}{2}+\frac{1}{2}\right)$ = 7

Alternatively, this question can be solved by using Ptolemy's theorem also.

Example 17 The adjacent sides AB, BC of a square of side 'a' units are tangent to a circle. The vertex D of the square lies on the circumference of the circle. The radius of the circle could be:

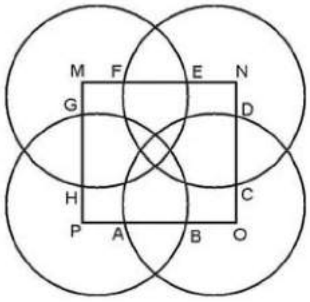
(a) a(2 – √2) (b) a(√2 – 1)
(c) a(√2 + 1.5) (d) a(√2 + 1)

Solution



OD = r, OB = r√2
Hence, r + r√2 = √2 a
Hence, r = a(2 –√2)

Example 18 M, N, O and P are centres of four intersecting circles each having a radius of 15 cm. If AB = 7 cm, CD = 5 cm, EF = 6 cm, GH = 8 cm, what is the perimeter of the quadrilateral MNOP?

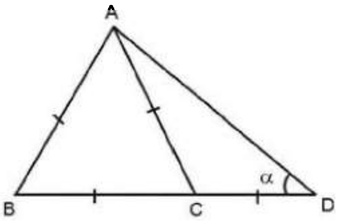


(a) 84 cm (b) 45 cm
(c) 94 cm (d) 124 cm

Solution MN = 30 – 6 = 24 cm

NO = 30 – 5 = 25 cm
OP = 30 – 7 = 23 cm
PM = 30 – 8 = 22 cm
So, the perimeter = 24 + 25 + 23 + 22 = 94 cm

Example 19

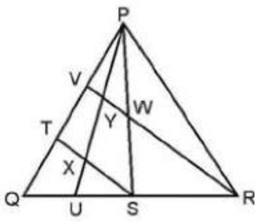


AB = BC = AC = CD. Find ∠a.

(a) 30° (b) 60°
(c) 15° (d) None of these

Solution ∠ACB = 60° ∠ACD = 180 – 60 = 120° ∠CAD = ∠CDA = 30° ∠α = 30°

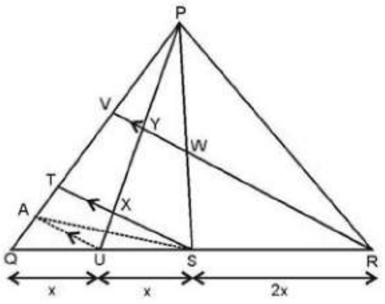
Example 20 In the figure shown here QS = SR, QU = SU, PW = WS and ST||RV. What is the value of the $\frac{\text{Area of } \Delta PSX}{\text{Area of } \Delta PQR}$?



(a) $\frac{1}{5}$ (b) $\frac{1}{7}$

(c) $\frac{1}{6}$ (d) $\frac{1}{9}$

Solution



Draw a line form U such that it is parallel to ST (and hence RV, also) joining AS, to get ΔQSA.

Now, let us first find the area of ΔSUX.

PW = WS (W is the mid point of PS)

Consider the ΔPXS, SX||YW and PX and PS are transversals to those parallel lines, we must have

$$\frac{PY}{YX}=\frac{PW}{WS}=\frac{1}{1}=1$$

⇒ PY = YX

Similarly we also get

PV = TV (in ΔPTS; VW||ST) and QA = AT (in ΔQTS: AU||ST) and QT = VT (in ΔVQR; ST||VR)

Combining all these QA = AT $=\frac{1}{2}(VT)+\frac{1}{2}(VP)$

Now, in ΔRUY

$$\frac{TS}{SR}=\frac{x}{2x}=\frac{1}{2}=\frac{UX}{XY}\Rightarrow UX=\frac{1}{2}(XY)$$

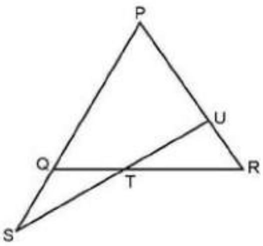
so, in ΔPUS, $UX=\frac{1}{2}XY=\frac{1}{2}PY\Rightarrow\frac{\text{Area }(\Delta SUX)}{\text{Area }(\Delta PUS)}=\frac{1}{5}$

$$\Rightarrow \text{Area }(\Delta SUX)=\frac{1}{5}\text{ Area }(\Delta PUS)$$

$=\frac{1}{5}\left(\frac{1}{4}\text{area }\Delta PQR\right)\Rightarrow \text{Area }(\Delta SUX)=\frac{1}{2}(\Delta SUX)$, then

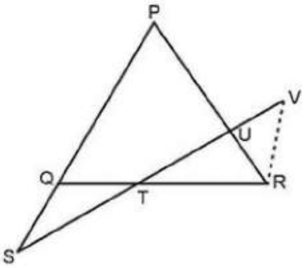
Area (ΔPSX) = Area (ΔPSU) − area (Δ SUX) $=\frac{1}{4}=\frac{1}{5}$ (ΔPQR)

Example 21 Through T, the mid-point of the side QR of a ΔPR, a straight line is drawn to meet PQ produced to S and PR at U, so that PU = PS. If length of UR = 2 units then the length of QS is:



(a) $2\sqrt{2}$ Units (b) $\sqrt{2}$ Units
(c) 2 Units (d) Cannot be determined

Solution We have QT = TR and PU = PS and UR = 2 units



Draw RV||PS that meets SU extended at V.

Now, in ΔQST and ΔTVR

∠QST = ∠TVR (alternate angles as PS||VR) and ∠QTS = ∠VTR

QT = TR

∴ DQST and DTVR are congruent.

∴ QS = VR ... (1)

Now ∠QST = ∠PUS = ∠VUR = ∠UVR

In ΔUVR, ∠VUR = ∠RVU

or, RV = UR = 2 ...(2)

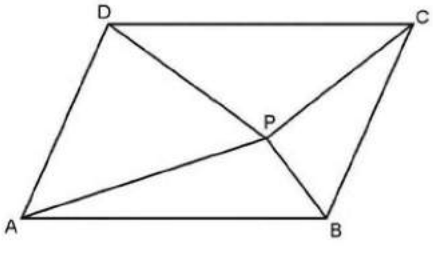
From (1) and (2)

QS = VR = UR = 2 units

Example 22 ABCD is a parallelogram and P is any point within it. If the area of the Parallelogram ABCD is 20 units, then what is the sum of the areas of the DPAB and DPCD?

(a) 5 units (b) 10 units
(c) 12 units (d) Cannot be determined

Solution



Let AB = CD = a and x, y be the lengths of the perpendiculars from P on AB and CD respectively, then

$$\text{Area of }(\Delta PAB+\Delta PCD)=\frac{1}{2}ax+\frac{1}{2}ay$$

$$=\frac{1}{2}a(x+y)=\frac{1}{2}ah$$

Given area of the parallelogram = 20 units

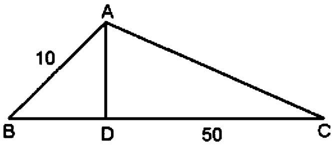
$$\text{Area of }(\Delta PAB+\Delta PCD)=\frac{1}{2}\times 20=10\text{ units}$$

Example 23 Nayantara bought a triangular piece of land of area 150 m^2. He took a piece of rope and measured the two sides of the plot and found the largest side to be 50 m and another side to be 10 m. What is the exact length of the third side?

(a) $40\sqrt{3}$ m (b) $30\sqrt{2}$ m

(c) $\sqrt{1560}$ m (d) 32 m

Solution



Let the triangle be ABC.

Now, area of $\Delta ABC = (1/2)\,(BC)\,(AD)$ (Where D is a point on BC such that $AD \perp BC$)

Now, AD has to be equal to $\dfrac{\text{Area} \vartriangle 2}{BC}$

$$AD = \frac{(150)(2)}{50}\text{ m} = 6\text{ m}$$

Now ΔBDA is a right-angled triangle

$\therefore BD^2 = AB^2 - AD^2 = 10^2 - 6^2 = 8^2\ m^2$

$\Rightarrow$ BD = 8 m and DC = BC – BD = 42 m

In a right-angled ΔADC, $AC^2 = AD^2 + DC^2 = (6)^2 + (42)^2$

$\Rightarrow AC = \sqrt{1800} = 30\sqrt{2}$

Alternatively, going through the options

$30\sqrt{2} = 42$

$40\sqrt{3} = 68$

$\sqrt{1560} = 39$

$\therefore$ Using the principle that the sum of any two sides of a triangle is greater than the third side. The two given sides are 50 and 10.

From the choices the above condition is satisfied only for choice (b).

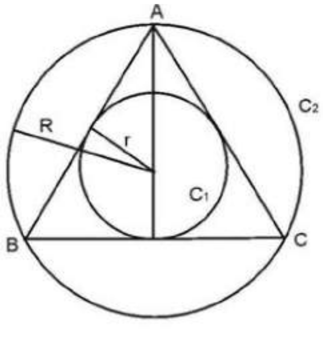
In Choice (a) 10 + 50 < 68
In Choice (c) 39 + 10 < 50
In Choice (d) 32 + 10 < 50

Example 24 C_1 and C_2 are two concentric circles with radii 5 cm and 9 cm respectively. If A, B and C are points on C_2 such that AB and AC are tangent to C_1 at how many points does BC intersect C_1?

(a) 0 (b) 1

(c) 2 (d) Cannot be determined

Solution

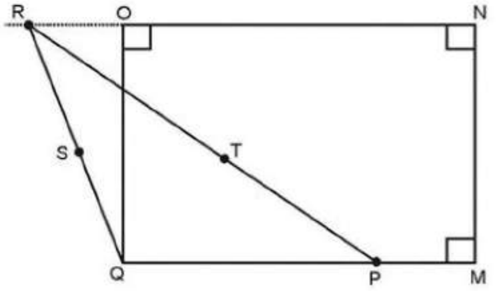


Consider two concentric circles with radii r and R (r < R). If $r = \dfrac{R}{2}$ and A, B and C are points on the outer circle such that AB and AC are tangent to the inner circle, then BC is also a tangent.

(In this case, C_1 and C_2 become the incircle and circumcircle of ABC respectively, which will be an equilateral triangle)

If $r < \dfrac{R}{2}$, BC does not intersect or touch the inner circle and if $r > \dfrac{R}{2}$ then BC intersects the inner circle at two points.

Example 25 A rectangle MNOQ is drawn and length 'NO' is extended to point R and a triangle QPR is drawn with $QP = \dfrac{2}{3}$ QM. Angle QRP = 45° and side QR $4\sqrt{7}$ cm, S and T are the midpoints of sides QR and PR respectively. If ST = 6 units, the area (in sq. cm) of the rectangle is



(a) 112 (b) 144

(c) 288 (d) 256

Solution Since the line joining the mid-points of two sides of a triangle is parallel and equal to half the third side, we have PQ = 2(ST), $\Rightarrow$ PQ = 12 cm

Since, $PQ = \dfrac{1}{5}$ QM

Now, $\angle ORP = 45°$

Draw $PV \perp ON$

In ΔRVP, $\tan 45° = \frac{PV}{VR} = \frac{PV}{OR+OV} = \frac{\text{breadth}}{12+OR}$

(Since OV = PQ = 12 cm) also, $\text{breadth}^2 + OR^2 = (QR)^2$

In ΔROQ, ⇒ Breadth = $x + 12$ where OR = x

$\Rightarrow (x + 12)^2 = \left(4\sqrt{17}\right)^2 - x^2$ (given that $QR = \left(4\sqrt{17}\right)$

$\Rightarrow 2x^2 + 24x - 128 = 0$

$\Rightarrow x^2 + 12x - 24 = 0$

$\Rightarrow x = 4$ or -16

Since $x > 0$, $x = 4$

Breadth = $x + 12$ i.e., 16 cm, and the area of MNOQ = 16 × 18 = 288 sq. cm.

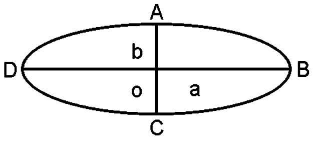
Example 26 ΔABC has sides AB, AC measuring 2001 and 1002 units respectively. How many such triangles are possible with all integral sides?

(a) 2001 (b) 1002 (c) 2003 (d) 1004

Solution Value of BC will lie in between 999 and 3003. Hence 999 < BC < 3003.

So, the total values possible for BC = 2003

Ellipse The path of a moving point which moves in such a way that its distance from a fixed point (focus) bears a constant ratio with its distance from a fixed line (directrix)



Given: OB = OD = a (Semi-major axis)

OA = Ol = b (Semi = minor axis)

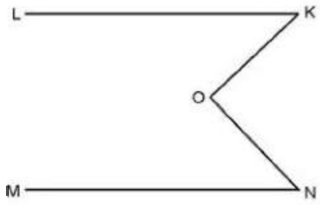
- Area = πab
- Perimeter = π(a + b)

Maths behind the formula (Area = πab) One way to see why the formula is true is to realize that the ellipse is just a unit circle that has been stretched by a factor 'a' in the x-direction and by factor 'b' in the y-direction. Hence the area of the ellipse is just $a \times b$ times the area of unit circle.

PRACTICE EXERCISES

WARM UP

Q.1. In the given figure, MN and KL are parallel lines.



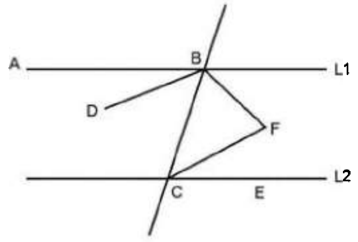
∠LKO = 70°, ∠KON = 100°

Find ∠MNO.

(a) 20° (b) 30°

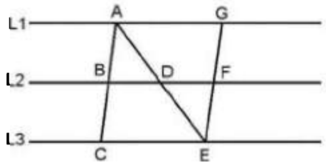
(c) 40° (d) 50°

Q.2. In the given figure, L1 and L2 are parallel lines. ∠ABC = 80°. If the lines BD and CF are parallel and ∠DBC = 30°, find ∠FCE.



(a) 30° (b) 45° (c) 50° (d) 60°

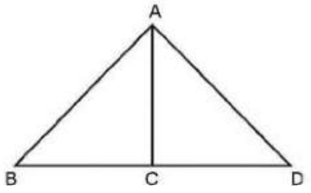
Q.3. The lines L1, L2 and L3 are parallel. If AB = 4 cm, BC = 6 cm, AD = 12 cm and FG = DE then find EF.



(a) 27cm (b) 36 cm
(c) 48 cm (d) None of these

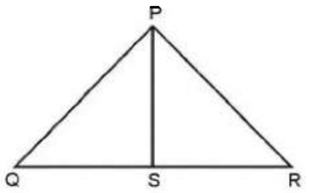
Q.4. The supplement of an angle is five times the angle. What is the measurement of the angle?
(a) 30° (b) 65° (c) 40° (d) 45°

Q.5. In the given figure, AB = AD. ∠ACB = 95° + ∠BAC and ∠BAD = 150°. Find ∠ACB.



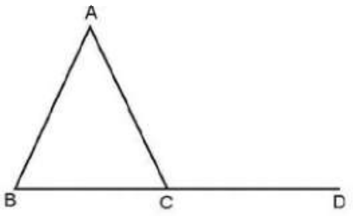
(a) 110° (b) 120° (c) 130° (d) 140°

Q.6. In the given figure, PS is the altitude drawn to the side QR of the triangle PQR. ∠PRS = ∠QPS – 20°. ∠PQS = 20°, ∠PRQ = 50°. What is the value of ∠QPR?



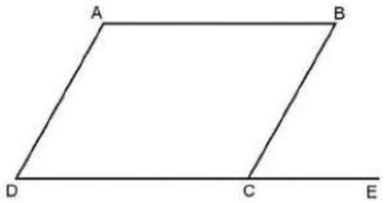
(a) 100° (b) 110° (c) 120° (d) 130°

Q.7. In ΔABC, ∠ACD = 100° and ∠ABC = 40°. Find ∠BAC



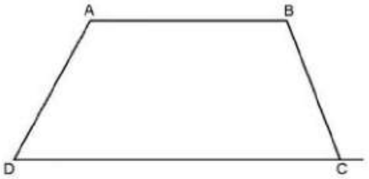
(a) 40° (b) 100° (c) 80° (d) 60°

Q.8. In the given figure, ABCD is a parallelogram. Find 2∠ABC – ADC if A = ∠135°.



(a) 50° (b) 60°
(c) 65° (d) 75°

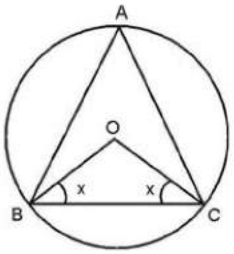
Q.9. ABCD is an isosceles trapezium with lines AB parallel to CD. If ∠DCB = 40°, ∠BAD equals



(a) 40° (b) 80° (c) 100° (d) 140°

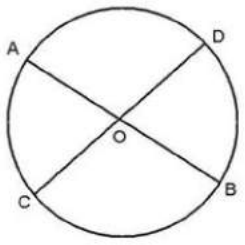
Q.10. If each interior angle of a regular polygon is 135°, find the number of diagonals.
(a) 54 (b) 48
(c) 20 (d) None of these

Q.11. In the given figure, ΔABC is circumscribed by a circle having its centre at O. If OB and OC are phe angle bisector of ∠B and ∠C respectively, what is the value of ∠BAC?



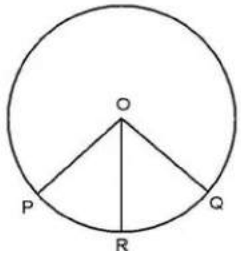
(a) 45° (b) 60° (c) 75° (d) 50°

Q.12. In the given figure, AB and CD are two chords of a circle intersecting at O. If AO = 4 cm and OB = 6 cm and OC = 3 cm. Find OD.



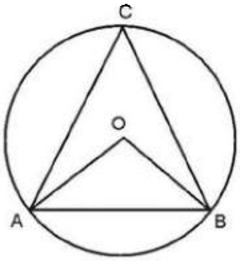
(a) 4 cm (b) 6 cm (c) 8 cm (d) 10 cm

Q.13. In the given figure, if 2 ∠POR = 3 ∠ROQ and ∠POQ = 100° then find ∠POR.



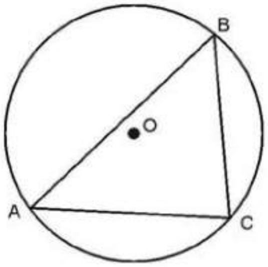
(a) 20° (b) 30° (c) 45° (d) 60°

Q.14. In the above figure, O is the centre of the circle, and ∠AOB = 120°. Find ∠ACB.



(a) 60° (b) 30°
(c) 45° (d) 50°

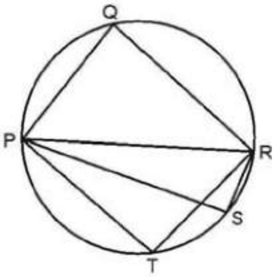
Q.15. In the given figure, O is the centre of the circle, ∠AOC = 160°. If AB = AC, then find ∠BAC.



(a) 45° (b) 50°
(c) 60° (d) 75°

Q.16. PQRS is a cyclic quadrilateral. If PQR is an equilateral triangle, find ∠RSP.
(a) 120° (b) 60°
(c) 30° (d) 150°

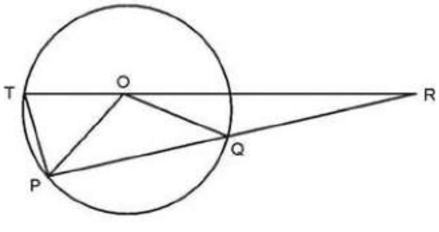
Q.17. In the given figure, PT = TR, ∠PQR = 100°. Find ∠PRT.



(a) 40° (b) 50°
(c) 60° (d) 70°

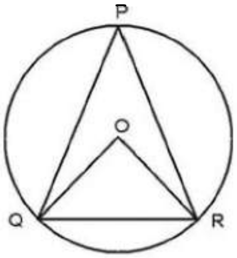
Q.18. In a triangle ABC, the incentre is at O. If ∠BOC = 100°, find ∠BAC.
(a) 10° (b) 20°
(c) 30° (d) 40°

Q.19. In the given figure, O is the center of the circle, OQ = QR and ∠QRO = 15°. Find ∠OQP.



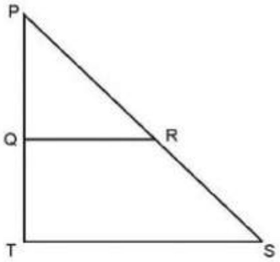
(a) 120° (b) 150° (c) 135° (d) 105°

Q.20. In the given figure, O is the centre of the circle and ∠OQP + ∠ORP = 70°. Find ∠ORQ.



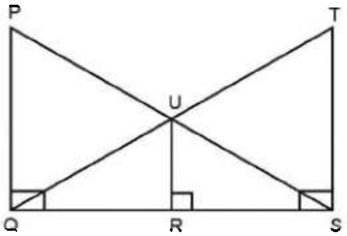
(a) 20° (b) 30°
(c) 40° (d) 50°

Q.21. In the given figure PQ × PS = PT × PR. If ∠PQR = ∠PST + 30° and ∠PTS = 100° then find ∠PRQ.



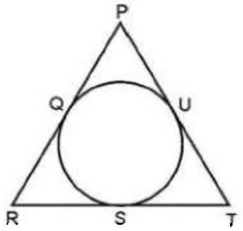
(a) 100° (b) 70° (c) 130° (d) 50°

Q.22. In the above figure, QR = 4 cm and RS = 12 cm. TS = 8 cm and QU is extended to T. Find PQ.



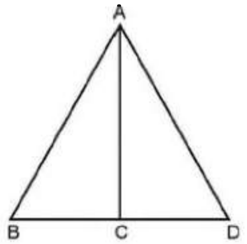
(a) 3 cm (b) 24 cm
(c) 21 cm (d) None of these

Q.23. In the given figure PR, PT and RT are tangents to the circle at Q, U and S respectively. PR = (RT+3) cm; PR = (PT+1) cm. If the perimeter of triangle RPT is 26 cm, what is value of QR + PT?



(a) 12 cm (b) 3 cm (c) 14 cm (d) 15 cm

Q.24. In the given figure, ∠CAB = 1/2 ∠ACD, ∠BAD = 85° and ∠ADC = 40°. Find ∠ACB.



(a) 45° (b) 55°
(c) 70° (d) 60°

Q.25. How many different regular polygons can be formed with the interior angle exceeding the exterior angle and the sum of the interior angles not exceeding 180°?
(a) 7 (b) 8
(c) 9 (d) None of these

FOUNDATION

Q.1. The ratio of the sides of ΔABC is 1:2:4. What is the ratio of the altitudes drawn onto these sides?
(a) 4:2:1 (b) 1:2:4
(c) 1:4:16 (d) None of these

Q.2. The largest angle of a triangle of sides 7 cm, 5 cm and 3 cm is
(a) 45° (b) 60° (c) 90° (d) 1200

Q.3. The three sides of a triangle measure 6 cm, 8 cm and 10 cm respectively. A rectangle equal in area to the triangle has a length of 8 cm. The perimeter of the rectangle is:
(a) 11 cm (b) 22 cm
(c) 16 cm (d) None of these

Q.4. X and Y are respectively two points on the sides DC and AD of parallelogram ABCD. The area of ΔABX is equal to
(a) $\frac{1}{3}\times$ area of ΔBYC
(b) Area of ΔBYC
(c) $\frac{1}{2}\times$ area of ΔBYC
(d) 2 × area of ΔBYC

Q.5. A rectangular enclosure 40 m × 36 m has a horse tethered to a corner with a rope of 14 m in length. What is the ratio of the respective areas it can graze, if it is outside the enclosure and if it is inside the enclosure?
(a) 2 (b) 2:3
(c) 1:4 (d) 3:1

Q.6. What is the area of the triangle having the sides 4, 6 and 12 units?
(a) 14.28 (b) 12.6
(c) 16.3 (d) Not possible

Q.7. Identical spherical balls are spread on a table top so as to form an equilateral triangle. How many balls are needed so that a side of the equilateral triangle contains n balls?
(a) $\frac{n(n+1)}{2}$ (b) $n^2 - 1$
(c) $n(n-1)$ (d) $n!$

Direction for questions 8 and 9: Read the passage below and solve the questions based on it.

The area of a square is equal to the area of a rectangle. Moreover, the perimeter of the square is also equal to the perimeter of the rectangle.

Q.8. The side of the square is equal to the:
(a) Length of the rectangle
(b) Breadth of the rectangle
(c) Cannot be determined
(d) Both [1] and [2]

Q.9. The length of the rectangle is equal to the:
(a) Breadth of the rectangle
(b) Side of the square
(c) Cannot be determined
(d) Both [1] and [2]

Q.10. A quadrilateral is inscribed in a circle. If an angle is inscribed in each of the segments outside the quadrilateral, then what is the sum of the four angles?

(a) 270° (b) 360°
(c) 540° (d) 720°

Q.11. In a triangle PQR, PQ = 6 cm, QR = 8 cm and PR = 10 cm. The length of the median bisecting the shortest side is (approx.):

(a) 10 cm (b) 8.5 cm
(c) 9 cm (d) None of these

Q.12. A square is inscribed in a circle which is inscribed in an equilateral triangle. If one side of the triangle is 'a', find the area of the square.

(a) $\frac{1^2}{3}$ (b) $\frac{a^2}{6}$
(c) $\frac{3a^2}{8}$ (d) $\frac{\pi a^2}{12}$

Q.13. A cyclic quadrilateral is such that two of its adjacent angles are divisible by 6 and 10 respectively. One of the remaining angles will necessarily be divisible by:

(a) 3 (b) 4
(c) 8 (d) None of these

Q.14. In the above question, if one of the remaining angles is necessarily divisible by 16, then its measure can be:

(a) 128° (b) 48°
(c) 132° (d) 112°

Q.15. The volume of spheres are proportional to the cubes of their radii. Two spheres of the same material weigh 3.6 kg and 2.7 kg and the radius of the smaller one is 2 cm. If the two were melted down and formed into a single sphere, what would be its radius?

(a) 4 cm (b) 4.3 cm
(c) 3 cm (d) 2.6 cm

Direction for questions 16 and 17: Read the passage below and solve the questions based on it.

$f(x)$ is the area of a square where, x is the side of a square. $g(x)$ is the perimeter of square where, x is the side of a square. $h(x, y)$ is the area of a rectangle where x is the length and y is the breadth. $i(x, y)$ is the perimeter of a rectangle where x is the length and y is the breadth.

Q.16. The value of $g[h\{f\{i\ (2,3)\}, 2\}$ is equal to

(a) 450 (b) 600
(c) 800 (d) 1000

Q.17. The value of $\frac{[f(3)-g(2)]}{h(4,2)-i(6,2)} \times \frac{g(6)}{f(1)}$ is equal to

(a) −3 (b) 2
(c) 1 (d) −4

Direction for questions 18 and 19: Read the passage below and solve the questions based on it.

A 10 m long piece of wire is cut into two pieces, one of which is bent into a circle and the other into the square enclosing it.

Q.18. The area of the square outside the circle is

(a) 1200 cm² (b) 2400 cm²
(c) 3600 cm² (d) 4200 cm²

Q.19. The ratio of the radius of the circle to the perimeter of the square is

(a) 8:1 (b) 1:8 (c) 4:1 (d) 1:4

Q.20. There are two regular polygons with the number of sides in the ratio of 4:5 and the interior angles in the ratio of 25:26. The number of sides in the first polygon are

(a) 8 (b) 10 (c) 12 (d) 15

Q.21. A square is inscribed in a semi circle of radius 10 cm. What is the area of the inscribed square? (Given that the side of the square is along the diameter of the semicircle.)

(a) 70 cm² (b) 50 cm²
(c) 25 cm² (d) 80 cm²

Q.22. ABCD is a parallelogram in which ∠B = 70°. Find the number of points X in the plane of the parallelogram such that it is equidistant from its vertices.

(a) zero (b) one (c) two (d) three

Q.23. There is a fan with 3 blades at 120° to each other whose central circular disc has an area of 3p cm² and a blade is $(20-\sqrt{3})$ cm long. If the tips of the blades are joined so as to form an equilateral triangle, what will be its area?

(a) 900 cm² (b) $900\sqrt{3}$ cm²
(c) $(900 + 9x)$ cm² (d) $(3x + 300)$ cm²

Q.24. Two circles of an equal radii are drawn, without any overlap, in a semicircle of radius 2 cm. If these are the largest possible circles that the semicircle can accommodate, what is the radius (in cm) of each of the circles?

(a) 0.414 (b) 0.828
(c) 0.172 (d) 0.586

Q.25. The sides of a regular octagon are extended to form a star. Find the measure of the internal angle at each point of the star.

(a) 45° (b) 90° (c) 135° (d) 60°

Q.26. PQRS is trapezium, in which PQ is parallel to RS, and PQ = 3 (RS). The diagonal of the trapezium intersect each other at X, then the ratio of ΔPXQ and ΔRXS is

(a) 6:1 (b) 3:1 (c) 9:1 (d) 7:1

Direction for questions 27 and 28: Read the passage below and solve the questions based on it.

All the angles of a quadrilateral form a GP and its common ratio is a natural number. Exactly two of the angles are obtuse and none of the angles is a right angle.

Q.27. What is the measurement of the largest angle?
(a) 124° (b) 156°
(c) 192° (d) None of these

Q.28. What is the measurement of the smallest angle?
(a) 18° (b) 24°
(c) 36° (d) None of these

Q.29. Two circles with centres C1 and C2 and radii 6 cm and 8 cm respectively cut each other at right angles. Find the length of the common chord.
(a) 10 cm (b) 4.8 cm
(c) 9.6 cm (d) 5 cm

Q.30. Let ABCDEF be a regular hexagon. What is the ratio of the area of the triangle ACE to that of the hexagon ABCDEF?
(a) 1/3 (b) 1/2 (c) 2/3 (d) 5/6

Q.31. The sides of a triangle are in the ratio of $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$. If the perimeter is 52 cm, then the length of the smallest side is
(a) 9 cm (b) 10 cm
(c) 11 cm (d) 12 cm

Q.32. A pond 100 m in diameter is surrounded by a circular grass walk-way 2 m wide. How many square metres of grass is the on the walk-way?
(a) 98 π (b) 100 π
(c) 204 π (d) 202 π

Q.33. A triangle and a parallelogram are constructed on the same base such that their areas are equal. If the altitude of the parallelogram is 100 m, then the altitude of the triangle is
(a) 100 m (b) 200 m
(c) $100\sqrt{2}$ m (d) $10\sqrt{2}$ m

Q.34. A rhombus OABC is drawn inside a circle, whose centre is at O, in such a way that the vertices A, B and C of the rhombus are on the circle. If the area of the rhombus is $32\sqrt{3}$ m², then the radius of the circle is
(a) 64 m (b) 8 m (c) 32 m (d) 46 m

Q.35. Let A be the area of a square inscribed in a circle of radius 'r' and let B be the area of a hexagon inscribed in the same circle. Then B/A equals:
(a) $2\sqrt{3}$ (b) $\frac{3\sqrt{3}}{4}$
(c) $\frac{2\sqrt{3}}{4}$ (d) None of these

Q.36. The dimensions of a rectangular box are in the ratio of 1:2:4 and the difference between the costs of covering it with the cloth and a sheet at the rate of Rs 20 and Rs 20.5 per sq m respectively is Rs 126. Find the dimensions of the box.
(a) 2 m, 6 m, 9 m (b) 6 m, 12 m, 24 m
(c) 1 m, 2 m, 4 m (d) None of these

Q.37. If the sides 50 m and 130 m of the triangular field meet at an angle of 72°, then find the area in which wheat is cultivated. (Sin 72° = 0.9510, Cos 72° = 0.309)
(a) 100 πm² (b) 125 πm²
(c) 160 πm² (d) None of these

Q.38. Four horses are tied on the four corners of a square field of 14 m length so that each horse can touch just the other two horses. They were able to graze in the area accessible to them for 11 days. For how many days is the ungrazed area sufficient for them?
(a) 3 days (b) 4 days
(c) 5 days (d) 2 days

Q.39. The ratio of the area of a square inscribed in a semicircle to that of the area of a square inscribed in the circle of the same radius is
(a) 2:1 (b) 2:3 (c) 2:5 (d) 2 : 7

Q.40. The area of a rectangle is given by $A = 8x^2 - 2x - 15$. If the length was larger than the breadth, then the length can be
(a) $(3x + 5)$ (b) $(5x + 3)$
(c) $(4x + 5)$ (d) $(2x - 3)$

Q.41. The ratio of the area of a square to that of the square drawn on the its the diagonal is
(a) 1:4 (b) 2:1 (c) 1:2 (d) 1:3

Q.42. An equilateral triangle had 7 cm long sides. Equal circles were drawn about the three vertices of the triangle so that each circle touched the other two. Calculate the area of space enclosed between these circles.
(a) 3.934 cm² (b) 1.967 cm²
(c) 3.9 cm² (d) 1.95 cm²

Q.43. What is the area of the triangle in which two of its medians 9 cm and 12 cm long intersect at the right angles?
(a) 72 (b) 60 (c) 56 (d) 48

Q.44. If one leg of an isosceles right-angled triangle is increased by 6 cm and that of the other leg decreased by 4 cm, then the area of the triangle decreases by 24 sq cm. Find the length of the leg of the original triangle.
(a) 36 cm (b) 30 cm
(c) 24 cm (d) None of these

Q.45. ABCD is a parallelogram in which $\angle D = 120°$. The bisector of $\angle D$ bisects the side AB. If the length of the bisector is 5 cm, then what is the perimeter of the parallelogram?

(a) 30 cm (b) 25 cm
(c) 40 cm (d) 15 cm

Q.46. Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the centre. Find the ungrazed area.

(a) 22 m^2 (b) 42 m^2(c) 84 m^2 (d) 168 m^2

Q.47. Euclid has a triangle in mind. Its longest side has length 20 and another of its sides has length 10. Its area is 80. What is the exact length of its third side?

(a) $\sqrt{260}$ (b) $\sqrt{250}$
(c) $\sqrt{240}$ (d) $\sqrt{270}$

Q.48. AB is the hypotenuse in the right angled triangle ABC. N is the point inside the triangle which divides the triangle in three equal parts (ΔABN, ΔCAN, ΔBCN). What is the distance between the circumcentre of this triangle from this point N?

(a) $\frac{AB}{4}$ (b) $\frac{AB}{6}$
(c) $\frac{AB}{3}$ (d) $\frac{2}{1+\sqrt{3}}$

Q.49. Two sides of a triangle are 4 and 5. Then, for the area of the triangle, which one of the following bounds is the sharpest?

(a) <10. (b) ≤ 10. (c) ≤ 8. (d) > 5.

Q.50. ABC is a triangle and P is a point inside it such that $\angle BPC = \angle CPA = \angle APB$. Then P is:

(a) The point of intersection of medians
(b) The incentre

MODERATE

(c) The circumcentre
(d) None of these

Q.1. The angles of a triangle are in the ratio of 4:1:1. Then the ratio of the largest side to the perimeter is

(a) $\frac{2}{3}$ (b) $\frac{1}{2+\sqrt{3}}$
(c) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (d) $\frac{2}{1+\sqrt{3}}$

Q.2. In ΔABC, P and Q are mid-points of sides AB and BC respectively, right angled at B, then

(a) $AQ^2 + CP^2 = AC^2$
(b) $AQ^2 + CP^2 = \frac{4}{5} AC^2$
(c) $AQ^2 + CP^2 = \frac{3}{4} AC^2$
(d) AQ2 + CP2 $= \frac{5}{4}$ AC2

Q.3. Three identical right angle cones with base radius r are placed on their bases so that each is touching the other two. The radius of the circle drawn through their vertices is:

(a) Smaller than r
(b) Equal to r
(c) Larger than r
(d) Depends on the height of the cones

Q.4. The line AB is 6 m, in length and is tangent to the inner of the two concentric circles at point C. It is known that the radii of the two circles are integers. The radius of the outer circle is-------, where A and B are points on the outer circle.

(a) 5 m (b) 4 m (c) 6 m (d) 3 m

Q.5. Under the usual 2-dimensional coordinate system the equation $|x| + |y| = 1$, where x and y are real numbers, represents

(a) A rhombus which is not a square
(b) A parallelogram which is not a rhombus
(c) A square whose sides are not parallel to the coordinate axes
(d) A square with sides parallel to the coordinate axis

Q.6. Two chords of lengths a and b of a circle subtend 60° and 90° angles at the centre respectively. Which of the following is correct?

(a) $b = \sqrt{2a}$ (b) $b = \sqrt{2b}$
(c) $a = 2b$ (d) $b = 2a$

Q.7. PQR is a right-angled triangle with $\angle Q = 90°$, S is the mid-point of PR, and QS $= \sqrt{117}$ cm. The sum of the length of sides PQ and QR is 30 cm. Area of ∇PQR is

(a) 216 cm^2 (b) 108 cm^2
(c) 54 cm^2 (d) 162 cm^2

Q.8. In a trapezium, the diagonals intersect at point O. The ratio of the length of one of the diagonals from one vertex of the trapezium to the point O to its entire length is 2:5. Find the ratio of its parallel sides (smaller side:larger side).

(a) 2 : 5 (b) 2 : 3 (c) 2:7 (d) 5 : 7

Q.9. A chord of length 32 cm is placed inside a circle of radius 20 cm and a point whose distance from the centre of the circle is 13 cm, is marked on the chord. Calculate the lengths of the segment of the chord.

(a) 21 cm and 11 cm (b) 19 cm and 13 cm
(c) 16 cm each (d) 18 cm and 14 cm

Q.10. The sum of the lengths of the hypotenuse and one of the perpendicular sides of a right angled triangle is L. When the area of this triangle is maximum, the angle between these two sides is:

(a) 45° (b) 22.5°
(c) 60° (d) None of these

Q.11. In a triangle, the lengths of the two larger sides are 8 cm and 7 cm. The angles of the triangle are in an arithmetic progression. The length of the remaining side can be:

(a) 3 cm
(b) 5 cm
(c) Either 3 cm or 5 cm
(d) None of these

Q.12. The biggest possible regular hexagon H is cut out of an equilateral triangle X. The biggest possible equilateral triangle Y is cut out from the hexagon H. What is the ratio of the areas of the equilateral triangles X and Y?

(a) 5 : 1 (b) 6 : 1 (c) 8 : 1 (d) 3 : 1

Q.13. Nine parallel chords are drawn in a circle of diameter 10 cm. If the distance between any two of the adjacent chords is 1 cm, which of the following statements is always true?

(a) One of the chords is the diameter of the circle.
(b) At least two of the chords must be of an equal length.
(c) The difference between the lengths of any two adjacent chords on the same side of the diameter is greater than 1 cm.
(d) None of these

Direction for questions 14 and 15: Read the passage below and solve the questions based on it.

There are three equal circles of unit radii touching each other.

Q.14. Find the area of the triangle circumscribing the three circles.

(a) $\sqrt{3}(\sqrt{3}+1)^2$ (b) $(\sqrt{3}+1)^2$
(c) $\sqrt{3}(\sqrt{3}+1)$ (d) None of these

Q.15. What would be the area of the remaining portion if the same three circles are circumscribed by another circle?

(a) $\pi\left(\frac{2}{\sqrt{3}}+1\right)^2$ (b) $\pi\left(\frac{2}{3}+1\right)^2-3\pi$
(c) $\left(\frac{3}{\sqrt{2}}+1\right)^2\pi-3\pi$ (d) $\left(\frac{1}{\sqrt{3}}+1\right)^2\pi-3\pi$

Q.16. A circle passes through the vertex A of an equilateral triangle ABC and is tangent to BC at it mid-point. Find the ratio in which the circle divides each of the sides AB and AC.

(a) 1 : 1 (b) 3 : 2 (c) 3 : 1 (d) 2 : 1

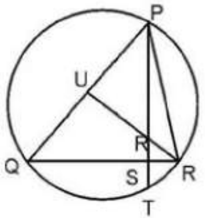
Q.17. In any quadrilateral ABCD, the diagonal AC and BD intersect at a point X. If E, F, G and H are the mid-points of AX, BX, CX and DX respectively, then what is the ratio of (EF + FG + GH + GE) to (AD + DC + CB + BA)?

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$
(c) $\frac{3}{4}$ (d) Data insufficient

Q.18. In ∇PQR, the incircle touches the sides QR, RP and PQ at T, U and S respectively. If the radius of the incircle is 4 units and QT, RU and PS are consecutive integers, then what is the area of the triangle (in sq. units)?

(a) 42 (b) 84 (c) 64 (d) 96

Q.19. If altitude PS meets the circumcircle of ∇PQR at T and H is the orthocenter, then what is the length of the line segment HT? (Given that $\angle RUP = \angle PSQ = 90°$)



(a) 4 cm (b) 5 cm (c) 6 cm (d) 8 cm

Q.20. A vertical lamp-post OP stands at the centre O of a square ABCD. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60°$ then the relationship between h and b can be expressed as

(a) $2b^2 = h^2$ (b) $2h^2 = b^2$
(c) $3b^2 = 2h^2$ (d) $3h^2 = 2b^2$

Q.21. Consider two different cloth-cutting processes. In the first one, *n* circular cloth pieces are cut from a square cloth piece of side a in the following steps: the original square of side a is divided into n smaller squares, not necessarily of the same size; then a circle of the maximum possible area is cut from each of the smaller squares. In the second process, only one circle of the maximum possible area is cut from the square of side a and the process ends there. The cloth pieces remaining after cutting the circles are scraped in both the process. The ratio of the total area of the scrap cloth generated in the former to that in the latter is

(a) 1:1 (b) $\sqrt{2}:1$

(c) $\frac{n(4-x)}{4n-x}$ (d) $\frac{4n-x}{n(4-x)}$

Q.22. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$ respectively, then the ratio of $\frac{P_1+P_2+P_3+\ldots..}{A_1+A_2+A_3+\ldots...}$ equals.

(a) $\frac{(2+\sqrt{2})}{a}$ (b) $\frac{2(2-\sqrt{2})}{a}$

(c) $\frac{2(2+\sqrt{2})}{a}$ (d) $\frac{2(1+2\sqrt{2})}{a}$

Q.23. One of the angles of a parallelogram is of 150°. Altitudes are drawn from the vertex of this angle. If these altitudes measure 6 cm and 8 cm, then find the perimeter of the parallelogram.

(a) 28 cm (b) 42 cm

(c) 56 cm (d) 64 cm

Q.24. A square, whose side is 2 meters, has its corners cut away so as to form an octagon with all sides equal. Then, the length of the each side of the octagon, in meters is:

(a) $\frac{\sqrt{2}}{\sqrt{2}+1}$ (b) $\frac{2}{\sqrt{2}-1}$

(c) $\frac{2}{\sqrt{2}-1}$ (d) $\frac{\sqrt{2}}{\sqrt{2}+1}$

Q.25. A certain city has a circular wall around it and this wall has four gates pointing north, south, east and west. A house stands outside the city three km north of the north gate, and it can be seen from a point nine km east of the south gate. What is the diameter of the wall that surrounds the city?

(a) 6 km (b) 9 km

(c) 12 km (d) None of these

Q.26. The perimeter of a right-angled triangle is four times the shortest side. The ratio of the other two sides is

(a) 5:6 (b) 3:4

(c) 4:5 (d) 2:3

Q.27. A circle is inscribed in an equilateral triangle and a square is inscribed in the circle. The ratio of the area of the triangle to the area of the square is

(a) $\sqrt{3}:\sqrt{2}$ (b) $3\sqrt{3}:2$

(c) $3:\sqrt{2}$ (d) $\sqrt{2}:1$

Q.28. The sides of a triangle are given to be $x^2 + x + 1$, $2x + 1$ and $x^2 - 1$. Then the largest of the three angles of the triangle is

(a) 75° (b) $\frac{x}{x+1}\pi$

(c) 120° (d) 135°

Q.29. ABC is a triangle inscribed in a circle. AD and AE are straight lines drawn from the vertex A to the base BC parallel to the tangents at B and C respectively. If AB = 5 cm AC = 6 cm, and CE = 9 cm, then the length of BD (in cm) equals.

(a) 7.5 (b) 10.8 (c) 7.0 (d) 6.25

Q.30. In a triangle ABC, the medians AM and CN to the sides BC and AB respectively, intersect at the point O. Let P be the mid-point of AC and let MP intersect CN at Q. If the area of the triangle OMQ is s square units, the area of ABC is

(a) 16s (b) 18s (c) 12s (d) 24s

Q.31. ABCD is a cyclic quadrilateral and the points A, B and C form an equilateral triangle. What is the sum of the lengths of line segments DA and DC?

(a) DB (b) DB/2 (c) √2 DB (d) DB/√2

Q.32. PQRS is a trapezium with PQ and RS parallel, PQ = 6 cm, QR = 5 cm, RS = 3 cm and PS = 4 cm. The area of PQRS is:

(a) 27 cm² (b) 12 cm²

(c) 18 cm² (d) None of these

Q.33. The side AB of a rectangle ABCD is a tangent to a circle which passes through the points C and D. The centre of the circle does not lie within the rectangle ABCD. If the length of the rectangle is twice the breadth, then what is the radius of the circle (in terms of breadth of the rectangle)?

(a) Breadth (2 − √2) (b) Breadth

(c) 2 × Breadth (d) None of these

Q.34. Let C_1 and C_2 be the inscribed and circumscribed circles of a triangle with sides 3 cm, 4 cm and 5 cm. The $\frac{\text{area of } C_1}{\text{area of } C_2}$ equals.

(a) $\frac{16}{25}$ (b) $\frac{4}{25}$ (c) $\frac{9}{25}$ (d) $\frac{9}{16}$

Q.35. The sides of a triangle are given by: $\sqrt{b^2+c^2}, \sqrt{c^2+a^2}$ and $\sqrt{a^2+b^2}$ where a, b, c are positive. Then the area of the triangle equals.

(a) $\frac{1}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$

(b) $\frac{1}{2}\sqrt{a^4+b^4+c^4}$

(c) $\frac{\sqrt{3}}{2}\sqrt{b^2c^2+c^2a^2+a^2b^2}$

(d) $\frac{\sqrt{3}}{2}(bc+ca+ab)$

Q.36. Let the bisector of the angle at C of a triangle ABC intersect the side AB in a point D. Then the geometric mean of CA and CB:

(a) Is less than CD
(b) Is equal to CD
(c) Is greater than CD
(d) Does not always satisfy any one of the foregoing property

Q.37. There is a pole at the centre of a stadium which is in the shape of an equilateral triangle. The angle subtended by any side of the stadium at the top of the tower is right-angled. What is the ratio of the height of the pole to the perimeter of the stadium?

(a) $\sqrt{6}$ (b) $\frac{\sqrt{6}}{18}$
(c) $\frac{\sqrt{6}}{3}$ (d) $\frac{\sqrt{6}}{4}$

Q.38. If in a triangle ABC with a, b, c denoting sides opposite to angles A, B and C respectively, a = 2 b and A = 3 B, then the triangle

(a) Is isosceles
(b) Is right-angled but not isosceles
(c) Is right-angled and isosceles
(d) Not necessarily be any of the above types

Q.39. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is (in cm)

(a) 24 (b) 25 (c) 15 (d) 20

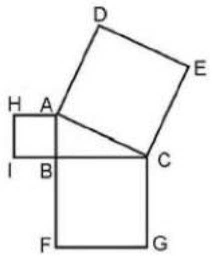
Q.40. PQRS is a trapezium with PQ and RS parallel. PQ = 6 cm, QR = 5 cm, RS = 3 cm, PS = 4 cm. The area of PQRS is

(a) 27 cm^2 (b) 12 cm^2
(c) 18 cm^2 (d) Cannot be determined

Q.41. If the number of square inches in the area of a square is equal to the number of inches in its circumference, then the diagonal of the square is equal to

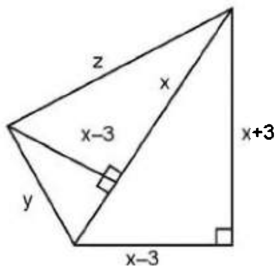
(a) 4 (b) $4\sqrt{2}$ (c) $3\sqrt{2}$ (d) $\sqrt{2}$

Q.42. All the three quadrilaterals ADEC, ABIH and BCGF are squares and ABC = 90°. If the area of ADEC = x^2 and the area of AHIB = y^2 ($x^2 > y^2$), then the area of BCGF is:



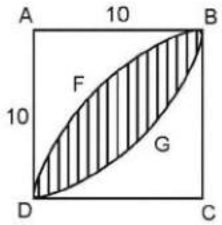
(a) $(x + y)(x - y)$ (b) $(x + y)^2$
(c) $(x - y)^2$ (d) None of these

Q.43. Based on the figure below, what is the value of x, if $y = 10$.



(a) 10 (b) 11
(c) 12 (d) None of these

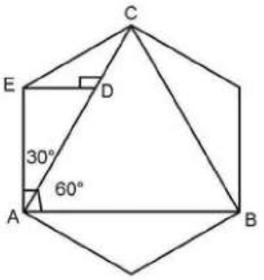
Q.44. In the figure, ABCD is a square with side 10. BFD is an arc of a circle with centre C. BGD is an arc of a circle with centre A. What is the area of the shaded region?



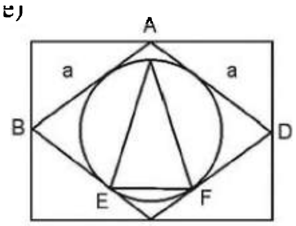
(a) $100\pi - 50$ (b) $100 - 25\pi$
(c) $50\pi - 100$ (d) $25\pi - 10$

Q.45. In the figure below, if the perimeter of ∇ABC is p, then the perimeter of the regular hexagon is

(a) $\frac{3p}{\sqrt{2}}$ (b) $\frac{\sqrt{2}p}{3}$
(c) $\frac{\sqrt{3}p}{2}$ (d) $\frac{2p}{\sqrt{3}}$



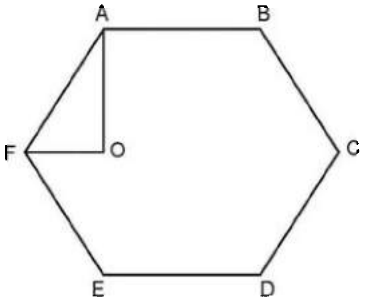
Q.46. What is the area of the inner equilateral triangle if the side of the outermost square is 'a'? (ABCD is a square)



(a) $\frac{3\sqrt{3}a^2}{32}$ (b) $\frac{3\sqrt{3}a^2}{64}$
(c) $\frac{5\sqrt{3}a^2}{32}$ (d) $\frac{\sqrt{3}a^2}{16}$

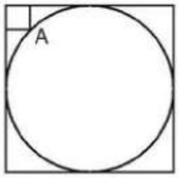
Q.47. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, then the number of concaves must be
(a) 20 (b) 0
(c) 21 (d) 22

Q.48. In the figure below, ABCDEF is a regular hexagon and AOF = 90°. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?



(a) $\frac{1}{2}$ (b) $\frac{1}{6}$
(c) $\frac{1}{24}$ (d) $\frac{1}{18}$

Q.49. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



(a) 10 cm (b) 40 cm
(c) 50 cm (d) None of these

Q.50. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was a 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?
(a) 16.665 (b) 16.565
(c) 15.465 (d) 14.365

ADVANCED

Q.1. A ball of diameter 15 cm is floating so that the top of the ball is 5 cm above the smooth surface (water) of the pond. What is the circumference in centimeters of the circle formed by the contact of the water surface with the ball?
(a) $10\sqrt{2}\pi$ (b) 50 p
(c) 10 p (d) $5\sqrt{2}\pi$

Q.2. Let S be as an arbitrary point on the side PQ of an acute-angled ∇PQR. Let T be the point of intersection of QR extended with the straight line PT drawn parallel to SR through P. Let U be the point of intersection of PR extended with the straight line QU drawn parallel to SR through Q. If PT = a and QU = b, then the length of SR is
(a) $\frac{a+b}{ab}$ (b) $\frac{a-b}{ab}$
(c) $\frac{ab}{a+b}$ (d) $\frac{ab}{a-b}$

Q.3. The municipal authorities of three cities A, B and C jointly constructed three straight roads connecting these cities. The area enclosed by three roads is $150\sqrt{39}$ km^2. The authorities of city B found the length of the road connecting it to the city A is 40 km and that to the city C is 50 km. What is the length of the road connecting the cities A and C, if it is the longest of the three roads?
(a) $20\sqrt{3}$ km (b) $30\sqrt{3}$ km
(c) $30\sqrt{6}$ km (d) $10\sqrt{55}$ km

Q.4. In an equilateral D, 3 coins of radii 1 unit each are kept in such a way that they touch each other and also the sides of the triangle. What is the area of the triangle (in sq. units)?
(a) $4 + 5\sqrt{2}$ (b) $6 + 4\sqrt{3}$
(c) $4 + 6\sqrt{3}$ (d) $3 + 8\sqrt{3}$

Q.5. In a scalene Δ, sum of all the sides can be at most 13 units. How many triangles are possible?
(a) 3 (b) 4 (c) 5 (d) 6

Q.6. Three circles of equal radii have been drawn inside an equilateral triangle, of side a, such that each circle touches the other two circles as well as two sides of the triangles. Then, the radius of each circle is

(a) $\frac{a}{2(\sqrt{3}+1)}$ (b) $\frac{a}{2(\sqrt{3}-1)}$

(c) $\frac{a}{\sqrt{3}-1}$ (d) $\frac{a}{\sqrt{3}-1}$

Q.7. Consider a circle with unit radius. There are seven adjacent sectors, $S_1, S_2, S_3, \ldots S_7$, in the circle such that their total area is $\frac{1}{8}$ th of the area of the circle. Further, the area of the jth sector is twice that of the (j – 1)th sector, for j = 2,… 7. What is the angle, in radians, subtended by the arc of S_1 at the centre of the circle?

(a) $\frac{\pi}{508}$ (b) $\frac{\pi}{2040}$

(c) $\frac{\pi}{1016}$ (d) $\frac{\pi}{1524}$

Q.8. In a triangle ABC, let C_1 be any point on the side AB other than A or B. Join CC_1. The line passing through A and parallel to CC_1 intersects the line BC extended at A_1. The line passing through B and parallel to CC_1 intersects the line AC extended at B_1. The lengths AA_1, BB_1, CC_1 are given to be *p*, *q*, *r* units respectively. Then

(a) $r = \frac{pq}{p+q}$ (b) $r = \frac{p+q}{4}$

(c) $r = \frac{\sqrt{pq}}{2}$ (d) None of these

Q.9. Two circles APQC and PBDQ intersect each other at the points P and Q and APB and CQD are two parallel straight lines. Then only one of the following statements is always true. Which one is it?
(a) ABDC is a cyclic quadrilateral
(b) AC is parallel to BD
(c) ABDC is a rectangle
(d) ΔACQ is a right angle

Q.10. Let s = $\{(x, y) : |x| + |y| = 2\}$. Then, the diameter of S is

(a) 2 (b) $4\sqrt{2}$

(c) 4 (d) $2\sqrt{2}$

Q.11. Through the centroid of an equilateral triangle, a line parallel to the base is drawn. On this line, an arbitrary point P is taken inside the triangle. Let h denote the distance of P from the base of the triangle. Let h_1 and h_2 be the distance of P from the other two sides of the triangle. Then

(a) $h = \frac{h_1 + h_2}{2}$ (b) $h = \sqrt{h_1 h_2}$

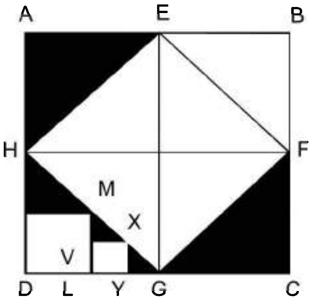
(c) $h = \frac{2h_1 h_2}{h_1 + h_2}$ (d) None of these

Q.12. Suppose ABCD is a cyclic quadrilateral within a circle of radius r. The bisector of the angle A cuts the circle at point P and the bisector of angle C cuts the circle at point Q. Then
(a) AP = 2r (b) PQ = 2r
(c) BQ = DP (d) PQ = AP

Q.13. Let A (x_1, y_1), B (x_2, y_2) C (x_3, y_3), D (x_4, y_4) be four points such that x_1, x_2, x_3, x_4 and y_1, y_2, y_3, y_4 are both in an AP. If Δ denotes the area of the quadrilateral ABCD, then
(a) Δ = 0
(b) Δ = 1
(c) Δ < 1
(d) Δ depends on the coordinates A, B, C, and D.

Direction for questions 14 to 17: Read the passage below and solve the questions based on it.

In the figure given below, EFGH is a square formed by joining the mid-points of the sides of the square ABCD. KMID and VXYL are the squares formed inside the right angled triangles HDG and MLG respectively. The side of the square ABCD is equal to '*a*' cm.



Q.14. What is the ratio of areas of ABCD and KMID?
(a) 4:1 (b) 8:1 (c) 16:1 (d) 12:1

Q.15. What is the ratio of areas of EFGH and VXYI?
(a) 32:1 (b) 64:1 (c) 16:1 (d) 8:1

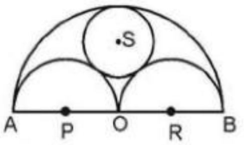
Q.16. What is the ratio of areas of the inscribed circles of DXVM, DMKH and DGDH?
(a) 1:4:16 (b) 1:2:4 (c) 1:3:9 (d) 1:2:√2

Q.17. What is the area of shaded region (in cm^2)?

(a) $\frac{5}{16}a^2$ (b) $\frac{19}{64}a^2$

(c) $\frac{7}{16}a^2$ (d) $\frac{27}{64}a^2$

Q.18. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi-circles with centres at P and R respectively and S is the centre of the circle touching the two semi-circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi-circle and the horse tied at S can graze within the circle centered at S. The percentage of the area of the semi-circles with diameter AB that cannot be grazed by the horses is nearest to

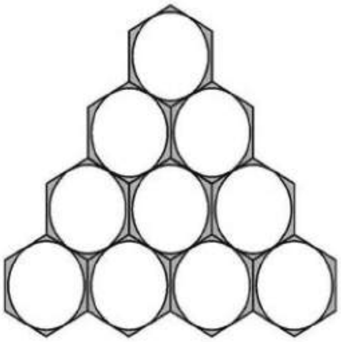


(a) 20 (b) 28
(c) 36 (d) 40

Q.19. Inradius of a right-angled ΔABC is 32 units and the sum of the length of its two sides other than the hypotenuse is 42 units. What is its circumradius?
(a) 26
(b) 52
(c) More than one value possible
(d) Triangle ABC is not possible

Direction for questions 20 and 21: Read the passage below and solve the questions based on it.

In the figure given above, each hexagon is regular and has side measuring 'a' cm. it is also given that the last row has 'n' such regular hexagons and all the hexagons are inscribed by circles.



Q.20. Find the ratio of area of the shaded region to the area of unshaded region.

(a) $\frac{3a^2}{2}[n(n+1)]\left(\sqrt{3}-\frac{\pi}{2}\right)$

(b) $\frac{a^2}{2}[n(n+1)]\left(\frac{3\sqrt{3}}{2}-\pi\right)$

(c) $\frac{3a^2}{2}[n(n+1)]\left(\sqrt{3}-\frac{\pi}{2}\right)$

(d) None of these

Q.21. Suppose the above figure is a 3-D figure and each hexagon is inscribed by a sphere of maximum volume. The height of the hexagonal figure is just enough to contain the sphere, then find the volume of the shaded region (there are 'n' hexagonal figures of side 'a' in the last row).

(a) $\frac{\sqrt{3}}{2}a^3\left(\frac{3\sqrt{3}}{2}-\pi\right)[n(n+1)]a^3$

(b) $\frac{\sqrt{3}}{4}a^3\left(\frac{3\sqrt{3}}{4}-\pi\right)[n(n+1)]$

(c) $\frac{1}{2}\frac{\sqrt{3}}{2}a^3\left(\frac{3\sqrt{3}}{4}-\pi\right)[n(n+1)]$

(d) $\frac{\sqrt{3}}{2}a^3(3\sqrt{3}-\pi)[n(n+1)]$

Q.22. If a sphere of the maximum volume is placed inside a hollow right circular cone with radius 'r' and slant height 'λ' such that the base of the cone touches the sphere, then the volume of the sphere is

(a) $\frac{4}{3}\pi\left(\frac{\ell+r}{\ell-r}\right)^3$ (b) $\frac{4}{3}\pi r^3\left(\frac{\ell-r}{\ell+r}\right)^{\frac{3}{2}}$

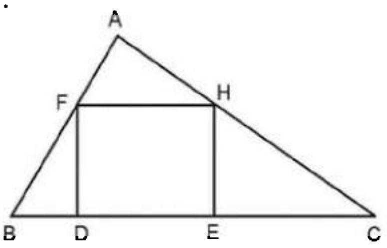
(c) $\frac{4}{3}\pi\left(\frac{\ell-r}{\ell+r}\right)^3$ (d) $\frac{4}{3}\pi r^3\left(\frac{\ell+r}{\ell-r}\right)^{\frac{3}{2}}$

Q.23. A hollow right circular cylinder of radius r and height 4r is standing vertically on a plane. If a solid right circular cone of radius 2r and height 6r is placed with its vertex down in the cylinder, then the volume of the portion of the cone outside the cylinder is

(a) $\frac{8}{3}\pi r^3$ (b) $2\pi r^3$

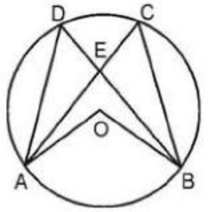
(c) $\frac{9}{8}\pi r^3$ (d) $7\pi r^3$

Q.24. Consider the following figure: AB = 10 cm, AC = 17 cm, BC = 21 cm and EHFD is a square. Find the length of the side of square (in cm).



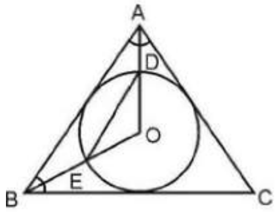
(a) 10.5 (b) 12
(c) 13.5 (d) None of these

Q.25. In the given figure, O is the centre of the circle and $\angle AOB = 120°$. What is the mesure of $\angle AEB$?



(a) 100° (b) 90°
(c) 110° (d) Cannot be determined

Q.26. For the equilateral triangle ABC, D & E are mid points of AO and OB, and DE = 3 units. The circumference of the circle is equal to



(a) 4π units (b) $\pi\sqrt{2}$ units
(c) $2\pi\sqrt{3}$units (d) None of these

Q.27. The length of the sides CB and CA of a triangle ABC are given by a and b, and the angle C is $2\pi/3$. The line CD bisects the angle C and meets AB at D. Then the length of CD is

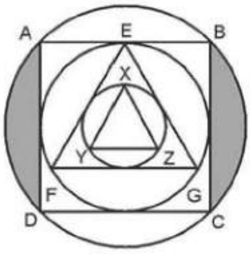
(a) $\dfrac{a^2+b^2}{2(a+b)}$ (b) $\dfrac{ab}{2(a+b)}$

(c) $\dfrac{1}{a+b}$ (d) $\dfrac{ab}{a+b}$

Q.28. For a regular octagon inscribed in a circle of radius 1 cm, the product of the distance from a fixed vertex to the other seven vertices is
(a) 4 (b) 8
(c) 12 (d) 16

Direction for questions 29 to 31: Read the passage below and solve the questions based on it.

In the figure given below a square ABCD is inscribed in a circle of radius 5 cm. The square ABCD is inscribed by a circle which is inscribed by a triangle EFG. The triangle EFG is inscribed by a circle which is again inscribed by a triangle XYZ.



Q.29. Find the ratio of the radius of the outermost circle to the radius of the radius of the circle inscribed in triangle XYZ.

(a) 4:1 (b) $2\sqrt{2}:1$

(c) $4\sqrt{2}:1$ (d) 8:1

Q.30. Find the ratio of the area of square ABCD and the area of triangle XYZ.

(a) $16:3\sqrt{3}$ (b) $64:3\sqrt{3}$

(c) 16:3 (d) $64:\sqrt{3}$

Q.31. Find the area of the shaded region (in cm^2).

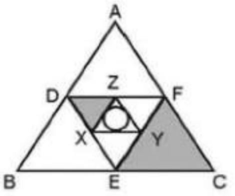
(a) $\dfrac{500\pi - 800 - 75\sqrt{3}}{16}$

(b) $\dfrac{900\pi - 1600 - 75\sqrt{3}}{}$

(c) $\dfrac{400\pi - 800 + 75\sqrt{3}}{32}$

(d) None of these

Q.32. Find the area of the shaded region, if D, E and F are the mid-points of the sides AB, BC and CA respectively and X, Y and Z the are midpoints of DE, EF and FD respectively. It is given that the circumradius of triangle ABC is $8\sqrt{3}$ cm and triangle XYZ is inscribed by a circle. (in cm^2)



(a) $48\sqrt{3}-3\pi$ (b) $48\sqrt{3}-\pi$

(c) $54\sqrt{3}-3\pi$ (d) $192\sqrt{3}-4\pi$

Q.33. The diameter of a right conical tent is 6 m. If a pole of length 2 m can be fixed in the tent at half the distance of the radius from the centre of the base, then the area of the canvas required is (in m^2)

(a) 10π (b) 12π

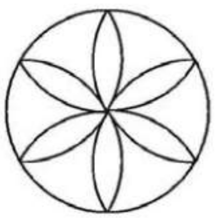
(c) 15π (d) 16π

Q.34. A pyramid has its base as an equilateral triangle, of each side being one m. Its slant edge is 3 m. The whole surface area of the pyramid is equal to.

(a) $\dfrac{\sqrt{3}+3\sqrt{13}}{4}m^2$ (b) $\dfrac{\sqrt{3}+2\sqrt{35}}{4}m^2$

(c) $\dfrac{\sqrt{3}+2\sqrt{13}}{4}$ (d) $\dfrac{\sqrt{3}+3\sqrt{35}}{4}m^2$

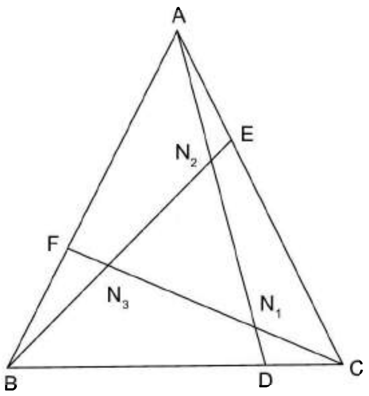
Q.35. In the figure given below a flower is inscribed in a circle of radius 1 cm, then find the area of the flower.



(a) $\left(\pi-\dfrac{3\sqrt{3}}{2}\right)\text{cm}^2$ (b) $(2\pi-3\sqrt{3})\text{cm}^2$

(c) $\left(\dfrac{5\pi}{3}-\dfrac{5\sqrt{3}}{2}\right)\text{cm}^2$ (d) $(4\pi-3\sqrt{3})\text{cm}^2$

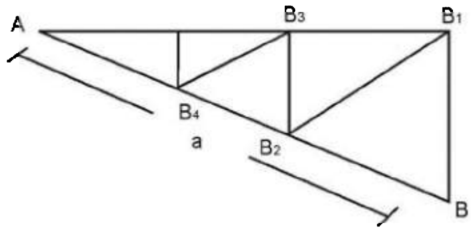
Q.36. In the figure given below CD, AE and BF are one third of their respective sides. It follows that $AN_2 : N_2N_1 : N_1D = 3:3:1$ and similarly for lines BE and CF. Then the area of triangle $N_1N_2N_3$ is



(a) $\dfrac{1}{7}\Delta ABC$ (b) $\dfrac{1}{6}\Delta ABC$

(c) $\dfrac{1}{9}\Delta ABC$ (d) $\dfrac{1}{10}\ \Delta ABC$

Q.37. In the figure given below, two rays are drawn through a point A at an angle of 30°. A point B is taken on one of them at a distance of 'a' from the point A. A perpendicular is drawn from the point B to the other ray, and another perpendicular is drawn from its foot to meet AB at another point from where the similar process is repeated indefinitely. The length of all such lines will be

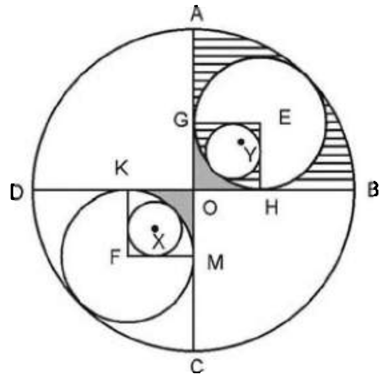


(a) $a(1-\sqrt{3})$ (b) $2a(2+\sqrt{3})$

(c) $\dfrac{3}{2a}$ (d) a

Direction for questions 38 to 40: Read the passage below and solve the questions based on it.

In the figure given below O is the centre of circles have radius (OB) + r. E and F are the centre of circles inscribed in the quarter circles AOB and COD respectively whereas X and Y are the centre of circles inscribed in the quarter circle KFM and GEH respectively.



Q.38. Find the area of the shaded region (not including the lined region).

(a) $\dfrac{3}{7}r^2(3-2\sqrt{2})$ (b) $\dfrac{3}{14}r^2(3-2\sqrt{2})$

(c) $3\dfrac{3}{14}r^2(3+2\sqrt{2})$ (d) None of these

Q.39. Find the ratio of the radii of the circle with centre Y, E and O.

(a) $1:(\sqrt{2}-1):(3+2\sqrt{2})$

(b) $1:(\sqrt{2}+1):(3-2\sqrt{2})$

(c) $1:(\sqrt{2}-1):(3-2\sqrt{2})$

(d) 1:2:4

Q.40. Find the area of the lined region.

(a) $\left(\dfrac{594\sqrt{2}-836}{14}\right)r^2$

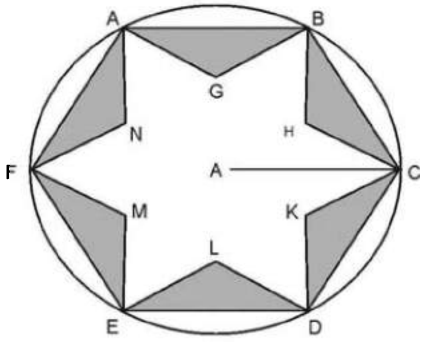
(b) $\left(\dfrac{600\sqrt{2}-836}{14}\right)r^2$

(c) $\left(\dfrac{594\sqrt{2}-845}{14}\right)r^2$

(d) $\left(\dfrac{600\sqrt{2}-845}{14}\right)r^2$

Direction for questions 41 to 43: Refer to the following information to answer the questions.

O is the centre of a circle having radiua (OC) = r. ABCDEF is a regular hexagon and AGBHCKDLEMFNA is a regular six pointed star.



Q.41. Find the ratio of the circumference of a circle to the perimeter of the hexagon ABCDEF to the perimeter of the star AGBHCKD LEMFNA.

(a) $\pi:\sqrt{3}$ (b) $\pi:3:2\sqrt{3}$

(c) $2\cdot\pi:6:4\sqrt{2}$ (d) $\pi:2\sqrt{3}$

Q.42. Find the ratio of the area of the shaded region to the area of the star.

(a) 1:1 (b) 1:2

(c) 1:3 (d) 2:3

Q.43. If the triangles ANG, BGH, CHK, DKL, ELM and FMN and the hexagon NGHKLM are inscribed by circles of the maximum area, then find the sum of the areas of all those circles.

(a) $\dfrac{\pi}{2}r^2$ (b) $\dfrac{\pi r^2}{3}$

(c) $\dfrac{5\pi r^2}{12}$ (d) $\dfrac{7\pi r^2}{12}$

Q.44. A triangle ABC with an obtuse angle B is inscribed in a circle. The altitude AD of the triangle is tangent to the circle. The side BC has length 12 cm and the segment BD has length 4 cm. Find the area of the triangle ABC (in cm^2).

(a) 48 (b) 60

(c) 72 (d) 36

Q.45. If the sum of the squares of the sides of a triangle is denoted by A and its perimeter by P, then which of the following is true about $\dfrac{A^2}{P}$?

(a) $\dfrac{A^2}{P}<2$ (b) $1\le\dfrac{A^2}{P}\le 4$

(c) $2\le\dfrac{A^2}{P}\le 3$ (d) $4\le\dfrac{A^2}{P}$

Q.46. A ring of diameter 10 cm is suspended from a point 12 cm vertically above the centre by six equal strings. The strings are attached to the circumference of the ring at equal intervals, thus keeping the ring in a horizontal plane. The value of cosine of the angle between the two adjacent strings lies in between

(a) 0 and 1/2 (b) 1/2 and 1

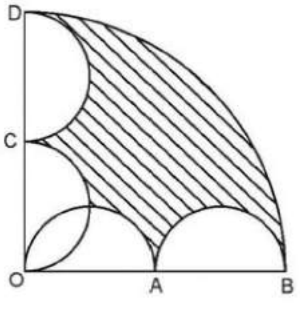
(c) −1/2 to 0 (d) −1 to −1/2

Q.47. A square of side 4 cm is drawn and a circle C_1 is inscribed in it. Now, four more circles C_2 C_3, C_4 and C_4, each of radius 2 cm, are drawn with centres as A, B, C and D respectively. In the resultant figure, what is the total area that is common to atleast two of the five circles (in sq. cms)?

(a) 6π−16 (b) 7π−16

(c) 8π−16 (d) 9π−16

Q.48. In the figure, find the area of the shaded portion, i.e., the portion of the quadrant DOB which is not included in the 4 semicircles. Given that OA = AB = OC = CD = 2 and ∠DOB = 90°.



(a) $5\pi - \frac{1}{2}$ (b) $5\pi - 1$

(c) $\frac{5\pi}{2} - \frac{1}{2}$ (d) $\frac{5\pi}{2} - 1$

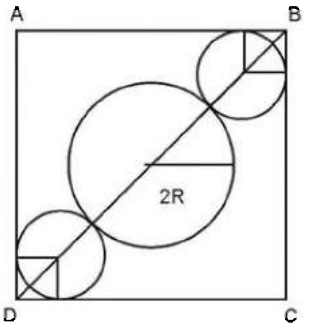
Q.49. Rajat cut out two identical triangular pieces of cardboard, each of area 300 sq. cm, and then placed them upon a table, one on top of the other such that the triangles completely coincide with each other. Now, if he rotated one of the two triangles by 180° about a vertical axis passing through its centroid, find the area that is common to both the triangles.

(a) 100 cm^2 (b) 150 cm^2

(c) 200 cm^2 (d) $133\frac{1}{3}$ cm^2

Q.50. See the figure given below:

Inside a square ABCD, three circles are drawn touching one another as shown in the figure. Radius of two smaller circles is R units and radius of the bigger circle is 2R units. Diagonal BD of the square passes through the centre of all the circles. What is the ratio of the radius of the smaller circles to the side of the square?



(a) 2:7 (b) 3:8

(c) 1:4 (d) None of these

ANSWERS

■■■

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(a)	(a)	(c)	(b)	(d)	(c)	(d)	(c)	(b)	(c)	(d)	(a)	(b)	(a)	(b)	(b)	(a)	(a)
Q. No.	21	22	23	24	25															
Answer	(b)	(d)	(d)	(c)	(b)															

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(b)	(b)	(d)	(d)	(d)	(d)	(d)	(c)	(b)	(b)	(a)	(d)	(d)	(c)	(a)	(d)	(b)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(a)	(b)	(a)	(b)	(c)	(c)	(b)	(c)	(b)	(d)	(c)	(b)	(b)	(b)	(d)	(d)	(a)	(c)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(c)	(b)	(a)	(d)	(a)	(b)	(a)	(b)	(b)	(d)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(c)	(a)	(c)	(a)	(b)	(b)	(a)	(c)	(c)	(d)	(d)	(a)	(b)	(c)	(a)	(b)	(c)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(a)	(c)	(c)	(a)	(b)	(c)	(b)	(c)	(d)	(d)	(a)	(c)	(b)	(b)	(a)	(c)	(b)	(b)	(a)	(c)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(b)	(a)	(d)	(c)	(d)	(d)	(c)	(a)	(c)	(d)										

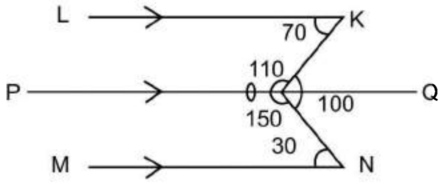
■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(b)	(b)	(c)	(a)	(c)	(a)	(b)	(c)	(a)	(c)	(a)	(c)	(a)	(a)	(b)	(b)	(d)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(d)	(b)	(d)	(d)	(d)	(c)	(d)	(b)	(c)	(b)	(d)	(b)	(c)	(d)	(b)	(a)	(c)	(a)	(c)	(d)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(b)	(b)	(c)	(a)	(c)	(b)	(c)	(d)	(c)	(d)										

HINTS AND SOLUTIONS

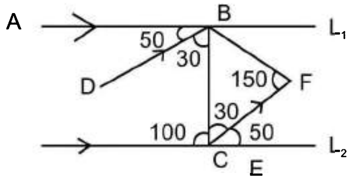
Warm up

1.


L
K
70
110
P
0
100
Q
150
30
M
N

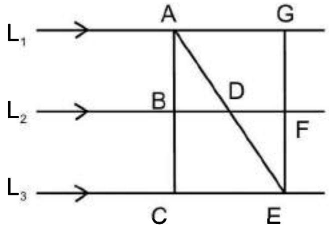
Hence, ∠MNO = 30º

2.


A
B
L₁
50
30
D
150
F
30
100
50
L₂
C
E

∠FCE=50º

3.


A
G
L₁
B
D
L₂
F
L₃
C
E

$$\frac{AB}{BC}=\frac{AD}{DE}\ \frac{4}{6}=\frac{12}{DE}$$

$$DE=\frac{12\times 6}{4}=18$$

$$\frac{ED}{EF}=\frac{DA}{FG},or,\frac{18}{EF}=\frac{12}{18}$$

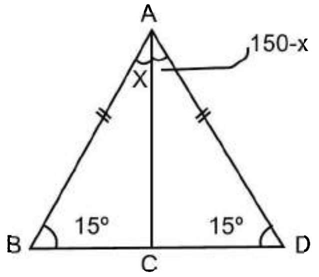
$$EF=24$$

4. $x + 5x = 180^\circ$

$6x = 180^\circ$

$x = 30^\circ$

5.


A
150-x
X
15°
15°
B
C
D

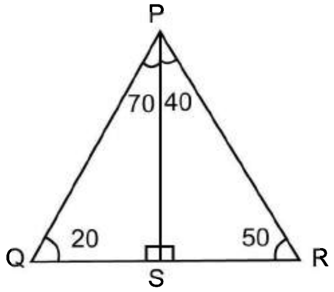
$150^\circ - x + 15^\circ = 95^\circ + x$

$2x = 70^\circ$

$x = 35^\circ$

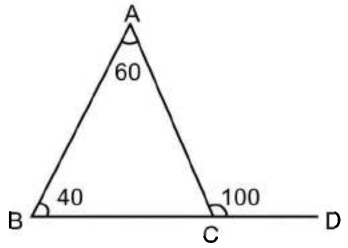
Therefore, $\angle ACB = 95^\circ + 35^\circ = 130^\circ$

6.

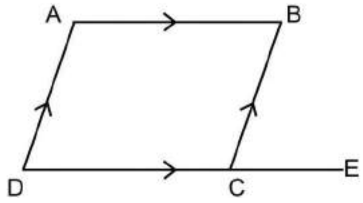

P
70
40
20
50
Q
S
R

$\angle QPR = 110^\circ$

7.


A
60
40
100
B
C
D

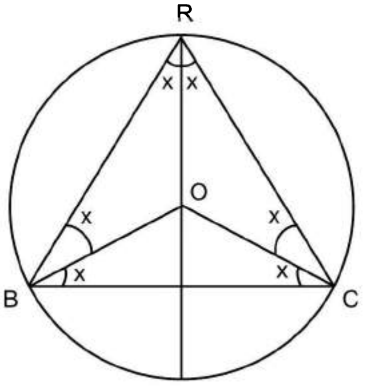
8.



Since A = 135°, So, B = D = 65°

We will find that (c) is the answer.

9.


R
x
x
O
x
x
x
x
B
C

Isosceles trapezium is always cyclic

The sum of opposite angles of a cyclic quadrilateral is 180°.

$\therefore \angle BCD + \angle BAD = 180^\circ$

$40^\circ + \angle BAD = 180^\circ$

$\angle BAD = 140^\circ$

10. $\dfrac{(2n-4)\times 90}{n} = 135$

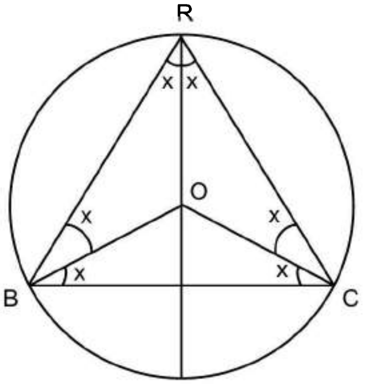
$(2n - 4)\, 2 = 3\, n$

$4n - 8 = 3n$

$n = 8$

No. of diagonals = $\frac{1}{2}n(n-3) = 20$

11.


R
x
x
O
x
x
x
x
B
C

$6x = 180^\circ$

$x = 30^\circ$

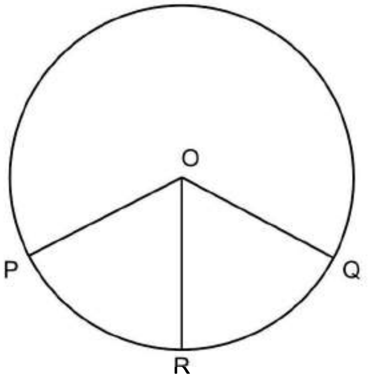
$\angle BAC = 2x = 2 \times 30 = 60^\circ$

12. AO × OB = OC × OD

or, 4 × 6 = 3 × OD

OD = 8 CM

13.



Let ∠POR = x

Therefore ∠ROQ = $\frac{2}{3}x$

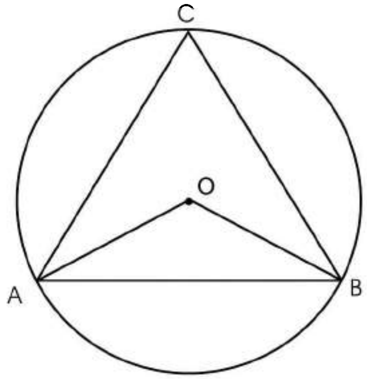
$$x+\frac{2}{3}x=100$$

$$\frac{5x}{3}=100$$

$x = 60°$

∴ Therefore ∠ POR = 60°

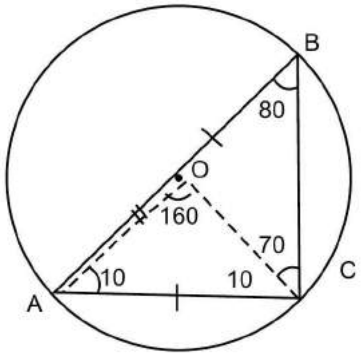
14.



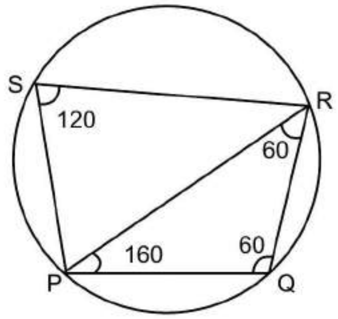
$\frac{1}{2}$ ∠ AOB = ∠ ACB

∠ ACB = 60°

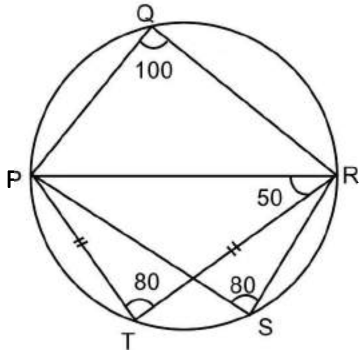
15.


B
80
O
160
70
10
10
A
C

16.

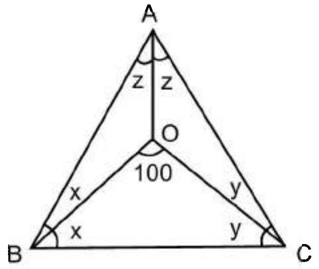

S
120
R
60
160
60
P
Q

17.


Q
100
P
R
50
80
80
T
S

∠ PRT = 50

18.


A
z
z
O
100
x
y
x
y
B
C

In ∠ BOC, $x + y = 80$

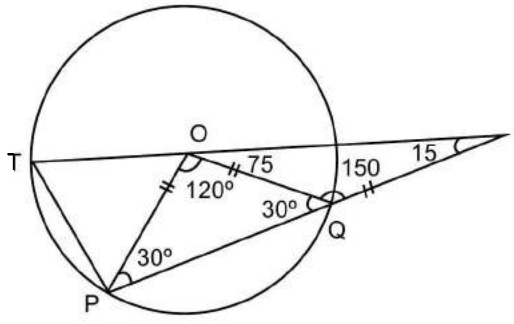
$2x + 2y = 160°$

Also, $2x + 2y + 2z = 180°$

$2z = 20°$

∠ BAC = $2z$ = 20°

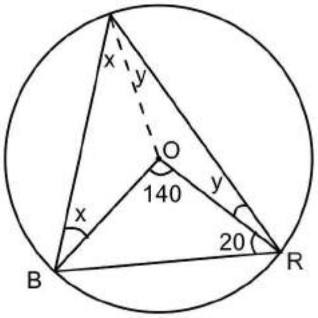
19.


O
T
75
150
15
120°
30°
Q
30°
P

∠ OQP = 30°

Therefore ∠ QOP = 120°

20.



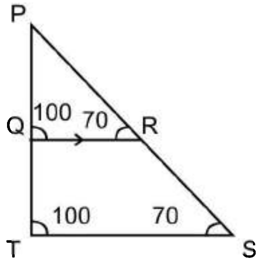
$\angle OQP + \angle ORP = 70^\circ$

$x + y = 70^\circ$

$\angle QPR = 70^\circ$

$\angle ORQ = 20^\circ$

21.



$PQ \times PS = PT \times PR$

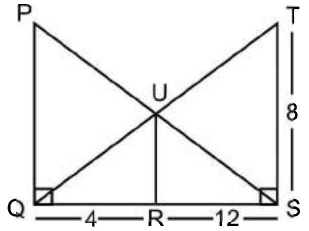
$QR \parallel TS$

$\angle PQR = \angle PST + 30^\circ$

$\angle PST = 70^\circ$

$\angle PRQ = 70^\circ$

22.



In $\angle QST$ and $\angle QRU$ ($\angle QST$ and $\angle QRU$ are similar triangles)

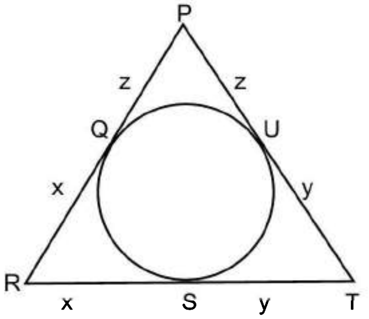
$$\frac{QR}{QS} = \frac{RU}{TS} \Rightarrow \frac{4}{16} = \frac{RU}{8} = RU = 2$$

$$\text{Also, } \frac{QR}{PQ} = \frac{RU}{TS} \Rightarrow \frac{4}{16} = \frac{RU}{8} = RU = 2$$

$$\frac{1}{PQ} = \frac{1}{2} - \frac{1}{8} = \frac{4-1}{8} = \frac{3}{8}$$

PQ = 8/3 cm

23.



$PR = RT + 3$

$x + z = x + y + 3$

$z - y = 3$ ………..(1)

Similarly, $z + x = z + y + 1$

Or, $x - y = 1$ ………..(2)

From 1 and 2

$z - 3 = x - 1$

$z - x = 2$ ………..(3)

Also, $2x + 2y + 2z = 26$

$x + y + z = 13$ ………..(4)

$3x + 1 = 13$

$3x = 12$

$x = 4$

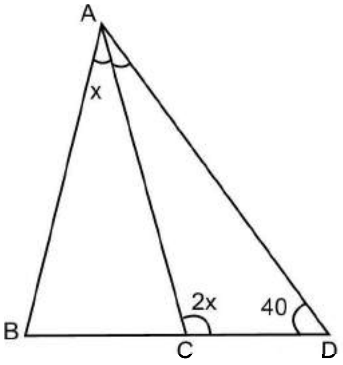
$z = 6$

and $y = 3$

$QR + PT = x + y + z$

$= 4 + 6 + 3 = 13$

24.



$85^\circ - x + 2x + 40^\circ = 180^\circ$

$x = 55^\circ$

$\angle ACB = 70^\circ$

25. Only 8 such regular polygons are possible.

Foundation

1. The triangle is not possible. So, (d) is the correct options
2. Clearly, triangle is obtuse. So (d) is the correct options

3. Clearly, triangle is a right angled triangle

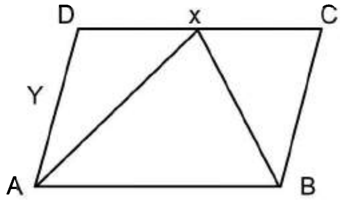
its area = $\frac{1}{2} \times 6 \times 8 = 24$ cm^2

Now, $8 \times b = 24$

$b = 3$

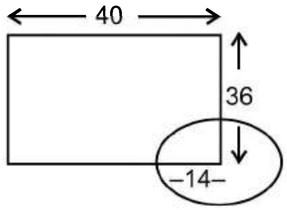
Required perimeter = 2 (8 + 3) = 22 cm

4.



We know that, between the same parallels and on the same base, the area of triangle is half the area of the parallelogram. Hence, Ans. (d)

5.



Required ratio = 3:1

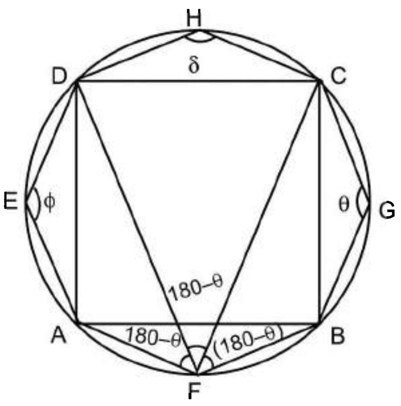
6. Triangle is not possible

7. Putting the value of n = 2, 3 and so on, we get (a) as answer

8. This is possible only when both length and breadth of rectangle are equal to the side of the square.

9. Clearly (d)

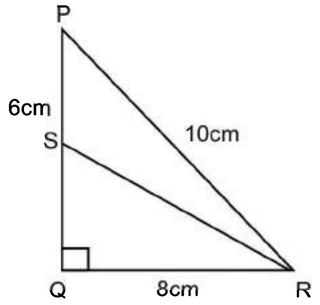
10.



Required sum

= $\theta + 180° - \theta + 180° - \theta + 180° - \theta + \theta + 8 = 540°$

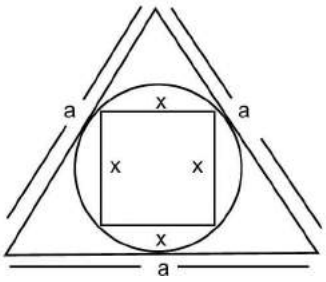
11.



Clearly, it is a right angle triangle at Q

$\text{RS} = \sqrt{8^2 + 3^2} = \sqrt{64 + 9} = \sqrt{73} \approx 8.5\text{cm}$

12.



Radius of circle $R = \dfrac{\frac{\sqrt{3}}{4}}{\frac{3a}{2}} = \dfrac{\sqrt{3}}{6}a$

Diagonal of square = diameter of circle

$= 2.\ \dfrac{\sqrt{3}}{6}a$

$\sqrt{2}x = \dfrac{\sqrt{3}}{3}a$

$x = \dfrac{\sqrt{3}}{3\sqrt{2}}a = \dfrac{\sqrt{6}}{6}a$

Area = $x^2 = \left[\dfrac{(\sqrt{6}a)}{6}\right]^2 = \dfrac{6}{36}a^2 = \dfrac{1}{6}a^2$

13. The two adjacent angles can be 30° and 60°, therefore others angle can be 150° and 120°

14. Going through options we find that only (d) satisfies the condition

15. 1 kg = 1000 cm^3

$2700 = k.2^3$

$k = \dfrac{2700}{8}$

$6300 = k\ .\ r^3$

$r^3 = \dfrac{6300}{k} = \dfrac{6300}{2700} = \dfrac{8}{3} \quad r^3 = \dfrac{56}{3}, \quad r = 2.6$ cm

16. $i\,(2,3) = 2\,(2+3) = 10$

$f(10) = 100$

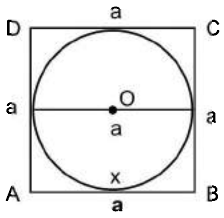
$h\,(100,2) = 200$

$g\,(200) = 80$

17. $$\frac{f(3)-g(2)}{h(4,2)-i(6,2)}\times\frac{g(6)}{f(1)}=\frac{9-8}{8-16}\times\frac{24}{1}$$

$$=\frac{1}{-8}\times 24=-3$$

18.



$4\,a + \pi\,a = 10$

$$a=\frac{10}{4+\pi}=1.4$$

Required Area $= (1.4)^2 - \pi\,(.7)^2$

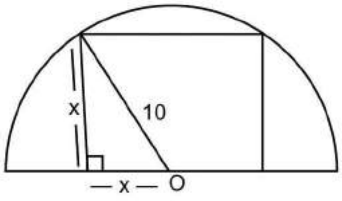
$$=2.56-\frac{22}{7}\times\frac{7}{10}\times\frac{7}{10}=196-1.54$$

$= 0.42\text{ m}^2 = 4200\text{ cm}^2$

19. $$\frac{.7}{4\times 1.42}=1.8$$

20. Going through option, we can easily eliminate (a), (b) and (d)
Hence, (c) is the answer

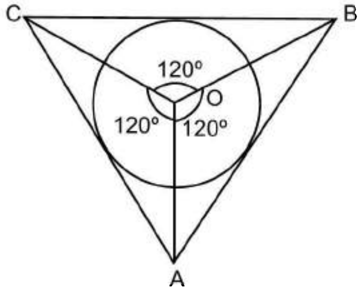
21.



$$x^2+\frac{x^2}{4}=100^{\circ}\ ,\ x^2 = 80^{\circ}$$

22. No such point is possible

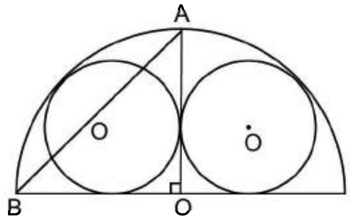
23.



Radius of disc $=\sqrt{3}$, OB = 20

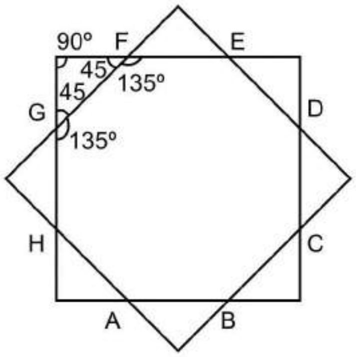
Use Sine law to find the length of AB = BC = AC

24.



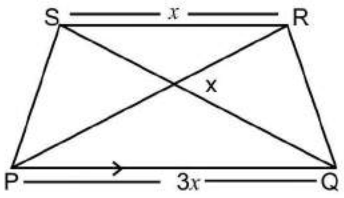
OA = OB = 2 cm, AB = 2 $\sqrt{2}$ cm

25.



Required angle = 90^0

26.



$$\frac{ar(\Delta P\times Q)}{ar(\Delta R\times S)}=\frac{PQ^2}{RS^2}=\frac{(3x)^2}{x^2}=9:1$$

27. Let the angle be x, xr, xr^2 and xr^3

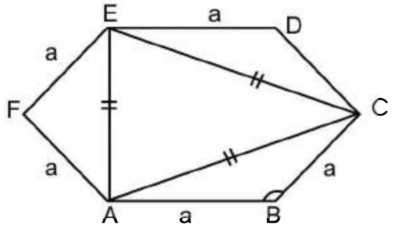
$x + xr + xr^2 + xr^3 = 360^{\circ}$

and $x\,(1 + r + r^2 + r^3) = 360^{\circ}$

therefore angles are 24°, 48°. 96°, 192° which satisfies the given condition.

Hence, answer to 27 is (c) and answer to 28 is (b)

30.



Clearly, Δ ACE is an equilateral triangle

$$\frac{AC}{Sin\,120^{\circ}}=\frac{a}{Sin\,30^{\circ}},or,\frac{AC}{Sin\,60^{\circ}}=\frac{a}{Sin\,30^{\circ}}$$

$$\rightarrow AC=\frac{aSin60^{\circ}}{Sin30^{\circ}}=\frac{a\cdot\sqrt{3/2}}{1/2}=\sqrt{3a}$$

$$\text{Required ratio} = \frac{\frac{\sqrt{3}}{4}(\sqrt{3}a)^2}{6.\frac{\sqrt{3}}{4}a^2}=\frac{3a^2}{6a^2}=\frac{1}{2}$$

31. $\frac{1}{2}:\frac{1}{3}:\frac{1}{4}=6:4:3$

$6x + 4x + 3x = 52$, or, $13x = 52$ and $x = 4$

Required length = 9 cm

32. Required Area = $\pi\left(\frac{104}{2}\right)^2-\pi.50^2$

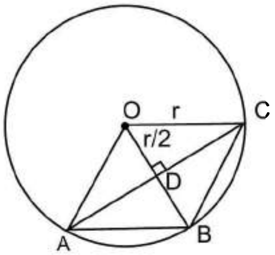
$=\pi.52^2-\pi.50^2=\pi(52-50)(52+50)$

$=\pi.2.102=204\pi$

33. Let h_1 = attitude of parallelogram and h_2 = altitude of triangle then $\frac{1}{2}h_2=h_1$

$h_2=2h_1=200\,m$

34.

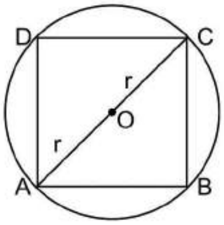


$$KD=\sqrt{r^2-\frac{r^2}{4}}=\frac{\sqrt{3}}{2}r$$

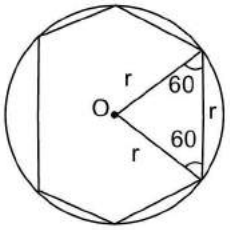
$\frac{1}{2}\cdot r\cdot\sqrt{3}r=32\times\sqrt{3}$, $r^2 = 32 \times 2$

$r^2 = 64$ and $r = 8$

35.



$A=\left(\sqrt{2}r\right)^2=2r^2$



$B=6\cdot\frac{\sqrt{3}}{4}r^2$

$$\frac{B}{A}=\frac{\frac{3}{2}\sqrt{3}r^2}{2r^2}=\frac{3\sqrt{3}}{4}$$

36. Let dimensions be x, $2x$ and $4x$

A/Q

$$20.5\times2\left[2x^2+8x^2+4x^2\right]-20\times2\left[2x^2+8x^2+4x^2\right]$$

= 120

or, $2 \times 14x^2 \times 0.5 = 126$

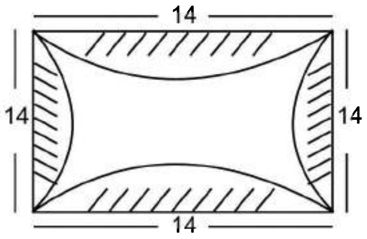
$x^2=\frac{63\times2}{14}\Rightarrow x=3$

therefore dimensions are 3, 6 and 12 meters.

37. Area = $\frac{1}{2}\cdot50\cdot130$ Sin 72°

$$=\frac{1}{2}\cdot50\cdot130\cdot\times0.9510=3090.75\,\text{m}^2$$

38.



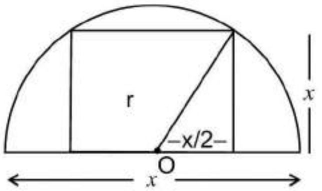
Area grazed by 4 horses = $2\cdot\pi\cdot7^2=308m^2$

Area ungrazed = $(196 - 112)$ m^2 = 84m^2

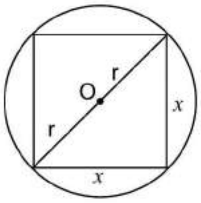
308 m^2 is grazed in – 11 days

84 m^2 is grazed in– 3 days

39.



Area of square = $\left(\frac{2}{\sqrt{5}}r\right)^2=\frac{4}{5}r^2$



Area = $2r^2$

Required ratio = $\dfrac{\frac{4}{5}r^2}{2r^2}$ = 2:5

40. $A = 8x^2 - 2x - 15$

$= 8x^2 - 12x + 10x - 15$

$= 4x(2x - 3) + 5(2x - 3)$

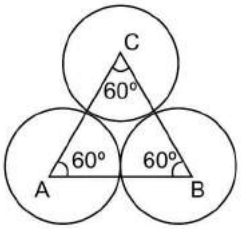
$A = (4x + 5)(2x - 3)$

Clearly, length = $(4x + 5)$

and breadth = $(2x - 3)$

41. Required ratio = $\dfrac{a^2}{(\sqrt{2}a)^2} = 1:2$

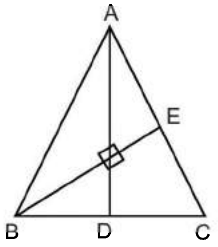
42.



Required area

$$= \frac{\sqrt{3}}{4}\cdot 7^2 - 3\cdot\frac{60}{360}\frac{22}{7}\times\left(\frac{7}{2}\right)^2 = 1.967\text{ cm}^2$$

43.



Area Δ AOB = $\frac{1}{2}\cdot 8.6 = 24\,\text{cm}^2$

Area of Δ BOD $= \frac{1}{2}\cdot 8.3 = 12\,\text{cm}^2$

Area of Δ ABD = 36 cm²

Area of ΔABC = 72 cm²

44. $\frac{1}{2}(x+6)(y-4) + 24 = \frac{1}{2}xy$

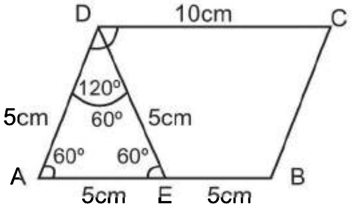
$\frac{1}{2}xy - \frac{1}{2}x\cdot 4 + \frac{1}{2}6\cdot y - \frac{1}{2}6.4 + 24 = \frac{1}{2}xy$

or, $-2x + 3y - 12 + 24 = 0$

$2x - 3y = 12$

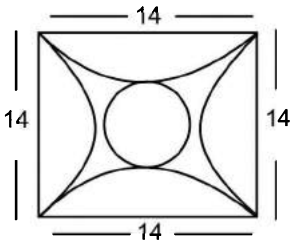
Now, putting the value of x or y from the options we get (d) as the answer.

45.



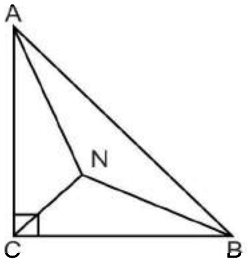
Perimeter = 30 cm

46.



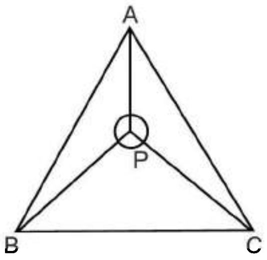
47. Going through option we get (a) as option

48.



49. Let AB = 4 and BC = 5 and AB is perpendicular to BC then Area = $\frac{1}{2}$ AB . AC $= \frac{1}{2}\cdot 4.5 = 10$

50.



First three options do not have the properties as given in the question

Moderate

1. ∠A:∠B:∠C : = 4:1:1 = x (let)

$4x + x + x = 180°$

$x = 30°$

Now, $\frac{\sin A}{\sin B} = \frac{A}{B}$ to solve get Ans. $\frac{A}{B} = \frac{\sqrt{3}}{1}$

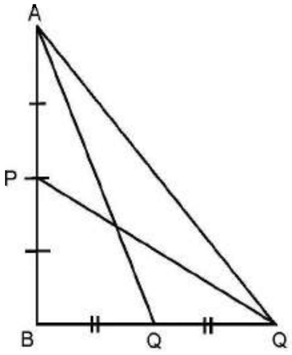
2. Use Pythagoras theorem, In right-angled ΔABQ

$$AQ^2 = AB^2 + \left(\frac{BC}{2}\right)^2 \qquad (1)$$

and in right-angled ΔCBP

$$CP^2 = \left(\frac{AB^2}{2}\right) + (BC)^2 \qquad (2)$$

add (1) and (2) to get the answer.

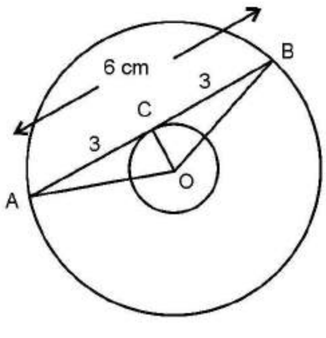


4. AC = 3 m

OC ⊥r AB

So, Δ AOI in right-angled triangle.

CO = 4 and AO = 5 cm



6. In ΔCOD

OC = OD = OB = OA = radius = r

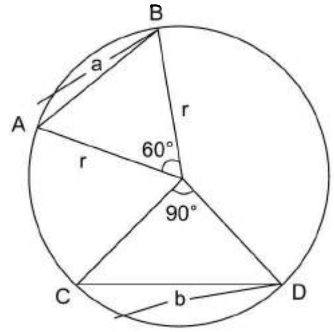
$b^2 = 2r^2$

$$r = \frac{b}{\sqrt{2}}$$

and ΔAOB= equilateral triangle

Hence, a = r

$r = \frac{b}{\sqrt{2}}$, hence $b = \sqrt{2a}$

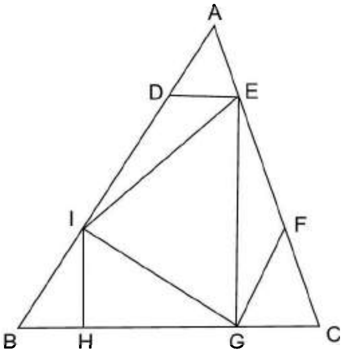


9. AQ = (16 + 5 = 21) cm and QB = 32 – 21 = 11 cm

12. If AB = 3a

then, DI = a = DE = EF = FG = GH = HI and IE = EG = IG = $\sqrt{3a}$

Now find the area.

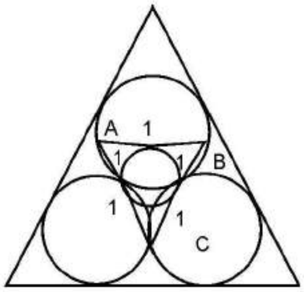


14. ΔABC is equilateral

tan 30 = $r/x = x = \sqrt{3r}$ (where r = 1)

AB = $2 + 2x = 2 + 2\sqrt{3} - 2\left(1+\sqrt{3}\right)$

$= \sqrt{3}\left(1+\sqrt{3}\right)^2$



15. Height of ΔABC = $\sqrt{\frac{3}{2}}a$

$\frac{\sqrt{3}}{2} \times 2h = h = \sqrt{3}$

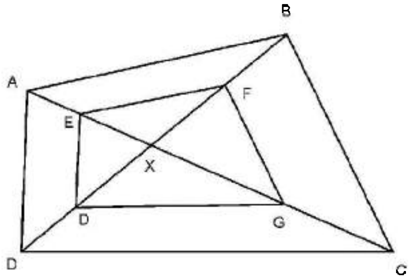
$\frac{2h}{3}$ is the radius of the circle inscribed in ABC

So radius of circumscribed circle

$$= \frac{2\sqrt{3}}{3} + 1 = \left(\frac{2}{\sqrt{3}} + 1\right)$$

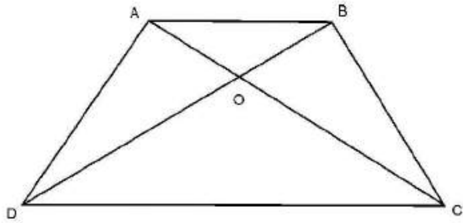
Hence, the required portion = $\pi\left(\frac{2}{\sqrt{3}} + 1\right)^2 - 3\pi$

17. This problem is based on mid-point theorem.

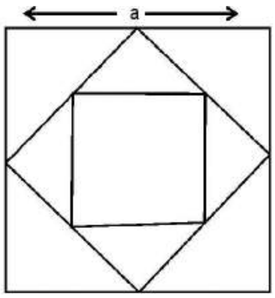


18. DAOB ~ DDOC

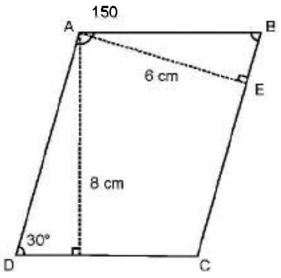
$$\therefore \frac{AO}{CO} = \frac{AB}{DC} = \frac{2}{3}$$



22. Find the sides of all the squares.
Then use geometric progression concept and get the answer.



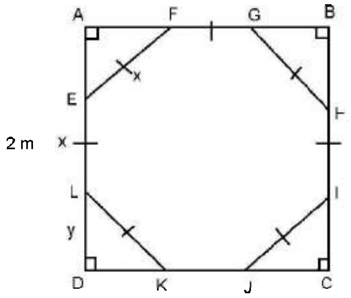
23. In ΔAEB, find AB and in ΔABF find AD.
then P = AB + BC + CD + AD.
Use this to get the answer.


150
A
B
6 cm
E
8 cm
30°
D
C

24. $x + 2y = 2$ (1)

In ΔAEF, $x^2 = 2y^2$ (2)

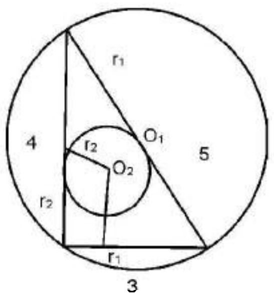
Use (1) and (2) to get the answer.


A
F
G
B
x
E
H
2 m
x
L
I
y
D
K
J
C

34. $r_1 = 2.5$ cm

r = 1 cm

Get the answer.


r₁
O₁
4
r₂
O₂
5
r₂
r₁
3

36. Use similarity theorem

40. Area of trapezium = $\frac{1}{2} \times \text{height}\ (l_1 + l_2)$

$SN \perp PQ$ and $RM \perp PQ$ gives RS = MN = 3, So, PN + QM = 3

Assume PN = x.

Then, in ΔPNS and ΔQMR (SN = RM)

$(SN)^2 = (RM)^2 = 4^2 - x^2 = 5^2 - (3 - x)^2$

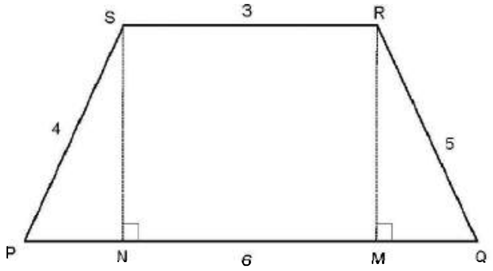
Finally, $x = 0$ = zero

That means,

$(SN)^2 = 4^2 - x^2$

SN = 4

So, Area = $\frac{1}{2} \times 4(3 + 6) = 18\ \text{cm}^2$


S
3
R
4
5
P
N
6
M
Q

Alternatively,
In trapezium PQRS, PN + MQ = 3. If we join ΔPSN and ΔMRQ through their altitude, it forms a triplet (3, 4, 5) of right-angled Δ making right angle at P, i.e., ∠SPQ = 90°. Then altitude = 4 cm.

Area of trapezium = $\frac{1}{2}\times(3+6)=18\text{ cm}^2$

41. Let side of square = a, then perimeter = $4a$

Area of square = a^2

According to the question, numerically $4a = a^2$
So, $a = 4$
Diagonal of square = $\sqrt{2}a = 4\sqrt{2}$

42. In square ADCE, area = x^2
So, side AC = x
In square AHIB, area = y^2

So, side AB = y

As $x^2 > y^2$ so, $x > y$

and in ΔABC (∠ABC = 90°)

$(AC)^2 - (AB)^2 = (BC)^2$

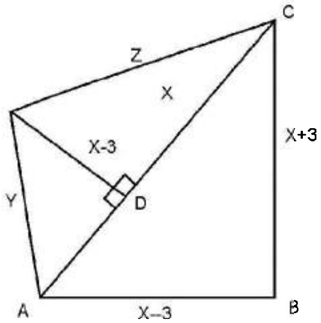
Hence area BCGF = $(AC)^2 - (AB)^2 = x^2 - y^2 = (x-y)(x+y)$

43. AD = $(y^2-(x-3)^2)^{½} = (100-(x-3)^2)^{½}$

DC = x

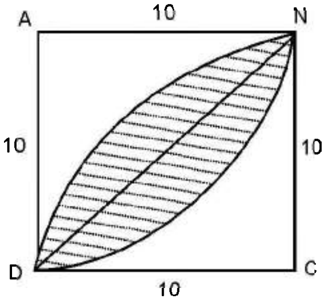
In ΔABC

$(x-3)^2 + (x+3)^2 = \left(x+\sqrt{100-(x-3)^2}\right)^2$



Now, putting the values of the given options, we can calculate the answer. It is none of these.
Alternatively, this question can be done very easily using the options.

44. In square ABCD
Area of shaded region = Area of sector BCD – Area of ΔBCD + Area of sector ABD – Area of ΔABD
Since, all sides of a square are same so,
Area of sectors and triangles will be the same having same radius (side of square) and angle 90°
So, area of shaded region = 2 (Area of Sector – Area of ΔABD)



$= 2\left(\frac{\pi}{4}(AB)^2 - \frac{1}{2}\times AD\times BA\right)$

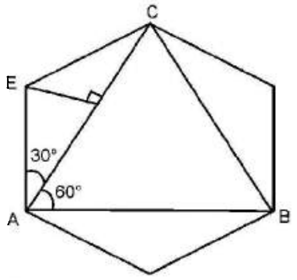
$= (AB)^2\left(\frac{\pi}{2}\right) = 100\left(\frac{\pi}{2}-1\right)$

$= 50\pi - 100$

45. As, hexagon is regular and AD = CD
So, ΔABC is equilateral triangle with

$AB = BC = AC = \frac{P}{3}$

$AD = \frac{P}{6}$



So,

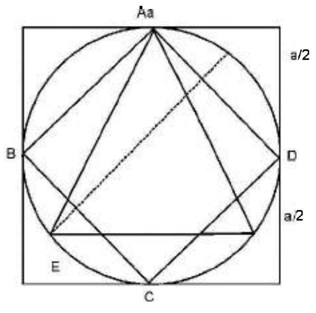
Cos 30° $\frac{\sqrt{3}}{2} = \frac{AD}{AE}$

$AE = AD\times\frac{2}{\sqrt{3}} = \frac{2p}{3\sqrt{3}}$

So, perimeter of hexagon = $6\times\frac{2p}{6\sqrt{3}} = \frac{2p}{\sqrt{3}}$

46. $AD = \left(\left(\frac{a}{2}\right)^2 + \left(\frac{a^2}{2}\right)\right)^{1/2}$

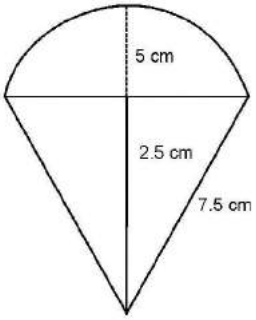
Then, radius of circle = $\frac{a}{2\sqrt{2}}$



Advanced

1. Radius of the circle cm $= \sqrt{(7.5)^2 = (2.5)^2} = 5\sqrt{2}$

Hence, circumference $=$ $10\sqrt{2}\pi$ cm.

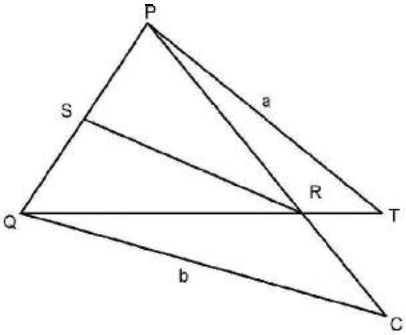


2. ΔPQU ~ PSR

$$\Rightarrow \frac{PS}{PW} = \frac{SR}{QU} \qquad (1)$$

ΔPQU ~ SQR

$$\Rightarrow= \frac{SR}{PQ} = \frac{SR}{PT} \qquad (2)$$



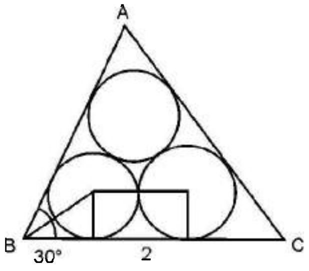
From (1) and (2)

PQ × SR = PS × QU = SQ × PT $\Rightarrow= \frac{SQ}{PS} = \frac{b}{a}$

Now use componendo and equation (1) to obtain SR $= \frac{ab}{a+b}$

3. Use options.

4. It can be seen that the side of the triangle is



7. $\frac{\pi}{8} = S,(2+4+8+16+32+64+128)$

Get S, from the above relation again $\frac{\pi\theta}{306°} = S_1$

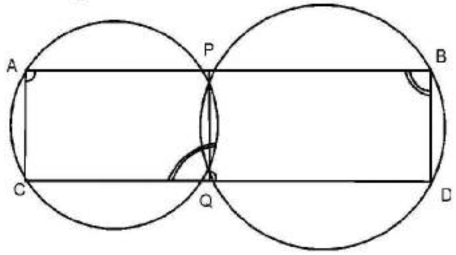
Now θ can easily be calculated from here.

9. From alternate segment theorem

∠PAC = ∠PQD and ∠PBD = ∠PQC

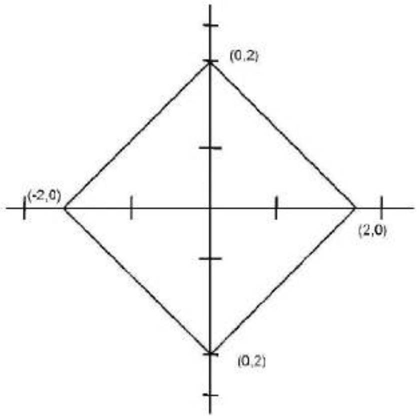
But ∠PQD and ∠PQC are supplementary.

Now since ∠PAC and ∠PBD are supplementary, hence AC is parallel to BD.



10. $x + y = 2$ when $x \geq 0$ and $y \geq 0$

$x + y = -2$ when $x \leq 0$ and $y \leq 0$

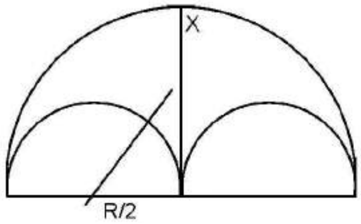


16. Ratio of radii $= \frac{a}{2\sqrt{2}} : \frac{a}{4\sqrt{2}} : \frac{a}{2\sqrt{2}}$

Ratio of areas $= 1:4:16$

18. $\left(\frac{R}{2} + x\right)^2 = \left(\frac{R}{2}\right)^2 + (R - x)^2$

x = R/3

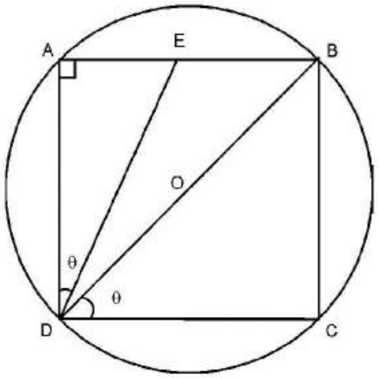


19. Let ∠BDC = θ

DB = 2r

BC = r

DE = r $\sqrt{3}$



Now area of ABCD $\frac{1}{2}\times\sqrt{3}\times r$

$= \frac{1}{2}\times\sqrt{3r}\times 2r\times\sin\theta$

$\Rightarrow \sin\theta = \frac{1}{2}$

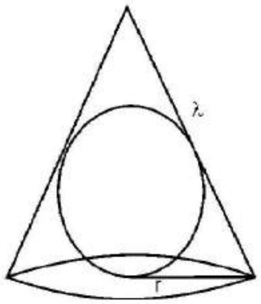
$\theta = 30^\circ$

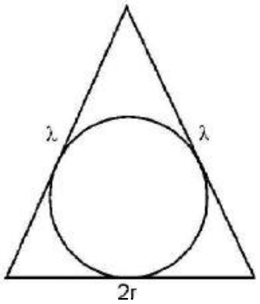
Now find AE.

21. Height of the octagonal prism will be the diameter of the sphere inscribed in it. Hence, the answer will be.

$$\frac{\sqrt{3}}{4}a^3\left(3\sqrt{3}-r\right)[n(n+1)]$$

22. Consider the biggest cross-section of the cone as a isosceles triangle therefore the circle inscribed in the triangle will be the biggest cross-section of the sphere.



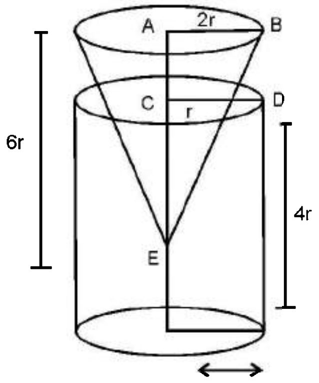


We know that in radius × semi perimeter = Area of the triangle.

A little calculation will lead to the answer, i.e.,

$$\frac{4}{3}\pi r^3\left(\frac{\lambda-r}{\lambda+r}\right)^{3/2}$$

23. ΔECD is similar to ΔEAB. Hence, EC = 3r



So the answer will be $7\pi r^3$.

24. Side of the square should always be less than the attitude of the triangle.

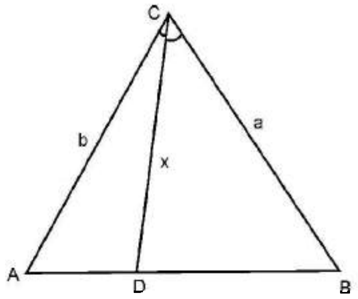
27. Let CD = x

Area ΔABC = ΔACD + ΔDCB

$\Rightarrow \frac{1}{2}ab\sin 120^\circ\, bx\sin 60^\circ + \frac{1}{2}ax\sin 60^\circ$

Now it can be observed that x

$x = \frac{ab}{a+b}$



28. We have to calculate

AH × AG × AF × AE × AD × AC × AB

AE = 2

∠AOB = 45°

Use Cosine formula and get AB (= AH). ∠AHG = 135°

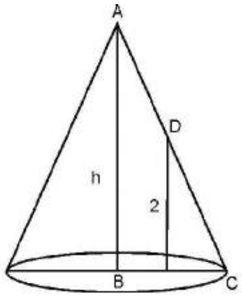
Again by Cosine formula get AG (= AC) and AF (= AD)

33. DE = 2

EC = 3/2

find DC and then find l

Thus, the answer will be 15 pm^2

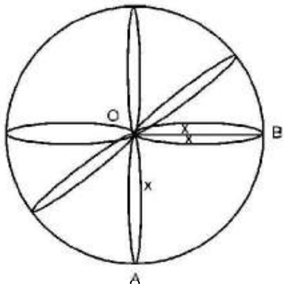


35. OA = OB = AB = 1 cm

    ∠OAB = 60°

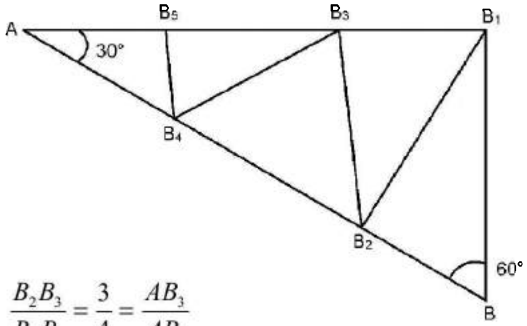
    area x = area of sector OAB – DOAB

    $12x$ = area of the flower



37. AB = a   BB, = a/2

    $B_2B = a/4$ $\Delta AB_2B_3 \sim \Delta ABB$,

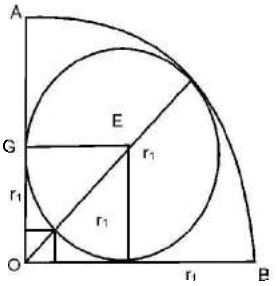


$$\frac{B_2B_3}{B_dB_1} = \frac{3}{4} = \frac{AB_3}{AB_1}$$

Now we have two infinite GP series having common ratio ¾. Calculate the sum of these series.

38. $r - r_1 = \sqrt{2}\, r_1$

    Now find r, and solve to get the answer.



41. Let OC = r

    ABCDEF is a regular hexagon

    ED = r

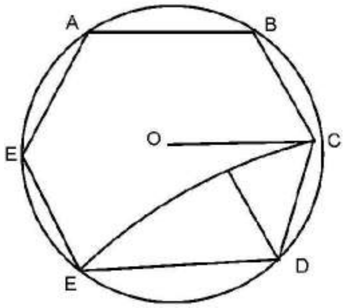
    Let EC = x

    In ΔEDC, using cosine formula we can get

    $x = \sqrt{3}\, r$

    Side of the star will be $r/\sqrt{3}$.

    Now the desired ratio can be easily calculated.



46. Consider the problem as a pyramid to the base of a regular hexagon.

    Now in Δ POR

    OP = 12 cm

    OR = 5 cm

    PR = 13 cm

    Now cosine formula will give the answer.

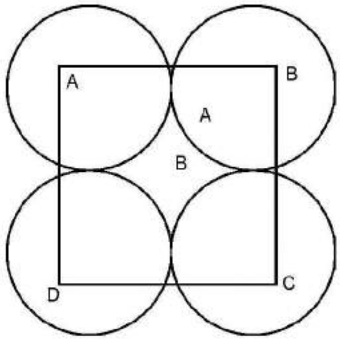
47. AB = 4 cm

    $$a = \frac{16 - 4\pi}{4} = 4 - \pi \text{ cm}$$

    area of common region = $b = \frac{4\pi}{4} - 4 + \pi$

    $b = 2\pi - 4$

    Total area = $4b = 8\pi - 16$



49. $$\frac{\sqrt{3}}{4}a^2 = 300 \text{ cm}^2$$

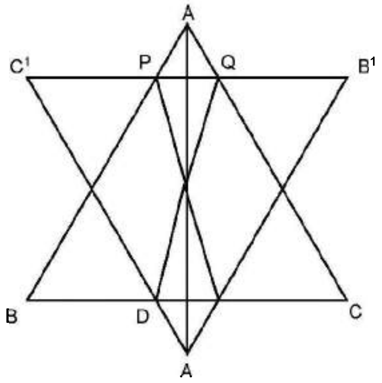
$$a = \sqrt{\frac{1200}{\sqrt{3}}}$$

So, AP will be equal to AB/3.

So the area common to both triangles

$$= 300 - 3\times\frac{\sqrt{3}}{4}\times\frac{1}{9}\times\frac{1200}{\sqrt{3}}$$

$$= 200 \text{ cm}^2$$



18

CHAPTER

Mensuration

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Relationship between 2-D and 3-D figures
- Definitions and properties
- Different structures
- Process of finding out volume, lateral surface area, curved surface area and total surface area
- Methods of solving questions

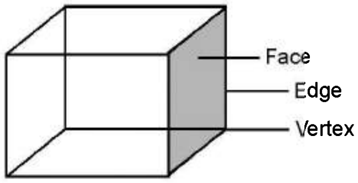
Introduction to the topic

There is a direct connection between geometry and mensuration. It can be seen through analogy that the relationship shared by 2-D and geometry is the same as the relationship shared by 3-D and mensuration in most cases. Mensuration has been an area from which questions have been asked in the CAT, albeit not as much as from geometry. A fair number of questions from this topic are being asked in the CAT. And therein lies the importance of this topic. Mensuration should be seen as a nice juxtaposition of 3-D figures which conforms to the properties of 2-D geometry as well as 3-D geometry.

SOLIDS

A solid has three dimensions, namely, length, breadth or width and height or thickness. The plane surfaces that binds it are called its faces and the solid so generated is known as a polyhedron.

The volume of any solid figure is the amount of space enclosed within its bounding faces. A solid has edges, vertices and faces which are shown in the figure given below:



A solid has two types of surface areas:

- *Lateral Surface Area (LSA)* LSA of a solid is the sum of the areas of all the surfaces it has except the top and the base.
- *Total Surface Area (TSA)* TSA of a solid is the sum of the lateral surface area and the areas of the base and the top.

Note In case of solids like the cube and cuboid, the lateral surface area consists of plane surface areas (i.e., area of all surfaces except the top and base) whereas in case of solids like cone and cylinder, it consists of curved surface areas (CSA). Thus, for such solids the LSA is also called CSA.

Euler's Rule

Euler's rule states that for any regular solid:
Number of faces (F) + Number of vertices (V) = Number of Edges (E) + 2

Cuboid

A cuboid is a rectangular solid having 6 rectangular faces. The opposite faces of a cuboid are equal rectangles. A cuboid has a length (l), breadth (b) and height (h).

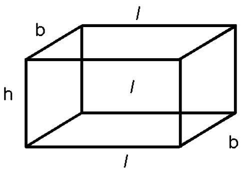


Fig. 1

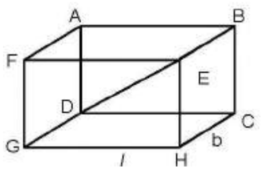


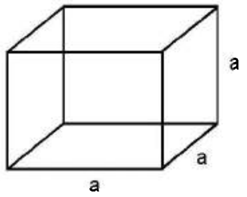
Fig. 2

In Fig. 2 ED is the diagonal of the cuboid. Moreover, the area of the surface GDCH is x, the area of the surface HEBC is y and the area of the surface GFEH is z.

- Volume = Area of base × Height = lbh
- Volume = $\sqrt{xyz}$
- Volume = $xh = yl = zb$
- Lateral surface area (LSA) or area of the 4 walls $= 2(l + b)h$
- Total surface area (TSA) $= 2(x + y + z) = 2(lb + bh + lh)$
- Diagonal = $\sqrt{l^2 + b^2 + h^2}$

CUBE

A cube is a solid figure having six faces. All the faces of a cube are equal squares (let us say of the side 'a'). Thus the length, breadth and height of a cube are equal.



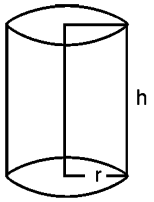
- Volume = a^3

(ii) Lateral surface area (LSA) or area of the 4 walls $= 4a^2$

(iii) Total surface area (TSA) $= 6a^2$

(iv) Diagonal = $a\sqrt{3}$

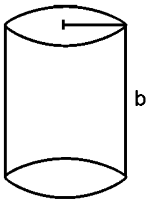
RIGHT CIRCULAR CYLINDER

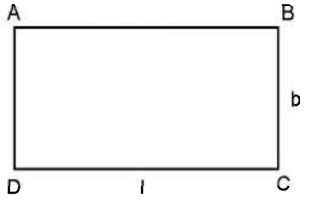


In the above figure, r is the radius of the base and h is the height of a right circular cylinder. A cylinder is generated by rotating a rectangle or a square by fixing one of its sides.

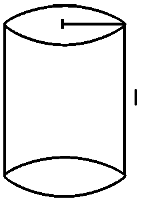
- Volume = area of base × height
  Volume = $\pi r^2 h$
- Curved surface area (CSA) = Perimeter of base × height
  LSA = $2\pi rh$
- Total surface area (TSA) = LSA + Area of the top + Area of the base
  TSA = $2\pi rh + \pi r^2 + \pi r^2$
  TSA = $2\pi r(r + h)$

Some important deductions





- If the above rectangular sheet of paper (ABCD) is rolled along its length to form a cylinder, then the radius (r) of the cylinder will be (L/2π) and its height will be b and Volume of this cylinder = $\frac{L^2b}{4\pi}$, where L is the length of the rectangle.
-

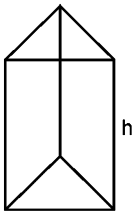


If the above rectangular sheet of paper (ABCD) is rolled along its breadth to form a cylinder, then the radius (r) of the cylinder will be $\frac{b}{2\pi}$ and its height will be L.

Volume of this cylinder = $\frac{b^2L}{4\pi}$

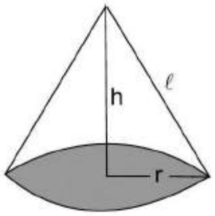
PRISM

A prism is a solid having identical and parallel top and bottom faces, i.e., they will be identical polygons of any number of sides. The side faces of a prism are rectangular and are known as lateral faces. The distance between two bases is known as the height or the length of the prism.



- Volume = Area of base × Height
- Lateral Surface Area (LSA) = Perimeter of the base × height
- Total surface Area (TSA) = LSA + (2 × Area of the base)

RIGHT CIRCULAR CONE



In the above figure 'r' is the radius of the base, h is the height and ℓ is the slant height of the right circular cone.

- Volume = $\frac{1}{3}$ × area of the base × height

  Volume = $\frac{1}{3}\pi r^2h$

(ii) Slant height = $\ell = \sqrt{r^2+h^2}$

(iii) Curved surface area (CSA) = $\pi r\ell$

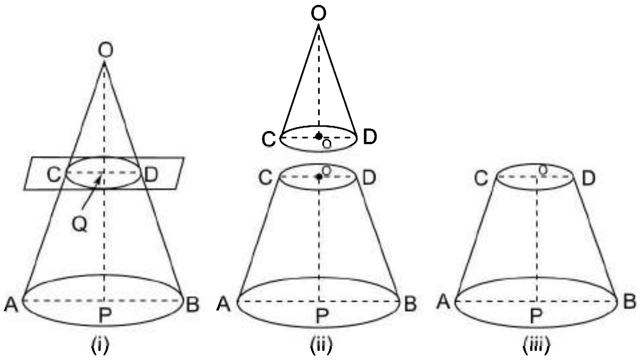
(iv) Total surface area (TSA) = (CSA + Area of the base)

TSA = $\pi r\ell + \pi r^2$

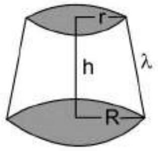
Frustum of Cone

A cone whose top portion is sliced off by a plane which is parallel to the base is called frustum of cone.

Formation of frustum:



However, for the sake of representing the formula, we will use another form of frustum right now as given below:



In the above figure, r is the radius of the base, h is the vertical height of the frustum and λ is the slant height of the frustum.

- Volume = $\frac{\pi h}{3}\left(R^2+r^2+Rr\right)$

(ii) Slant height = $\lambda = \sqrt{(R-r)^2+h^2}$

(iii) Curved Surface Area (CSA) = $\pi(R+r)\lambda$

(iv) Total Surface Area (TSA) = CSA + Area of the Top + Area of the base

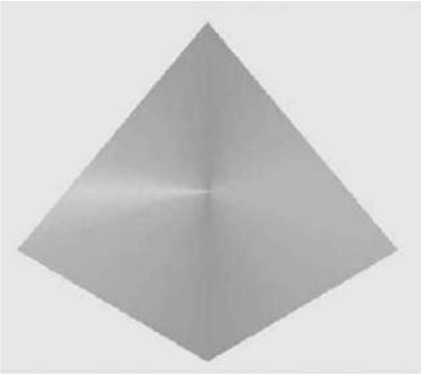
TSA = $\pi(R+r)\lambda + \pi r^2 + \pi R^2$

TSA = $\pi(R\lambda + r\lambda + r^2 + R^2)$

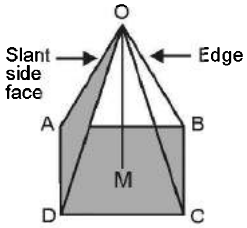
(v) To find the height (H) of original cone.

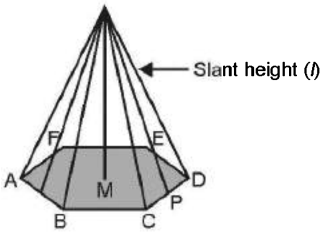
$$H = \frac{Rh}{R-r}$$

PYRAMID



A pyramid is a solid having an n-sided polygon at its base. The side faces of a pyramid are triangular with the top as a point.





In the above figures OM is the height of the pyramid.

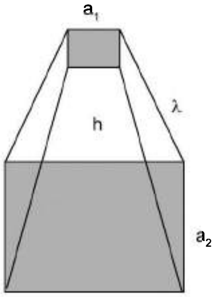
- Volume = $\frac{1}{3}$ × Area of the base × height

(ii) Lateral Surface Area (LSA) = $\frac{1}{2}$ × (Perimeter of the base) × slant height

(iii) Total Surface Area (TSA) = LSA + Area of the base

Frustum of Pyramid

A pyramid whose top portion is sliced off by a plane which is parallel to the base is called the frustum of a pyramid.



In the above figure, A_1 is the area of the top face of the frustum, A_2 is the area of the bottom face of the frustum, h is the height of the frustum and l is the slant height of the frustum.

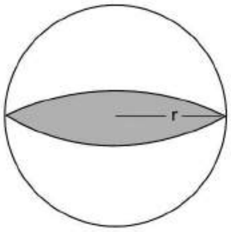
- Volume = $\frac{1}{3}h\left(A_1 + A_2 + \sqrt{A_1A_2}\right)$

(ii) Lateral Surface Area (LSA) = $\frac{1}{2}(P_1 + P_2)\,\ell$

where P_1 and P_2 are perimeters of the top and the bottom faces.

(iii) Total surface Area (TSA) = ℓS.A. + A_1 + A_2

SPHERE

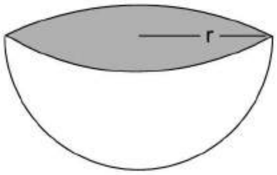


In the above figure r is the radius of the sphere.

- Volume = $\frac{4}{3}\pi r^3$

(ii) Surface Area = $4\pi r^2$

HEMISPHERE



- Volume = $\frac{2}{3}\pi r^3$

(ii) Curved surface area (CSA) = $2\pi r^2$

(iii) Total surface area (TSA) = LSA + Area of the top face (read circle)

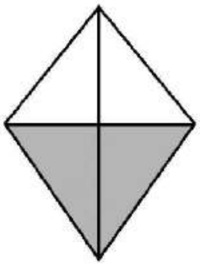
TSA = $2\pi r^2 + \pi r^2$

TSA = $3\pi r^2$

SOME MORE SOLIDS

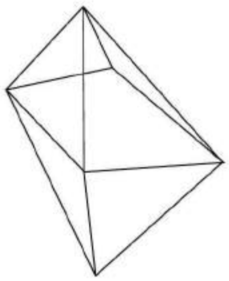
Tetrahedron

A tetrahedron is a solid which has 4 faces. All the faces of a tetrahedron are equilateral triangles. A tetrahedron has 4 vertices and 6 edges.



Octahedron

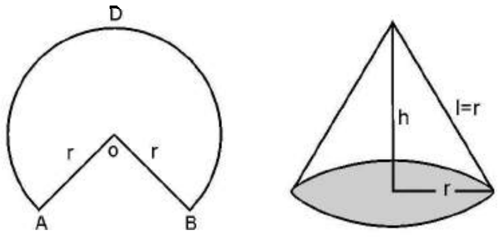
An octahedron is a solid which has 8 faces. All the faces of an octahedron are equilateral triangles. An octahedron has 6 vertices and 12 edges.



Inscribed and circumscribed solids

- If a sphere of the maximum volume is inscribed in a cube of edge 'a', then the radius of the sphere = $\frac{a}{2}$
- If a cube of the maximum volume is inscribed in a sphere of radius 'r', then the edge of the cube = $\frac{2r}{\sqrt{3}}$
- If a cube of the maximum volume is inscribed in a hemisphere of radius 'r', then the edge of the cube = $\sqrt{\frac{2}{3}} \times r$

Some important deductions



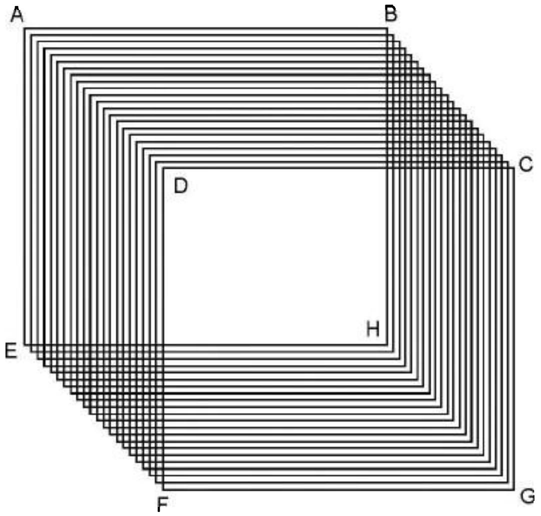
If a cone is made by a sector of a circle (AOBD), then the following two things must be remembered

- The area of the sector of a circle (AOBD) = The CSA of the cone
- Radius of the cirlce (r) = Slant height (l) of the cone.

VISUAL MENSURATION

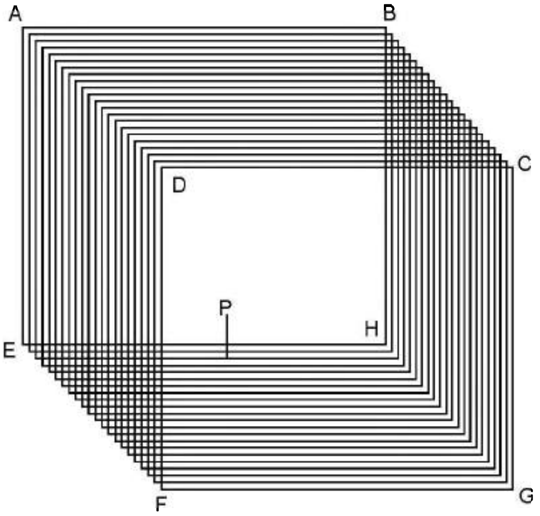
Under this section of mensuration, we will be required to visualize some of the unknown dimensions of any structure with the help of the given dimension. Some of the typical examples of Visual Mensuration are given below:

Given is a cube ABCDEFGH of side length 'a' units. Its top face is ABCD and its bottom face is EFGH. Since the side length of this cube is 'a' units, so, AB = BC = CD = AD = AE = EF = FD = FG = GH = EH = BH = CG = 'a'



Minimum length between vertex A and vertex G We can find the minimum distance between these two vertices in the following two ways:

- *Aerial distance* Aerial distance can be understood by assuming that there is a fly at vertex A and it has to reach vertex G through the minimum possible distance. This distance will be the diagonal distance between the vertices A and G $= \sqrt{a^2 + a^2 + a^2} = a\sqrt{3}$
- *Physical distance* Physical distance can be understood by assuming that there is an ant at vertex A and it has to reach vertex G through the minimum possible distance. Since it cannot fly, so ant will cover this distance along the two faces viz., face ABEH and face EFGH or face ADFE and face CDFG or face ABCD and face CDFG or face ABCD and face BCGH.



The shortest possible distance can be the diagonal only. Let us assume that the ant is going via face ABEH and face EFGH. So, the ant will cover first the diagonal distance between A and P, where P is the mid point of side EH, and then from P to vertex G.

Now $AP^2 = EP^2 + GP^2 = \dfrac{a\sqrt{5}}{2}$

And $GP^2 = HP^2 + GH^2 = \dfrac{a\sqrt{5}}{2}$

Hence, the minimum possible physical distance

$= 2\dfrac{a\sqrt{5}}{2} = a\sqrt{5}$

Let us find out why the distance AE + EG cannot be the shortest?

First calculate AE and EG.

AE = a

$EG = \sqrt{a^2 + a^2} = a\sqrt{2}$

So, $AE + EG = a + \sqrt{a^2 + a^2} = a\sqrt{2} + a = a\,(1 + \sqrt{2})$

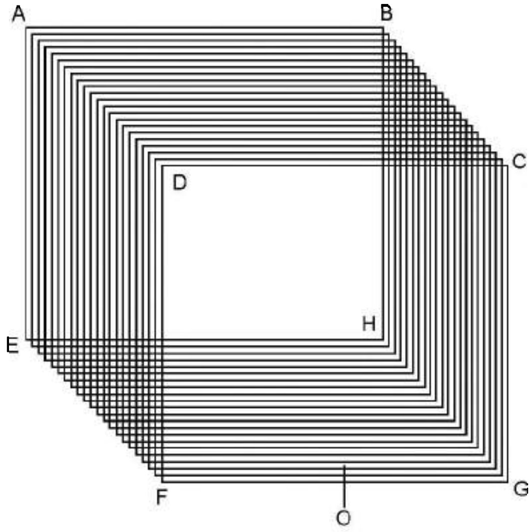
And obviously, $a\sqrt{5} < a(1 + \sqrt{2})$[For $a>0$]

We can understand this phenomenon by having a bit of *mental mapping*. As we have seen earlier that the minimum possible distance can be the diagonal distance only. Now let us cut open the face ABHE by making a cut mark at EH so that the faces ABHE and EFGH are in a plane, lying horizontal on the ground. Now the minimum possible distance between A and G will be the diagonal of the newly formed rectangle AFGB. This diagonal will pass through the mid-point of EH.

So, $AG^2 = AF^2 + FG^2 = (2a)^2 + (a)^2 = 5a^2$

$AG = a\sqrt{5}$

Minimum length between vertex A and vertex O, where O is the mid-point of FG

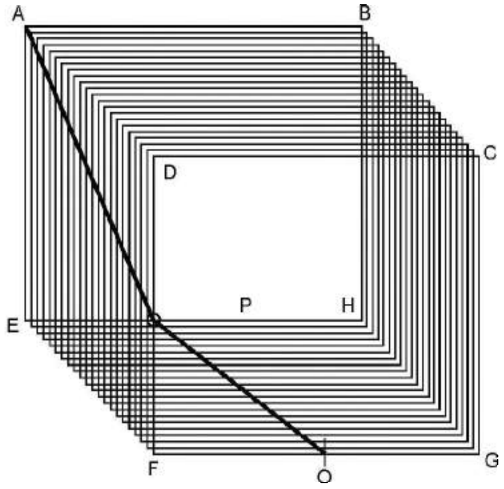


- *Aerial distance* We can see this situation vis-à-vis a cuboid of side lengths 'a' units, 'a' units and $\dfrac{a}{2}$ units.

  Minimum aerial distance between A and O

  $= \sqrt{\dfrac{a^2}{4} + a^2 + a^2} = \dfrac{3a}{2}$

- *Physical distance* Let us assume that the ant is moving through the faces ABHE and EFGH. Ant will first go the point Q, where Q is the mid-point of E and P. And then the ant will cover QO.

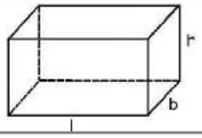
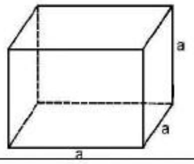
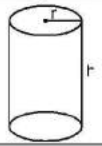
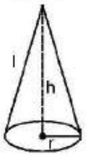
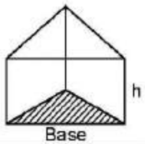
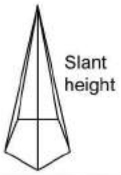
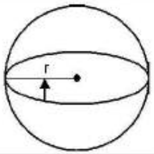
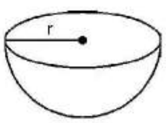
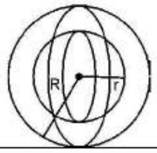
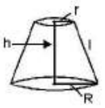


$AQ^2 = AE^2 + EQ^2 = a^2 + \dfrac{a^2}{16} = \dfrac{17a^2}{16}$

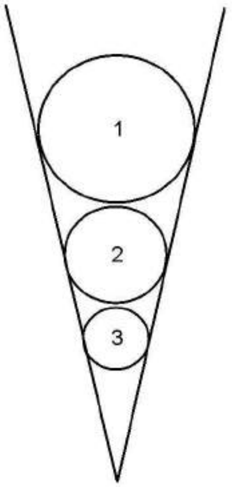
So, $AQ = QO = \dfrac{a\sqrt{17}}{4}$

So, the minimum distance between the vertex A and point $O = AQ + QO = \dfrac{a\sqrt{17}}{2}$.

FORMULAE

S.No.	*Name*	*Figure*	*Nomenclature*	*Volume*	*Curved/* **Lateral**	**Total** *Surface Area Surface Area*
1.	Cuboid		l = length b = breadth h = height	lbh	2(l+b) h	2(lb+bh+hl)
2.	Cube		a = edge/side	a^3	$4a^2$	$6a^2$
3.	Right Circular Cylinder		r = radius of base h = height of the cylinder	$\pi r^2 h$	$2\pi rh$	$2\pi r\,(r+h)$
4.	Right Circular Cone		r = radius h = height l = slant height $l=\sqrt{r^2+h^2}$	$\frac{\pi r^2 h}{3}$	πrl	$\pi r\,(l+r)$
5.	Right Triangular Prism		–	Area of the base × height	Perimeter of the base × height	Lateral surface area + 2 (area of base)
6.	Right Pyramid		– height	1/3 × area of the base × of the base × slant height	½ × perimeter the base × slant height	Lateral surface area + area of base
7.	Sphere		r = radius	$\frac{4}{3}\pi r^3$	–	$4\pi r^2$
8.	Hemisphere		r = radius	$\frac{2}{3}\pi r^3$	$2\pi r^2$	$3\pi r^2$
9.	Spherical shell		r = inner radius R = outer radius	$\frac{4}{3}\pi(R^3-r^3)$	–	$4\pi(R^2+r^2)$
10.	Frustum of a Cone		–	–	–	Lateral surface area + Area of top + Area of base

Example 1 Three spheres are kept inside a cone, as given in the figure. Spheres are touching both the slant sides of the cone and the adjacent spheres. If the radius of the 1st sphere and the 3rd sphere are 5 units and 20 units respectively, find the radius of the 2nd sphere.



Solution We can see in Fig. 1, that the AOA′P, BOB′Q and COC′R will be similar.

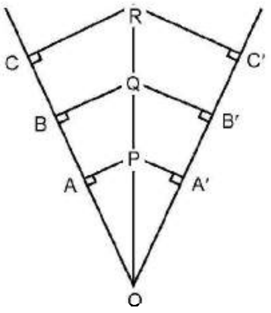
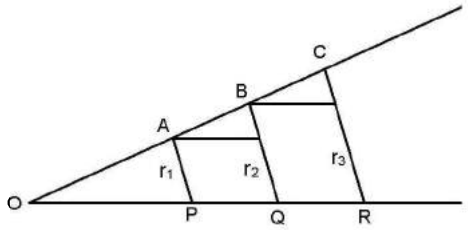


Fig. 1.



In Fig. 2, $(r_2 - r_1)/(r_2 + r_1) = (r_3 - r_2)/(r_3 + r_2) = K$
Using Componendo and Dividendo,
$r_2/r_1 = r_3/r_2$

Hence, the three radii are in a GP.

So, $r_2/20 = 5/r_2$, hence $r_2 = 10$ units.

Result of this question can be used as a formula also.

SOLVED EXAMPLES

Example 2 The diameter of a metallic sphere is 6 cm. The sphere is melted and drawn into a wire of uniform circular cross-section. If the length of the wire is 36 m, find its radius.

Solution The diameter of the metallic sphere is 6 cm. Hence, radius of the sphere is 3 cm. Now, let the radius of the cross–section of the wire be r cm. And as we know, metallic sphere is converted into a cylindrical shaped wire, then their volumes will be equal.

$$\text{So, } \frac{4}{3} \times \pi \times 3^3 = \pi \times r^2 \times 3600$$

$$\text{or, } 4 \times 9\,\pi \times 3^3 = 3600\,\pi r^2$$

$$\text{This gives } r^2 = 0.01$$

$$\text{i.e., } r = 0.1$$

Example 3 Sardar Sarovar Dam which is rectangular in shape, can produce electricity only if the height of the water level in it is atleast 7cm. Now the water is pumped in at the rate of 5 km per hour through a pipe of diameter 14 cm into the dam area of dimensions 50 m × 44 m. In what time the dam will be able to produce electricity?

Solution The volume of water flowing through the cylindrical pipe in one hour at the rate of 5 km (5000 m) per hour

$$= \frac{22}{7} \times \frac{7}{100} \times \frac{7}{100} \times 5000\,\text{m}^3$$

[Since radius = 7 cm = $\frac{7}{100}$ m

$= 77\ \text{m}^3$

Thus, 77 m^3 of water will fall into the tank in 1 h. Since the level of the water is required to be raised by 7 cm i.e., $\frac{7}{100}$ m,

Volume of the required quantity of water = $50 \times 44 \times \frac{7}{100}\ \text{m}^3 = 154\ \text{m}^3$

Since 77 m^3 of water falls into the tank in 1 h, therefore, 154 m^3 of water will fall into the dam in $\frac{154}{77}$ h, i.e., 2 h.

So, the level of water will rise by 7 cm in 2 h.

Example 4 A right angled triangle ABC, whose two sides other than the hypotonuse are 15 cm and 20 cm. The triangle is made to revolve about its hypotonuse. Find the volume of the double cone so formed.

Solution Let ΔABC be the right angled triangle right angled at A, whose sides AB and AC measures 15 cm and 20 cm, respectively.

The length of the side BC (hypotenuse)

$= \sqrt{15^2+20^2}$ cm = 25

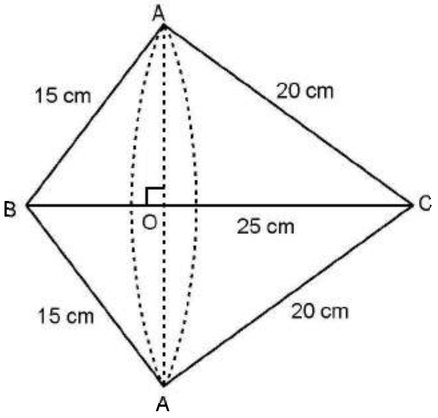
Here, AO (and A'O) is the radius of the common base of the double cone formed by revolving the ΔABC about BC.

Height of the cone BBA' is BO and the slant height is 15 cm.

Height of the cone CAA' is CO and the slant height is 20 cm

Using AA similarity,

Now, ΔAOB ~ ΔCAB



Therefore, $\frac{AO}{20}=\frac{15}{25}$

This gives $AO=\frac{20\times15}{25}\text{ cm}=12\text{ cm}$

Also, $\frac{BO}{15}=\frac{15}{25}$ cm

This gives $BO=\frac{15\times15}{25}\text{cm}=9\text{cm}$

Thus, CO = 25 cm – 9 cm = 16 cm

Now, volume of the double cone

$$=\left(\frac{1}{3}\times3.14\times12^2\times9+\frac{1}{3}\times3.14\times12^2\times16\right)\text{cm}^2$$

$$=\frac{3.14}{3}\times12^2\times(9+16)\text{cm}^2$$

= 3768 cm^3

Example 5 When shopping in Big Bazar, I saw a peculiar solid toy in the form of a hemisphere surmounted by a right circular cone. Height of the cone was 2 cm and the diameter of the base was 4 cm. If a right circular cylinder circumscribed the solid, find out how much more space will it have, provided the height of the cone was 2 cm and diameter of the base was 4 cm respectively?

Solution See the figure below.

Assume BPC be the hemisphere and ABC is the cone standing on the base of the hemisphere.

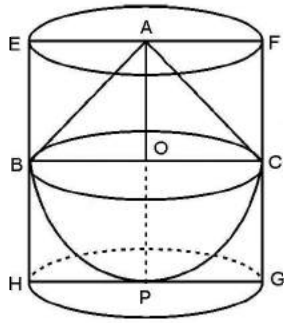
Radius BO of the hemisphere (as well as of the cone) = $\frac{1}{2}\times 4$ cm = 2 cm.

Now, let the right circular cylinder EFGH circumscribe the given solid.

Radius of the base of the right circular cylinder = HP = BO = 2 cm.

Height of the cylinder = AP = AO + OP = 2 cm + 2 cm = 4 cm

Now, volume of the right circular cylinder – volume of the solid



$$=\left[\pi\times2^2\times4-\left(\frac{2}{3}\times\pi2^3+\frac{1}{3}\times\pi\times2^3\right)\right]\text{cm}^3$$

= (16π – 8π) cm^3

= 8π cm^3

Hence, the right circular cylinder is having 8p cm^3 more space than the solid.

Example 6 A toy consists of a base that is the section of a sphere and a conical top. The volume of the conical top is 30π sq. units and its height is 10 units. The total height of the toy is 19 units. The volume of the sphere (in cubic units), from which the base has been extracted, is

(a) $\frac{256}{3}\pi$ (b) $\frac{64}{3}\pi$

(c) $\frac{108}{3}\pi$ (d) $\frac{500}{3}\pi$

Solution Height of the cone = 10 units

Volume of the cone = 30π cubic units

= diameter of the cross section from where the sphere has been sectioned = 6 units

$=r+\sqrt{2^2-3^2}=9$

= r = 5 units

= volume of the original sphere from which the base has been sectioned = $\frac{4}{3}\pi r^3=\frac{500}{3}\pi$

PRACTICE EXERCISES

WARM UP

Q.1. If the side of a cube is increased by 100%, find by what percentage the surface area of the cube is increased?
(a) 150% (b) 200% (c) 300% (d) 350%

Q.2. A banquet hall has the dimensions 30 m×12 m×6 m. Each person should get 8 m^3 of space. Find the number of persons who can be accommodated in this hall.
(a) 240 (b) 250 (c) 270 (d) 300

Q.3. Find the height of the cylinder whose volume is 511 cm^3 and the area of the base is 36.5 cm^2.
(a) 3.5 cm (b) 10.5 cm
(c) 14 cm (d) None of these

Q.4. The ratio between the curved surface area and the total surface area is 2:3 and the total surface is 924 cm^2. What is the volume of the cylinder?
(a) 2156 cm3 (b) 2183 cm^3
(c) 2492 cm^3 (d) None of these

Q.5. If the curved surface area of a cylinder is 1320 cm^2 and its base radius is 21 cm, then what is its total surface area?
(a) 4092 cm2 (b) 2084 cm^2
(c) 5104 cm^2 (d) None of these

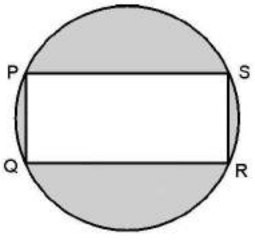
Q.6. The radius of an iron rod is decreased to one fourth of its actual radius. If its volume remains constant, then the length will become
(a) 2 times (b) 12 times
(c) 8 times (d) 16 times

Q.7. A reservoir is in the shape of a frustum of a right circular cone. It is 8 m wide at the top and 4 m wide at the bottom. If it is 6 m deep, then what is its volume?
(a) 224 m^3 (b) 176 m^3
(c) 204 m^3 (d) None of these

Q.8. A spherical metal ball of 6 cm radius is melted and recast into three spherical balls. The radii of two of these balls are 3 cm and 4 cm. What is the radius of the third ball?
(a) 4.5 cm (b) 5 cm
(c) 6 cm (d) 7 cm

Q.9. If the diagonals of a rhombus are 18 cm and 24 cm respectively, then find its perimeter.
(a) 15 cm (b) 42 cm
(c) 60 cm (d) 68 cm

Q.10. In the figure shown below, PQRS is a rectangle of the dimension 8 cm × 6 cm and is inscribed in a circle. Find the area of the shaded portion.



(a) 44 cm^2 (b) 34.25 cm^2
(c) 32.50 cm^2 (d) None of these

Q.11. The radius of the base and the volume of the right circular cone are doubled. What is the ratio of the length of the larger cone to that of the smaller cone?
(a) 1:4 (b) 1:2
(c) 1:3 (d) 4:1

Q.12. A cone and a hemisphere have equal base radius and equal volumes. The ratio of their heights is:
(a) 3:1 (b) 2:1
(c) 4:1 (d) None of these

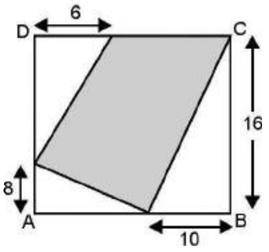
Q.13. If the right circular cone is cut into three solids of volumes V_1, V_2 and V_3 by two cuts which are parallel to the base and trisects the altitude, then V_1:V_2:V_3 is
(a) 1:2:3 (b) 1:4:6
(c) 1:6:9 (d) None of these

Q.14. Water flows at the rate of 10 m per min from a cylindrical pipe of radius 2.5 mm. A conical vessel whose diameter is 40 cm and depth 24 cm is filled with water flowing from this pipe. The time taken to fill the conical vessel is
(a) Less than 30 min
(b) Less than 50 min but more than 30 min
(c) More than 50 min but less than 75 min
(d) More than 75 min

Q.15. If a regular square pyramid has a base of side 8 cm and height of 30 cm, then what is its volume (in cc)?
(a) 360 (b) 480
(c) 640 (d) 800

Q.16. A cylinder circumscribes a sphere. The ratio of their volumes is
(a) 2 : 1 (b) 3 : 2 (c) 4 : 3 (d) 6 : 5

Q.17. Find the area of the shaded region in the given figure of square ABCD.



(a) 128 cm^2 (b) 184 cm^2
(c) 154 cm^2 (d) 168 cm^2

Q.18. An open box is made of wood 2 cm thick. Its internal dimension is 86 cm × 46 cm × 38 cm. What is the cost of painting the outer surface of this box @ Rs 10 per m^2?
(a) Rs 12.35 (b) Rs 8.85
(c) Rs 15.70 (d) Rs 16.50

Q.19. A copper wire when bent in the form of a square, encloses an area of 121 m^2. If the same wire is bent to form a circle, the area enclosed by it would be
(a) 124 m^2 (b) 136 m^2
(c) 154 m^2 (d) 184 m^2

Q.20. A spherical ball was painted black. After getting painted, it was cut into 4 similar pieces. What is ratio of the painted area to the non-painted area?
(a) 1 : 1 (b) 1 : 2
(c) 3 : 1 (d) 3 : 2

Q.21. Volume of a sphere, a cube, a tetrahedron and an octahedron is same. Find for which of the following structures, it will have the maximum surface area?
(a) Sphere (b) Cube
(c) Tetrahedron (d) Octahedron

Q.22. The ratio of diagonals of the two cubes is 3 : 2. What is the ratio of the surface areas of these two cubes respectively?
(a) 27 : 8 (b) 3 : 2
(c) 9 : 4 (d) 16 : 9

FOUNDATION

Q.1. A rectangular classroom has to be painted covering all its surfaces excluding floor. The cost of painting a similar room is Rs 1872. But its length, breadth and height are 10% less, 10% more and 10% less than the dimensions of the classroom respectively. How much will it cost to paint the classroom if its length, breadth and height are in the ratio of 1:2:3?
(a) Rs 1,956 (b) Rs 2,000
(c) Rs 2,100 (d) Cannot be determined

Q.2. The top of a conical container has a circumference of 308 m. Water flows in at a rate of 12 m^3 every 2 s. When will the container be half filled, if its depth is 12 m?
(a) 42 min (b) 68 min
(c) 54 min (d) 82 min

Q.3. What is the radius of a spherical ball in inches which is formed by melting a cylinder of base diameter 8 inches and height 160 inches, if the conversion wastage results in a 10% weight loss?
(a) 6 (b) 8
(c) 12 (d) 16

Q.4. A solid wooden toy is in the shape of a right circular cone mounted on the base of a hemisphere. If the radius of the hemisphere is 4.2 cm and the total height of the toy is 10.2 cm, find the volume of the wood needed to construct another such toy.
(a) 104 cm^3 (b) 162 cm^3
(c) 427 cm^3 (d) 266 cm^3

Q.5. A well is dug 20 ft deep and the mud which came out is used to build a wall of width 1 ft around the well on the ground. If the height of the wall around the well is 5 ft, then what is the radius of the well?
(a) $\frac{\sqrt{5}+1}{4}$ (b) $\sqrt{5}$ (c) 1 (d) $\frac{1}{4}$

Q.6. The foot of a ladder 15 m long, is 9 m from a house and its top reaches the upper part of a circular window. When the foot of the ladder is drawn to a distance of 3 m away from the house, the top reaches the lower edge of the window. The area of the window is
(a) $5\frac{2}{3}$ m^2 (b) 6 m^2
(c) $7\frac{1}{14}$ m^2 (d) $8\frac{2}{3}$ m^2

Q.7. A brick wall is to be built 20 m long, 3 m high and 40 cm thick. It has a door 3 m by 2 m. Suppose each brick is to be 15 cm long, $7\frac{1}{2}$ cm broad and 5 cm thick, how much will be the cost of the bricks at the rate of Rs 800 per thousand bricks?

(a) Rs 55,270 (b) Rs 66,230
(c) Rs 30,720 (d) Rs 15,750

Q.8. A rectangular field is of the dimension 15.4 m × 12.1 m. A circular well of 0.7 m radius and 3 m depth is dug in the field. The mud, dug out from the well, is spread in the field. By how much would the level of the field rise?

(a) 1 cm (b) 2.5 cm
(c) 3.5 cm (d) 4 cm

Q.9. The dimension of a room is thrice as long as it is height, but only twice as long as it is wide. The total cost of painting its walls at the rate of Rs 2.50 m^2 is Rs 360. What is the cost of laying a carpet on its floor at the rate of Rs 3/m^2?

(a) Rs 81 (b) Rs 125
(c) Rs 216 (d) Rs 260

Q.10. A man, by walking diametrically across a circular grass plot, finds that it has taken 45 s less than if he had kept to the path around the outside if he walks at the rate of 80 m per minute, the diameter of the grass plot is

(a) 35 m (b) 65 m
(c) 105 m (d) 145 m

Q.11. Within a rectangular courtyard of length 60 feet, a graveled path, 3 feet wide, is laid down along all the four sides. The cost of gravelling the path is Rs 2 per sq ft^2. If the path had been twice as wide, the gravel would have cost Rs 984 more. The width of the courtyard is

(a) 24 feet (b) 40 feet
(c) 45 feet (d) 54 feet

Q.12. A vessel 2 m long, 1 m wide and 1.5 m deep contains 2 m^3 water. How many bricks of 20 cm by 10 cm by 7.5 cm can be put in it so that water does not overflow provided that a brick is supposed to absorb 1/7 of its own volume of water?

(a) 666 (b) 111
(c) 555 (d) None of these

Q.13. In Snehans Apartment there is a playground of the dimension $7\frac{1}{2}$ by 7 m. In the centre of the ground, there is a flower-bed cutout of the dimension 3 m by $2\frac{1}{2}$ m. What fraction of the whole ground is occupied by the flower-bed?

(a) $\frac{3}{7}$ (b) $\frac{3}{5}$ (c) $\frac{1}{7}$ (d) $\frac{1}{11}$

Q.14. The height of a room is $\frac{1}{5}$ th of the sum of its length and breadth. The cost of preparing its wall @ Rs 4 per m^2 is Rs 640. What is the height of the room?

(a) 4 m (b) 5 m
(c) 6 m (d) 7 m

Q.15. A cylindrical container of 32 cm height and 18 cm radius is filled with sand. Now all this sand is used to form a conical heap of sand. If the height of the conical heap is 24 cm, what is the radius of its base?

(a) 12 cm (b) 24 cm
(c) 36 cm (d) 48 cm

Q.16. The total area of the four walls of a room is 150 m^2. If the area of the floor is 50 m^2 and the width of the floor is 3 m, then find the height of the room.

(a) 2.6 m (b) 3.8 m
(c) 5.42 m (d) 7.32 m

Q.17. Anoop has a wooden box, each sides of which is an integer (in cm) and whose volume (neglecting the thickness of the box) is 216 cm^3. What is the minimum possible amount that Anoop will need to spend in order to get the outside of the box painted at the rate of Rs 2 per cm^2?

(a) Rs 320 (b) Rs 225
(c) Rs 236 (d) Rs 432

Direction for questions 18 and 19: Read the passage below and solve the questions based on it.

There is a rectangular table of the dimension 2.20 m × 1.4 m. A rectangular tablecloth of the dimension 2.4 m × 0.8 m is spread unfolded on the table so that the longer sides of the cloth and the table are parallel.

Q.18. If the larger side of the tablecloth and the smaller side of the table are parallel, then what fraction of the area of the top of the table remains uncovered by the cloth?

(a) 3/7 (b) 4/9
(c) 7/11 (d) None of these

Q.19. In the previous question, what fraction of the cloth hangs perpendicularly over the edge of the table?

(a) 5/9 (b) 2/3
(c) 7/13 (d) 5/12

Q.20. A solid consists of a right circular cylinder with the exact fitting right circular cone placed on its top. The height of the cone is N. If the ratio of the total volume of the solid and volume of the cone is 3 : 1, then the height of the circular cylinder is

(a) h/2 (b) 3h
(c) h/3 (d) None of these

Direction for questions 21 and 22: Read the passage below and solve the questions based on it.

There is an equilateral triangle of side 'a' units. Now we join any two sides of the triangle to form a cone.

Q.21. What is the slant height of the cone formed?

(a) $\frac{a}{2\pi}$ (b) a (c) $\frac{a}{2}$ (d) 2 πa

Q.22. What is the radius of the cone formed?

(a) a (b) $\frac{a}{4}$

(c) $\frac{a}{\pi}$ (d) None of these

Q.23. The area of three adjacent faces of a cuboidal box are *p*, *q* and *r* square cm. What is the volume of this box?

(a) $(p + q + r)$ cm³ (b) $\sqrt{pqr}$ cm^3

(c) $\sqrt[3]{pqr}$ cm² (d) pqr cm³

Q.24. Three equal cubes of unit side length are placed adjacent to each other in a row. Find the ratio of the total surface area of the new cuboid to that of the sum of the surface areas of all the three cubes.

(a) 3:5 (b) 4:5

(c) 6:7 (d) 7:9

Q.25. A classroom is to be built to accommodate 70 students. It should be done in such a way that for every student there is 2.2 m^2 of floor and 11 m^3 of space. If the length of the room is 14 m, then find the breadth and height of the room.

(a) 12 m 5.5 m (b) 11 m 5 m

(c) 13 m 6 m (d) 11 m 4 m

Q.26. Length, breadth and height of a rectangular parallelopiped are in the ratio of 6:5:4 and its total surface area is 33,300 m². Find length, breadth and height of the parallelopiped (in cm).

(a) 42, 35, 28 (b) 60, 50, 40

(c) 102, 85, 68 (d) 90, 75, 60

Q.27. A rectangular tank is of dimension 30 m × 20 m. Water is being flown into it through a square pipe of side length 5 cm. Find the speed of the water if the level of water in the tank rises by 1 m in 8 h?

(a) 30 km/h (b) 36 km/h

(c) km/h (d) None of these

Q.28. A cube of side length 3 cm weighs 12 kg. What is the weight of the similar cube of same material whose side length is 12 cm?

(a) 768 kg (b) 678 kg

(c) 964 kg (d) 864 kg

Q.29. A cube of side length 4 cm is cut into cubes of side 1 cm. Find the ratio of the sum of the surface area of all the small cubes to that of the large cube.

(a) 1:16 (b) 2:3

(c) 4:1 (d) 6:1

Q.30. A cylindrical well of depth 12 m with internal ratio of 1.75 m is dug up. The mud, so obtained, is spread evenly to form a platform of dimension 10.5 m × 8.8 m. What is the height of the platform?

(a) 2.25 m (b) 3.25 m

(c) 1.25 m (d) 4.25 m

Q.31. A hollow spherical ball of outer diameter 120 mm is cut into two equal hemispheres. One of the hemispheres is filled with honey and the total surface area of the other hemisphere is $389^5/_7$ cm^2. What is the volume of the honey put in the 1st hemi-sphere?

(a) 128 cm^3 (b) 134 cm^3

(c) 136 cm^3 (d) 138 cm^3

Q.32. The diameter of a cylindrical vessel is made twice as large. How should the height be changed so that it still contains the same quantity of water?

(a) $\frac{1}{2}$ × height of the two cylinders

(b) $\frac{1}{4}$ × height of the original cylinder

(c) Let it be the same

(d) None of these

Q.33. Two shopkeepers are offering Pepsi in two different types of cans. The 1st shopkeeper offers cylindrical cans with a circular base of radius 5 cm and 2nd shopkeeper offers the cans having a square base of length 10 cm. If height of both the cans is the same, then what is the difference in their capacities?

(a) 350 cm^3 (b) 450 cm^3

(c) 250 cm^3 (d) None of these

Q.34. A bucket is a standard example of frustum of a cone. The circumference of such a bucket of one end is 48 cm and of the other end is 34 cm. If the height of the bucket is 10 cm, its volume (in cm^3) is:

(a) 5400 (b) 1350 (c) 2700 (d) 4050

Q.35. The sum of the length, width and depth of a cuboid is *s* and its diagonal is *d*. What is its surface area?

(a) s^2 (b) d^2 (c) $s^2 - d^2$ (d) $s^2 + d^2$

Q.36. A solid right circular cylinder with height 10 cm and radius of the base 6 cm is given. A right circular cone of the same height and base is removed from this cylinder. Find the volume (in cm³) of the remaining solid.

(a) 855.5 (b) 754.3

(c) 1294 (d) None of these

Q.37. At the Amul cooperative society, right cylindrical vessels were used to keep milk. But after the unceremonious exit of Mr Durien, it was decided that instead of right cylindrical vessels, right cones having the same diameter and height would be used. What would be the ratio of the number of the right cones to the number of right cylinders needed to hold the same quantity of milk?

(a) 2:1 (b) 3:1 (c) 4:1 (d) 5:1

Q.38. The radius of a right cylinder is doubled and its height is halved. What is the ratio between the new curved surface area and the previous curved surface area of the cylinder?

(a) 1:1 (b) 2:1 (c) 3:3 (d) 2:3

Q.39. A solid cone is converted into a solid cylinder of the same radius. If the height of the cylinder is 5 m, what is the height of the cone?

(a) 25 m (b) 15 m (c) 20 m (d) 10 m

Q.40. The radii of a cylinder and a cone are equal and the height of the cylinder is equal to the slant height of the cone. Find the ratio of the curved surfaces of the cylinder and the cone.

(a) 1:1 (b) 2:1 (c) 3:1 (d) 4:1

Q.41. A right circular cone is placed inside a cube such that the edges of the base of the cone are touching the edges of one of the faces of the cube and the vertex is touching the opposite face of the cube. If the volume of the cube is 343 cm^3, what is the volume of the cone (approx in cm^3)?

(a) 90 (b) 60 (c) 80 (d) 85

Q.42. When Natraj circus came to Patna for its show at Sonepur mela last year, I observed quite a few things. Its tent was cylindrical to a height of 3 m and conical above it and its diameter was 105 m and slant height of the conical portion was 53 m. But I was not able to calculate the length of the canvas 5 m wide to make the tent. Can you please bail me out by telling this?

(a) 1857 m (b) 1647 m
(c) 1947 m (d) 1847 m

Q.43. A cone and a cylinder have their height in the ratio of 3:2 and the radii of their bases in the ratio of 4:3. Find the ratio of their volumes.

(a) 9:1 (b) 9:2 (c) 8:9 (d) 3:1

Q.44. A cylindrical structure standing on its base with radius 1.5 m and height 5 m is cut with a saw in such a way that the cutting planes goes through all the points at a distance of 0.625 m from the base. Find the volume of the remaining piece.

(a) 5.62π (b) 9.24π
(c) 9.04π (d) None of these

Q.45. The height of a right circular cylinder is 6 m. Three times the sum of the areas of its two circular faces is twice the area of its curved surface. The radius of the base is

(a) 4 m (b) 2 m
(c) 6 m (d) 1.5 m

Q.46. A solid cone kept on its base is cut at 2/3rd of its height along a plane parallel to its circular base. The base radius and the slant height are 14 cm and 50 cm respectively. What is the ratio of the portion cut-out from the solid to the volume of the remaining solid?

(a) 1:20 (b) 1:25
(c) 1:36 (b) None of these

Q.47. A big cube of side 6 cm is formed by putting together 216 small identical cubes each of side 1 cm. Now, if the corner cubes in the topmost layer of the big cube are removed, what will be the impact on the total surface area of the big cube?

(a) Will decrease
(b) Will increase
(c) Will remain unchanged
(d) Cannot be determined

Q.48. Mr Sinha and Mr Verma are two brothers. They inherited their paternal land in a rectangular form having the same area but different dimensions. For Mr Sinha, rectangular plot, the length is increased by 10% and the breadth is decreased by 10%. For Mr Verma's rectangular plot, the length is decreased by 10% and the breadth is increased by 10%. Which of the following is true about the areas of the two rectangular plots now?

(a) The area of the first rectangle is 1% more than that of the second.
(b) The area of the second rectangle is 1% more than that of the first.
(c) The area of the two rectangles is the same.
(d) We cannot comment unless we know the dimensions of the individual rectangles.

Q.49. A big solid sphere of diameter 10 cm is melted and made into solid spheres of diameters 2 cm. What is the percentage increase/decrease in the surface area of smaller spheres over that of the big sphere?

(a) 200% increase (b) 400% increase
(c) 400% decrease (d) None of these

Q.50. A cone is made of a sector of a circle of radius 21 cm and an angle of 60°. What is the total surface are of the cone (in cm^2)?

(a) 269.5 (b) 308
(c) 312.5 (d) 231

MODERATE

Q.1. There is a right circular cone with base radius 3 units and height 4 units. The surface of this right circular cone is painted. It is then cut into two parts by a plane parallel to the base so that the volume of the top part (the small cone) divided by the volume of the frustum equals the painted area of the top part divided by the painted area of the bottom part. The height of the small cone is

(a) 7/3 (b) 5/4
(c) 5/2 (d) None of these

Direction for questions 2 and 3: Read the passage below and solve the questions based on it.

My grandfather owned plenty of land, which easily encompassed an area of 1000 × 200 m^2. He wanted to give some part of it to his servant Ramu. But he did not gift it directly. He supplied the material that could form a fence of length 100 m only. Then he allowed Ramu to take any part with four sides that could be encased with the help of the given fencing material.

Q.2. What is the maximum possible land Ramu can take away from my grandfather? (All the four sides are fenced).

(a) 100 m^2 (b) 2500 m^2
(c) 750 m^2 (d) 625 m^2

Q.3. Ramu had intimate knowledge of the land. Hence, he selected the site having a natural fencing of rocks on one side, because he could utilize the given material only on three sides of the plot. What is the maximum possible land that Ramu can claim now?

(a) 125 m^2 (b) 2500 m^2
(c) 625 m^2 (d) None of these

Q.4 The diameter of a road-roller is 42 cm and its length is 100 cm. It takes 400 complete revolutions moving once over to level the stretch of the road. If the cost of leveling is Rs 100 per m^2 then the total cost of leveling works out to

(a) Rs 52,800 (b) Rs 5,280
(c) Rs 5,28,000 (d) Rs 528

Q.5. Inside a triangular garden, there is a flower bed in the form of a similar triangle. Around the flower bed runs a uniform path of such a width that the sides of the garden are double of the corresponding sides of the flower bed. The areas of the path and the flower bed are in the ratio

(a) 1:1 (b) 4:1
(c) 1:3 (d) 3:1

Q.6. ABCD is a square of side 10 cm. What is the area of the least-sized square that may be inscribed in ABCD with its vertices on the sides of ABCD?

(a) 0 cm^2 (b) 25 cm^2
(c) 50 cm^2 (d) 66.66 cm^2

Q.7. A cylinder of the maximum possible size is made out of a solid wooden cube. How much material is lost in this process (approx.)?

(a) 20% (b) 22%
(c) 24% (d) Cannot be determined

Q.8. Two friends, ND and SD invited friends for a party. A hemispherical bowl full of ice cream of diameter 18 cm was served for dessert. Each of them had 3 scoops of the ice cream and it was exactly sufficient for all of them. If the ice cream was served in a hemispherical scoop of radius 1 cm, how many friends did they have for their party?

(a) 240 (b) 241 (c) 242 (d) 243

Q.9. The cost of white washing one m^2 is Rs 50. What will be the maximum amount saved in painting the room in the most economical way, if the sum of the length, breadth and height is 21 m and all the sides are integers (floor is not to be white washed)?

(a) Rs 1,08,050 (b) Rs 8,400
(c) Rs 9,300 (d) Rs 8,540

Q.10. A small bird is taking rest at the centre of the base of a hemispherical cage. Suddenly it stands up, flies to the topmost point in the cage, then in a straight line to the cage door at the intersection of the curved surface and the base. In the process it covers a total distance of 482 cm. What is the radius of the hemisphere?

(a) 100 cm (b) 150 cm
(c) 200 cm (d) 50 cm

Q.11. A cube is inscribed in a hemisphere of radius R, such that four of its vertices lie on the base of the hemisphere and the other four touch the hemispherical surface of the half-sphere. What is the volume of the cube?

(a) $0.25R^3$ (b) $0.67\sqrt{\frac{2}{3}}R^3$
(c) $0.5\sqrt{\frac{2}{3}R^3}$ (d) $0.67R^3$

Q.12. The biggest possible cube is taken out of a right solid cylinder of radius 15 cm and height 20 cm respectively. What will be the volume (in cm^3) of the cube?

(a) 375 (b) 800
(c) $375\sqrt{2}$ (d) None of these

Direction for questions 13 and 14: Read the passage below and solve the questions based on it.

A larger cube is made up of $N = n^3$ smaller cubes.
Let A = number of smaller cubes with no exposed surfaces.
B = number of smaller cubes with one exposed surfaces.
C = number of smaller cubes with two exposed surfaces.
D = number of smaller cubes with three exposed surfaces.

Q.13. What is the ratio of the number of unexposed smaller cubes to the total number of cubes?

(a) $\frac{1}{27}$ (b) $\frac{(n-2)}{n}$
(c) $\left(\frac{n-2}{n}\right)^3$ (d) $\frac{n}{(n-2)}$

Q.14. What is the total number of smaller cubes in a bigger cube for which A = B?

(a) 512 (b) 256 (c) 1000 (d) 729

Q.15. A ball is fitted into the mouth of an open cone whose base radius is equal to that of the ball. How far can the ball definitely go inside the cone?

(a) Half the ball will fit into the cone
(b) Less than half the ball will fit into the cone
(c) More than half the ball will fit into the cone
(d) One fourth of the ball will fit into the cone

Q.16. A well, 4 m in diameter and 35 m deep, is being dug out and the excavated soil is transported away in a truck trolley of size 5 m × 2 m × 0.5 m. Since the soil that is taken out is loose it occupies 20% more space. How many trips will the truck have to make to clear the excavated soil, if the truck can be filled only up to 80% of its height?

(a) 106 (b) 132
(c) 425 (d) 528

Q.17. The product of all the twelve edges of a rectangular box is equal to:

i. The fourth power of the volume of the box
ii. The product of the areas of the six faces of the box
iii. Sum of the squares of the areas of the six faces of the box

(a) None of the above three values
(b) The figure given by exactly one of the three statements
(c) The figure given by exactly two of the three statements
(d) The figure given by all the three statements

Q.18. A right circular cone is given with a height 6 cm and its slant side is making an angle of 60° with its base. Now a sphere of the maximum volume is placed inside this cone. If the radius of this sphere is 2.33 cm, then find the volume of the cone.

(a) 18π (b) 15π
(c) 24π (d) Cannot be determined

Q.19. A solid sphere of radius 7 cm is melted to form a number of small cones and cylinders. The requirement is such that the number of cones should be twice the number of cylinders. Also the radius of the cone must be equal to its height, which should be equal to the radius of the cylinder and also be half the height of the cylinder. If the height of one such cylinder is 4 cm, find the maximum number of cones which can be made out of the sphere.

(a) 18 (b) 21 (c) 36 (d) 42

Q.20. Let A and B be two solid spheres such that the surface area of B is 300% greater than the surface area of A. The volume of A is found to be K% lower than the volume of B. Then the value of K must be

(a) 85.5 (b) 92.5 (c) 90.5 (d) 87.5

Q.21. A large cube is formed from the material obtained by melting three smaller cubes of 3, 4 and 5 cm side. What is the ratio of the total surface areas of the smaller cubes and the large cube?

(a) 2:1 (b) 3:2 (c) 25:18 (d) 27:20

Q.22. The volume of a cube is V. Then the total length of its edges is

(a) $6V^3$ (b) $8\sqrt{V}$ (c) $12V^{2/3}$ (d) $12V^{1/3}$

Q.23. The height of a cone is 30 cm. A small cone is cut off at the top by a plane parallel to the base. The volume of this smaller cone is 1/27 of the given cone. What is the height of the smaller cone?

(a) 13.5 cm (b) 11 cm
(c) 10 cm (d) 12 cm

Q.24. The maximum distance between two points of the unit cube is

(a) $\sqrt{2}+1$ (b) $\sqrt{2}$
(c) $\sqrt{3}$ (d) $\sqrt{2}+\sqrt{3}$

Q.25. A sealed cylindrical drum of radius r is 9% filled with paint. If the drum is tilted to rest on its side, the fraction of its curved surface area (not counting the flat sides) that will be under the paint is

(a) Less than $\frac{1}{12}$

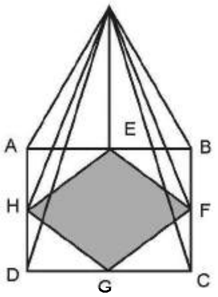
(b) Between $\frac{1}{12}$ and $\frac{1}{6}$

(c) Between $\frac{1}{6}$ and $\frac{1}{4}$

(d) Greater then $\frac{1}{4}$

Direction for questions 26 and 27: Read the passage below and solve the questions based on it.

In the figure given below, the base of the pyramid is a square ABCD of side 10 cm, another pyramid with base EFGH is formed where the points, E, F, G and H are obtained by joining the mid-points of the sides of the square ABCD. The height of the pyramid is 30 cm.



Q.26. Find the ratio of the volumes of pyramid with ABCD as base to the pyramid with EFGH as base.

(a) $\sqrt{2}:1$ (b) $2\sqrt{2}:1$
(c) 4:1 (d) 2:1

Q.27. Find the volume of the space left

(a) 1500 cm^3 (b) 500 cm^3
(c) 1000 cm^3 (d) 800 cm^3

Q.28. All five faces of a regular pyramid with a square base are found to be of the same area. The height of the pyramid is 3 cm. The total area of all its surfaces (in cm^2) is

(a) 8 (b) 10 (c) 12 (d) 16

Direction for questions 29 and 30: Read the passage below and solve the questions based on it.

A right-angled triangle of sides 5 m, 12 m and 13 m is made to spin about its hypotonuse.

Q.29. What kind of figure will be formed with this?

(a) Square (b) Double cone
(c) Sphere (d) None of these

Q.30. What is the volume of the figure thus formed?

(a) 270 m^3 (b) 290 m^3
(c) 320 m^3 (d) None of these

Q.31. A solid cube is cut into two halves by a plane passing through exactly two corners of the cube. What is the ratio of the total surface area of both the halves put together and the original total surface area of the cube?

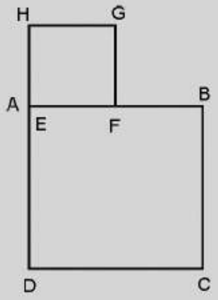
(a) $\sqrt{3}:\sqrt{3}+\sqrt{2}$ (b) $\sqrt{6}+1:\sqrt{6}$
(c) $6:6+\sqrt{6}$ (d) $3+\sqrt{2}:3$

Q.32. Sanjay wound a piece of string over a cylindrical block around its curved surface, and found that the string could be wound for exactly half a round more than an integral number of rounds. He also figured that if the string had been five times as long, the string would have had wound for exactly 10 cm more than an integral number of the rounds. Find the radius of the cylindrical block. (Assume that all windings are done in the same plane and neglect any increase in the radius due to the winding of the string).

(a) $2^{6}/_{22}$ cm (b) $3^{11}/_{22}$ cm
(c) $3^{4}/_{22}$ cm (d) $3^{9}/_{22}$ cm

Direction for questions 33 and 34: Go through the figure given below and solve the questions based on it.

Consider the figure given below:



A cube with surface EFGH is put on a cube with a surface ABCD in such a way that vertices E and A of the two blocks coincide and side EF coincides with side AB. EF = 1 unit and AB = 2 units. Now, keeping vertex F fixed, the smaller cube is rotated along the larger cube till G coincides with B once. Then keeping G fixed, the smaller block is again rotated till GH coincides with BC.

Q.33. How much is the aerial distance traversed by point E?

(a) $\sqrt{2}\pi$ units (b) π units
(c) $\sqrt{2}\pi + \frac{1}{2}\pi$ units (d) None of these

Q.34. In the above question, what is the total distance covered by point E?

(a) π units (b) $(1 + \pi)$ units
(c) $(1 + \sqrt{\pi})$ units (d) None of these

Q.35. How many cuboids of different dimensions can be made assembling 100 identical cubes?

(a) 9 (b) 8
(c) 12 (d) 10

Q.36. A cuboid is of the dimension $2 \times 2 \times 3$ units. Of the six surfaces of this cuboid, there are exactly x squares and exactly y rectangles (rectangles which are not squares). Find the product of $x \times y$.

(a) 8 (b) 12
(c) 6 (d) None of these

Q.37. A box without a lid has a height of x cms, a square base of side y cm and a volume of 500 cm^3. For what value of x (in cms), the least quantity of the material will be needed to construct the box?

(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) 5 (d) $5\sqrt{2}$

Q.38. Under the Indian Posts and Telegraph Act 1885, any package in the form of a right circular cylinder will not be accepted if the sum of its height and the diameter of its base exceeds 10 inches. The height (in inches) of a package of maximum volume that would be accepted is

(a) 10/3 (b) 20/3 (c) 10 (d)20

Q.39. A convex pentagon IJKLM is inscribed in a circle. If the angles subtended by the sides IJ, JK, KL, LM at the centre are 50°, 80°, 60° respectively, what is the measurement of the smallest interior angle of the pentagon?

(a) 120° (b) 95°
(c) 85° (d) None of these

Q.40. A right circular cone, with radius to height ratio as 12 : 5, is cut parallel to its base to get a smaller cone and a frustum. If the height of the smaller cone to the height of the frustum are in the ratio of 3 : 1, by what percentage is the combined total surface area of the smaller cone and frustum more with respect to the original cube?

(a) 22% (b) 32%
(c) 46% (d) None of these

ADVANCED

Q.1. A cuboid of length 20 m, breadth 15 m and height 12 m is lying on a table. The cuboid is cut into *two equal halves* by a plane which is perpendicular to the base and passes through a pair of diagonally opposite points of that surface. Then, a second cut is made by a plane which is parallel to the surface of the table again dividing the cuboid into *two equal halves*. Now this cuboid is divided into four pieces. Out of these four pieces, one piece is now removed from its place. What is the total surface area of the remaining portion of the cuboid?

(a) 1290 m^2 (b) 1380 m^2
(c) 1440 m^2 (d) Cannot be determined

Q.2. A metal is made up of a cylindrical base and a conical top with the base of radius 5 cm. The ratio of height of the cone and the cylinder is 2:3. A cylindrical hole is drilled through the metal solid with height two-third the height of the metal solid. What should be the radius of the hole, so that the volume of the hole is 1/3rd the volume of the metal solid after drilling?

(a) $\sqrt{45}$ cm (b) $\sqrt{\frac{55}{8}}$ cm
(c) $\sqrt{35}$ cm (d) $\sqrt{65}$ cm

Q.3. Sixteen cylindrical cans of Coke, each with a radius of 1 cm, are placed inside a wooden carton, four in a row. If the cans touch the adjacent cans and/or the walls of the box, then which of the following could be the internal area of the bottom of the carton (in cm^2)?

(a) 16 (b) 32
(c) 64 (d) None of these

Q.4. The areas of three adjacent faces of a cuboid are a, b and c. If the volume of the cuboid is N, then N^2 is equal to

(a) abc (b) $(ab + bc + ac)$
(c) $1/c$ (d) None of these

Q.5. A square tin sheet of side 12 inches is converted into a box with open top in the following steps: The sheet is placed horizontally. Then, equal-sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?

(a) 3 (b) 4 (c) 1 (d) 2

Q.6. A rectangular piece of cardboard 18 cm × 24 cm is made into an open box by cutting a square of 5 cm side from each corner and building up the side. What is the volume of the box (in cm^3)?

(a) 560 (b) 432
(c) 216 (d) None of these

Q.7. Consider the following function

E = f(A) = $\sqrt{\frac{A}{7}}$ where length of the edge E of a pyramid and the surface area A of the pyramid are the different terms used. How much longer is the edge of the pyramid with a surface area 3087 square units than the edge of one with a surface area 2023 square units?

(a) 1064 (b) 152
(c) 4 (d) None of these

Q.8. The ratio of the volumes of the two cylinders are a : b, and their heights are in the ratio c:d. What is the ratio of their diameters?

(a) $\frac{ad}{bc}$ (b) $\frac{d^2}{c^2}$

(c) $\sqrt{\frac{ad}{bc}}$ (d) $\sqrt{\frac{a}{b}\times\frac{c}{d}}$

Q.9. From a solid right circular cone made of iron with base of radius 2 cm and height 5 cm, a hemisphere of diameter 2 cm and its centre, coinciding with the centre of the base of the cone, is cut out. The structure so obtained is then dropped in a right circular cylinder whose inner radius is 3 cm and inner height is 10 cm. Water is then poured into the cylinder to fill it up to its brim. What is the volume of the water required to fill it?

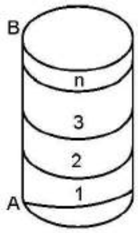
(a) 84π cm³ (b) $\frac{250\pi}{3}$ cm³

(c) $\frac{270\pi}{4}$ cm³ (d) None of these

Direction for questions 10 to 12: Read the passage below and solve the questions based on it.

Consider a cylinder of height h cm and radius $r=\frac{2}{\pi}$ cm as shown in the figure (not drawn to scale). A string of a certain length, when wound on its cylindrical surface, starting at a point A and ending at point B, gives a maximum of n turns (in other words, the string length is the minimum length to wind n turns).

Q.10. What is the vertical spacing in cm between consecutive turns?



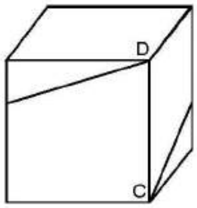
(a) $\frac{h}{n}$

(b) $\frac{h}{\sqrt{n}}$

(c) $\frac{h}{n^2}$

(d) Cannot be determined with the given information.

Q.11. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string in cm, is



(a) $\sqrt{2}n$ (b) $\sqrt{17}n$ (c) n (d) $\sqrt{13}n$

Q.12. In the setup of the previous two questions, how is h related to n?

(a) $h=\sqrt{2}n$ (b) $h=\sqrt{17}n$
(c) $h=n$ (d) $h=\sqrt{13}n$

Q.13. An octagonal prism, of height 20 cm and a side of the base as $\sqrt{4\left(2-\sqrt{2}\right)}$ cm, is cut from its base to the top along the edges into two similar parts of equal volume. What is the total surface area of each part?

(a) $80+160\sqrt{2-\sqrt{2}}+4\sqrt{2}$ cm²
(b) $80+40\sqrt{2-\sqrt{2}}$ cm²
(c) $80+160\sqrt{2-\sqrt{2}}+8\sqrt{2}$ cm²
(d) None of these

Direction for questions 14 to 16: Read the passage below and solve the questions based on it.

There are 300 coins, each coin having radius 2 cm and height 1 cm. The coins are so kept that each coin touches the other two. The base has 3 coins and the figure is built upon this base.

Q.14. Find the volume of the region enclosed by the coins.

(a) $600\left(2\sqrt{3}-\pi\right)$cm³ (b) $400\left(2\sqrt{3}-\pi\right)$cm³

(c) $100\left(2\sqrt{3}-\pi\right)$cm³ (d) $200\left(2\sqrt{3}-\pi\right)$cm³

Q.15. Find the number of spheres of the maximum volume that can be accommodated in the above region.

(a) 324 (b) 323
(c) 162 (d) 161

Q16. Find the volume of the prism circumscribing the whole structure of the coins.

(a) $100\sqrt{3}\left(7+4\sqrt{3}\right)\text{cm}^3$

(b) $2400\sqrt{3}\left(2+\sqrt{3}\right)\text{cm}^3$

(c) $800\sqrt{3}\left(2+\sqrt{3}\right)\text{cm}^3$

(b) None of these

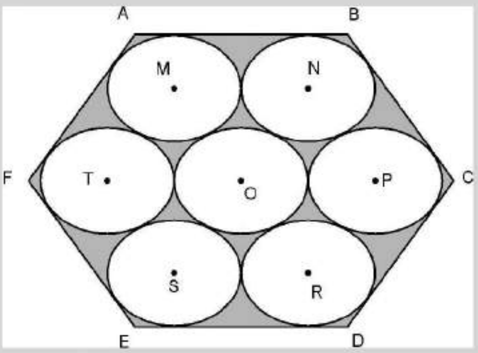
Q.17. A cylinder of height 'h' cm and radius 6/p cm is wound around with a string of width 'x' cm. The string covers the lateral surface area of the cylinder completely without keeping any space between two turns. What is the required length of the string.

(a) $\frac{12h}{x}$cm (b) $\frac{12x}{h}\times$cm

(c) 12 h cm (d) $\frac{6h}{x}$cm

Direction for questions 18 to 20: Read the passage below and solve the questions based on it.

In the figure given below, a regular hexagon ABCDEF is inscribed by 7 circles. The radius of each circle is equal to 6 cm. The centres of the circles touching the sides are in, M, N, P, R, S, and T whereas the centre of the central circle is O.



Q.18. Find the ratio of the perimeter of the hexagon circumscribing the circle with centre O, the perimeter of hexagon formed by joining the points M, N, P, R, S and T and the perimeter of the hexagon ABCDEF.

(a) 6:12:(12 + 2√3 (b) 2:2√3:(2√3 + 1)

(c) 1:√3:(√3+1) (d) 1:2:3

Q.19. Find the area of the shaded region (in cm^2).

(a) 136.83 (b) 129.63

(c) 139.83 (d) 138.83

Q.20. If the above figure is a 3-D figure and seven spherical balls of radius 6 cm each are tightly arranged in a hexagonal box in a single layer, then what will be the volume of the box unoccupied by the balls in cm^3?

(a) $3456\sqrt{3}+5184-252\pi$

(b) $3456\sqrt{3}+5184-288\pi$

(c) $576\sqrt{3}+864-288\pi$

(d) None of these

Q.21. Four spheres each of radius 10 cm lie on a horizontal table so that the centres of the spheres form a square of side 20 cm. A fifth sphere also of radius 10 cm is placed on them so that it touches each of these spheres without disturbing them. How many cm above the table is the centre of the fifth sphere?

(a) $10\sqrt{6}$ (b) $10\left(1+\sqrt{2}\right)$

(c) $10\left(1+\sqrt{3}\right)$ (d) $10\left(4-\sqrt{2}\right)$

Q.22. A cube with an edge 'a' is cut from the angles by planes so that a regular octagon remains from each face, then the volume of the polygon so obtained will be

(a) $a^3\left(7\sqrt{2}-9\right)$ (b) $\frac{7}{3}a^3\left(\sqrt{2}-1\right)$

(c) $7a^3\left(\sqrt{2}-1\right)$ (d) $\frac{a^3}{3}\left(7\sqrt{2}-9\right)$

Q.23. A square of side x cm is cut from each corner of a rectangular sheet of metal of 10 cm by 14 cm. The resulting projections are folded up and the seams welded to construct an open box. What is the volume of the box thus obtained?

(a) $4x^3 - 48x^2 + 140x$ (b) $4x^3 + 48x^2 + 140x$

(c) $x^3 + 24x^2 + 140x$ (d) None of these

Q.24. A sphere of radius 13 cm is cut by a plane whose distance from the centre of the sphere is 5 cm. What is the circumference of the plane circular section?

(a) 10π cm (b) 12π cm

(c) 24π cm (d) 26π cm

Q.25. The square of side 1 cm are cut from four corners of a sheet of tin (having length = l and breadth = b) in order to form an open box. If the whole sheet of tin was rolled along its length to form a cylinder, then the volume of the cylinder is equal to (343/4) cm^3. Find the volume of the box. (l and b are integers)

(a) 154 cm^3 (b) 100 cm^3

(c) 126 cm^3 (c) Insufficient data

Q.26. Two spheres of radii 6 cm and 1cm are inscribed in a right circular cone. The bigger sphere touches the smaller one and also the base of the cone. What is the height of the cone?

(a) 14 cm (b) 12 cm

(c) 85/6 cm (d) 72/5 cm

Q.27. If the total length of the diagonals of a cube is 12 cm, what is the total length of the edges of the cube?

(a) $6\sqrt{3}$ cm (b) 12 cm

(c) $12\sqrt{3}$ cm (d) 15 cm

Q.28. A cylindrical vessel open at the top contains water up to 1/3 of its height. A heavy sphere whose diameter is equal to the height of the cylinder is placed into the vessel touching its curved surface from all sides, then the water.

(a) Level will rise to half of its height

(b) Level will rise to 3/4th its height

(c) Level will rise to its height

(d) Water will overflow

Q.29. A sphere of maximum possible volume is to be completely immersed into a cylindrical container of radius 'a' containing water up to a height '$2a$'. What is the minimum height of the jar so that no water spills out of it?

(a) $\frac{10a}{3}$ (b) $\frac{11a}{3}$ (c) $\frac{12a}{3}$ (d) $\frac{13a}{3}$

Q.30. In a rectangular parallelopiped A, a, b and c are the lengths of the diagonals on three faces having a corner in common. Let d be the length of the largest diagonal that can be drawn in A. Then

(a) $2d^2 = a^2 + b^2 + c^2$

(b) $d^2 = a^2 + b^2 + c^2$

(c) $d^2 = 2(a^2 + b^2 + c^2)$

(d) None of these

Direction for questions 31 to 33: Read the passage below and solve the questions based on it.

A right-angled triangle with base and height measuring 15 cm and 20 cm is rotated along its hypotonuse. The structure so formed is placed by two spheres such that they are just touching the sides of the cone.

Q.31. Find the volume of the structure formed before the double cone is placed (in cm^3)?

(a) 1200π (b) 3600π

(c) 1000π (d) None of these

Q.32. Find the sum of the volumes of both the sphere (in cm^3).

(a) $\frac{364}{3}\pi$ (b) $\frac{140}{3}\pi$

(c) 12π (d) 47π

Q.33. Find the curved surface area of the figure (in sq. units).

(a) 840π (b) 400π

(c) 420π (d) None of these

Q.34. A square hole of cross-sectional area 4 cm^2 is drilled across a cube with its length parallel to a side of the cube. If an edge of the cube measures 5 cm, what is the total surface area of the body so formed?

(a) 158 cm^2 (b) 190 cm^2

(c) 166 cm^2 (d) 182 cm^2

Q.35. The radius and the height of a right solid circular cone are r and h respectively. A conical cavity of radius r/2 and height h/2 is cut out of the cone. What is the whole surface area of the rest of the portion?

(a) $\frac{\pi r}{4}\left(5\sqrt{(r^2+h^2)}\right)+3r$

(b) $\frac{5\pi r}{4}\sqrt{r^2+h^2}$

(c) $\frac{3\pi r}{4}\left(\sqrt{r^2+h^2}+r\right)$

(d) $\frac{3\pi r}{7}\left(\sqrt{r^3+h^2}+r\right)$

Q.36. John Nash, an avid mathematician, had his room constructed such that the floor of the room was an equilateral triangle in shape instead of the usual rectangular shape. One day he brought home a bird and tied it to one end of a string and then tied the other end of the string to one of the corners of his room. The next day, he untied the other end of the string from the corner of the room and tied it to a point exactly at the center of the floor of the room. Assuming that the dimensions of the room are relatively large compared to the length of the string, find the number of times, by which the maximum possible space in which the bird can fly, increase.

(a) 4 (b) 5

(c) 6 (d) 7

Direction for questions 37 to 40: Read the passage below and solve the questions based on it.

Homes of three of my friends–Bango, Mango and Tatto–are located at the three vertices of the municipal park near my house. This park is triangular in shape. Given below are the directions to reach each of their homes from my home:

- Bango's house–100 m to the east and 150 m to the north.
- Mango's house–200 m to the east and 50 m to the north.
- Tatto's house–50 m to the west and 25 m to the north.
- Neglect dimensions of all the house while calculating the distances.

Q.37. What is the area of the municipal park?

(a) 27,500 m^2 (b) 11,500 m^2

(c) 13,750 m^2 (d) Cannot be determined

Q.38. There is a highway (a straight road) passing through the houses of Bango and Mango. Tatto starts from his house and takes the shortest possible route to reach the highway. What is the distance travelled by Tatto?

(a) $\frac{125}{\sqrt{3}}$ (b) $\frac{275}{\sqrt{2}}$

(c) $\frac{150}{\sqrt{2}}$ (d) None of these

Q.39. Travelling along one of the following routes will take me from my house to the point (on the highway) where Tatto reached in the previous question. Which one is it?

(a) 100 m to the east, 200 m to the north

(b) 87.5 m to the east and 62.5 m to the north

(c) 67.5 m to the east and 167.5 m to the north

(d) None of these

Q.40. What is the approximate distance between Mango's house and the point Tatto reached in Q. 39?

(a) $\frac{225}{\sqrt{2}}$ m (b) $\frac{275}{\sqrt{2}}$ m

(c) $\frac{150}{\sqrt{2}}$ m (d) $\frac{325}{\sqrt{2}}$ m

ANSWERS

■■■

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(c)	(c)	(a)	(a)	(d)	(b)	(b)	(c)	(d)	(b)	(b)	(d)	(c)	(c)	(b)	(a)	(c)	(c)	(a)
Q. No.	21	22																		
Answer	(c)	(c)																		

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(c)	(d)	(a)	(c)	(c)	(b)	(c)	(c)	(b)	(d)	(c)	(a)	(c)	(b)	(d)	(c)	(d)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(d)	(b)	(d)	(b)	(d)	(a)	(a)	(c)	(c)	(b)	(b)	(b)	(d)	(c)	(b)	(b)	(a)	(b)	(b)
Q. No.	41	42	43	44	45	46	47	48	49	50										
Answer	(a)	(c)	(c)	(d)	(a)	(d)	(c)	(c)	(b)	(a)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(d)	(a)	(d)	(c)	(b)	(b)	(c)	(c)	(b)	(d)	(c)	(a)	(b)	(b)	(c)	(c)	(d)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(c)	(d)	(c)	(c)	(a)	(d)	(a)	(c)	(b)	(d)	(d)	(c)	(c)	(b)	(b)	(a)	(c)	(a)	(b)	(d)

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(c)	(a)	(d)	(a)	(c)	(c)	(a)	(a)	(b)	(c)	(d)	(d)	(d)	(c)	(a)	(c)	(d)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Answer	(b)	(b)	(a)	(c)	(b)	(d)	(c)	(c)	(a)	(a)	(a)	(b)	(c)	(d)	(a)	(b)	(c)	(b)	(b)	(a)

HINTS AND SOLUTIONS

Moderate

1. Take the total surface area of the initial cone into consideration and then proceed.

6. The vertices of the smaller square will be on the mid-points of the sides of the larger square.

9. The amount saved would be maximum only when the difference of maximum area and minimum area to be painted is maximum.

11. Let ABCDEFGH be the cube of side a and O be the centre of the hemisphere.
AC = √2 a
OD = OC = R
Let P be the mid-point of AC
OP = a
Now in ΔAOC

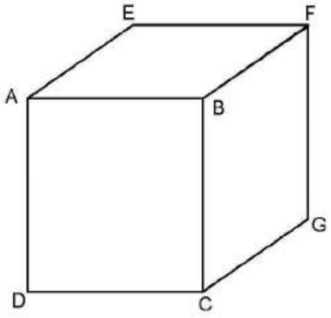
$$R^2 = a^2 + \frac{a^2}{2} \qquad \therefore a = \sqrt{\frac{2}{3}}R$$

13. Take a value of *n*, say 3 and then check options.

14. Go through options.

20. Get a relation between the radii of the two spheres.

24. The distance from any vertex at the base of the cube to the vertex that is perpendicular along height to the diametrically opposite vertex is required.
We have to calculate DF.



28. Equate the area of the square ABCD and triangle PDC and find a relation between the slant height and the length of the base of the pyramid

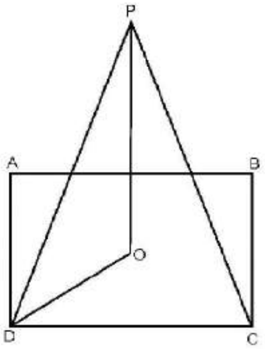
$$a^2 = \frac{a}{4}\sqrt{4x^2 - x^2}$$

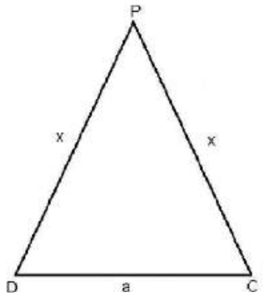
$$16a^2 = 4x^2 - a^2$$

$$\Rightarrow 17a^2 = 4x^2$$

$$\frac{a}{x} = \frac{2}{\sqrt{17}}$$

Let $a = 2\pi$; $x = \sqrt{17}\,\text{P}$





Now in Δ POD

$\text{OD} = \sqrt{2}P$ $\text{PD} = \sqrt{17}P$

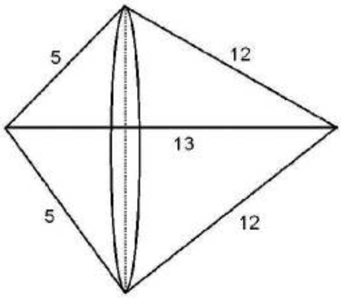
$\text{AO} = \sqrt{15}P$ But AO = 3 ⇒ $\text{P} = \frac{3}{\sqrt{15}}$

$2\text{P} = a = \frac{6}{\sqrt{15}}$

Area of the base = $\frac{36}{15}$

Total surface area = $\frac{36}{15} \times 5 = 12$

29.



A structure will be formed like the figure given.

32. In the first operation, the string that is extra after winding an integral number of times will be equal to 10 cm.

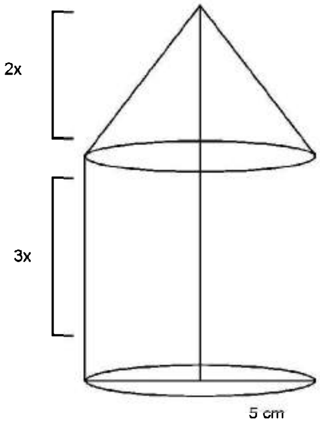
35. Use factors of 100.

38. Volume is maximum when radius is equal to height.

Advanced

1. Since we don't know that the cut is made parallel to which face, we cannot determine the surface area.

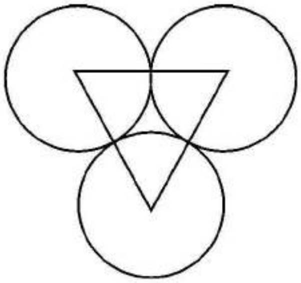
2. Suppose the height of the cone is 6 cm and that of cylinder is 9 cm. Thus, the height of the hole is10 cm now a straightforward calculation leads to the answer.



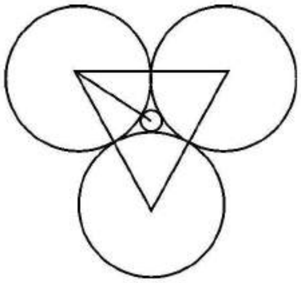
10. If the cylinder is cut along its length, it becomes a rectangle.

12. Equate the lengths of strings obtained from the above two questions.

14. Volume of hollow space = area of the triangle × height (100 cm) – ½ volume of the cylinder of base radius 2 cm and height 100 cm



15.

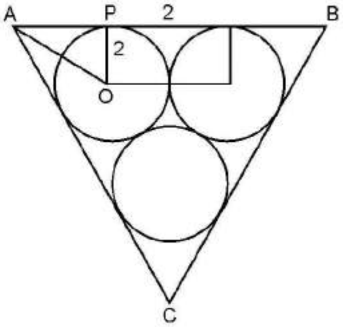


Volume of the hollow space = Area of the triangle – ½ Volume of the cylinder of base radius 2 cm and height 100 cm.

16. r = Radius of sphere

$$\frac{V_3}{2} \times 2\frac{2}{3} - 2\left(\frac{4}{\sqrt{3}-2}\right) \text{cm}$$

16.



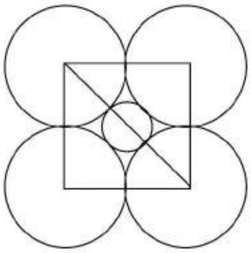
∠AOP = 30°

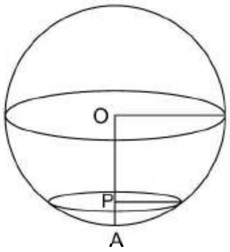
OP = 2 cm

Calculate AP

20. The 3D shape of the hexagon will be a hexagonal prism.

21. OA = 10

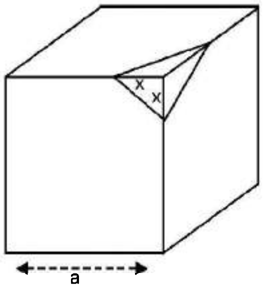




OA = 10

To find the value of PA, go through the options now.

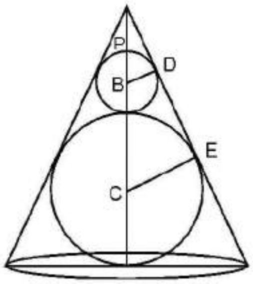
22. In this question we need to cut eight identical pyramids having a base of equilateral triangle of side equal to the side of the newly-formed octagon.



$A = 2x + \sqrt{2}x$

25. The length of the rectangle will be equal to the circumference of the base of the cylinder.

26. Let AP = x
$\Delta ABD \sim \Delta ACE$
Now x can be calculated.



36. Consider the length of the string less than or equal to the inradius of the floor. At the corner of the floor you will find a sixth part of a hemisphere and at the centre it will be a hemisphere.

Questions 37 to 40

Plot these points and direction on a graphical plane for a clear interpretation.

19

CHAPTER

Co-Ordinate Geometry

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Types of sets and methods of their formation
- Definitions and properties
- Maxima and minima in set theory
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

The method of denoting points with the help of coordinates was proposed by French Mathematician Descartes. Further, he also proposed that lines and curves are nothing but the collection of points, and hence can be represented by equations derived out of Coordinate Geometry. To honour his work, the coordinates of a point are often referred to as its Cartesian coordinates.

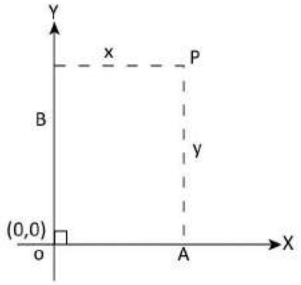
CONSIDER THE FOLLOWING CASE

Suppose there is an ant at point *O* in the figure and it wants to go to point *P*.

One way to reach P is that the ant travels along *OX*, reaches A and then travels along *AP* and reaches *P*.

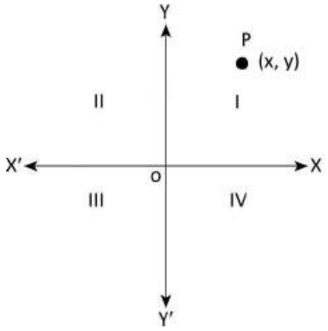
Hence, it first covers a distance '*X*' horizontally and then covers a distance '*Y*' vertically.

1st of all, we will assume a reference point, which is at a distance of 0 unit from both the X-axis and Y-axis and will call this Origin.



If we assume '*O*' as the origin, the distances *X* and *Y* are the distances of this point *P* from Y–axis and X-axis respectively. These are known as coordinates of point P and it is written as *P*(*X*, *Y*).

CO-ORIDNATE AXES AND REPRESENTATION OF A POINT



The figure given alongwith is called the *X*-*Y* Cartesian plane. The line XOX' is called the X-axis and YOY' the Y-axis.

If $P(x, y)$ is a point in this plane, then x is the X- coordinate of *P* or abscissa of *P* and y is called the Y-coordinate of P or the ordinate of *P.*

Remember that X-coordinate of the point is the distance of the point from Y-axis and Y-coordinate of the point is the distance of the point from X-axis.

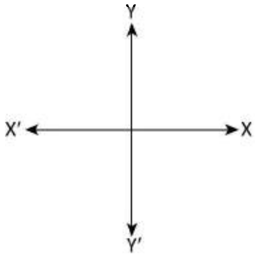
The X-Y Cartesian plane is divided into four equal parts called Quadrants (I, II, III, IV).

SIGN CONVENTION

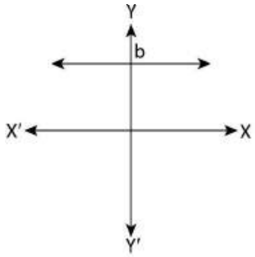
	1st Quadrant	*2nd Quadrant*	*3rd Quadrant*	*4th Quadrant*
X-axis	+ve	–ve	–ve	+ve
Y-axis	+ve	+ve	–ve	–ve

EQUATION AND GRAPH OF CO-ORDINATE AXES

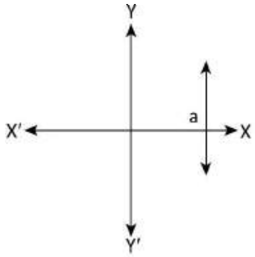
1. Equation of *X* and *Y* axes are $Y = 0$ and $x = 0$ respectively.



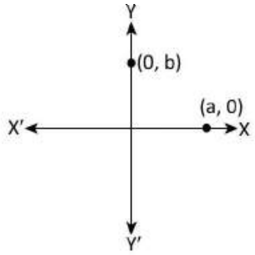
2. Equation of a line parallel to *X* axis is $Y = b$ (*b* is constant).



3. Equation of a line parallel to *Y* axis is $X = a$ (*a* is constant).



4. Any point on the X axis can be taken as (*a*, 0) and any point on the Y axis can be taken as (0, *b*).



5. To find out X and Y intercepts of a line, we will put Y = 0 and X = 0 respectively in the equation of the line.

Some standard formulae

1. Distance between two points:
   If there are two points $A\ (x_1,\ y_1)$ and $B\ (x_2, y_2)$ on the XY plane, then the distance between them is given by

$$AB = d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

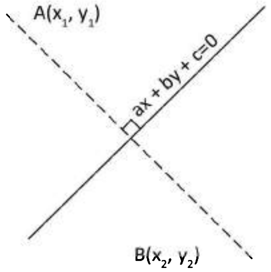
2. Division of a line segment:
   i. Internal - The Coordinates of a point P which divides the line joining $A\ (x_1\ y_1)$ and $B\ (x_2, y_2)$ internally in a ratio $l : m$ are given by

$$x = \frac{lx_2 + mx_1}{l+m}, y = \frac{ly_2 + my_1}{l+m}$$

   ii. External:
   The coordinates of a point P which divides the line joining the point $A(x_1, y_1)$ and $B(x_2, y_2)$ externally in the ratio $l : m$ are given by

$$x = \frac{lx_2 - mx_1}{l-m}, y = \frac{ly_2 - my_1}{l-m}$$

THE IMAGE OF A POINT ALONG THE MIRROR PLACE ON A STRAIGHT LINE



The image of $A\ (x_1, y_1)$ with respect to the line mirror $ax + by + c = 0$ be $B(x_2, y_2)$ is given by

$$\frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b}$$

$$= \frac{-2(ax_1 + by_1 + c)}{(a^2 + b^2)}$$

FOOT OF THE PERPENDICULAR

If the foot of the perpendicular from (x_1, y_1) to the line $lx + my + n = 0$ is (h, k) then

$$\frac{h - x_1}{l} = \frac{k - y_1}{m} = \frac{-(lx_1 + my_1 + n)}{l^2 + m^2}$$

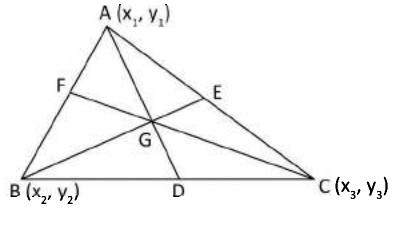
CENTROID OF A TRIANGLE

The point at which the medians of a triangle intersect is called the centroid of the triangle.

Let ABC be a given triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.

Since D is the mid-point of BC, its coordinates are $[(x_2 + x_3)/2, (x_2 + x_3)/2]$

Let G (x, y) be a point dividing AD in the ratio 2:1.



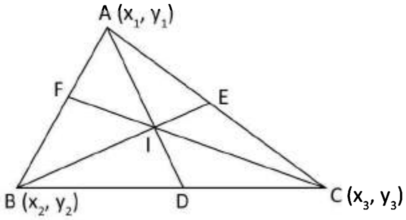
$$\text{Then, } x = \frac{2\left(\frac{x_2 + x_3}{2}\right) + 1.x_1}{(2+1)} = \left(\frac{x_1 + x_2 + x_3}{3}\right)$$

$$\text{and } \quad y = \frac{2\left(\frac{y_2 + y_3}{2}\right) + 1.y_1}{(2+1)} = \left(\frac{y_1 + y_2 + y_3}{3}\right)$$

Similarly, the coordinates of a point which divides BE in the ratio 2:1 as well as those of the point which divides CF in the ratio 2:1 are

$$\left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}\right)$$

INCENTRE OF A TRIANGLE

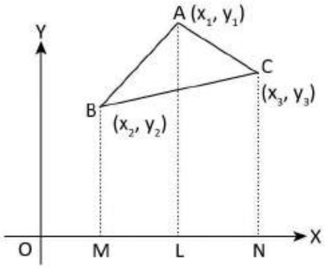


The point at which the bisectors of the angles of a triangle intersect, is called the incentre of the triangle. From geometry, we know that the bisector of an angle of a triangle divides the opposite side in the ratio of length of remaining sides. Hence, the bisectors of the angle of ΔABC are concurrent, and meet at a point, called incentre.

$$\left(\frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c}\right)$$

Area of triangle

Let ABC be a given triangle whose vertices are $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$.



Then the area of the triangle is given by

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right].$$

If we interchange the order of any two vertices of the ΔABC, we obtain a negative value of the area. However, the area shall always be taken to be positive.

EQUATION OF A CURVE

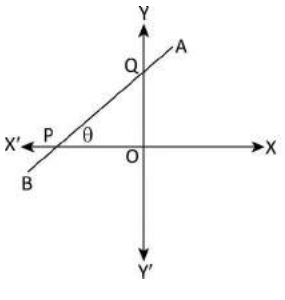
An equation in two variables X and Y with the degree of the equation being equal to or more than two is called the equation of a curve. If the graph of that equation plotted on the XY Cartesian plane, it will give a shape of a curve, and not a straight line

For example, $x^2 + y^2 = 16, y = x^2$.

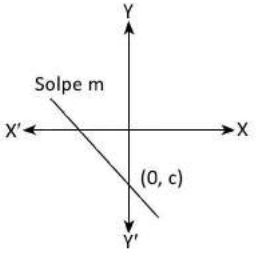
STRAIGHT LINE

Any equation with the degree of equation being one is known as an equation of straight line. General equation of straight line is given by $aX + bY + c = 0$, where X and Y are variables and a, b, c are constants.

Any point lying on this line will satisfy the equation of the line.



If AB is a straight line on the XY plane, then the angle θ which the line makes with the X axis in the anticlockwise direction is called the inclination of the line and tangent of this angle θ (tan θ) is called the slope of the line AB. It is denoted by 'm'. The lengths OP and OQ are respectively known as the intercepts on X-axis and Y-axis, made by the line.



DIFFERENT FORMS OF REPRESENTING A STRAIGHT LINE

1. Slope- intercept form

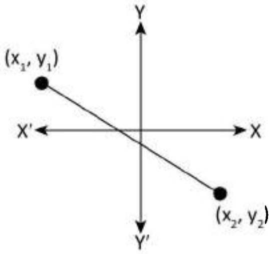
$$y = mx + c$$

If 'm' is the slope of the line and 'c' the intercept made by the line on Y axis, the equation is

$$y = mx + c$$

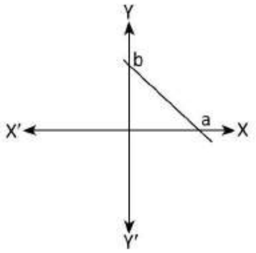
2. Point-slope form
If 'm' is the slope of the line and it passes through the point, the equation is (x_1, y_1), then the equation of the line is given by:

$$y - y_1 = m\,(x - x_1)$$



3. Two-point form
If the line passes through two points (X_1, Y_1) and (X_2, Y_2) the equation is

$$(Y - Y_1) = \frac{Y_2 - Y_1}{X_2 - X_1}(X - X_1)$$



Using Point-slope form and Two-point form, we can find out the formula for slope also. Comparing the two equations, we get m = $\frac{Y_2 - Y_1}{X_2 - X_1}$

4. Slope-intercept form
   If the line makes an intercept of '*a*' units on X-axis and b units on Y-axis, then the equation is:

$$\frac{X}{a}+\frac{Y}{b}=1$$

Angle between two intersecting lines

The angle between two lines whose slopes are m_1 and m_2 is given by a formula such that $\tan , = \frac{m_1-m_2}{1+m_1m_2}$ (where θ is the angle between the lines)

Condition for two straight lines to be parallel

It can be visualized that two straight lines can be parallel only if they make an equal inclination with the X-axis. This will, in turn, ensure that their slopes are equal.

The lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are parallel if and only if $m_1 = m_2$.

Condition for two straight lines to be perpendicular

The lines $y = m_1x + c_1$ and $y = m_2x + c_2$ are perpendicular if and only if $m_1\,m_2 = -1$.

The equation of a line parallel to a given line $ax + by + c = 0$ will be $ax + by + k = 0$, where k is any constant which can be found by additional information given in the question.

The equation of a line perpendicular to a given line $ax + by + c = 0$ will be $bx - ay + k = 0$, where k is any constant which can be found by additional information given in the question.

Point of intersection of two lines

The coordinates of the point of intersection of the two intersecting lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

$$\left(\frac{b_1c_2-b_2c_1}{a_1b_2-a_2b_1},\frac{a_2c_1-a_1c_2}{a_1b_2-a_2b_1}\right)$$

However, to obtain the point of intersection, we are required to just solve the equations of the straight lines given as we do in the case of Simultaneous Equations.

Condition of concurrency of three lines

Three lines are said to be concurrent, if they pass through a common point i.e. if they meet at a point. The condition for three lines $a_1x + b_1y + c_1 = 0$, $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ is $a_1\,(b_2c_3 - b_3c_2) + b_1\,(c_2a_3 - c_3a_2) + c_1(a_2b_3 - a_3b_2) = 0$

Length of perpendicular

The length of perpendicular (p) from (X_1, Y_1) on the line $AX + BY + C = 0$ is:

$$P = \left|\frac{AX_1+BY_1+C}{\sqrt{A^2+B^2}}\right|$$

Distance between two parallel lines

The distance between two parallel lines $AX + BY + C_1 = 0$ and $AX + BY + C_2 = 0$ is given by

$$\left|\frac{C_1-C_2}{\sqrt{A^2+B^2}}\right|$$

Conditions for points to be collinear

If three points A, B, C are co-linear, then any one of the following conditions should be true :

i. Area of triangle $ABC = 0$
ii. Slope of AB = slope of BC = slope of AC
iii. $AB + BC = AC$

Depending upon the points given, we can use any one of three to check if the points are collinear. It should also be mentioned that if one of these conditions is true, then other two will definitely be true.

WORKED OUT PROBLEMS

Example 1 Find the coordinates of the point which divides the line segment joining the point (5, –2) and (9,6) in the ratio 3:1.

Solution Let the required point be (x, y). Then,

$$x=\left(\frac{3\times9+1\times5}{3+1}\right)=8$$

and

$$y=\left(\frac{3\times6+1\times(-2)}{3+1}\right)=4$$

The coordinates of the required point are (8, 4).

Example 2 Find the ratio in which the point (2, y) divides the join of (–4, 3) and (6, 3) and hence find the value of y.

Solution Let the required ratio be k : 1.

Then, $$2=\frac{6k-4\times1}{k+1}\Rightarrow k=\frac{3}{2}$$

∴ The required ratio is $\frac{3}{2}$:1, i.e., 3:2.

Also, $$y=\frac{3\times3+2\times3}{3+2}=3$$

When asked for ratio m : n, for convenience we take ratio as m/n : 1 or k : 1.

Example 3 Two vertices of a triangle are (–1, 4) and (5, 2) and its centroid is (0, –3). Find the third vertex.

Solution Let the third vertex be (x, y). Then

$$\frac{x+(-1)+5}{3}=0 \text{ and } \frac{y+4+2}{3}=-3$$

$\therefore\ x=-4$ and $y=-15$

Hence the third vertex of the triangle is (–4, –15).

Example 4 Find the co-ordinates of incentre of the triangle whose vertices are (4, –2), (–2, 4) and (5, 5).

Solution $a=BC=\sqrt{(5+2)^2+(5-4)^2}=\sqrt{50}=5\sqrt{2}$

$$b=AC=\sqrt{(5-4)^2+(5+2)^2}=\sqrt{50}=5\sqrt{2}$$

and $c=AB=\sqrt{(-2-4)^2+(4+2)^2}=\sqrt{72}=\sqrt{2}6$

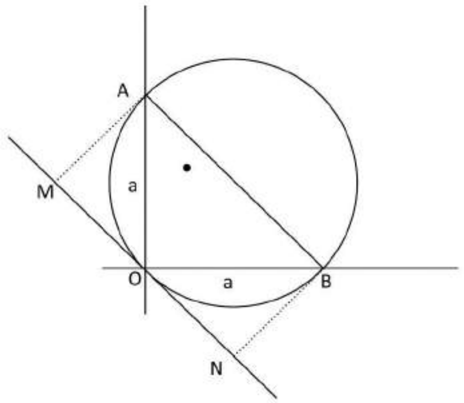
Let (x, y) be the coordinates of incentre of ΔABC, Then,

$$x=\frac{ax_1+bx_2+cx_3}{a+b+c}=\left(\frac{20\sqrt{2}-10\sqrt{2}+30\sqrt{2}}{5\sqrt{2}+5\sqrt{2}+6\sqrt{2}}\right)=\frac{40\sqrt{2}}{16\sqrt{2}}=\frac{5}{2}$$

$\therefore$ The coordinates of the incentre are $\left(\frac{5}{2},\frac{5}{2}\right)$

Example 5 A line makes equal intercepts of length 'a' on the coordinate axes, intersecting the X axis and Y-axis at A and B respectively. A circle is circumscribed about the triangle OAB, where O is the origin of the coordinate system. A tangent is drawn to this circle at the point O, the sum of the perpendicular distances of the vertices A, B and O from this tangent is:

Solution



$$\text{AM + BN + OO} = \frac{a}{\sqrt{2}}+\frac{a}{\sqrt{2}}+0=\sqrt{2}a$$

Example 6 What is the area of the triangle whose vertices are (4, 4), (3, –16) and (3, –2)?

Solution Let $x_1=4, x_2=3, x_3=3$ and $y_1=4, y_2=-16, y_3=-2$.

Then area of the EXAMPLE ven triangle

$$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}\left[4.\{-16-(-2)\}+3.(-2-4)+3.\{4-(-16)\}\right]$$

$$=\frac{1}{2}\left[4.\{-14+3(-6)\}+3\times 20\right]=\frac{1}{2}[-56-18+60]=-7$$

Since area of a triangle cannot be negative, so area = 7 sq. units.

Example 7 What is the area of the triangle formed by the points (–5, 7), (–4, 5) and (1, –5)?

Solution Let $x_1=-5, x_2=-4, x_3=1$ and $y_1=7, y_2=5, y_3=-5$.

Area of the triangle formed by given points

$$=\frac{1}{2}\left[x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right]$$

$$=\frac{1}{2}[(-5)\{5-(-5)\}+(-4)(-5-7)+(7-5)]$$

$$=0$$

Hence the EXAMPLE ven points are not forming any triangle, rather they are collinear.

Example 8 Find the equation of the straight line which passes through (3,4) and the sum of whose X and Y intercepts is 14.

Solution Let the intercepts made by the line x-axis and y-axis and y-axis be α and $(14-\alpha)$ respectively.

Then, its equation is $\frac{X}{\alpha}+\frac{Y}{14-\alpha}=1$ ...(i)

Since it passes through (3,4), we have:

$$\frac{3}{\alpha}+\frac{4}{14-\alpha}=1 \Rightarrow \alpha^2-13\alpha+42=0 \Rightarrow (\alpha-6)(\alpha-7)=0.$$

$\alpha=6$ and $\alpha=7$.

So, the required equation is: $\frac{x}{6}+\frac{y}{8}=1$ or $\frac{x}{7}+\frac{y}{7}=1$ i.e. $4x+3y=24$ or $x+y=7$.

Example 9 What is the equation of a line which passes through the point (–1, 3) and is perpendicular to the straight-line $5x+3y+1=0$?

Solution The equation of any line perpendicular to the line $5x+3y+1=0$ is

$3x-5y+K=0$ ... (i)

Since the required line passes through the point (–1, 3), we have

$3\times(-1)-5\times 3+K=0$, or, $K=18$

Hence, the required equation is $3x-4y+18=0$.

Example 10 What is the point of intersection of the lines $2x + 3y = 5$ and $3x - 4y = 10$?

Solution To find out the point of intersection, we just need to solve the simultaneous equations.

$2x + 3y = 5$ ……………………………..(1)

$3x - 4y = 10$ …………………………(2)

Multiplying equation (1) by 3 and equation (2) by 2.

$2x + 3y = 5$ becomes $6x + 9y = 15$ (equation 3) And $3x - 4y = 10$ becomes $6x - 8y = 20$ (equation 4).

Doing equation (3) – equation (4) gives us:

$$\begin{array}{l} 6x+9y=15 \\ 6x-8y=20 \\ \underline{-\quad +\qquad\quad} \\ \qquad 17y=35 \end{array}$$

Hence $y = 35 / 17$. This gives us $x = -10/17$

So point of intersection = –10/17, 35/17.

PRACTICE EXERCISES

■■■

FOUNDATION

Q.1. The coordinates of the diagonals of a square are (2, 0) and (0, 5). What is the area of the square?

(a) 29sq.units (b) $\frac{\sqrt{29}}{\sqrt{2}}$sq.units

(c) $\frac{29}{4}$sq.units (d) 14.5sq.units

Q.2. Let P and Q be points on the line joining A (–2, 5) and B (3, 1) such that AP = PQ = QB. Then the mid-point of PQ is

(a) (1/2, 3) (b) (–1/2, 4)

(c) (2, 3) (d) (–1, 4)

Q.3. If each of the point $(x_1, 4),(-2, y_1)$ lies on the line joining the points (2, –1),(5, –3), then the point $p(x_1,y_1)$ lies on the line

(a) $6(x + y) - 25 = 0$ (b) $2x + 6y + 1 = 0$

(c) $2x + 3y - 6 = 0$ (d) $6(x + y) + 25 = 0$

Q4. Find the area enclosed by the x-axis, y-axis and the graph $y = |x| - 4$ in the first quadrant (in sq. units).

(a) 8 (b) 16

(c) 14 (d) None of these

Q.5. The points (–a, –b),(0, 0),(a, b) and (a^2, ab) are

(a) collinear

(b) vertices of a parallelogram

(c) vertices of a rectangle

(d) none of these

Q.6. The points (0, 8/3), (1, 3) and (82, 30) are vertices of

(a) An obtuse angled triangle

(b) an acute angled triangle

(c) a right angled triangle

(d) none of these

Q.7. The points (0, –1), (–2, 3), (6, 7) and (8, 3) are

(a) collinear

(b) vertices of a parallelogram which is not a rectangle

(c) vertices of a rectangle, which is not a square

(d) none of these.

Q.8. Find the area enclosed by the graph $y = |X + 3|$ with the coordinate axes in square units.

(a) 9 (b) 4.5

(c) 0 (d) 12

Q.9. The coordinates of the middle points of the sides of a triangle are (4, 2) (3, 3) and (2, 2) then the coordinates of its centroid are

(a) (3, 7/3) (b) (3, 3)

(c) (4, 3) (d) none of these.

Q.10. The incentre of the triangle whose vertices are (–36, 7), (20, 7) and (0, –8) is

(a) (0, –1) (b) (–1, 0)

(c) (1/2, 1) (d) none of these.

Q.11. The triangle with vertices at (2, 4) (2, 6) and $(2+\sqrt{3}, 5)$ is

(a) right angled (b) right angled and isosceles

(c) equilateral (d) obtuse angled

Q.12. The area of the triangle with vertices at the point $(a, b + c)$, $(b, c + a)$, $(c, a + b)$ is

(a) 0 (b) $a + b + c$

(c) $ab + bc + ca$ (d) none of these.

Q.13. The nearest point on the line $3x - 4y = 25$ from the origin is

(a) (–4, 5) (b) (3, –4)

(c) (3, 4) (d) (3, 5)

Q.14. If three vertices of a rhombus taken in order are (2, –1), (3, 4) and (–2, 3), then the fourth vertex is

(a) (–3, –2) (b) (3, 2)

(c) (2, 3) (d) (1, 2)

Q.15. If a,b,c be in AP, then $ax + by + c = 0$ represents

(a) a single line

(b) a family of concurrent lines

(c) a family of parallel lines

(d) none of these

Q.16. S_1 and S_2 are two light sources. Both the sources are originated from a single point (0, 1) and travel along a linear path. S_1 passes through (1, –2) and S_2 passes through (–1, 2). The path of S_1 & S_2 are:

(a) $x + 3y = 1, x + y = 1$

(b) $3x + y = 1, x - y = 1$

(c) $3x + y = 1, x + y = 1$

(d) $x - 3y = 1, x + 3y = 1$

Q17. A ray of light passing through the point (1, 2) is reflected on the x-axis at a point *P*, and then passes through the point (5, 3). Then the abscissa of the point *P* is

(a) $2\frac{1}{5}$; (b) $2\frac{2}{5}$;

(c) $2\frac{4}{5}$; (d) $2\frac{3}{5}$;

Q.18. Find the area of the region enclosed by the graphs of $y + \frac{1}{2}(|x-1| + (x-1))$ and $x = 4$ above the x-axis.

(a) 9 (b) 6

(c) 4.5 (d) 4.5

MODERATE

Q.1. There is a straight line *S* perpendicular to the line $5x - y = 1$. The area of the triangle formed by the line *S* and the coordinate axes is 5 square units. Which of the following is the equation of straight line *S*?

(a) $x + 5y = 5\sqrt{2}$ (b) $x + 5y + 5\sqrt{2} = 0$

(c) Both 1 and 2 (d) None of these

Q.2. Find the distance of a point (3, 5) from the line $x + y = 2$, measured along a line making an angle 45° with the positive x-axis. (The anti clockwise direction is positive angle)

(a) $\sqrt{2}$ unit (b) $2\sqrt{2}$ unit

(c) $3\sqrt{2}$ units (d) $4\sqrt{2}$ units

Q.3. On an axis expressed in kilometers, the co-ordinates of 2 points are A (17, 11) and B (22, 23). In how much time does a man travelling at 91 kmph cover the shortest possible distance between the two points A and B?

(a) $6\frac{2}{7}$ min (b) $8\frac{3}{7}$ min

(c) $8\frac{4}{7}$ min (d) $8\frac{6}{7}$ min

Q.4. Find the distance between the parallel lines $3x + 4y - 7 = 0$ and $6x + 8y + 3 = 0$.

(a) 4 units (b) 11/10 units

(c) 17/10 units (d) 13/10 units

Q.5. The two vertices of the base of an isosceles triangle are (2p, 0) and (0, p). We also know that the equation of one of the sides of this triangle is $X = 2p$. What is the area of this isosceles triangle?

(a) 3.5 p^2 (b) 6.7 p^2

(c) 2.5 p^2 (d) 2 p^2

Q.6. Given four lines whose equations are $x + 2y - 3 = 0$, $2x + 3y - 4 = 0$, $3x + 4y - 7 = 0$ and $4x + 5y - 6 = 0$, then the lines are

(a) concurrent

(b) sides of a square

(c) sides of a rhombus

(d) none of these.

Q.7. Points on the line $x + y = 4$ that lie at a unit distance from the line $4x + 3y - 10 = 0$ are

(a) (3, 1) and (–7, 11) (b) (–3, 7) and (2, 2)

(c) (–3, 7) and (–7, 11) (d) none of these.

Q.8. The number of points on the lines $4x - 3y + 7 = 0$ or $x - y + 3 = 0$ which are at a distance 5 units from the point (2, 5) are

(a) 1 (b) 2

(c) 3 (d) 4

Q.9. If one vertex of an equilateral triangle is at (2, –1) and the base is $x + y - 2 = 0$, then the length of each side is

(a) $\sqrt{3/2}$ (b) $\sqrt{2/3}$

(c) 2/3 (d) 3/2

Q.10. If the points (1, 3) and (5, 1) are two opposite vertices of a rectangle and the other two vertices lie on the line $y = 2x + c$, then the value of *c* is

(a) 4 (b) –4

(c) 2 (d) None of these

Q.11. In the above question the coordinates of the other two vertices are

(a) (2, 0), (4, –4) (b) (2, 4), (4, 0)
(c) (–2, 0), (–5, 1) (d) None of these

Q.12. If the vertices of diagonal of a square are (–2, 4) and (–2, 2), then its other two vertices are at
(a) (1, –1), (5, 1) (b) (1, 1), (5, –1)
(c) (1, 1), (–5, 1) (d) None of these

Q.13. If the sum of the distances of a point from two perpendicular lines in a plane is 1, then it will form a structure-
(a) square (b) circle
(c) straight line (d) two intersecting lines.

Q.14. If a, b, c are in AP, then $ax + by + c = 0$ will always pass through a fixed point whose coordinates are
(a) (1, –2) (b) (–1, 2)
(c) (1, 2) (d) (–1, –2)

Q.15. The points (2, 1), (8, 5) and (x, 7) lie on a straight line. The value of x is
(a) 10; (b) 11;
(c) 12; (d) $11\frac{2}{3}$

Q16. The area of the bounded region enclosed between the curves $y^3 = x^2$ and $y = 2 - x^2$ is
(a) $2\frac{4}{25}$; (b) $1\frac{1}{15}$;
(c) $2\frac{2}{15}$; (d) $2\frac{14}{15}$

Q17. The area of the figure formed by the lines $ax \pm by \pm c = 0$ is
(a) $\frac{c^2}{ab}$ (b) $\frac{2c^2}{ab}$
(c) $\frac{c^2}{2ab}$ (d) None of these

Q.18. The algebraic sum of the perpendicular distances from $A(x_1, y_1)$, $B(x_2, y_2)$ and $A(x_3, y_3)$ to a variable line is zero, then the line passes through
(a) the orthocentre of ∇ABC
(b) the centroid of ∇ABC
(c) the circumcentre of ∇ABC
(d) None of these

Q.19. Length of the median from B on AC where A (–1, 3), B (1, –1), C (5, 1) is
(a) $\sqrt{18}$ (b) $\sqrt{10}$
(c) $2\sqrt{3}$ (d) 4

Q.20. If A and B are two points having coordinates (3, 4) and (5, –2) respectively and P is a point such that PA = and area of triangle PAB = 10 square units, then the coordinates of P are
(a) (7, 4) or (13, 2) (b) (7, 2) or (13, 4)
(c) (2, 7) or (4, 13) (d) (2, 7) or (4, 13)

Q.21. If A and B are two fixed points, then the locus of a point which moves in such a way that the angle APB is a right angle is
(a) a circle (b) an ellipse
(c) a parabola (d) none of these.

Q.22. A point P with coordinates (x, y) is said to be good if both x and y are positive integers. The number of good points on the curve xy = 27027 is
(a) 8 (b) 16
(c) 32 (d) 64

Q.23. The point (4,1) undergoes the following three transformations successively
I. Reflection about the line $y = x$
II. Translation through a distance 2 units along the positive direction of x-axis.
III. Rotation through an angle $\pi/4$ about the origin in the clockwise direction.

The final position of the point is given by the coordinates
(a) $\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}$ (b) $(-2, 7\sqrt{2})$
(c) $\left(-\frac{1}{\sqrt{2}}, \frac{7}{\sqrt{2}}\right)$ (d) $(\sqrt{2}, 7\sqrt{2})$

Q.24. Line L has intercepts a and b on the coordinate axes, When the axes are rotated through a given angle; keeping the origin fixed, the same line has intercepts p and q. Then
(a) $a^2 + b^2 = p^2 + q^2$ (b) $\frac{1}{a^2} + \frac{1}{b^2} = \frac{1}{p^2} + \frac{1}{q^2}$
(c) $a^2 + p^2 = b^2 + q^2$ (d) $\frac{1}{a^2} + \frac{1}{p^2} = \frac{1}{b^2} + \frac{1}{q^2}$

ANSWERS

■■■

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Answer	(d)	(a)	(b)	(a)	(a)	(d)	(c)	(b)	(a)	(b)	(a)	(a)	(b)	(a)	(b)	(c)	(d)	(c)

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(c)	(c)	(c)	(c)	(d)	(a)	(d)	(a)	(b)	(a)	(c)	(a)	(a)	(b)	(c)	(b)	(b)	(b)	(b)
Q. No.	21	22	23	24																
Answer	(a)	(c)	(a)	(b)																

BENCHMARKING TEST 4

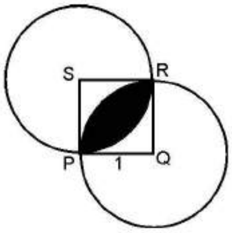
This test paper contains 30 questions of 4 marks each. One-third of the mark allotted to a particular question will be deducted in case of wrong answer.

Time Given: 70 Minutes **Total Marks: 120** **Qualifying Marks: 11**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
82+	99+
77	98.5+
71	98
51	92
37	85
27	80

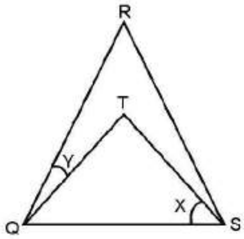
Q.1. Consider the figure given below:



PQRS is a square of side 1 unit and Q and S are the centers of the two circles. The area of the shaded portion is

(a) $\frac{\pi}{2}$ (b) $\frac{1}{2}$ (c) $\frac{\pi}{4}-\frac{1}{2}$ (d) $\frac{\pi}{2}-1$

Q.2. In the figure given below, if QRS is an equilateral triangle and TRS is an isosceles triangle and $x = 47°$, then the value of y (in degrees) will be

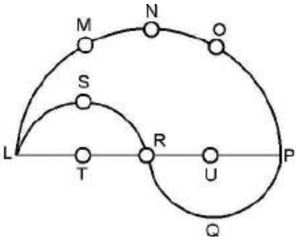


(a) 13° (b) 23° (c) 33° (d) 43°

Q.3. If the number representing the volume and the surface area of a cube are equal, then the length of the edge of the cube in terms of the unit of measurement will be

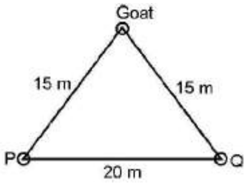
(a) 3 (b) 4 (c 5 (d) 6

Q.4. LMNOP is semicircle with the centre at R and diameter LP. LSR and RQP are also semicircles with centres at T and U respectively and diameters LR = RP = ½ LP. The ratio of the perimeter of LMNOP and LSRQP is

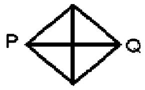


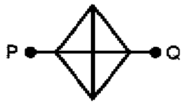
(a) 0.75:1 (b) 1:1 (c) 1:0.75 (d) 1.25:1

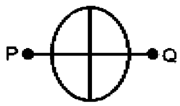
Q.5. A goat is tied to two poles P and Q with ropes that are 15 m long. P and Q are 20 m apart as shown in the given diagram:

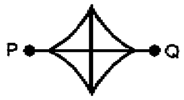


Which one of the following shaded portions indicates the total area over which the goat can graze?

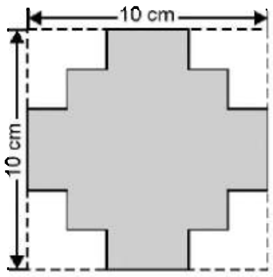
(a)


(b)


(c)


(d)


Q.6. In the given figure, all line segments of the shaded portion are of the same length and at right angles to each other. The same can be cut out of a board of side 10 cm. What is the area of the shaded portion?



(a) 36 cm^2 (b) 48 cm^2
(c) 52 cm^2 (d) 64 cm^2

Q.7. An equilateral triangle plate is to be cut into n number of identical small equilateral triangle plates. Which of the following can be a possible value of *n*?

(a) 196 (b) 216 (c) 256 (d) 296

Q.8. In an isosceles trapezium ABCD, P and Q are the midpoints of diagonals BD and AC respectively. PQ = 4 cm, the perpendicular drawn from A on BD is 5 cm and that from C on BD is 6 cm and each of the diagonals measure 8 cm. Find the length of the sides AB and CD, given that the height of the trapezium is 4 cm?

(a) 12, 7 (b) 7, 15
(c) 5, 6 (d) None of these

Q.9. Consider a cylinder of radius 7 cm and height 14 cm. There is an intelligent ant that needs to travel on the surface of the cylinder. In order to travel to any point on the surface of the cylinder, the ant always chooses the shortest path. What is the distance, along the outer surface of the cylinder, of the point on the top of the cylinder for which the ant requires maximum time to reach?

(a) $\sqrt{340}$ (b) $\sqrt{680}$
(c) $\sqrt{240}$ (d) None of these

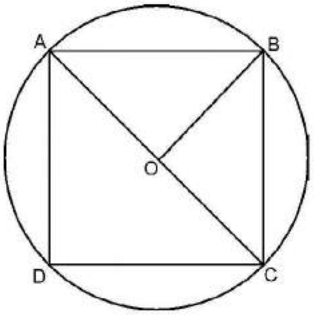
Q.10. There are 2 identical hollow cubical vessel A and B. A contains 64 identical solid spherical metallic balls. B contains 27 identical spherical balls of the same metal. In each cube the balls are packed in such a manner that no more balls can be packed (of course balls in A will not be the same as balls in B). Which cube is heavier?

(a) A (b) B
(c) A = B (d) Cannot be determined

Q.11. There is a fan with 3 blades at 120° to each other. Its central circular disc has an area of 3À cm^2 and length of the blade is $\left(20-\sqrt{3}\right)$ cm. If the tips of the blades are joined so as to form an equilateral triangle, what will be its area?

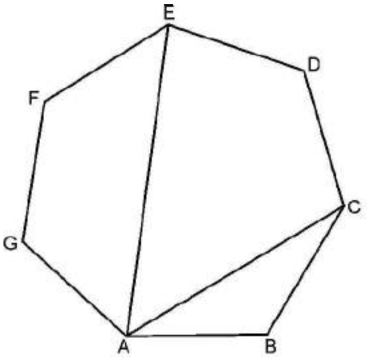
(a) 900 cm^2 (b) $300\sqrt{3}$ m^2
(c) 900 + 9 cm^2 (d) None of these

Q.12. In the given figure, radius of the circle is 2.5 cm. O is the centre of the circle and OB bisects angle ABC. What can be the maximum area of ABCD?



(a) 2.5 (b) 12.5
(c) 12 (d) None of these

Q.13. In the given regular heptagon ABCDEFG, AB = *a*, AC = *b*, AE = *c*. Which of the following statements is true?



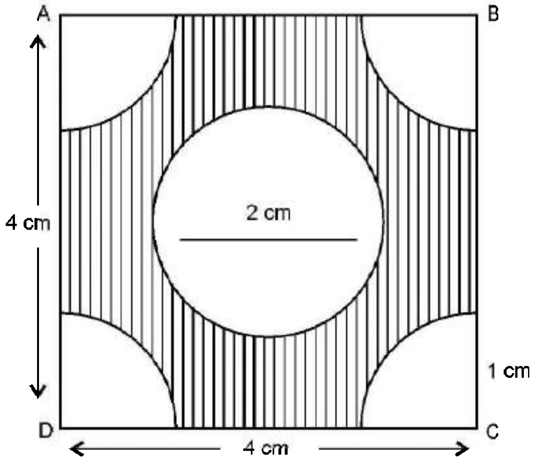
(a) $c = b - a$ (b) $\frac{1}{b} = \frac{1}{c} + \frac{1}{a}$

(c) $\frac{1}{a} = \frac{1}{b} - \frac{1}{c}$ (d) None of these

Q.14. An insect wants to travel from one bottom corner to the opposite top corner inside a room of dimension $6 \times 8 \times 10$ units. What is the minimum distance that the insect needs to travel?
[Common sense hint – Insect can fly]

(a) 10√2 units (b) 20√2 units
(c) 15√2 units (d) 25√2 units

Q.15. In the figure given below, ABCD is a square of side length 4 units which has four symmetric cuts at all its corners. Find the area of the shaded portion.



(a) 0.6 sq units (b) 0.3 sq units
(c) 0.9 sq units (d) None of these

Q.16. If $(9p^2 + 4)$, $(2p + 7)$ and $(8p^2 - 1)$ are the lengths of three sides of a triangle (p>1). Which of the following cannot be a value of p?

(a) 2 (b) √2
(c) 2√2 (d) None of these

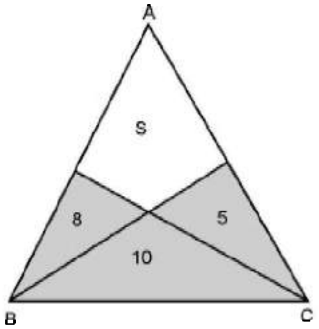
Q.17. AB is a line segment and P is its mid-point. Semicircles are drawn with AP, PB and AB as diameters on the same side of the line AB. A circle is drawn to touch all the three semicircles. Find the radius of this circle.

(a) 1/4 AB (b) 1/6 AB
(c) 1/7 AB (d) 1/8 AB

Q.18. In two circles C_1 and C_2, arcs of the same length subtend angles of 70° and 80° at their respective centers. What is the ratio of their radii (in the same order)?

(a) 7 : 8 (b 8 : 7
(c) 8 : 15 (d) Cannot be determined

Q.19. Consider the figure given below:



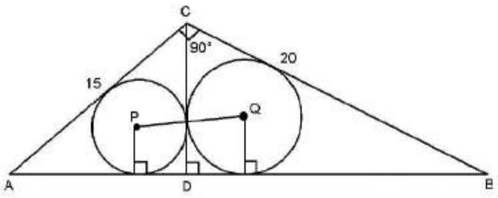
Δ ABC is divided into four parts by straight lines from two of its vertices. The areas of the three triangular parts are 8 sq units, 5 sq units and 10 sq units. What is the area of the remaining part S (in sq units)?

(a) 32 (b) 40
(c) 54 (d) None of these

Q.20. In a convex quadrilateral ABCD, the diagonal BD bisects neither the angle ABC nor the angle CDA. A point P lies inside ABCD and satisfies ∠PBC = ∠DBA and ∠PDC = ∠BDA. ABCD is cyclic quadrilateral if and only if

(a) AP = CP (b) AB = BC
(c) BP = AP (d) None of these

Q.21.



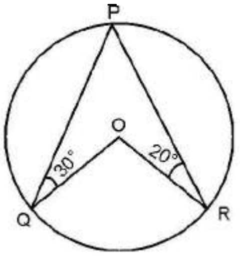
In the above figure, ACB is a right-angled triangle. CD is the altitude. Circles are inscribed within the ΔACD and ΔBCD. P and Q are the centres of the circles. The distance PQ is

(a) 5 (b) $\sqrt{50}$ (c) 7 (d) 8

Q.22. If a, b and c are the sides of a triangle, and $a^2 + b^2 + c^2 = bc + ca + ab$ then the triangle is

(a) Equilateral (b) Isosceles
(c) Right-angled (d) Obtuse-angled

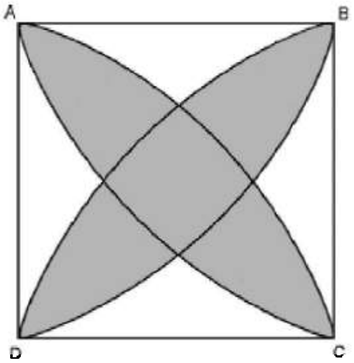
Q.23.



In the figure given about $\angle OQP = 30°$ and $\angle ORP = 20°$, then $\angle QOR$ is equal to

(a) 100° (b) 120° (c) 130° (d) 140°

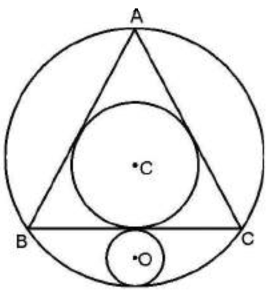
Q.24. In the figure given below, ABCD is a square of side length $4\sqrt{2}$ units. Shaded parts are the sectors of the different circles with A, B, C and D as the centers. What is the area (in sq. units) of the shaded region?



(a) $4\pi + 16$ (b) $8(\sqrt{2} + 1)\pi$

(c) $16\sqrt{2}\pi$ (d) None of these

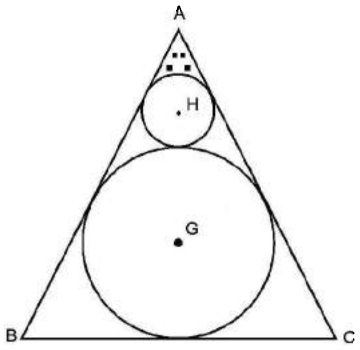
Q.25. In the figure given below, ABC is an equilateral Δ of side length 3 units and circle with centre C is the in-circle. Another circle with centre O is drawn in such a way that side BC of the triangle ABC is tangent to the circle and circumcircle is having a point of tangency with this circle. What will be the length of the radius of the circle with centre O?



(a) $\sqrt{3}$ (b) $\frac{\sqrt{3}+1}{2}$

(c) $\frac{\sqrt{3}-1}{2}$ (d) None of these

Q.26. In the figure given below, ΔABC is an equilateral triangle with side length 3 units. Triangles are arranged inside this triangle in such a way that the circle with centre G is the incircle of the triangle ABC and the circle with centre H is formed on the circle with centre H and so on. The next circle will be created on this circle with centre H, and so on the infinite circles are created one on the top of other.



What is the sum of the radius of all such circles formed?

(a) 4.5 units (b) 2.25 units

(c) 3 units (d) 1.5 units

Q.27. An octagon is inscribed in a circle. One set of alternate vertices forms a square of area 5 units. The other set forms a rectangle area of 4 units. What is the maximum possible area for the octagon (in sq. units)?

(a) $3\sqrt{5}$ (b) $\frac{3}{\sqrt{5}}$

(c) $5\sqrt{3}$ (d) None of these

Q.28. What is the largest number of the quadrilaterals formed by four adjacent vertices of an convex polygon of n sides that can have an inscribed circle?

(a) n (b) $n/2$

(c) $n/3$ (d) None of these

Q.29. There are two circles with centres at A and B respectively. The circle with center A has a radius of 8 units and the circle with center B has a radius of 6 units and the distance AB is 12 units. Both the circles meet at points P and S. A line through P meets the circles again at Q and R (with Q on the larger circle) in such a way that QP = PR. Find the length of QP.

(a) $\sqrt{160}$ units (b) $\sqrt{140}$ units

(c) $\sqrt{80}$ units (d) None of these

Q.30. A circle of 4 units is taken. Now n circles of the same radii are inserted in this circle ($1 \le n \le 10$, where n is a natural number) in such a way that they are encompassing the maximum possible areas of the circle and are inside the bigger circle along its circumference. [Obviously for $n = 1$, radius of the inside circle will be same as the radius of outside circle. Similarly for $n = 2$, radius of the inside circle will be half of the the outside bigger circle and so on]. For how many values of n, radius of the circle will be an integer

(a) 2 (b) 3 (c) 4 (d) 5

ANSWERS

Benchmarking Tests

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(d)	(b)	(b)	(c)	(c)	(b)	(b)	(a)	(b)	(b)	(d)	(a)	(b)	(c)	(b)	(b)	(d)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(b)	(a)	(a)	(d)	(d)	(b)	(a)	(b)	(d)	(b)										

Module 5

Miscellaneous

- Set Theory
- Miscellaneous
- Managerial Decision Making

20

CHAPTER

Set Theory

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Types of sets and methods of their formation
- Definitions and properties
- Maxima and minima in set theory
- Kinds of questions asked in the CAT
- Methods of solving questions

Introduction to the topic

The concept of set theory has always been an important topic for the CAT, both in terms of direct and indirect application of this concept and as a principle of counting. Questions from this topic are asked very regularly in the CAT and other major B-school exams. However, this concept is still in an evolutionary phase, as can be seen from the last few years' QA paper for this examination. The questions from this topic are not conventional. Maxima and minima can emerge as important concept of set theory.

SET THEORY

A set is a collection of well-defined objects. The members of a set can be literally anything like letters of English alphabet, or different types of alphabets, or name of the countries, or numbers, or marks obtained by a student.

Given, capital letters are representing a standard set.

A = (a, b, c, d, e, f) – The first six letters of english alphabet

B = (US, China, Japan, India) – The names of the top four countries in terms of their GDP

C = (2, 4, 6, 8, 10) – The first five even natural numbers

Here, A, B and C are different sets which are representing different group of objects.

In this chapter, we will confine ourselves with

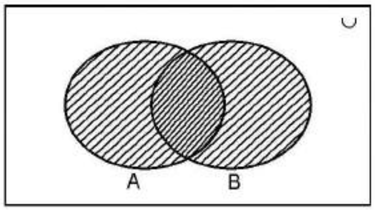
- Type of sets
- Solving techniques
- Maxima and minima

Different ways of representing a set:

1. **Roster Method** With the help of this method, a set is represented by all the elements of it written under the brackets separated by commas.
   For example, A = {1, 2, 3, 4, 5}
2. **Set Builder Method** With the help of this method, a set is represented by the common property of all its elements. It is written as
   A = {x| x, $p(x)$ holds}
   A = (x:x, $p(x)$ holds}; where $p(x)$ is the common property shared by all the elements of set A.
   For example, A = {x e N |$x \leq 5$), which can be written with the roster method as A = {1, 2, 3, 4, 5}

Types of Sets

1. *Empty or Null Set* A set having no element is known as a empty or a null set and it is denoted as ϕ or { }.
   For example, A = set of even prime numbers excluding 2.
2. *Singleton set* A set having only one element is known as a singleton set.
   For example, A = set of even prime number/s.
3. *Finite set* A set having a countable number of elements is known as the finite set.
   For example, A = set of odd numbers from 100 to 890.
4. *Infinite set* A set whose elements cannot be counted is known as infinite set.
   For example, A = set of all irrational numbers between 2 and 3.
5. *Equal sets* Two sets are said to be equal sets if all the elements of set A are included in set B and all the elements of set B are included in set A. If two sets A and B are equal then it is represented by A = B and if A and B are not equal then it is written as A^1 B, that is, all the elements of set A are not included in set B and all the elements of set B are not included in set A.
   For example, A = {a, b, c} and B = {c, b, a} are equal sets. Hence, in this case we can write Set A = Set B or simply A = B.
6. *Subsets Set* A is said to be the subset of another set B if all the elements of set A are included in set B. 'Set A is subset of Set B' is shown by A $\subseteq$ B. We can now say that every element of set A is a member of Set B.
   For example, If A = {a, b, c} and B = {a, b,c,d,e} then A $\subseteq$ B, or A is a subset of B.
   Some important results on subsets
   - Every set is a subset of itself.
   - Every set has an empty set as its subset.
   - Total number of subsets of a set having n elements in 2^n.
7. *Universal Set* A set which contains all the sets in a given context is a universal set.
   For example, when we are using sets containing natural numbers, N is the universal set.
   If A = {a, b, c,}, B = {b, c, d}, C = {c, d}
   Then we can take u = {a, b, c, d} as universal set.

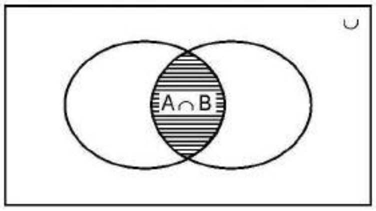


8. *Power Set* The collection of all the subsets of a set is known as the power set of that set. If A is the set, then a set containing all the subsets of A is known as the power set of A. It is denoted by P(A).
   Let A = {1, 2}, then the number of subsets of this set will be 2^2 and the subsets are {}, {1}, {2} and {1, 2} and the set containing all these four sets is known as the power set represented as P(A).
9. *Venn Diagram* Swiss mathematician Euler first gave the idea of representing sets by a diagram. Later on, British mathematician Venn brought this into practice. So, it is known as Euler–Venn diagram or simply Venn diagram. In this way of representing sets we use a closed curve, generally a circle, to denote sets and their operations.

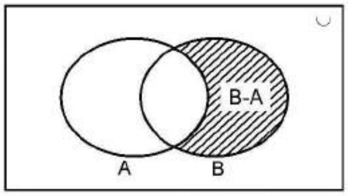
OPERATIONS ON SETS

1. *Union of Sets* If two sets are A and B, then union of A and B is defined as the set that have all the elements which belong to either A or B or both. It is represented by A $\cup$ B.

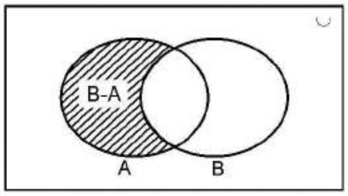
2. *Intersection of Sets* If two sets are A and B, then the intersection of A and B is defined as the set that have all the elements which belong to both A and B.
   It is represented by $A \cap B$.
   For example, find $A \cup B$ and $A \cap B$ if $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 4, 6\}$
   $A \cup B = \{1, 2, 3, 4, 5, 6\}$
   $A \cap B = \{2, 4\}$



3. *Disjoint Sets* Two sets are said to be disjoint if $A \cap B = 0$, that is, not a single element is common to both of these two sets.
   For example, If A = {Set of all odd numbers} and B = {Set of all even numbers} then set A and set B are disjoint sets.



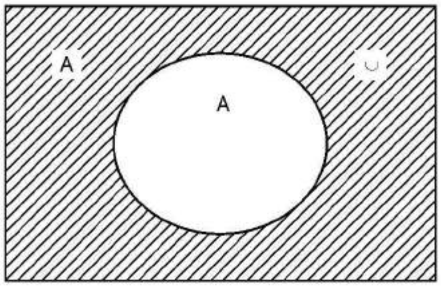
4. *Difference of Sets* For two sets A and B, A – B is the set of all those elements of A that do not belong to B. Similarly, B – A is the set of all those elements of B that do not belong to A.
   For example, $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 8\}$, then $A - B = \{1, 3, 5, 6,\}$ and $B - A = \{8\}$



5. *Complement of a Set* If U is the universal set and a set A is such that $A \subseteq U$ then complement of the set A is defined as U – A and represented as A’ or A^c
   For example, U = {Set of all prime numbers} and A = {Set of all even prime numbers} then U – A = {Set of all odd prime numbers}

ALGEBRAIC LAWS IN SET THEORY

1. *Commutative Laws* For any two sets A and B
   $A \cup B = B \cup A$ and $A \cap B = B \cap A$



2. *Associative Laws* For any three sets A, B and C,
   $(A \cup B) \cup C = A \cup (B \cup C)$ and
   $A \cap (B \cap C) = (A \cap B) \cap C$
3. *Distributive laws* For any three sets A, B and C,
   $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ and
   $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
4. *Identity laws*
   $A \cup \phi = A$, where ϕ is the null set
   $A \cap U = A$, where U is the universal set
5. *De-Morgan’s law* If A and B are two sets then
   $(A \cup B)' = A' \cap B'$
   $(A \cap B)' = A' \cup B'$
6. $A \cup A = A$ and $A \cap A = A$

Some Important Results

If A and B are any two sets, then

1. $A - B = A \cap B'$
2. $B - A = B \cap A'$
3. $(A - B) \cup B = A \cup B$
4. $(A - B) \cap B = f$
5. $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

Some Important Results on the Number of Elements in Sets

If A, B and C are three finite sets then

1. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
2. $n(A \cup B) = n(A) + n(B)$ if set A and set B are two disjoint sets.
3. $n(A - B) = n(A) - n(A \cap B)$
4. $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$
5. $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$
6. $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

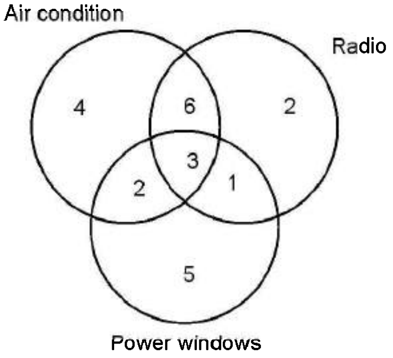
Example 1 A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options—air conditioning, radio and power windows—were already installed. The survey found:

15 had air conditioning
2 had air conditioning and power windows but no radios
12 had radio
6 had air conditioning and radio but no power windows
11 had power windows
4 had radio and power windows
3 had all three options

What is the number of cars that had none of the options?

(a) 4 (b) 3 (c) 1 (d) 2

Solution From the given conditions, we have



When we add all the values, we get a total of 23 cars.

So, 2 cars don't have the air conditioning or the radio, or the power windows.

Example 2 If A = { 1, 3, 5, 7}, B = { 1, 2, 3, 4}

What is the value of (A U B) and (A ∩ B)?

Solution (A U B) = {1, 2, 3, 4, 5, 7}

(A ∩ B) = {1, 3}

Example 3 How many numbers from 1 to 100 are not divisible by either 2 or 4 or 5?

Solution Let us first understand the meaning of the statement given in the question—

It is given that the numbers are from 1 to 100, so while counting we will include both the limits, i.e., 1 and 100. Had this been "How many numbers in between 1 to 100 are ... ", then we would not have included either 1 or 100.

Now to solve this question, we will first find out the number of numbers from 1 to 100 which are divisible by either 2 or 5 (since all the numbers which are not divisible by 2 will not be divisible by 4 also, so we do not need to find the numbers divisible by 4). And then we will subtract this from the total number of numbers i.e., 100. It can be seen below:

Total number of numbers = numbers which are divisible + numbers which are not divisible

So, $n(2 \cup 5) = n(2) + n(5) - n(2 \cap 5)$

Now, $n(2) = 50$

$n(5) = 20$

$n(2 \cap 5) = 10$

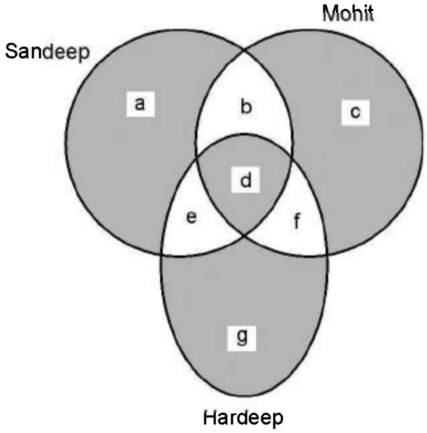
$n(2 \cup 5) = 50 + 20 - 10 = 60$

Numbers which are not divisible = total number of numbers-numbers which are divisible

= 100 – 60 = 40

Example 4 Sandeep, Mohit and Hardeep are three financial analysts working at Due North Consultants. The sum of the number of projects handled by Sandeep and Hardeep individually is equal to the number of projects in which Mohit is involved. All three consultants are involved together in 6 projects. Sandeep works with Mohit in 14 projects. Hardeep has 2 projects with Mohit but without Sandeep, and 3 projects with Sandeep but without Mohit. The total number of projects is one less than twice the number of projects in which more than one consultant is involved. In how many projects is Mohit involved alone?

Solution Let us see the situation given above through the Venn-diagram:



$a + g = b + d + f + c$ ... (i)

And $d = 6$

$b + d = 14$, so, $b = 8$.

$f = 2$ and $e = 3$

And finally, $a + b + c + d + e + f + g$

$= 2(b + d + e + f) - 1$ ... (ii)

Using (i) and (ii),

c = Number of projects handled by Mohit alone = 1

MAXIMA AND MINIMA IN SET THEORY

Possibility of maxima and minima occurs in a set theory due to the fact that intersections of two or more than two values, in most of the cases, have a finite range. In other words, they can lie in between *x* and *y*, where *x* and *y* can be any number depending upon the given conditions.

Let us understand this with the following example:

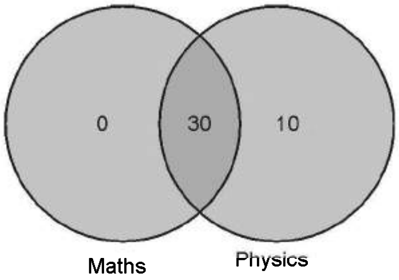
Example 5 In a class of 50 students, a test for 2 subjects maths and physics was conducted. 30 students passed in maths and 40 passed in physics.

- What is the maximum number of students who passed in both the subjects?
- What is the minimum number of students who passed in both the subjects?

Solution

- Since 30 students passed in maths, so we can assume that these are the same 30 students out of 40 students who passed physics also. So, the maximum number of students who passed in both the subjects = 30. (Remember that in this case there was no restriction regarding the number of students who passed any of the subject. Had it been given that all these 50 students are passing atleast one subject, then we would not have been in a position to do this.)

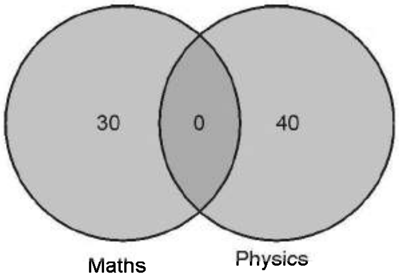
The Venn-diagram representation of the same is given below.



- To find out the minimum number of students who passed in both the subjects, intersection (math ∩ physics) should be minimum, i.e., individually math and physics each should be maximum.

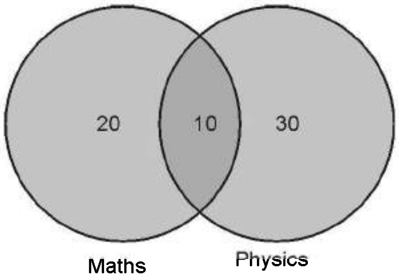
Let us do a bit of hit and trial before we finally move on to do it with its proper solution.

If we take (math ∩ physics) = 0, then the situation would be like



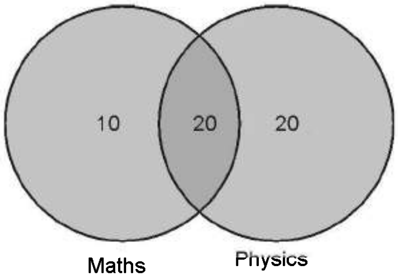
It can be seen that total number of students in this situation = 30 + 40 = 70, which is not true.

Similarly, if we take (math ∩ physics) = 10, then the situation would be like



It can be seen that total number of students in this situation = 20 + 10 + 30 = 60, which is again not true.

Hence, now we can conclude that since the number of students = 50, so (math ∩ physics) = 20. And this is the minimum value of (math ∩ physics).



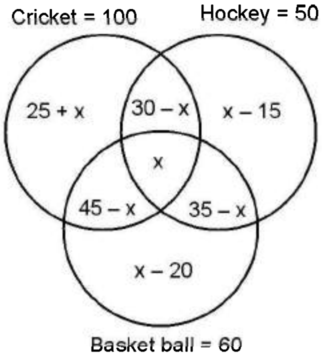
Alternatively, question (ii) can be done by the following set theory formula also: n(maths U physics) = n (maths) + n(physics) – n(maths ∩ physics)

> **Remember** In case of two sets, by using this formula of set theory we get the minimum value, but if we use the same formula in case of three sets, then we obtain the maximum value.

Example 6 There are 200 students in a school. Out of these, 100 students play cricket, 50 students play hockey and 60 students play basketball. 30 students play both cricket and hockey, 35 students play both hockey and basketball, and 45 students play both basketball and cricket.

- What is the maximum number of students who play at least one game?
- What is the maximum number of students who play all the 3 games?
- What is the minimum number of students playing at least one game?
- What is the minimum number of students playing all the 3 games?

Solution



Consider the venn diagram given above:

At first we will convert all the values in terms of x, which can be seen above.

Since the number of students cannot be negative,

$x - 15 \geq 0$

$\therefore\ x - 20 \geq 0$

So, iv. For the minimum number of students playing all three games, i.e., $x = 20$

For the maximum value of x, again none of the categories should have –ve number of students.

$\therefore 30 - x \geq 0$

$x \leq 30$

If x is more than 30, $30 - x$ would be –ve which is not possible.

Total number of students playing at least one game,

$= 100 + x - 15 + 35 - x + x - 20$

$= 100 + x$

So, the minimum number of students playing at least one game = 100 + 20 = 120

Hence, the maximum number of students playing at least one game = 100 + 30 = 130

Example 7 In a class of 50 students, 70% students pass in QA and 60% pass in RC. What is the minimum percentage of students who pass in both the papers?

Solution We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Assuming that $n(Q)$ = number of students passing QA and $n(R)$ = number of students passing RC.

Hence, $n(Q \cup R) = n(Q) + n(R) - n(Q \cap R)$

Or, $n(Q \cap R) = n(Q) + n(R) - n(Q \cup R) = 70\% + 60\% - n(Q \cup R)$

Now the minimum value of $n(Q \cap R)$ will occur for the maximum value of $n(Q \cup R)$. The maximum possible value of $n(Q \cup R) = 100\%$

So, the minimum value of $n(Q \cap R) = 30\%$

Example 8 In a class of 50 students, 70% students pass in QA, 90% pass in EU and 60% pass in RC. What is the minimum percentage of students who pass in all the papers?

Solution We can do this question by using either the formula for three sets, or we can simply keep on applying the formula for two sets required a number of times.

We know that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

Assuming that $n(Q)$ = number of students passing QA, $n(E)$ = number of students passing EU and $n(R)$ = number of students passing RC.

Hence, $n(Q \cup R) = n(Q) + n(R) - n(Q \cap R)$

Or, $n(Q \cap R) = n(Q) + n(R) - n(Q \cup R) = 70\% + 60\% - n(Q \cup R)$

Now the minimum value of $n(Q \cap R)$ will occur for the maximum value of $n(Q \cup R)$. The maximum possible value of $n(Q \cup R) = 100\%$

So, the minimum value of $n(Q \cap R) = 30\%$

Now, the minimum value of EU, QA and RC will be obtained by finding the minimum of EU and $(QA \cap RC)$.

The minimum value of EU and $(QA \cap RC) = 90\% + 30\% - 100\% = 20\%$.

Example 9 In a class of 50 students, 70% students pass in QA, 90% pass in EU, 85% pass in DI and 60% pass in RC. What is the minimum percentage of students who pass in all the papers?

Solution This question is just the extension of the earlier question.

The minimum value of EU and $(QA \cap RC) = 90\% + 30\% - 100\% = 20\%$.

The minimum value of DI and EU and $(QA \cap RC) = 20\% + 85\% - 100\% = 5\%$

PRACTICE EXERCISES

WARM UP

Direction for questions 1 to 5: Read the passage below and solve the questions based on it.

A newspaper agent deliver the morning papers to 240 families. 90 buy only *Times of India*. 15 buy *Times of India* and *The Telegraph*. 100 buy The Telegraph. 30 buy *The Telegraph* and *The Hindu*. 30 buy only *The Hindu*. No family buys all the three papers.

Q.1. How many buy the *Times of India*?
(a) 120 (b) 110 (c) 125 (d) 150

Q.2. How many buy The *Telegraph*?
(a) 50 (b) 55 (c) 45 (d) 65

Q.3. How many people read only one paper?
(a) 165 (b) 175 (c) 145 (d) 195

Q.4. Which newspaper is the most popular?
(a) The Hindu (b) The Telegraph
(c) Times of India (d) None of these

Q.5. Which is the least popular newspaper?
(a) The Hindu (b) The Telegraph
(c) Times of India (d) None of these

Direction for questions 6 and 7: Read the passage below and solve the questions based on it.

Of the 400 members of a sport club, 144 play squash (S), 135 play badminton (B), 156 play cricket (C). Each member plays at least one game. The members who play badminton and cricket both must play squash also. For every three members who play at least two games, there are two members who play all the three games.

Q.6. Find the number of members who play all the three games.
(a) 14 (b) 23
(c) 11 (d) None of these

Q.7. Find the number of members who play only Squash.
(a) 125 (b) 122
(c) 101 (d) 123

Direction for questions 8 and 9: Read the passage below and solve the questions based on it.

The following are the statistics of 22 students of Career Zone who appeared for CAT. 15 students got calls from the IIMA, 12 students got calls from the IIMB and 8 students got calls from IIMC. 6 students got calls from the IIMA and IIMB, 7 students got calls from the IIMB and the IIMC and 4 students got calls from the IIMA and the IIMC if 4 students got calls from all three IIMs

Q.8. How many students of Career Zone got a call from IIMA only?
(a) 12 (b) 9
(c) 7 (d) None of these

Q.9. How many students of Career Zone got a call from IIMC only?
(a) 3 (b) 4 (c) 1 (d) 2

Direction for questions 10 to 12: Read the passage below and solve the questions based on it.

In order to study the breakfast habits of the people in the city of Longkong, a survey of 400 families was conducted. Of these 400 families, 156 had cereals, 288 had eggs and 300 had toast. It was found that 212 families had egg and toast and 104 had cereal and toast, and 128 had egg and cereal and 84 had all three.

Q.10. How many families had neither toast nor cereal nor egg?
(a) 9 (b) 16 (c) 32 (d) 64

Q.11. How many had egg or toast but not cereal?
(a) 196 (b) 252 (c) 228 (d) 238

Q.12. How many families had only two items?
(a) 192 (b) 182 (c) 212 (d) 312

Q.13. In a survey of 100 people, it was found that 60 people read the India Today, 55 read the Outlook and 10 read neither of the two magazines. How many people read both the magazines?
(a) 45 (b) 35
(c) 25 (d) None of these

Q.14. In a class of 50 students, 28 like pop music, and 20 like classical music. If 10 students like one of the two kinds of music, find the number of the students who like both kinds of music?
(a) 8 (b) 5 (c) 7 (d) 6

FOUNDATION

Direction for questions 1 to 5: Read the passage below and solve the questions based on it.

During the census 2001, 8,000 people were surveyed in an extremely backward village, where most of the people were suffering from a certain number of diseases. In that village, 1,264 people were suffering from japanese fever and dehydration, 2,976 people were suffering from dehydration, 3,472 people from japanese fever and cholera, 4,720 people from cholera and 1,008 people from cholera and dehydration. 300 people suffered from all the three given diseases.

Q.1. Find the number of persons who suffered from dehydration only.
(a) 586 (b) 368 (c) 492 (d) None of these

Q.2. Find the number of persons who suffered from dehydration or cholera but not japanese fever.
(a) 2552 (b) 2252 (c) 3516 (d) 3216

Q.3. Find the number of persons who suffered from japanese fever but neither cholera nor dehydration.
(a) 1258 (b) 1256
(c) 1254 (d) Cannot be determined

Q.4. Find the number of persons who suffered from dehydration but not cholera.
(a) 2268 (b) 2976 (c) 1968 (d) 2676

Q.5. Find the number of persons who suffered from cholera but not dehydration.
(a) 3712 (b) 4012
(c) 4920 (d) Cannot be determined

Q.6. In a class of 120 students, students are participating in the Sudoku or crossword classes. The ratio of students who participate in Sudoku to the number of students who participate in crossword is 2 : 1. 45 students do not participate in any of these two. What is the number of students who participate in Sudoku?
(a) 50 (b) 80 (c) 70 (d) 84

Q.7. In a group of 120 persons, there are 80 Biharis and 40 Punjabis. Further, 70 persons in the group are muslims and remaining are hindus. Then the number of Biharis who are also muslims in the group is
(a) Between 30 and 70
(b) Exactly 20
(c) Between 15 and 25
(d) Between 20 and 25

Q.8. 30 monkeys went to a picnic. 25 monkeys chose to irritate buffaloes, and 20 chose to irritate cows. How many monkeys chose to irritate both buffaloes and cows?
(a) 10 (b) 15 (c) 5 (d) 20

Q.9. A surveys shows that 89% of the Australians like butter whereas 79% like chocolate. It is known that they like at least one of the two. What is the percentage of Australians who like both butter and chocolate?
(a) 13% (b) 15%
(c) 11% (d) None of these

Q.10. The market research department of a certain breakfast cereal firm interviewing 100 persons found that on a certain morning for breakfast
i. 72 had cereals, 39 had fruits, 75 had toast.
ii. 32 had fruits and cereal, 53 had cereal and toast, 26 had toast and fruits
iii. 21 had all three.
How many of those interviewed had neither cereal nor toast nor fruits?
(a) 1 (b) 2
(c) 4 (d) None of these

Q.11. If S and T are two sets such that S has 21 elements, T has 32 elements and S∩T has 11 elements, how many elements does S∩T have?
(a) 36 (b) 38 (c) 42 (d) 40

Q.12. In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea. Then what is the number of persons who take coffee but not tea?
(a) 11 (b) 19
(c) 23 (d) Cannot be determined

Direction for questions 13 to 16: Read the passage below and solve the questions based on it.

Out of 10,000 persons surveyed, 3,700 liked city A, 4,000 liked city B, and 5,000 liked city C. 700 persons liked city A and B, 1,200 liked A and C, and 1,000 liked city B and C. Each person liked at least one city.

Q.13. Find the number of persons who liked all the three cities.
(a) 100 (b) 200
(c) 300 (d) 400

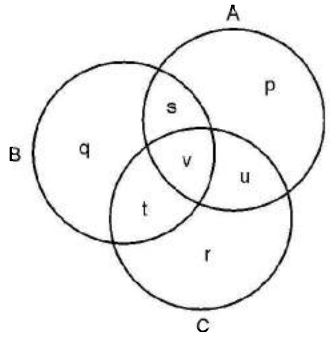
Q.14. Find the number of persons who liked at least two cities as a percentage of number of persons who liked exactly one city.
(a) 32.4% (b) 29.8%
(c) 20.83% (d) 33.33%

Q.15. Find the number of persons who liked exactly two cities as a percentage of the number of persons who liked at least one city.
(a) 21.5% (b) 19.16%
(c) 25.5% (d) None of these

Q.16. Find the number of persons who liked city A and city B but not city C.
(a) 400 (b) 5500
(c) 600 (d) None of these

Q.17. A group of 50 students appeared for the two examinations-physics and mathematics. 38 students passed in physics and 37 students passed in mathematics. If 30 students passed in both subject, what is the number of students who failed in both the subjects?
(a) 2 (b) 3 (c) 4 (d) 5

Q.18. In the given diagram, circle A represents teachers who can teach physics, circle B represents teachers who can teach chemistry and circle C represents those who can teach mathematics. Among the regions marked *p*, *q*, *r*, ... the one which represents teachers, who can teach physics and mathematics but not chemistry, is



(a) v (b) u (c) s (d) t

Q.19. In a group of 70 people, 37 like coffee, 52 like tea and each person likes at least one of the two drinks. How many people like both coffee and tea?
(a) 19 (b) 21 (c) 17 (d) 15

Direction for questions 20 and 21: Read the passage below and solve the questions based on it.

In a school with 727 students, 600 students take mathematics and 173 students take both mathematics and physics. Each student is taking at least one of the two subjects.

Q.20. How many students took physics?
(a) 250 (b) 300 (c) 280 (d) 290

Q.21. How many students took physics only?

(a) 300 (b) 280

(c) 127 (d) 173

Q.22. Of the members of three athletic teams in a certain school, 21 are on the basketball team, 26 on the hockey team, and 29 on the football team. 14 play hockey and football, 12 play football and basketball, 15 play hockey and football. 8 are on all the three teams. How many members are there altogether?

(a) 38 (b) 47

(c) 51 (d) 43

Direction for questions 23 and 24: Read the passage below and solve the questions based on it.

In a survey of 500 students, it was found that 255 play table tennis, 195 play lawn tennis, 115 play tennis, 45 play table tennis and tennis, 70 play table tennis and lawn tennis, 50 play lawn tennis and tennis. There are 80 students who do not play any of the three games.

Q.23. What is the number of students who play all the three games?

(a) 30 (b) 20

(c) 50 (d) None of these

Q.24. What is the number of students who play exactly one game?

(a) 295 (b) 300

(c) 325 (d) None of these

Q.25. In a party 70% of the people drank coffee, 65% drank soft drinks, 27% did not drink anything, whereas 248 people drank both. What is the number of people who attended the party?

(a) 400 (b) 600

(c) 800 (d) 900

Direction for questions 26 to 29: Read the passage below and solve the questions based on it.

In a class of 106 students, each student studies at least one of the three subjects maths, political science and chemistry. 48 of them study maths, 51 political science and 53 chemistry. 16 study maths and political science, 17 study maths and chemistry and 18 study political science and chemistry.

Q.26. What is the number of students who study, exactly two subjects?

(a) 31 (b) 32

(c) 33 (d) 36

Q.27. What is the number of students who study more than one subject?

(a) 39 (b) 41

(c) 40 (d) 42

Q.28. The number of students who study all the three subjects is

(a) 5 (b) 6

(c) 7 (d) 4

Q.29. The number of students who study exactly one subject is

(a) 45 (b) 55

(c) 65 (d) 70

Q.30. In a survey conducted in Patna, it was found that $\frac{3}{4}$th of the town owns colour TV, 85% of the people own refrigerator and every 4 in 5 in the town own music system. What is the minimum percentage of people who have all the three?

(a) 30% (b) 40%

(c) 55% (d) None of these

MODERATE

Q.1. In a referendum about three proposals, 78% of the people were against at least one of the proposals. 50% of the people were against proposal 1st, 30% against proposal 2nd and 20% against proposal 3rd. If 5% of the people were against all the three proposals, what percentage of people were against more than one of the three proposals?

(a) 10 (b) 12

(c) 17 (d) 22

Direction for questions 2 to 4: Read the passage below and solve the questions based on it.

In a recent survey conducted by Cable TV Association among the people who watch DD, Zee and Star TV, it is found that 80% of the people watched DD, 22% watched Star TV and 15% watched zee.

Q.2. What is the maximum percentage of people who can watch all the three channels?

(a) 12.5% (b) 8.5%
(c) 15% (d) Data insufficient

Q.3. If 5% of the people watched DD and Zee, 10% watched Star TV and DD, then what percentage of the people surveyed watched Star TV and Zee only?
(a) 2% (b) 5%
(c) 8.5% (d) Cannot be determined

Q.4. In the previous question, what percentage of people watched all the three channels?
(a) 3.5% (b) 0%
(c) 8.5% (d) Cannot be determined

Q.5. 80,000 students appeared in the last year's CAT consisting of three sections QA, DI and EU. 24,000 students passed in QA; 31,000 students passed in DI; 35,000 students passed in EU. Further 49,000 students passed in at least one of QA and DI; 59,000 students passed in at least one of DI and EU, 54,000 students passed in at least one of QA and EU. Calls for IIMs are sent to only those students who passed in all the three sections in CAT. If only 2,000 students got the IIM calls, then what is the numbers of students who failed in all three papers (either you can pass or fail)?
(a) 6,000 (b) 8,000
(c) 10,000 (d) 4,000

Direction for questions 6 to 9: Read the passage below and solve the questions based on it.

CAT 2010 was having four sections namely QA, LR, EU and RC. Following is the data pertaining to CAT 2010: Answer the following questions based on the followings information:

• The number of students who appeared	1867
• The number of students who passed in exactly two sections	232
• The number of students who passed in exactly one section	132
• The number of students who failed in all four sections	292
• The number of students who failed in QA only	200
• The number of students who failed in LR only	177
• The number of students who failed in EU only	108
• The number of students who failed in RC only	211

A student is assumed to be pass in CAT Examination if he is passed in all the four sections. Further, a student can either pass or fail in the exam.

Q.6. What is the number of students who passed CAT 2010?
(a) 508 (b) 1011
(c) 812 (d) None of these

Q.7. What is the number of students who failed because of failing in more than two sections?
(a) 108 (b) 324
(c) 672 (d) None of these

Q.8. What is the number of students who failed in QA?
(a) 200 (b) 492
(c) 711 (d) Cannot be determined

Q.9. What is the number of students who failed in EU?
(a) 400 (b) 108
(c) 732 (d) Cannot be determined

Direction for questions 10 to 13: Read the passage below and solve the questions based on it.

5% of the passengers who boarded Delhi-Lucknow Shatabdi express on 27th April 2005, do not like coffee, tea and lassi and 10% like all the three. 20% like coffee and tea, 25% like lassi and coffee and 25% like lassi and tea. 55% like coffee, 50% like tea, and 50% like lassi.

Q.10. The passengers who like only coffee is greater than the passengers who like only lassi by
(a) 25% (b) 100% (c) 75% (d) 0 %

Q.11. The percentage of passengers who like both tea and lassi but not coffee, is
(a) 15 (b) 25 (c) 40 (d) 75

Q.12. The percentage of passengers who like at least 2 of the coffee, tea and lassi, is
(a) 30 (b) 45 (c) 50 (d) 60

Q.13. If the number of passengers is 180, then the number of passengers who like lassi only, is
(a) 10 (b) 18 (c) 27 (d) 36

Q.14. Sandeep, Mohit and Hardeep are three financial analysts working at due North consultants. The sum of the number of projects handled by Sandeep and Hardeep individually is equal to the number of projects in which Mohit is involved. All three consultants are involved together in 6 projects. Sandeep works with Mohit in 14 projects. Hardeep has 2 projects with Mohit but without Sandeep, and 3 projects with Sandeep but without Mohit. The total number of projects is one less than twice the number of projects in which more than one consultant is involved. In how many projects Sandeep alone is involved?
(a) 1 (b) 0
(c) 3 (d) Cannot be determined

Q.15. There are a total of 78 persons in a big joint family. Each one of them watch atleast one out of Ramayana, Mahabharat, or Chanakya. Of these, 36 watch Ramayana, 48 watch Mahabharat and 32 watch Channakya. If 14 people watch both Ramayana and Mahabharat, 20 people watch both Mahabharat and

Chanakya and 12 people watch both Ramayana and Chanakya, find the ratio of the number of people who watch only Ramayana to the number of people who watch only Chanakya?

(a) 9:7 (b) 5:2 (c) 5:3 (d) 7:3

Q.16. In a class of 200 students, 70 played chess, 60 played hockey, and 80 played football. Thirty played chess and football, 30 played hockey and football, 40 played chess and hockey. If 130 people played at least one game, what is the number of people who play all the three games?

(a) 40 (b) 20
(c) 10 (d) None of these

Q.17. In a club, all the members are free to vote for one, two or three of the candidates. 20% of the members did not vote. 38% of the total members voted for at least two candidates. What percentage of the members voted for either one or three candidates if 10% of the total members who voted, voted for all the three candidates?

(a) 40% (b) 44%
(c) 36% (d) None of these

Q.18. In a class of 25 students, at least one of mathematics or statistics is taken by everybody. 12 have taken mathematics, 8 have taken mathematics but not statistics. Find the difference in the number of students who have taken mathematics and statistics and those who have taken statistics but not maths?

(a) 13 (b) 9
(c) 5 (d) Cannot be determined

Q.19. In a recent survey (conducted by HLL) of 1,000 houses, washing machine, vacuum cleaners and refrigerators were counted. Each house had at least one of these products. 400 had no refrigerators, 380 had no vacuum cleaners, 542 had no washing machines. 294 had both a vacuum cleaner and washing machine, 277 had both a vacuum cleaner and a refrigerator, and 120 had both a refrigerator and a washing machine. How many had only a vacuum cleaner?

(a) 132 (b) 234
(c) 342 (d) None of these

Q.20. A group of tourists (104 in all) speak Bengali or Spanish or Chinese. Out of these, 66 are Bengali speaking, 74 are Spanish speaking, 64 Chinese speaking, 44 speak Bengali and Spanish, 40 speak Bengali and Chinese, and 50 speak Chinese and Spanish. What is the number of tourists speaking all the three languages?

(a) 24 (b) 34
(c) 44 (d) 54

Q.21. In the above question the number of people speaking neither Spanish nor English, is

(a) 6 (b) 16
(c) 22 (d) Cannot be determined

ADVANCED

Q.1. We define, for two sets A and B, the following operations as:

A – B is the set of all elements of set A that are not the elements of set B,

$A \cup B$ is the set of all elements that appear in A or B or both A and B,

$A \cap B$ is the set of all elements that are common to both sets A and B.

If P, Q and R are three sets, then P-(Q-R) is equivalent to

(a) $(P\text{-}Q) \cup (P \cap R)$ (b) $P\text{-}(Q \cup R)$
(c) $P\text{-}(Q \cap R)$ (d) $(P\text{-}Q) \cup (P\text{-}R)$

Q.2. We define, for two sets A and B, the following operations as:

A – B is the set of all elements of set A that are not the elements of set B,

$A \cup B$ is the set of all elements that appear in A or B or both A and B,

$A \cap B$ is the set of all elements that are common to both sets A and B.

If P, Q and R are three sets, then (P-Q) – (Q-R) is equivalent to

(a) $P \cap (Q \cap R)$ (b) $P\text{-}(Q \cup R)$
(c) $(P \cap Q) \cup R$ (d) None of these

Q.3. A survey was conducted regarding the hobbies of a class having 200 students. The survey revealed that 85% of the students have played cricket, 80% have played basketball, 75% have played hockey, and 70% have played tennis. What is the minimum number of students who must have played all the four games?

(a) 15 (b) 25
(c) 40 (d) None of these

Q.4. There are three groups: A = {all men}, B = {all women}, and C = {men and women}, who have four members in common, and each pair of groups has 5 members in common. If each of the group has only eight members and if the common members are men only, what is the minimum number of men among the three groups taken together?

(a) 13 (b) 14
(c) 15 (d) None of these

Direction for questions 5 and 6: Read the passage below and solve these questions based on it:

Following is the result of an examination.

Out of 1,000 students who appeared

(a) 658 failed in physics
(b) 166 failed in physics and chemistry
(c) 372 failed in chemistry, 434 failed in physics and biology
(d) 590 failed in biology, 126 failed in biology and chemistry

Find the number of students who failed in (assuming that none of them passed in all the subjects)

Q.5. Chemistry but not physics.
(a) 312 (b) 226
(c) 266 (d) 206

Q.6. Physics or biology but not in chemistry.
(a) 560 (b) 710
(c) 620 (d) None of these

Direction for questions 7 and 8: Read the passage below and solve these questions based on it.

During break, 123 students go to the school canteen, which sells pastries, ice cream, and patties. 36 students buy patties and, 10 students buy only pastries. 15 students buy ice cream and patties, 10 students buy ice cream and pastries, 4 students buy pastries and patties but not ice cream and 11 students buy ice cream and patties but not pastries. 42 students buy ice cream.

Q.7. How many students buy nothing at all?
(a) 40 (b) 50 (c) 55 (d) 60

Q.8. How many students buy at least two items?
(a) 27 (b) 30 (c) 25 (d) 35

Direction for questions 9 to 12: Read the passage below and solve these questions based on it:

A and B are two sets (for e.g., A = mothers, B = women). The elements that could belong to both the sets (for e.g., women who are mothers) is given by the set $C = A \times B$. The elements which could belong to either set A or B or both is indicated by the set $D = A \cup B$. A set that does not contain any elements is known as a null set, represented by ϕ (for e.g., if none of the women in the set B is a mother, then $C = A \times B$ is a null set or $C = \phi$.

Let N signify the set of all the vertebrates; M the set of all the mammals; D the set of all the dogs; F the set of all the fish; A the set of all the alsatian and P is a dog named Party.

Q.9. Given that $X = M \times D$ such that $X = D$, then which of the following is true?
(a) All dogs are mammals
(b) Some dogs are mammals
(c) $X = \phi$
(d) All mammals are dog

Q.10. Given that $Y = F \times (D \times N) = \phi$. Which of the following is true?
(a) All fish are vertebrates
(b) All dogs are vertebrates
(c) Some fish are dogs
(d) None of these

Q.11. If $Z = (P \times D) \cup M$, then which of the following is true?
(a) The elements of Z consist of Party, the dog or any other mammal
(b) Z implies any dog or animal
(c) Z implies Party or any dog that is a mammal
(d) Z is a null set

Q.12. If $P \times A = \phi$, and $P \cup A = D$, then which of the following is true?
(a) Party and alsatians are dogs
(b) Party is an alsatian
(c) Party is not an alsatian
(d) D is a null set

Q.13. In a class of 200 students, 70 played cricket, 60 played hockey, and 80 played football. Thirty played cricket and football, 30 played hockey and football, 40 played cricket and hockey.
Find the maximum number of people playing all the three games and also the minimum number of people playing at least one game?
(a) 200,100 (b) 30, 110
(c) 30, 120 (d) None of these

Q.14. If class with n students is organized into four groups keeping the following conditions:

- Each student belongs to exactly two groups and
- Each pair of groups has exactly one student in common.

What is the value of n?
(a) $n = 11$ (b) $n = 7$
(c) $n = 9$ (d) None of these

Q.15. A survey show that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples then find the range of x.
(a) $0 \le x \le 23\%$ (b) $0 \le x \le 39\%$
(c) $4 \le x \le 35\%$ (d) $6 \le x \le 33\%$

Q.16. In Ranchi, only two newspapers *The Hindustan* and *The Indian Express* are published. It is known that 25% of the city population reads *The Hindustan* and 20% reads *The Indian Express*, while 8% read both newspapers. It is also known that 30% of those who read *The Hindustan* but not *The Indian Express* look into advertisements while 50% of those who read both

The Hindustan and *The Indian Express* look into advertisement. What percentage of the population reads an advertisement?

(a) 8.25% (b) 6.5%
(c) 11.75% (d) Cannot be determined

Q.17. In a certain examination of 3 subjects, 35 students failed in maths, 40 in physics and 40 in chemistry. 20 failed in maths and physics, 17 in physics and chemistry, 15 in maths and chemistry. If 132 students did not fail in any subject, what is the maximum number of students that may have appeared for the examination?

(a) 95 (b) 200 (c) 210 (d) 246

Direction for questions 18 to 20: Read the passage below and solve these questions based on it:

There are 120 players, having a chest number 1 through 120, seated in a circular pattern in the same order. Since the selector is involved in game fixing, he tries to ensure that none of the right candidate for right game should go. For this to happen, he starts selecting the players in a very arbitrary way for four games namely shot-put, javelin throw, discus throw and sprint: Player numbered 1 is selected for sprint and after that, every 11th player is selected for sprint. The counting continues around the circle repeatedly until it ends at player number 1. Similarly, starting with player number 1, every 9th player is selected for shot–put, every 7th player for javelin throw and every 8th player for discus throw.

Q.18. How many players are not selected for any of the four events?

(a) 45 (b) 65
(c) 33 (d) None of these

Q.19. How many players are selected for all the four events?

(a) 1 (b) 3
(c) 5 (d) None of these

Q.20. How many players were selected neither for discus throw nor shot-put?

(a) 12 (b) 14
(c) 16 (d) None of these

ANSWERS

■■■

■■■ Warm Up

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Answer	(c)	(b)	(b)	(c)	(b)	(a)	(d)	(b)	(c)	(b)	(c)	(a)	(c)	(a)	(d)

■■■ Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(d)	(c)	(a)	(a)	(a)	(b)	(c)	(c)	(c)	(d)	(b)	(d)	(b)	(a)	(d)	(b)	(a)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(c)	(d)	(a)	(c)	(a)	(d)	(b)	(a)	(c)	(b)										

■■■ Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(c)	(d)	(d)	(a)	(d)	(d)	(d)	(d)	(b)	(a)	(c)	(b)	(d)	(c)	(b)	(d)	(b)	(d)	(b)
Q. No.	21																			
Answer	(b)																			

■■■ Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(d)	(d)	(d)	(d)	(d)	(b)	(c)	(a)	(c)	(a)	(c)	(b)	(d)	(b)	(d)	(c)	(a)	(b)	(d)

HINTS AND SOLUTIONS

Moderate

1. $n(1 \cup 2 \cup 3) = n(1) + n(2) + n(3) - n(1 \cap 2) - n(2 \cap 3) - n(1 \cap 3) + n(1 \cap 2 \cap 3)$

   Or, $78\% = 50\% + 30\% + 20\% - n(1 \cap 2) - n(2 \cap 3) - n(1 \cap 3) + 5\%$

   $n(1 \cap 2) + n(2 \cap 3) + n(1 \cap 3) = 27\%$

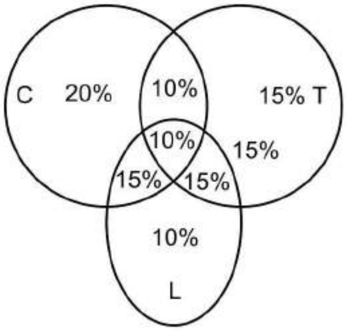
   Only $[n(1 \cap 2) + n(2 \cap 3) + n(1 \cap 3)] = 27\% - 15\% = 12\%$, since $n(1 \cap 2 \cap 3)$ is present in all the three.

   Percentage of people against more than one of the three proposals = Only $[n(1 \cap 2) + n(2 \cap 3) + n(1 \cap 3)] + n(1 \cap 2 \cap 3) = 12\% + 5\% = 17\%$

2. To find out the maximum percentage of people watching all the three channels, we can assume that the people who watch ZEE, which is minimum among all the three channels, are watching all the three channels. Any value of percentage of people more than this 15% will make the percentage of people watching ZEE negative. Hence, maximum percentage of people watching all the three channels = 15%

4. In this question, we can talk about a range of percentages who watch all the three channels. We cannot answer definitely what percentage of people watched all the three channels?

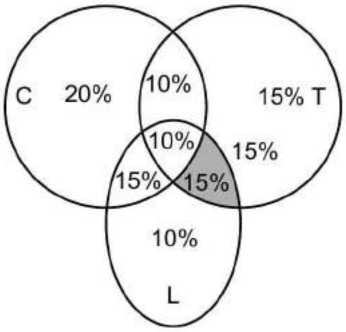
Questions 10 to 13



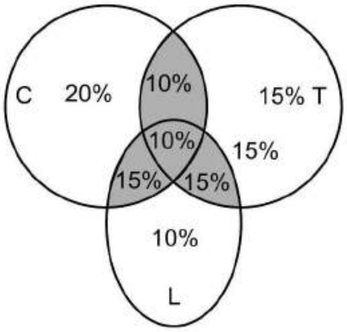
Where C = Coffee, T = Tea and L = Lassi.

10. The passengers who like only coffee = 20% and the passengers who like only lassi = 10%.

11. It can be seen that the percentage of passengers who like both tea and lassi but not coffee = 15%. This is the figure representing this area:



12. The percentage of passengers who like at least 2 of the coffee, tea and lassi can be seen in the below figure:



13. 10% of the people likes only lassi. So, the number of persons = 18

14. Go through the worked-out examples section.

15. Use the formula: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

17. The statement "12 have taken mathematics, 8 have taken mathematics but not statistics" means that 4 students have taken mathematics and statistics both.

    Total number of students = 25. Out of this, 8 have taken only mathematics, 4 have taken both. Hence 13 have taken only statistics.

    So, the difference = 13 – 4 = 9

18. Solution to this question requires only the application of: $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

Advanced

[illegible]

It is a simple application of the formula
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - c(A \cap B) - n(B \cap C) - (A \cap C) + (A \cap B \cap C)$

[illegible]

It is a simple application of the formula
$n(A \cup B \cup C) = n(A) + n(B) + n(C) - c(A \cap B) - n(B \cap C) - (A \cap C) + (A \cap B \cap C)$

9. $X = M \times D \rightarrow$ Mammals who are dogs. Now since $X = D$, so all the dogs are mammals.

15. $n(C) = 63\%$
$n(A) = 76\%$
$n(C \cup A) = n(C) + n(A) - n(C \cap A)$
$100\% = 63\% + 76\% - X\%$
$X\% = 39\%$

18. n(sprint) = 19
n(shot) = 23
n(javelin = 28
n(discus) = 25
n(sprint ∩ short) = 2
n(shot ∩ javelin) = 3
n(javelin ∩ discus) = 3
n(discus ∩ sprint) = 0
n(sprint ∩ shot ∩ javelin ∩ discus) = 0

21

CHAPTER

Miscellaneous

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- Concept of Calendars, Inequality and Height and Distance
- Definitions and properties
- Kind of questions which are asked at CAT
- Method of solving questions

Introduction to the topic

If we go into the history of CAT, we will conclude that CAT has never spared the above collection of miscellaneous concepts while asking questions. 1-3 questions have been asked from these topics every year.

Let us see each one of these concepts one-by-one.

CALENDAR

We know that

Any non-leap year contains 365 days = 52 weeks + 1 day

And leap-year contains 366 days = 52 weeks + 2 days

This 1-day and 2 days extra added to any year create changes in the calendar and this is the reason why calendar of Nth year will not be same as N + 1th year.

Before we proceed ahead, we should be very clear about two things:

Which years are leap years?

It takes the earth about 365.2422 days to go around the sun, but a normal calendar year is only 365 days. The extra fraction of a day added up four times makes four years (or, four revolution of earth around sun) 1460.9688 days, but four calendar years would only be 1460 days. That 0.9688 is almost a whole day, so every four years we add an extra day to our calendar, February 29. We call that year leap year. To make things easier, leap years are always divisible by four: 2004 and 2008 will both be leap years.

For hundreds of years, people used a calendar called the Julian calendar that followed this rule, adding a leap year every four years. However, because 0.9688 isn't *exactly* a whole day, the Julian calendar slowly began to disagree with the real seasons. In 1582, Pope Gregory fixed this problem by ordering everyone to use a new set of rules. These rules are named the Gregorian calendar, after him. They work like this:

The Gregorian Calendar

Rule	Examples
Every fourth year is a leap year.	2004, 2008, and 2012 are leap years.
However, every centenary year is *not* a leap year.	1900 and 2100 are *not* leap years.
In case of centuries, every four hundred years, there's a leap year after all.	2000 and 2400 are leap years.

In layman terms, all the century years divisible by 400 will be leap years and all the non-century years divisible by 4 will be leap years. So, leap year next to 2096 AD is 2104 AD and not 2100 AD.

Because 2000, 4000, 6000, etc. are leap years and 1000, 3000, 5000, etc. are not, the number of leap days in each millennium alternates between 242 and 243, with the first, third, etc. millennia (i.e., 1–1000, 2001–3000, etc.) having 242 leap days, and the second, fourth, etc. (i.e., 1001–2000, 3001–4000, etc.) having 243 leap days.

How the days of consecutive years change?

Due to any non-leap year, calendar of next year go ahead by 1 day and due to any leap year, calendar of next year goes ahead by 2 days, but this change in calendar will be there only before 29th February.

It can be seen through the example given below:

	1991	1992	1993
1st January	Sunday	Monday	Wednesday
28th February	Tuesday	Wednesday	Friday
1st March	Wednesday	Friday	Saturday

In the above example, we have assumed that 1st January of 1991 is Sunday. 1991 and 1993 are non-leap years and 1992 is a leap year.

If now we try to find out the symmetricity of calendars, we can see this in the following way:

i. For any leap-year

Let us see, for example, the case of 1972.

Year	1972	1973	1974	1975	1976
Excess days	2	1	1	1	2

Since no. of excess days are 7, so the days of the year 1972 and year 1977 will be same from 1st of January and 28th of February.

ii. For any (leap-year+1) year

Year	1973	1974	1975	1976	1977	1978
Excess days	1	1	1	2	1	1

Since no. of excess days are 7, so calendar of year 1973 and 1979 will be same for whole year.

iii. For any (leap-year + 2) year

Year	1974	1975	1976	1977	1978	1979
Excess days	1	1	2	1	1	1

Since excess days are 7, so calendar of year 1974 and 1980 will be same till 28th of February.

iv. For any (leap-year + 3) year

Year	1975	1976	1977	1978	1979	1980	1981	1982	1983	1984	1985
Excess days	1	2	1	1	1	2	1	1	1	2	1

Since no. of excess days are 14, so calendar of year 1975 and 1986 will be same for whole year.

This whole mechanism can be summed up in following way:

Nature of year	No. of years after which 1st January will be same
Leap year	5
Leap year + 1	6
Leap year + 2	6
Leap year + 3	11

So, if 1st January of 1972 and 1st January of 1977 will be on same day.

If 1st January of 1973 and 1st January of 1979 will be on same day and so on.

Exception

No century year, which is not a leap year, should be included in this calculation.

Example 1 Sum of dates of last Monday of previous month and 1st Thursday of next month is 38. If both the dates are of the same year, then which month is the current month?

Solution Sum of dates of last Monday of previous month and 1st Thursday of next month is 38 is possible only if last Monday is 31st and 1st Thursday is 7th. (Since if we take 30 + 8 = 38, then 30 can be last Monday of any month but 8th can not be the 1st Thursday of any month)

So, 31st of last month is a Monday. Hence 7th of current month, 14th of current month, 21st of current month and 28th of current month will be a Monday.

Now, if current month is a month with 30 days, then 5th of next month will be a Monday, so 7th of next month cannot be a Thursday.

If current month is a month with 31 days, then 4th of next month will be a Monday, so 7th of next month will be a Thursday.

Finally we can conclude that previous month and current month, both are having 31 days. Since both the dates are of the same year, so current month is August.

Finding day of a date by using a reference date

Let us see this with the help of an example: If 9th Dec of 1972 is Sunday, then which day it will be on 14th December 1998?

Process: There are several processes to do this calculation: 1. Year method, 2. Days method, 3. Actual calculation method

1. **Year Method –** We use the above given table to find out about any of the years.
   9.12.1972 – Sunday
   1.1.1973 – Tuesday (It is a Leap year + 1 year)
   So, 1.1.1979 – Tuesday, (It is a Leap year + 3 year)
   So, 1.1.1990 – Tuesday, (It is a Leap year + 2 year)
   So, 1.1.1996 – Tuesday
   Now, we can find out all the next years one-by-one.
   1.1.1997 – Thursday
   1.1.1998 – Friday – 31.12.1998 – 24.12.1998 – 17.12.1998
   So, 14.12.1998 – Tuesday
2. **Days method –** We use the no. of excess days every year to find out the no. of days calendar will move ahead by.
   1.1.1973 – Tuesday
   Due to 1973, calendar will go ahead by 1 day, similarly due to 1974 – 1 day, due to 1975 – 1 day and due to 1976- 2 days.
   So, in four years, calendar will go ahead by 5 days.
   Using unitary method, in four years, calendar will move ahead by 5 days.
   So, in 24 years calendar will move ahead by 30 days.
   Hence calendar will move ahead by 2 days.
   So, 1.1.1997 will be two days ahead of Tuesday i.e. Thursday.
   Now, it is calculation as given in **Year Method.**
3. **Actual Calculation method –** With the help of this method, we can find the actual day of any date of 20th century. To use this method effectively, we need to remember the Month Codes of all the months.
   Let us learn this method by finding the date of 15th August 1947 –
   At 1st, add the Date + Month code of August + Last two digits of year + $\left[\frac{\text{Last two digits of year}}{4}\right]$
   (Where $[x]$ represents the greatest integer value of x.)
   So, corresponding to 15th August 1947 – 15 + 3 + 47 + 11 = 76
   Now, divide this value by 7 to find out the remainder.
   If the remainder is 0 → then it is a Saturday
   If the remainder is 1 → then it is a Sunday
   If the remainder is 2 → then it is a Monday
   If the remainder is 3 → then it is a Tuesday
   If the remainder is 4 → then it is a Wednesday
   If the remainder is 5 → then it is a Thursday
   If the remainder is 6 → then it is a Friday
   Here, remainder is 6, so 15th August 1947 was a Friday. (It should have been 'Free'day)

List of Month Code:

	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Leap Year	0	3	4	0	2	5	0	3	6	1	4	6
Non-leap year	1	4	4	0	2	5	0	3	6	1	4	6

INEQUALITIES

While discussing Equations and Inequations in Polynomials, we have seen what Inequations are. However while discussing Inequalities here, our focus will be to discuss this concept in isolation with Equations, for real numbers only.

Tools of Inequality

'>' means 'greater than'
'<' means 'less than'
'≥' means 'greater than or equal to'
'≤' means 'less than or equal to'

If 'N' is any real number, then value of 'N' will be either positive or negative or zero.

When 'N' is positive, we write $N > 0$; which is read 'N is greater than zero'.

Similarly When 'N' is negative, we write $N < 0$; which is read as 'N is less than zero'. If 'N' is zero, we write $N = 0$ and in this case, 'N' is neither positive nor negative.

The two signs '>' and '<' are called the signs of inequalities.

Standard Definitions

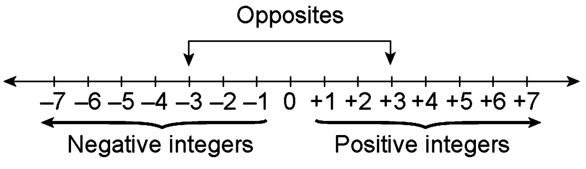
For any set of real numbers M and N,

(i) M is said to be greater than N when $M - N$ is positive
⇒ $M > N$ when $M - N > 0$ and
As we can see, 10 is greater then 5 because $10 - 5 = 5$ and 5 is greater than zero.
(ii) M is said to be less than N when $M - N$ is negative.
⇒ $M < N$ when $M - N < 0$.
As we can see, -10 is less than -5 because $-10 - (-5) = -5$ and -5 is less than zero.

However, in case of numbers inequalities can be understood through Number Line also.

Number Line

The number line is used to represent the set of real numbers. Below is the brief representation of the number line:



Basics of Inequalities

It is quite pertinent here to understand some of the very basic properties related to inequalities. These properties should be seen as the building block of the concepts of Inequalities. Assume all the numbers used here are real numbers.

For any two real numbers M and N, either $M > N$ or $M < N$ or $M = N$.

If $M > N$, then $N < M$.

- If $M > N$ and $N > P$, then $M > P$.
- If $M < N$ and $N < P$, then $M < P$.
- If $M > N$, then $M \pm c > b \pm c$.
- If $M > N$ and $P > 0$ then $MP > NP$.
- If $M > N$ and $P < 0$, then $MP < NP$.
- If $M < N$ and $P > 0$, then $MP < NP$.
- If $M < N$ and $P < 0$, then $MP > NP$.
- If $M > N$ and $P > Q$, then $M + P > N + Q$.
- If $M < N$ and $P < Q$, then $M + P < N + Q$.
- However if $M > N$ and $P < Q$, Or, $M < N$ and $P > Q$ then we cannot comment about the inequality between $(M + P)$ and $(N + Q)$.
- If $M > N$ and $P > Q$, then we cannot infer the inequality sign between $(M - N)$ and $(P - Q)$. Depending on the values of M, N, P and Q, it is possible to have $(M - N) > (P - Q)$, $(M - N) = (P - Q)$ or $(M - N) < (P - Q)$
- The square of any real number is always greater than or equal to 0.
- The square of any NON-ZERO real number is always greater than 0.
- If $N > 0$, then $-N < 0$ and if $M > N$, then $-M < -N$.
- If M and N are positive numbers and $M > N$, then
  (i) $1/M < 1/N$
  (ii) $M/P > N/P$ if $P > 0$ and
  (iii) $M/P < N/P$ if $P < 0$
- For any two positive numbers M and N

  If $M > N$ then $M^2 > N^2$.
  If $M^2 > N^2$, then $M > N$.
  If $M > N$, then for any positive value of n, $M^n > N^n$.
- **For two positive numbers M and N**
  If $M/N < 1$ then $M < N$
  If $M/N = 1$ then $M = N$
  If $M/N > 1$ then $M > N$
- **Relationship between a number and its square root**
  Let N be a natural number.

  $\sqrt{N} = N$, for $N = 0$ or $N = 1$
  $\sqrt{N} > N$, for $0 < N < 1$
  $\sqrt{N} < N$, for $N > 1$
- Let A, G and H be the Arithmetic Mean, Geometric Mean and Harmonic Mean of n positive real numbers, then $A \geq G \geq H$. Equality occurs only when all the numbers are equal.
- If the sum of two positive quantities is given, their product is greatest when they are equal; and if the product of two positive quantities is given, their sum is least when they are equal.
- For any positive number, the sum of the number and its reciprocal is always greater than or equal to 2,

  i.e., $x + \frac{1}{x} \geq 2$ where $x > 0$.

  The equality in this relationship will occur only when $x = 1$.

Cauchy–Schwarz Inequality

If $a_1, a_2, \ldots\ldots a_n$ and $b_1\, b_2 \ldots b_n$ are $2n$ real numbers, then

$$(a_1b_1 + a_2b_2 + \ldots.. + a_nb_n)^2$$
$$\leq \left(a_1^2 + a_2^2 + \ldots\ldots.a_n^2\right)\left(b_1^2 + b_2^2\ldots\ldots\ldots + b_n^2\right)$$

With the equality holding if and only if

$$\frac{a_1}{b_1} = \frac{a_2}{b_2} = \ldots\ldots = \frac{a_n}{b_n}$$

Despite all the points given above, however, we should not let the LOGIC die. Most of the questions asked in CAT can be solved by using options and we won't be in need of using any concept of Inequalities. But this should not be seen as a case in support of not-going through concepts.

Example 2 If $R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then (CAT 05)

(a) $0 < R \leq 0.1$ (b) $0.1 < R \leq 0.5$
(c) $0.5 < R \leq 1.0$ (d) $R > 10$

Solution $\frac{30^{65} - 29^{65}}{30^{64} + 29^{64}} > 1$

As, $30^{65} - 29^{65} > 30^{64} + 29^{64}$

Or, $30^{64}\,(30 - 1) > 29^{64}\,(29 + 1)$

Or, $30^{64} \times 29 > 29^{64} \times 30$

Or, $30^{63} > 29^{63}$

Hence option (4)

Example 3 If $13x + 1 < 2z$, and $z + 3 = 5y^2$, then (CAT 03)

1. x is necessarily less than y.
2. x is necessarily greater then y.
3. x is necessarily equal to y.
4. None of the above is necessarily true.

Solution We have

$13x + 1 < 2z$ .... (1)

and $z + 3 = 5y^2$ .... (2)

From (1) and (2) we get

$13x + 1 < 2\,(5y^2 - 3)$

$\Rightarrow$ $13x + 1 < 10y^2 - 6$

$\Rightarrow$ $13x < 10y^2 - 7$

$\Rightarrow$ $x < \frac{10y^2 - 7}{13}$

If $y = 1$ then, we get $x < 0.230$

i.e. $y > x$.

If $y = 2$. Then, we get $x < 2.538$.

i.e. $x > y$. This is not possible. Hence correct answer is (4).

Example 4 If $|b| \geq 1$ and $x = -|a|\,b$, then which one of the following is necessarily true? (CAT 03)

(a) $a - xb < 0$ (b) $a - xb \geq 0$
(c) $a - xb > 0$ (d) $a - xb \leq 0$

Solution Lets start assuming the values of a and b.

Assume $b = 2$ and $a = 1/2$. Then $x = -1$.

Thus options 1 and 4 have been eliminated.

Assuming $a = -1$ and $b = -1$, we get $x = 1$. Putting the values in options 1 and 2, we get **Ans.(2)**

HEIGHTS AND DISTANCES

General Theory on Heights and Distances

Angle of elevation: If the object is above the horizontal level of the eyes i.e. if it is above the eye level), we have to turn our head upwards to vie the object. In this process, our eyes move through an angle. Such an angle is called the angle of elevation of the object from our eyes.

Angle of depression: Suppose a boy, standing on the roof of a building, observes a ball lying on the ground at some distance form the building. In this case, he has to move his head downwards to view the ball. In this process, his eyes again move through an angle. Such an angle is called the angle of depression of the object from the location of his eyes (figure.)

SOME TRIGONOMETRIC RATIOS AND FORMULAE

	0°	30°	45°	60°	90°
Sin	0	1/2	1/√2	√3/2	1
Cos	1	√3/2	1/√2	1/2	0
Tan	0	1/√3	1	√3	∞

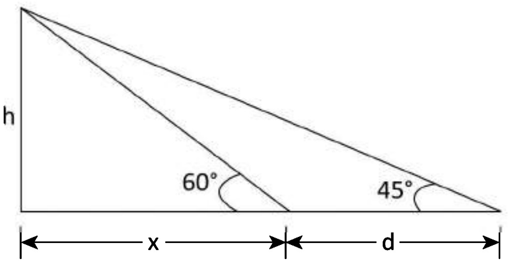
$\sin(-\theta) = -\sin\theta$; $\cos(-\theta) = \cos\theta$; $\tan(-\theta) = -\tan\theta)$;

$\cot(-\theta) = -\cot\theta$; $\sec(-\theta) = \sec\theta$; $\mathrm{cosec}(-\theta) = -\mathrm{cosec}\theta)$;

$\sin(90^0 - \theta) = \cos\theta$; $\cos(90^0 - \theta) = \sin\theta$

$\sin 2\theta = 2\sin\theta.\mathrm{Cos}\theta$

$\cos 2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$



Example 5 A car is being driven, in a straight line and at a uniform speed, towards the base of a vertical tower. The top of the tower is observed from the car and, in the process, it takes 10 minutes for the angle of elevation to change from 45° to 60°. After how much more time will this car reach the base of the tower?

(a) $5(\sqrt{3}+1)$ (b) $6(\sqrt{3}+\sqrt{2})$

(c) $7(\sqrt{3}-1)$ (d) $8(\sqrt{3}-2)$

Solution

$$\tan 45^\circ = \frac{h}{x+d}$$

or, $h = x + d$

Similarly, $\tan 60^\circ = \frac{h}{x}$

or, $h = x\sqrt{3}$

Hence, $x + d = x\sqrt{3}$,

So, $x(\sqrt{3}-1) = d$

Car takes 10 mins in covering '*d*' distance, so car will take $10/(\sqrt{3}-1)$ min in covering '*x*'[$x = d/(\sqrt{3}-1)$] distance.

Now, $t = 10/(\sqrt{3}-1)$

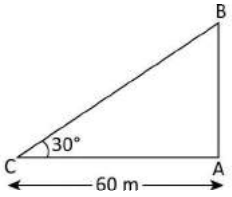
Multiplying both numerator and denominator by $(\sqrt{3}+1)$, we get $t = 5(\sqrt{3}+1)$

Example 6 The angle of elevation of the top of a tower from a point 60m from its foot is 30^0. What is the height of the tower?

Solution Let *AB* be the tower with its foot at *A*.

Let *C* be the point of observation.

Given $\angle ACB = 30^0$ and $AC = 60$m



From right triangle BAC:

$$\frac{AB}{AC} = \tan 30^0$$

$$\Rightarrow \ AB = 60 \tan 30^0 = 60\left(\frac{1}{\sqrt{3}}\right) = 20\sqrt{3}$$

Example 7 Two pillars of equal height stand on either side of a road. At a point on the road between the pillars the elevation of the tops of the pillars are 60^0 and 30^0. Find the height of the pillars if it is given that the width of the road is 150m.

Solution $\tan 60^0 = \sqrt{3} = h/x$

$\tan 30^0 = 1/\sqrt{3} = h/(150 - X)$

Solving we get: $x = 37.5$m, $h = 64.95$m.

PRACTICE EXERCISES

FOUNDATION

Q.1. The total number of terms of in the expansion of $(x + a)^{100} + (x - a)^{100}$ after simplification is

(a) 202 (b) 51

(c) 50 (d) None of these

Q.2. The number of terms of terms in the expansion $(x + y + z)^{10}$ is

(a) 11 (b) 33

(c) 66 (d) None of these

Q.3. In the expansion of $(1 + x)^{50}$, the sum of the coefficients of odd powers of *x* is

(a) 0 (b) 2^{49}

(c) 2^{50} (d) 2^{51}.

Q.4. What is the no. of leap years in between any 100 consecutive years?

(a) 24 (b) 25

(c) 26 (d) cannot be determined

Q.5. Year 200X is a leap year. Which of the following year will definitely be a leap year again?

(a) 200X + 4 (b) 200X + 5

(c) 200X + 6 (d) cannot be determined

Q.6. June 10, 1979, was a SUNDAY. Then May 10, 1972, was a

(a) Wednesday (b) Thursday

(c) Tuesday (d) Friday

Q.7. In a certain year, the month of January had exactly four Wednesdays and four Sundays. Then January 1 of that year was a

(a) Monday (b) Thursday
(c) Friday (d) Saturday

Q.8. A. I was born in a century in which the last year of the previous century was leap year.
B. I was born in the first year after the first leap year of the century.
C. I was born after 1200 A.D. and before 2000 A.D.
What is the year of my birth?

(a) 1405 (b) 1505
(c) 1605 (d) 1705

Q.9. If $a \leq 22$ and $b \geq 12$, then which of the following is always true?

(a) $a > b$ (b) $a - b > 10$
(c) $a - b \leq 10$ (d) $a + b \geq 34$

Q.10. If x, y and z are such that $x < y$ and $z > 0$, then

(a) $\frac{x}{y} > 1$ (b) $\frac{z}{x} < \frac{z}{y}$
(c) $x > \frac{y}{z}$ (d) $xz < yz$

Q.11. Which of the following satisfies the condition: $\frac{1}{x} > -10$

(a) $x + 2 = 0$ (b) $x + 3 = 0$
(c) $x + 5 = 0$ (d) None of these

Q.12. Which of the following satisfies the condition: $\frac{1}{x} < -10$

(a) $x - 2 = 0$ (b) $x - 3 = 0$
(c) $x - 5 = 20$ (d) None of these

[illegible]

Q.13. $x + 2 < \sqrt{x+14}$

(a) $-2 \leq x < 14$ (b) $-14 \leq x < 2$
(c) $x \leq -2$ or $x > 14$ (d) $x < -14$ or $x > 2$

Q.14. $(x-1)(3-x)(x-2)^2 > 0$

(a) $1 < x < 3$ (b) $1 < x < 3, x \neq 2$
(c) $-1 < x < 3, x \neq 2$ (d) $-1 < x < 3$

Q.15. $\frac{x}{x-5} > \frac{1}{2}$

(a) $-5 < x < 5$ (b) $x < -5$ or $x > 5$
(c) $-5 < x \leq 0$ (d) $0 < x \leq 5$

Q.16. $\frac{3}{x-2} < 1$

(a) $2 < x < 5$ (b) $-2 \leq x < 5$
(c) $-5 \leq x \leq 2$ (d) $x < 2$ or $x > 5$

Q.17. For how many values of x is the following equation true $|2x + 3| + x = 10$?

(a) 1 (b) 2
(c) 0 (d) None of these

Q.18. From the top of a lighthouse 60 metres high with its base on the sea level, the angle of depression of a boat is 15°. Find the distance of the boat from the foot of the lighthouse.

(a) $60(2+\sqrt{3})$m (b) $60(1+\sqrt{3})$m
(c) $60\sqrt{3}$m (d) $100(2+\sqrt{3})$m

Q.19. From the top of a cliff 200 metres high, the angles of depression of the top and bottom of a tower are observed to be 30°and 60° respectively. Find the height of the tower.

(a) $400\sqrt{3}$ m (b) $200\sqrt{3}$ m
(c) $100\sqrt{3}$ m (d) None of these

Q.20. The angle of elevation of a moon when the length of the shadow is equal to its height is

(a) 30° (b) 60°
(c) 45° (d) none of these

Q.21. The ratio of the length of a rod and its shadow is $1 : \sqrt{3}$. The angle of elevation of the sun is

(a) 30° (b) 45°
(c) 60° (d) 90°

Q.22. A tower stands on a horizontal plane. A man on the ground 100 m from the base of the tower finds the angle of elevation of the top of the tower to be 30°. What is the height of the tower?

(a) 100 m (b) $100\sqrt{3}$
(c) $100/\sqrt{3}$ (d) None of these

Q.23. When the sun is 30° above the horizontal, the length of shadow cast by a building 50 m high is

(a) $\frac{50}{\sqrt{3}}$ m (b) $50\sqrt{3}$ m
(c) 25 m (d) $25\sqrt{3}$ m

Q.24. A pole being broken by the wind, the top a struck the ground at an angle of 30° and a distance of the 21 m from the foot of the pole. Find the total height of the pole.

(a) 21 m 2.$21/\sqrt{3}$
(c) $21\sqrt{3}$ m (d) None of these

Q.25. The shadow of a tower standing on a level plane is found to be 50 m longer when the altitude of the sun is 30° than when it is 60°. Find the height of the tower.

(a) $20\sqrt{3}$m (b) $25\sqrt{3}$ m
(c) $25/\sqrt{3}$ m (d) None of these

Q.26. From the top of a cliff 25 m high, the angle of elevation of a tower is found to be equal to the angle of depression of the foot of the tower. Find the height of the tower.

(a) 40 m (b) 48 m
(c) 50 m (d) 52 m

Q.27. When the length of the shadow of a pole is equal to the height of the pole, then the elevation of source of light is
(a) 30° (b) 45°
(c) 60° (d) 75°

Q.28. If I set out from my office at quarter past six and reach the station at half past seven, I will be ten minutes earlier than the person who is fifteen minutes late to the train. At what time does the train start?
(a) 7.25 p.m. (b) 7:35 p.m.
(c) 7:40 p.m. (d) 7:20 p.m.

Q.29. If June 8, 2006 is Thursday, then which day would be 15th August 1947?
(a) Sunday (b) Saturday
(c) Friday (d) Thursday

Q.30. 50 kids are standing in a circle and one of them is having a ball. The ball is passed counter clockwise, skipping 0, 1, 2, 3, 4 and so on kids respectively, i.e., the 1st kid gives it to the 2nd kid, then 2nd kid gives it to 4th kid, then 4th kid gives it to 7th kid and so on. After how many hands change will the ball be back to the 1st kid?
(a) 12 (b) 20
(c) 22 (d) None of these

MODERATE

Q.1. Jappi and Diksha are playing a card game involving green and red cards. They get 3 points every time they draw a green card and 5 points every time they draw a red card. The winner is the first one to reach 50 points. At one stage Diksha has 41 points. Which of the following could not be the total number of cards drawn by Diksha at that stage?
(a) 7 (b) 12
(c) 10 (d) None of these

Q.2. If the rate of change of the length of a rectangle is 5 m per second and the rate of change of breadth of the same rectangle is 3 m per second, then the rate of change of area of this rectangle, when length = 10m and breadth = 2 m, is:
(a) 30 m^2 per second (b) 20 m^2 per second
(c) 50 m^2 per second (d) 40 m^2 per second

Q.3. A cube of side of 4 cm is painted on all 6 faces with the same colour. It is then broken up into smaller cubes, each of side 1 cm. What is the proportion of the number cubes with no paint to the number of cubes with exactly one side painted to those with exactly two sides painted?
(a) 1:2:3 (b) 3:6:8
(c) 1:3:4 (d) 1:3:3

Q.4. The following data are available for the monsoon season of the Hyderabad racing club. The data is for a total of '*y*' days.

- There were races on 11 days – morning or evening.
- Whenever there was a race in the morning, there was no race in the evening.
- There were 8 mornings without any race.
- There were 5 evenings without any race.

What is the value of *y*?
(a) 10 (b) 12
(c) 14 (d) Cannot be determined

Q.5. A gathering of a certain number of families consists of people to two generations only. It is known that the number of families is less than the number of girls, the number of girls is less than the number of boys and that the number of boys is less than the number of parents. If the minimum number of single parent families is two, then what is the minimum number of families in the gathering, given that no family has more than three children?
(a) 4 (b) 5
(c) 3 (d) None of these

Q.6. 1st January of Year 19XY and 1st January of year 19PQ are same day. It is also given that no other year before 19PQ has its 1st January on the same day as that of 1st January of 19XY.What is the minimum possible value of | PQ-XY|, where PQ and XY are the last two digits of the year?
(a) 5 (b) 6
(c) 11 (d) 13

Q.7. In the above question, what is maximum possible value of | PQ-XY|, where PQ and XY are the last two digits of the year?
(a) 5 (b) 6
(c) 11 (d) 13

Direction for questions 8 and 9: Read the passage below and solve the questions based on it.

January 1 of the year 1 of a century falls on a Sunday. Let Year *X* be the earliest year by which January 1 has fallen on all days of the week.

Q.8. What is year X?

(a) 7 (b) 10

(c) 8 (d) 9

Q.9. Which of the following occurs the most number of times by year X (including year X)?

(a) Mondays

(b) Sundays, Mondays and Wednesdays

(c) Mondays and Wednesdays

(d) None of these

Q.10. Given that $-3 \le a \le 2$, $0.5 \le b \le 3$ and $-2.5 \le 0.5 - 0.5$ If $x = \frac{a^2}{bc}$, then which of the following is necessarily true?

(a) $-16 \le x \le 0$ (b) $-6 \le x \le 16$

(c) $-18 \le x \le 35$ (d) $-36 \le x \le 0$

Q.11. Given that $-3 \le x \le 4$ and $1 \le y \le 6$. Which of the following expressions can have the least possible value?

(a) $x^2y + y^2x$ (b) $xy^3 - yx^2$

(c) $x^2y^2 - xy^3$ (d) $xy^2 + y^2x^3$

Q.12. n is a real number. It is given that $n!^2 > n^n$. For what values of n relationship holds good?

(a) All even nos. (b) All natural nos.

(c) All whole nos. (d) None of these

Q.13. If $|x^2 - 7x + 12| > x^2 - 7x + 12$, then

(a) $x \le 3$ or $x \ge 4$

(b) $3 \le x \le 4$;

(c) $3 < x < 4$

(d) x can take any value except $x = 3$ and $x = 4$.

Q.14. The inequality

$$\frac{|x|^2 - |x| - 2}{2|x| - |x|^2 - 2} > 2$$

holds if and only if

(a) $-1 < x < -2/3$ or $2/3 < x < 1$;

(b) $-1 < x < 1$;

(c) $\frac{2}{3} < x < 1$;

(d) $x > 1$ or $x < -1$ or $-\frac{2}{3} < x < \frac{2}{3}$

Q.15. For natural nos. N, the inequality $2^N > N^2$ holds when

(a) $N > 2$

(b) $N = 1$ and $N > 4$

(c) $N > 1$

(d) None of these

Q.16. What are the values of x satisfying

$|x - 2| \le 2$ and $|x + 3| \ge 4$.

(a) $1 < x < 4$

(b) $1 \le x \le 4$

(c) $1 \le x$ or $x \ge 4$

(d) $1 < x$ or $x > 4$

[redacted]

Q.17. $\frac{x^2 - 5x + 6}{|x| + 7} < 0$

(a) $2 \le x < 5$ (b) $-2 \le x \le 3$

(c) $2 < x < 3$ (d) $2 \le x \le 3$

Q.18. $|x^2 + 3x| + x^2 - 2 \ge 0$

(a) $-1/2 \le x < ½$ (b) $x \le -½$ or $x \ge ½$

(c) $x \ge ½$ and $x \le ½$ (d) None of these

Q.19. $\frac{x^2 - 5x + 6}{x^2 + x + 1} < 0$

(a) $2 < x < 3$ (b) $x < 2$ or $x > 3$

(c) $2 \le x < 3$ (d) $2 \le x \le 3$

Q.20. In a group of 120 persons there are 80 Bengalis and 40 Gujratis. Further, 70 persons in the group are Muslims and the remaining are Hindus. Then the number of Bengali Muslims in the group is

(a) between 10 and 14

(b) between15 and 19

(c) exactly 20

(d) 25 or more

Q.21. 300 persons are participating in a meeting in India, out of which 120 are foreigners, and the rest are Indians. Out of the Indians, there are 110 men who are not judges; 160 are men or judges, and 35 are women judges. There are no foreign judges. How many Indian women attended the meeting?

(a) 60 (b) 45

(c) 55 (d) 40

Q.22. There are six persons: A, B, C, D, E and F.

A has 3 items more than C.
D has 4 items less than B.
E has 6 items less than F.
C has 2 items more than E.
F has 3 items more than D.

Which one of the following figures can not be equal to the total number of items possessed by all the six persons?

(a) 41 (b) 53

(c) 47 (d) 58

Q.23. The angle of elevation, of the top of a tower from a point 20 m away from its base is 45°. The height of the tower is

(a) 10 m (b) 20 m

(c) 40 m (d) $200\sqrt{3}$ m

Q.24. The angle of elevation of the sun when the length of the shadow of a pole is $\sqrt{3}$ times the height of the pole will be

(a) 30° (b) 60°
(c) 90° (d) 45°

Q.25. A person walking along a straight road towards a hill observes at two points, distance $\sqrt{3}$ km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is

(a) $\frac{3}{2}$ km (b) $\sqrt{\frac{2}{3}}$ km
(c) $\frac{\sqrt{3}+1}{2}$ km (d) $\sqrt{3}$ km

Q.26. The tops of two poles of height 20 m and 14 m of are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is

(a) 12m (b) 10m
(c) 8m (d) None of these

Q.27. The angles of elevation of the top of a tower from points at distance A and B from the base and in the same line with it are complementary. If $A > B$, then height of the tower is

(a) $\sqrt{B} - A$ (b) $\sqrt{B}/A$
(c) $\sqrt{B} + A$ (d) $\sqrt{BA}$

Q.28. A man walks 10m towards a lamp post and notices that the angle of elevation of the top of the post increases from 30^0 to 45^0. Find the height of the post.

(a) 10 m (b) $5(\sqrt{3}+1)$ m
(c) $5(\sqrt{3}-1)$ m (d) None of these

Q.29. A 6-ft-tall man finds that the angle of elevation of the top of a 24-ft-high tower and the angle of depression of its base are complementary angles. The distance of the man from the pillar is

(a) $2\sqrt{3}$ft. (b) $8\sqrt{3}$ft.
(c) $6\sqrt{3}$ft. (d) None of these

Q.30. A vertical pole PO is standing at the centre O of a square ABCD. If AC subtends an angle 90^0 at the top, P, of the pole then the angle subtended by a side of the square at P is

(a) 45^0 (b) 30^0
(c) 60^0 (d) None of these

Q.31. A vertical lamp-post of height 9 meters stands at the corner of rectangular field. The angel of evaluation of its top from the fare farthest corner is 30^0, while from another corner it is 45^0. The area of the field is

(a) $81\sqrt{2}$ metre (b) 9 metre
(c) 81 metre (d) 9 metre

Q.32. A vertical lamp-post, 6 m high, stands at a distance of 2m from a wall, 4 m high. A 1.5 m,-tall man starts to walk away form the wall on the other side of the wall, in line with the lamp – post. The maximum distance to which the man can walk remaining in the shadow is

(a) $\frac{5}{2}$m (b) $\frac{3}{2}$m
(c) 4 m (d) None of these

Direction for questions 33 and 34: Read the passage below and solve the questions based on it.

Mayank had left his home at the age of about 14 years. He remembers that the day was Monday. Since then he has been fasting on every Tuesday. Today he is celebrating his 60th birth anniversary in a five-star hotel with his friends. As today is Tuesday again, so he is not taking anything except wine. At the end of the party he discloses that it is his 2400th Tuesday of fasting.

Q.33. Today is 9th Oct, 2001. On which date had he left his home?

(a) 10.10.1956 (b) 8.8.1955
(c) 10.10.1955 (d) 8.8.1955

Q.34. He was born on

(a) Sunday (b) Monday
(c) Wednesday (d)Thursday

Q.35. All the possible two-character codes are formed using the digits 0 to 9 and the capital letters of the English alphabet A to Z. How many such codes are there which, upon writing the code on a piece of paper and then turning the paper around to read to code upside down will be mistaken for another code? There is no confusion between zero and the letter O or the number 1 and the letter I.

(a) 72 (b) 90
(c) 94 (d) 100

ADVANCED

Q.1. Let us express date in the DDMMYYYY, where DD represents the day of the month, MM represents the month, YYYY represents the year. If the last possible date in the 20th century with all the eight digits (in the date as expressed in the above format) being odd is a Sunday, then what day of the week will the first date of the 21st century with all the eight digits even will be?

(a) Monday (b) Saturday
(c) Friday (d) Tuesday

Q.2. A computer science student was using the following algorithm on his computer:

```
Step 1 : Read A
Step 2 : $B = 1$
Step 3 : If $A > 1$, then $B = B \times A$
Step 4 : If $A \le 1$, then print B and stop
Step 5 : $A = A - 1$
Step 6 : go to step 3
```

If A is a non-negative integer, what is the student trying to find using the algorithm?

(a) Square root of A
(b) Factorial of A
(c) Sum of first A natural numbers
(d) None of the above

Direction for questions 3 to 5: Read the passage below and solve the questions based on it.

Pankaj has six friends, and during a certain vacation he met them at several dinners. He found that he dined with all six friends exactly one day; with every five of them of 2 days; with every four of them of 3 days; with every three of them of 4 days; with every two of them of 5 days. Further, every friend was present at 8 dinners and every friend was absent at 8 dinners.

Q.3. How many dinners did Pankaj have during the vacation?

(a) 220 (b) 173
(c) 16 (d) 12

Q.4. How many dinners did Pankaj have alone during the vacation?

(a) 6 (b) 1
(c) 7 (d) 2

Q.5. When Pankaj had dinner with every three of them of 4 days and every two of them of 5 days, what was the minimum number of friends who had dinner twice in these 9 days?

(a) 2 (b) 4
(c) 6 (d) 8

Q.6. A club with x members is organized into four committees according to the following rules:

1. Each member belongs to exactly two committees.
2. Each pair of committees has exactly one member in common.

Then which of the following is true regarding the value of x?

(a) $x = 4$ (b) $x = 6$;
(c) $x = 8$ (d) x cannot be determined.

Q.7. In a football league, a particular team played 60 games in a season. The team never lost three games consecutively and never won five games consecutively in that season. If N is the number of games the team won in that season, then N satisfies

(a) $24 \le N \le 50$ (b) $20 \le N \le 48$
(c) $12 \le N \le 40$ (d) $18 \le N \le 42$.

Q.8. From a group of seven persons, seven committee are formed. Any two committees have exactly one member in common. Each person is in exactly three committees. Then

(a) at least one committee must have more than three members;
(b) each committee must have exactly three members;
(c) each committee must have more than three members;
(d) nothing can be said about the sizes of the committees.

Q.9. How many integers satisfy the inequality

$$\frac{(x-1)(x+4)(x+2)}{(x-3)(x+8)(x+3)} < 0?$$

(a) 2 (b) 4
(c) 6 (d) Infinite

Q.10. If x satisfies $|x-1| + |x-2| + |x-3| \ge 6$, then

(a) $0 \le x \le 4$ (b) $x \le -2$ or $x \ge 4$
(c) $x \le 0$ or $x \ge 4$ (d) none of these

Q.11. For real values of x, the expression $\frac{(x-c)(x-b)}{(x-a)}$ will assume all real values provided

(a) $a \le c \le b$ (b) $b \le a \le c$
(c) $b \le c \le a$ (d) $a \le b \le c$

Q.12. Let a_n = product of the first n natural numbers. Then for all $n \ge N$

(a) $n^n \ge a_n$ (b) $\left(\frac{n+1}{2}\right)^n \ge n!$
(c) $n^n \ge a_{n+1}$ (d) None of these

Q.13. Solve for the values of y:
$\frac{z}{y} < 4, z \times y > 3, -3 < z < 3$
(a) $y < -\frac{\sqrt{3}}{2}$ (b) $y > \frac{\sqrt{3}}{2}$
(c) all the real values (d) None of these

Q.14. If $a^2 + b^2 + c^2 = 1$, then $ab + bc + ca$ lies in between(including)
(a) –2 and 1 (b) –1 and –2
(c) 0 and 1 (d) None of these

Q.15. If a, b, c and d are real numbers such that $b > 0$, $d > 0$ and $\frac{a}{b} < \frac{c}{d}$, then which of the following is true?
(a) $\frac{a}{b} < \frac{a-c}{b+d} < \frac{c}{d}$ (b) $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$
(c) $\frac{a}{b} < \frac{a-c}{b-d} < \frac{c}{d}$ (d) $\frac{a}{b} < \frac{a+c}{b-d} < \frac{c}{d}$

Q.16. Let x, y, z, w be positive real numbers, which satisfy the two conditions that
(i) If $x > y$, then $z > w$ and
(ii) If $x > z$, then $y < w$

One of the following statements given below is a valid conclusion. Which one is it?
(a) If $x < y$ then $z < w$
(b) If $x < z$ then $y > w$
(c) If $x > y + z$ then $z < y$
(d) If $x > y + z$ then $z > y$

Q.17. If $xy + x + y = 2$, then which of the following is true?
(a) $\left(\frac{x+1}{y+1}\right) > 0$ (b) $\left(\frac{x+2}{y+2}\right) > 0$
(c) $\frac{x-1}{y-1} > 0$ (d) $\frac{x-2}{y-2} > 0$

Q.18. Let $N = \frac{3 \times 5 \times 7 \times \dots\dots\dots \times 99}{2 \times 4 \times 6 \times \dots\dots\dots \times 100}$

Then which of the following is true regarding the value of N?
(a) $1/3 < N < 1/2$ (b) $1/5 < N < 1/4$
(c) $1/15 < N < 1/10$ (d) $1/10 < N < 1/5$

Q.19. The number of points (x, y) satisfying (i) $3y - 4x = 20$ (ii) $x^2 + y^2 \leq 16$ is
(a) 0; (b) 1;
(c) 2; (d) infinite

Q.20. A sequence of terms, a_1, a_2, a_3 …. a_i, is generated as follows. The first term (a_1) is chosen as (a) To obtain every subsequent term a coin is tossed. If a heads (H) appears, the next term is obtained by adding 2 to the previous term. If a tails (T) appears, only 1 in added to obtain the next term. For example, the sequence corresponding to HTH is 1, 3, 4, 6.

If $a_{11} = 15$, what is the maximum number of consecutive tails that could have appeared up to that term?
(a) 6 (b) 9
(c) 7 (d) Cannot be determined.

Q.21. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 15 seconds, the angle of elevation changes to 30°. If the jet plane is flying at a constant height of $1500\sqrt{3}$m find the speed of jet plane.
(a) 720 kmph (b) 600 kmph
(c) 540 kmph (d) None of these

Q.22. A club has 108 members. Two third of them are men and the rest are women. All members are married except for 9 women members. How many married women are there is the club?
(a) 20 (b) 24
(c) 27 (d) 30

Q.23. I see a bird flying away at a constant speed of 1.7568 kmph in the sky. The angle of elevation is 60°. After $\frac{1}{2}$ minute, I see the bird again and this time the angle of elevation is 45°. What is the perpendicular (horizontal) distance of the bird from me?
(a) 10 m (b) $10\sqrt{3}$ m
(c) 20 m (d) $20\sqrt{3}$ m

Q.24. Amitabh makes a road trip to the Hooghly beach. If he averaged his speed 5kmph more, he could have decreased his time by 10%. On the other hand, if he averaged his speed 5 kmph less, the journey to the beach would have taken an extra hour. What is the distance from Amitabh's home to the Hooghly beach?
(a) 380 kms (b) 420 kms
(c) 275 kms (d) 360 kms

Direction for questions 25 and 26: Read the passage below and solve the questions based on it.

Two Supersonic planes are flying in a straight line horizontally on different altitudes such that the distance between them is 200 km and the pilot of upper plane observes the lower plane at an angle of depression of 30°. After half an hour, the angle of depression becomes 60°.

Q.25. Find the distance between them after half an hour with respect to initial point of reference.
(a) $\frac{100}{\sqrt{3}}$km (b) $200\sqrt{3}$km
(c) $\frac{200}{\sqrt{3}}$km (d) Cannot be determined

Q.26. If they are travelling in opposite directions and the ratio of speed of upper plane to speed of lower plane is 2:1, find the speed of lower plane.

(c) $\frac{400}{\sqrt{3}}$ km/hr
(b) $\frac{200}{\sqrt{3}}$ km/hr
(c) $\frac{800}{\sqrt{3}}$ km/hr
(d) Cannot be determined.

Q.27. The length of a ladder exactly equals the height of a wall. If the ladder is placed on a 2 ft. tall stool placed 10ft. away from the wall, then its tip can just touch the top of the wall. The height of the wall is
(a) 15ft.
(b) 26ft.
(c) 28ft.
(d) 32ft.

Q.28. A tree of height 21m on a roadside broke at a certain height and fell in such a way that its top touched the other edge of the road. If the breadth of the road is 12m, then the height above the ground at which the tree broke was
(a) 12m
(b) 6m
(c) 9m
(d) None of these

Q.29. If a flagstaff of 6 metres high placed on the top of a tower throws a shadow of $2\sqrt{3}$ metres along the ground then the angle(in degrees) that the sun makes with the ground is
(a) 30°
(b) 60°
(c) 45°
(d) none of these

Q.30. In a chess tournament, each of the 5 players plays against every other player. No game results in a draw and the winner of each game gets one point and the loser gets zero. Then which one of the following sequences cannot represent the scores of the five players?
(a) 3,3,2,1,1
(b) 3,2,2,2,1
(c) 2,2,2,2,2
(d) 4, 4,1,1,0

Q.31. We consider the relation "a person x shakes hand with a person *y*". Obviously, if *x* shakes hand *y*, they *y* shakes hand with *x*. In a gathering of 99 persons, one of the following statements is always true, considering 0 to be an even number. Which one is it?
(a) There is at least one person who shakes hand exactly with an odd number of persons.
(b) There is at least one person who shakes hand exactly with an even number of persons.
(c) There is even number of persons who shake hand exactly with an even number of persons.
(d) None of the above.

Q.32. Let P, Q, R, S and T be statements such that if P is true then both Q and S are true, and if both R and S are true then T is false. We then have:
(a) If T is true then both P and R must be true.
(b) If T is true then both P and R must be false.
(c) If T is true then at least one of P and R must be true.
(d) If T is true then at least one of P and R must be false.

Q.33. If A, B, C and D are statements such that if at least one of A and B is true, then at least one of C and D must be true. Further, both A and C are false. Then
(a) If D is false then B is false
(b) Both B and D are false
(c) Both B and D are true
(d) If D is true then B is true

Q.34. If x, y and z are eliminated from the equations $\frac{x}{y}=\frac{a+1}{a-1}, \frac{y}{z}=\frac{b+1}{b-1}$ and, $\frac{z}{x}=\frac{c+1}{c-1}$ then which of the following is true regarding the relationship between a, b and c?
(a) $a + b + c = -1$
(b) $abc = -1$
(c) $ab + bc + ca = -1$
(d) $a^2 + b^2 + c^2 = -1$

ANSWERS

Foundation

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(b)	(d)	(d)	(a)	(b)	(c)	(c)	(d)	(d)	(d)	(b)	(b)	(b)	(d)	(b)	(a)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(a)	(c)	(b)	(c)	(b)	(c)	(b)	(a)	(c)	(d)										

Moderate

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(d)	(b)	(b)	(a)	(c)	(b)	(b)	(a)	(a)	(d)	(c)	(a)	(b)	(b)	(c)	(d)	(a)	(d)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
Answer	(c)	(b)	(b)	(a)	(a)	(a)	(d)	(b)	(c)	(c)	(a)	(a)	(c)	(d)	(c)					

Advanced

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(b)	(a)	(d)	(d)	(b)	(b)	(b)	(c)	(b)	(a)	(d)	(d)	(b)	(d)	(a)	(c)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34						
Answer	(a)	(c)	(d)	(d)	(d)	(b)	(b)	(d)	(b)	(d)	(b)	(d)	(a)	(c)						

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CHAPTER

Managerial Decision Making

LEARNING OBJECTIVES

After completion of this chapter, you should have a thorough understanding of the following:

- When decisions are required to be taken
- Some of the typical decisions making situations
- Some of the standard questions and methods to solve the same

Introduction to the topic

Decision making is the process by which managers respond to opportunities and threats by analysing options, and making decisions about goals and courses of action.

WHEN DECISIONS ARE MADE

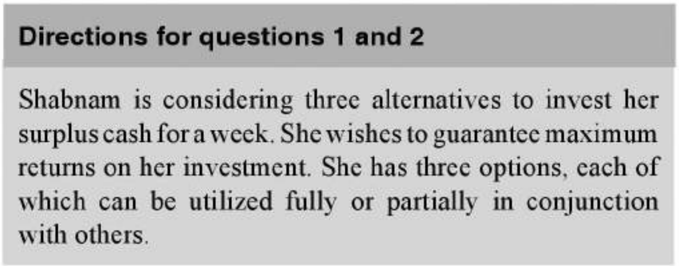
Decisions in response to opportunities: managers respond to ways to improve organizational performance.

Decisions in response to threats: occurs when managers are impacted by adverse events to the organization.

Types of Decision Making

- **Programmed Decisions—are routine decisions and follow almost automatic process. Following are some of the attributes of Programmed decisions:**
- Managers have made decision many times before.
- There are rules or guidelines to follow.
- Example: Deciding to reorder office supplies.
- **Non-programmed Decisions—are required to be taken when some new situation has arisen, or a unusual situations that have not been often addressed has arisen.**
- No rules or established norms to follow since the situation is new to the organization. In some of the cases, managers take the cue of decisions made by other firms facing similar situations. However, these "other firms" may be the closest competitor and hence following their strategy may not produce the desired results.
- Hence, these decisions are made based on qualitative information, quantitative information, intuition, and judgment.
- **Example:** Should the firm invest in a new echnology?
- Rather, looking from an organizational perspective, if we look on a personal level, question asked in CAT 2007 was a similar situation of decision making.

Let us have a look at the questions from CAT 2007:



Directions for questions 1 and 2

Shabnam is considering three alternatives to invest her surplus cash for a week. She wishes to guarantee maximum returns on her investment. She has three options, each of which can be utilized fully or partially in conjunction with others.

Option A: Invest in a public sector bank. It promises a return of +0.10%

Option B: Invest in mutual funds of ABC Ltd. A rise in the stock market will result in a return of +5%, while a fall will entail a return of –3%.

Option C: Invest in mutual funds of CBA Ltd. A rise in the stock market will result in a return of –2.5%, while a fall will entail of +2%.

Q.1. The maximum guaranteed return to Shabnam is

(a) 0.20% (b) 0.15%
(c) 0.30% (d) 0.25%
(e) 0.10%

Q.2. What strategy will maximize the guaranteed return to Shabnam?

(a) 64% in option B and 36% in option C
(b) 1/3 in each of the three options
(c) 30% in option A, 32% in option B and 38% in option C
(d) 100% in option A
(e) 36% in option B and 64% in option C

(Solution to these questions given at the end in CAT section).

In this chapter, we would be discussing the following types/tools of decision making:

1. Basics of Linear programming
2. Scheduling problems

BASICS OF LINEAR PROGRAMMING (LP)

At the onset, I would like to clarify that we are not getting into the depth of formulation of Linear programming. Our focus is to learn the methods of solving the questions using the concepts of LP.

Example 1 There is a carpenter who sells only tables (with two legs only) or chairs (with four legs only). The top part of table and top part of chair (the wooden plank) are same and one wooden plank is required to configure one chair or one table. Carpenter gets the legs and top wooden plank separately and then assembles it to make either chair or table.

Its also given that Profit per table = Rs 1000/unit and profit per chair = Rs 1500/unit.

Q.1. If there is limited supply of 1200 legs and top wooden plank, which is more profitable to produce—table or chair to maximize the overall profit?

Solution In this situation, for any limited supply of legs, it is more profitable to make table (if at all production to be made). We can simply see it through the concepts of average that in case of table, profit per leg = Rs 500, and for chair profit per leg = Rs 375/leg. We are not considering the top wooden plank because number of unit required for each of table and chair is same = 1 top wooden plank.

Now we are adding another aspect to this situation:

Carpenter takes 4 hours to assemble a table and 3 hours to assemble a chair.

Q.2. If there is limited supply of 1000 man-hour labor, which is more profitable to produce—table or chair to maximize the overall profit?

Solution Calculating again the per unit time profit of table and chair,

Profit per hour for table = Rs 250 and

Profit per hour for chair = Rs 500. Hence it is more profitable to produce chair.

Q.3. If we club the information given in Q.1 and Q.2, then which is more profitable to produce—table or chair to maximize the overall profit?

Solution It is difficult to tell by only looking at the average profit per leg or per unit of man-hour labor because we are getting contradictory results. It shows that there will be some midway solution, possibly some units of table and some units of chair.

Concept of Linear Programming comes handy in this situation. As I have stated earlier too, we will be concerned only with our primary purpose of getting the solution of our question, and not the concepts of LP in depth.

FORMULATION OF LP

It is simple linear equation representation of the constraints given in the question in terms of limited supply of legs and man-hour labour.

Assume that 'x' table and 'y' chairs are to be made.

Equation for legs ⇒

$2x + 4y \le 1200$ (we are taking '$\le$' and not '=' sign because we may not be using all the units supplied)

Equation for man-hour labour ⇒

$4x + 3y \le 1000$ (we are taking '$\le$' and not '=' sign because we may not be using all the units supplied)

Total profit = $1000\,x + 1500\,y$

Objective is to maximize total profit ⇒ maximize (1000 x + 1500 y)

Process to solve:

Step 1: Solve all the pairs of equations (convert the inequations into equation by putting "=" sign instead of inequality sign) formed from the constraints (the way we solve simultaneous equations).

Step 2: Put all the point of intersection obtained into total profit one by one.

Step 3: The point which gives maximum total profit will give the units to be made.

In this question, we have only one pair of equation formed from the constraints viz., of legs and man-hour labour.

$2x + 4y = 1200$ ---------------------------(1)

$4x + 3y = 1000$ ---------------------------(2)

Solving these two equations, we get $y = 280$, $x = 40$
Hence carpenter should make 40 tables and 280 chairs to maximize his profit.

Total profit = $1000x + 1500y = 1000 \times 40 + 1500 \times 280 = 460{,}000$

Please note that if at all a solution can be found in case of LP, the solution has to be in the 1st quadrant ($x \ge 0, y \ge 0$).

SCHEDULING

Scheduling specifies the order in which the given tasks should be performed. In other words, it gives the precedence relationship among the tasks—which task will be done first, which task will be done second and so on. In a production environment, scheduling becomes very critical owing to the cost constraint and the offerings of the competitor. Even at B-Schools, scheduling constitute a substantial part of the course curriculum.

For the sake of CAT preparation, CAT has asked scheduling problems quite a few times in the last couple of years. We will now learn the methods of scheduling through some typical problems.

Example 2 Go through the following passage and solve the questions based on it.

Heartkare hospital is specialized in heart related surgeries. Hospital boasts of the following facilities and conditions of surgery:

(i) surgery is performed by one of the 12 senior doctors assisted by one of the seven junior doctors. Senior doctors generally take about one hour to prepare for and perform each surgery, and they operate on four patients per day.

(ii) Senior doctors day ends at 4 p.m., although they can be asked to come every 14th night and every 10th weekend.

(iii) Each patient goes through a series of tests and reporting before going for surgery. Tests are done between 9 a.m. to 3.30 p.m. (Monday to Friday), 10 a.m. to 2 a.m. (on Saturday).

(iv) Patients come to hospital between 1 p.m. to 3 p.m. the day before their surgery.

(v) 1st operation begins at 7.30 a.m.

(vi) The hospital is having five operating rooms, a patient recovery room, a laboratory, and five test examination rooms. Hospital performs 150 surgeries per week on an average with patients staying at the hospital for three days. Although surgeries are performed only five days a week, hospital is needed to be in operation for whole of the week to take care of recovering patients. Table above gives the current scenario of number of beds required in the hospital:

Current Scenario Beds required : Operations with 90 beds (30 patients per day)*
*** - Patients stay in the hospital for three days requiring beds.**

Check in day		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Monday		30	30	30				
Tuesday			30	30	30			
Wednesday				30	30	30		
Thursday					30	30	30	
Friday								
Saturday								
Sunday		30	30					30
Total		60	90	90	90	60	30	30

For example, 30 patients checked in the hospital on Tuesday and they stayed on Tuesday–Wednesday–Thursday. On Thursday, they were discharged from the hospital.

Hospital management is seriously considering now to expand its number of surgeries to generate more revenue. There are two options before the management right now: 1. Start surgeries on one more day (Saturday) ⇒ Leading to a flat 20% rise in surgeries done.

2. Add one more premises to increase the number of beds by 50%.

   Assume that there is sufficient demand of the facilities of the hospital.

Q.1. What is the 'bed' utilization of the hospital?

Q.2. How would the above given table look like if management decides to perform surgery on Saturday too (assume 30 surgeries per day still)? What will be the 'bed' utilization in that case?

Q.3. What problems can occur, if any, if management decides to add another floor to expand the operation (Assume that in one operating room, only one surgery can be done at one point of time)?

Solution

Q.1. Hospital is not able to use its beds on all the days at its maximum level. Only on three days of the week—Tuesday, Wednesday and Thursday—all the beds in the hospital are utilized. For two of the remaining days of the week, only 30 beds are utilized and for the left two days 60 beds are utilized.

Hence 'bed' utilization for the whole week = 450/(90×7) = 450 / 630 = 71.4%

Q.2. Following will be the occupancy table in the new case:

Proposed Scenario

Check in day		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Monday		30	30	30				
Tuesday			30	30	30			
Wednesday				30	30	30		
Thursday					30	30	30	
Friday						30		
Saturday							30	
Sunday		30	30					60
Total		60	90	90	90	90	60	60

New 'bed' utilization for the whole week = 540/(90×7) = 540 / 630 = 85.7%

Q.3. If the capacity of the number of beds in the hospital is increased by 50%, then on an average 45 operations should have been done using the existing resources.

The resources are:

Operation room, senior doctors and junior doctors.

Now, Operation starts at 7.30 a.m. and goes till 4 p.m.

In this period of 8.30 hours × 5 operation room, a maximum of 40 operations can be done. And this lack of available operating room is the bottleneck. Hence to fully utilize the beds, we need to increase the operating hours by 30 mins making it 9 hours per day (from 7.30 a.m. to 4.30 p.m. for example), and then we can do 45 operations daily on an average.

After increasing 30 mins of operating time per day, new bed utilization chart will look like:

Scenario

Check in day		Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
Monday		45	45	45				
Tuesday			45	45	45			
Wednesday				45	45	45		
Thursday					45	45	45	
Friday								
Saturday								
Sunday		45	45					45
Total		90	135	135	135	90	45	45

'Bed' utilization percentage for the whole week now = 675 /(135×7) = 71.4%

Example 3 Read the following passage and answer the questions based on it.

'Noodles' the fast food joint at North Campus, Delhi is famous for its value-for-money Chinese food. To understand and expand their business, management at 'Noodles' analyzed its processes and came out with following:

Customer enters→ Process 1→ Process 2 → Process 3 → Process 4 → Customer leaves

Any customer order has to go through all the four processes as given above.

Following is the time taken to do the processes:

Process 1 – 1 minute/order

Process 2 – 4 minutes/order

Process 3 – 3 minutes/order

Process 4 – 2 minutes/order

Noodles is open for 8 hours per day. A particular employee can work only in one process.

Q.4. What is the maximum number of customers that 'Noodles' can service daily?

Q.5. It has been found that only one new employee can be added to 'Noodles' looking upon cost-benefit analysis. Which process the employee should be added to? How many customers can be serviced after this addition of new employee?

Solution

Q.4. Total number of customers that can be serviced per day through different processes:

Process 1 – 480

Process 2 – 120

Process 3 – 160

Process 4 – 240

Since process 2 can service only 120 customers, hence total number of customers serviced = Min (480, 120, 160, 240) = 120

Q.5. Obviously process 2 is the bottleneck, so the new employee should be added to process 2. After adding new employee, total number of customers serviced/day = 160

PRACTICE EXERCISES

WARM UP

Direction for questions 1 to 3: Read the passage below and solve the questions based on it.

MUL is considering establishing a new factory for making its new C segment of cars—either at towns Ludhiana or Saharsa or Goa, or perhaps even at all the three towns. It is also considering building at most one new servicing station but the choice of location for servicing station is restricted to the city where a new factory is built i.e., servicing station can be built only at a town where factory is being built. Objective of the MUL is to get maximum benefit out of this whole exercise of establishing the factories and servicing station.

Following table gives the cost-benefit scenario for the two towns:

	Benefit INR (After deducting the cost)	Cost INR
Factory at Ludhiana	90 crores	60 crores
Factory at Saharsa	50 crores	30 crores
Factory at Goa	60 crores	40 crores
Service station at Saharsa	40 crores	20 crores
Service station at Goa	60 crores	30 crores
Service station at Ludhiana	40 crores	50 crores

MUL can invest a maximum of INR 100 crores for establishing the factories and service stations.

Q.1. At what location the factory should be built?

Q.2. At what location the service station should be built?

Q.3. What can be the maximum benefit (after deducting the cost) that MUL can derive?

Direction for questions 4 and 5: Read the passage below and solve the questions based on it.

An investment bank is considering 7 plans for investment. Following table gives the investment required and profit generated (after deducting the investment) in INR (Indian National Rupees) for each of the seven investment plans:

	Investment Required	Profit (after deducting the investment)
Investment plan 1	500	1600
Investment plan 2	250	800
Investment plan 3	350	1000
Investment plan 4	600	2000
Investment plan 5	700	2200
Investment plan 6	450	1200
Investment plan 7	300	800

Bank can investment a maximum of INR 1500 in all the investment plans added together.

Q.4. Which investment plans should be chosen to maximize the profit (after deducting the investment) by the bank?

Q.5. What maximum profit (after deducting the investment) can be generated under the given constraints?

Direction for questions 6 to 10: Read the passage below and solve the questions based on it.

ABC Limited is evaluating new copying machines to replace the current copier of the company. Three machines are available with the following estimates of operating costs. The labour costs on the copying operation is payable in proportion to the number of copies actually made:

Particulars	Copier A	Copier B	Copier C
Material cost per copy	0.15	0.12	0.06
Labour cost per copy	0.21	0.12	0.06
Annual lease rental	9000	17400	30000

(All figures are in Rs)

Q.6. Which of the copiers should be chosen by the company? Would your answer change for the different number of copies made per year?

Q.7. At what number of copies made per annum will be total cost of copier A and total cost of copier B per annum be same?

Q.8. If the total number of copies made is 45,000, then which copier should be chosen?

Q.9. What is the minimum number of copies made per annum so that copier C becomes the most profitable option?

Q.10. Company estimates that the copying volume will be 80,000 per annum. However the actual number of copies made is only 60,000. Company has already opted for copier B. What is the resultant loss (if any) with respect to the most efficient copier for this volume category?

Q.11. A company manufactures four variants of the same product and in the final part of the manufacturing process there are assembly, polishing and packing operations. For each variant the time required for these operations is shown below (in minutes) as is the profit per unit sold.

	Assembly	Polish	Pack	Profit (INR)
Variant 1	2	3	2	1.50
2	4	2	3	2.50
3	3	3	2	3.00
4	7	4	5	4.50

Given the current labour force, the company estimates that, each year, they have 100000 minutes of assembly time, 50000 minutes of polishing time and 60000 minutes of packing time available. How many of each variant should the company make per year and what would the associated profit?

HINTS AND SOLUTIONS

1. Make factory at Saharsa and Goa.
2. Make service station at Goa.
3. 170 crores.
4. Investment plans 1, 5 and 7.
5. INR 46,000.
6. Company should use different copiers for different volume of copies made:

   For 0 – 70, 000 copies – Copier A

   For 70, 000 – 1,05,000 copies – Copier B

   For 1,05,000+ copies – Copier C
7. At 70, 000 copies.
8. Copier A
9. 1,05,001 copies (For 1,05,000 copies, both copiers B and C are equally profitable.)
10. Rs 1200 (most efficient copier for 60,000 copies = copier A)
11. Total profit = Rs 58,000.

    And number of units made $X_1 = 0$, $X_2 = 16000$, $X_3 = 6000$ and $X_4 = 0$

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PART

2

Data Interpretation

Module 1

- Data Interpretation

Module 2

- Test Your Mettle

Module 3

- Data Sufficiency

Module 1

Data Interpretation

1

CHAPTER

Introduction to Data Interpretation

LEARNING OBJECTIVES

- What do we understand with data
- What is data interpretation
- Different ways of representing the data
- Problem solving techniques

WHAT IS DATA INTERPRETATION?

Data Interpretation is the act of transforming data with the objective of extracting useful information and facilitating conclusions on the basis of the given data. Depending upon the type of data and the questions, we might be required to apply certain statistical tools with various methods to represent the given data.

Before we move ahead to see and learn the various methods of representing data, let us understand some basics.

WHAT IS DATA?

Data is a means to represent facts, concepts or instructions in a formalized manner suitable for communication, interpretation or processing by humans or other automatic modes.

In other words, propositions like measurements or observations of a variable, which may comprise numbers, words or images can be quantified with the help of data.

Data can be a numerical or non-numerical fact and figure related to a particular social, economic or political event etc.

Let us see some examples of data:

Yesterday the Sensex closed at 14,004 points.

25% of the total population of India is below The poverty line.

In the last general elections in India, 40% of the total seats were won by regional parties. Data can be further understood to be an agent to cause the sentiments and even sometimes it can be effect also.

If the data given is distinct and separate, i.e. It can be counted (1, 2, 3….), then it is known as Discrete Data. For example, population of a country, production of cars in a manufacturing plant.

However, if the data takes any value within a finite or infinite interval, then it is known as a Continuous Variable. For example, the percentage of sugar in mangoes or weight/height etc., are continuous variables.

Significance of Organized Data

It is said, “Trust only two—God and data.” Since God cannnot be everywhere to prove a point, we have to resort to data to validate points or make inferences. The significance of data can be further illustrated by the fact that data is the basis of an argument. In fact, data is the starting point for most arguments.

However, data if not properly managed is not of much use. People at the higher echelons of any organization will have no time to go through the details of all the reports. The need for organized data becomes more pertinent because data can be used to describe a current situation with respect to its future possibility. Data can establish a relationship between different phenomenon like expenditure government in the various sectors of its annual budget vis-a-vis the priority sectors; The per capita income of different countries in relation to the number of deaths due to a particular disease.

For maximum utilization of data, it should be formatted properly for easy interpretation and deduction. However, in CAT and other B-School entrance examinations, we should not expect easy-to-comprehend data. Rather, the students should be prepared to find data which is difficult to interrelate or comprehend.

Different Ways of Representing Data

Data representation in the DI section is primarily of two types:

Narration Based

Also known as caselets, these questions often involve stories that define a situation and give details of various parameters involved; including their inter-relationships.

E.g., Mittal has recently acquired four companies, viz., Bank of Bozoland (BOB), My Own Bank (MOB), Zany Obliterated Bank (ZOB) and Dogmatically Obscure Bank (DOB). He noticed that the sales of DOB are half than that of BOB, whereas, the profits of DOB are double than that of BOB. The expenses of ZOB are Rs 3 crores less than that of DOB, whereas, the profits of MOB is Rs 1 crore less than that of ZOB. The expenses of BOB are three times than that of DOB. It is also known that the sales of ZOB are Rs 15 crore or one-fourth of MOB's sales. All the figures are for 1992–93. An insider further informs Mittal that the sales of DOB are Rs 10 crores more than that of ZOB and the expenses of BOB are 90% of its own sales. Sales – Expenses = Profit

Q. The total sales of all the four companies is (Rs crores):
(a) 200 (b) 150
(c) 125 (d) 160

Pictorial

This is the most common form of data representation. In such problems, data is presented in various pictorial forms such as line graphs, bar diagrams, line charts etc.

The important point to remember pertaining to all these questions is the fact that each and every question asked in the CAT is based on some logic and reasoning meant to check your aptitude. Few questions that involve numbers may also require a basic level of calculation skills.

Let us see the various pictorial representation of data:

Table

Tabular method is the most fundamental way of representing data. In fact, most of the different kinds of data presentation formats like the bar charts, line charts etc. originate from the table. In other words, presenting the data in a tabular format is the first step in forming other types of data presentation formats.

E.g., the table given below shows the break-up of the percentage of people of different age groups frequenting bars in 4 different metro cities viz., Delhi, Hyderabad, Bangalore and Patna in the year 2002.

Cities	**Percentage break-up for age groups (Years) in 2006**						
	Up to 15	**15-20**	**20-25**	**25-30**	**30-35**	**35-40**	**Above 40**
Delhi	8	13	24	21	11	17	6
Hyderabad	3	8	35	23	10	16	5
Bangalore	4	21	27	11	8	14	15
Patna	1	7	43	32	9	5	3

The tabular format is considered to be the most versatile data presentation method. All data which can be expressed in any other format can also be expressed in the format of a table. On the other hand, it is quite possible that data that can be presented in a tabular format, cannot be presented in any other format like the pie chart etc.

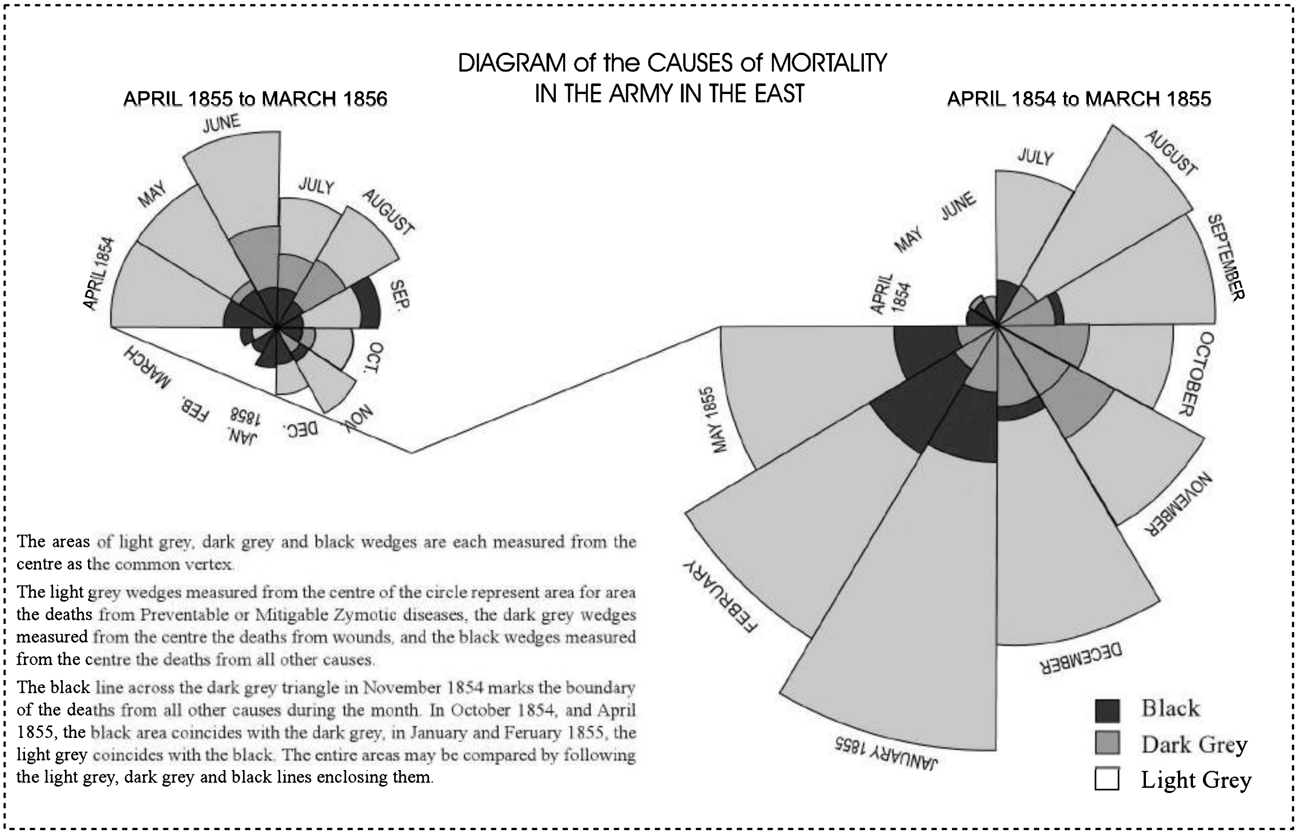
Pie Chart

Pie charts are a typical type of data representation where data is represented as a part of a circle. The circle represents the total value (or 100%), and the different parts represent certain proportions (or percentage) of the total. In a pie chart, the arc length of each sector (and in turn its central angle and area), is proportional to the part it represents.

The origin of the pie chart is traced back to Florence Nightingale in 1858. This was the year when she presented a paper on the causes of deaths in her army in the eastern part of the world.

Following is the pie chart originally developed by Florence Nightingale in the year 1858. (Actually called by her as the 'Polar Area Diagram'.)

The below graphic gives the number of deaths that occured from diseases that could have been prevented (in light grey), those that were the results of wounds (in dark grey) and those due to other causes (in black).



The areas of light grey, dark grey and black wedges are each measured from the centre as the common vertex.

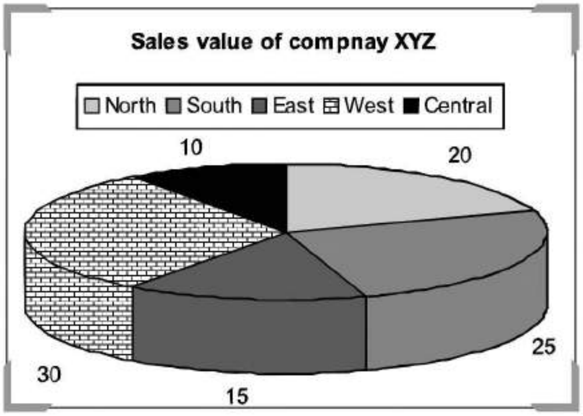
The light grey wedges measured from the centre of the circle represent area for area the deaths from Preventable or Mitigable Zymotic diseases, the dark grey wedges measured from the centre the deaths from wounds, and the black wedges measured from the centre the deaths from all other causes.

The black line across the dark grey triangle in November 1854 marks the boundary of the deaths from all other causes during the month. In October 1854, and April 1855, the black area coincides with the dark grey, in January and Feruary 1855, the light grey coincides with the black. The entire areas may be compared by following the light grey, dark grey and black lines enclosing them.

There are two approaches constructing a pie chart from any given data:

(A) Degree Approach:
The central angle in a circle represents 360°, so any part or segment in a pie chart is calculated as a proportion of 360°.

(B) Percentage Approach:
In this case, any part or segment in a pie chart is calculated as a part of 100%.



If we convert the same pie chart into the degree format, we will be required to do the following conversions:

Total = 100% = 360°
Hence 1% = 3.6°
Central = 10% = 36°
North = 20% = 72°
South = 25% = 90°
East = 15% = 54°
West = 30% = 108°

Limitations of Pie Charts

Despite the pie chart being one of the most important ways to represent data, it is marred by limitations of its own:

- Pie charts can be used only when the sum of all categories is given, for example if the categories represent proportions or percentage of a total.
- A single pie chart can represent only one continuous variable.

Significance of Pie Charts

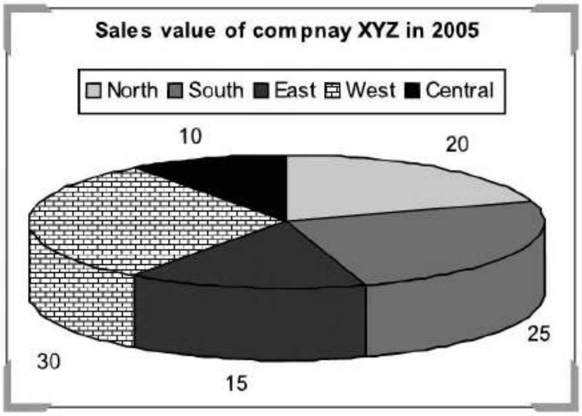
The pie chart has gained prominence due to the following reasons:

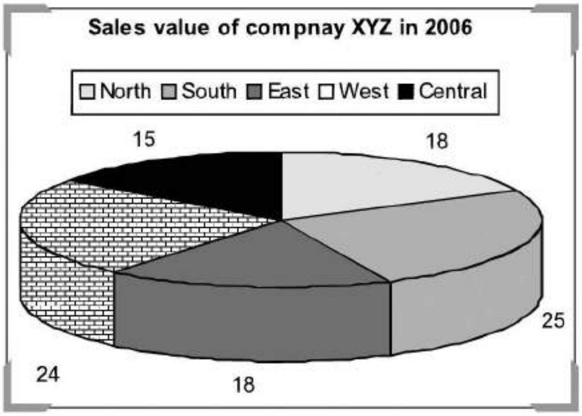
- In a pie chart, we get a clear picture of the contribution of different sectors to the build up of the total. E.g., presentation of budgets.

- Comparing two pie charts is easier than comparing two bar charts or any other format of data representation.

Example 1

Let us see the following data:





Sales value in 2005 = Rs 180 crores

Sales value in 2006 = Rs 204 crores

What is the percentage increase in the sales value of the East zone?

Solution

There are two percentage increases (A) The total sales value of company XYZ is increasing. (B) The percentage contribution of the East zone is increasing.

Percentage increase in The total sales value of the company XYZ = 30%

Percentage increase in the percentage contribution of the East zone = 20%

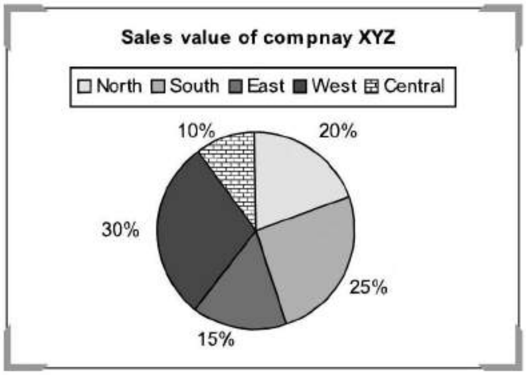
Hence, the net percentage increase = 56%
(Successive increase of 20% and 30%)

Types of Pie Charts

There are two types of pie charts:

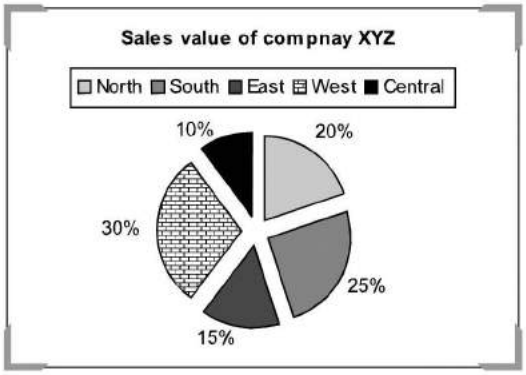
(A) Normal Pie Chart

This displays the contribution of each component of the pie.



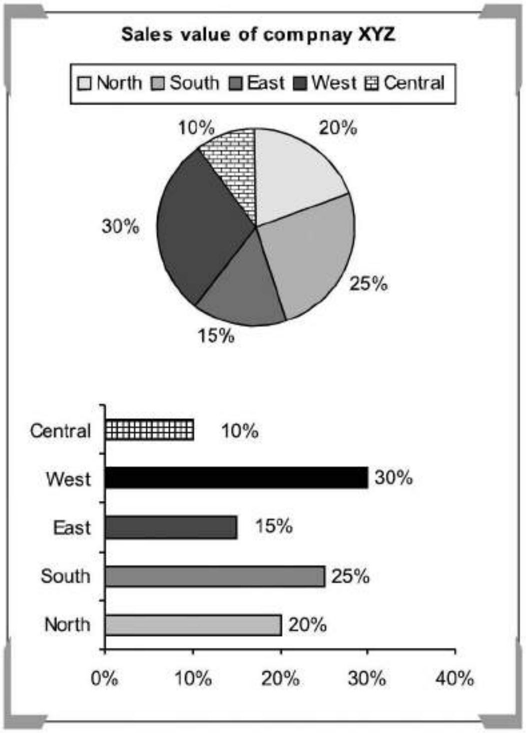
(B) Exploded Pie Chart

This pie chart has all the characteristics of a normal pie chart, the only addition is that the contribution of individual segments is highlighted.



Bar Chart/Bar Graph

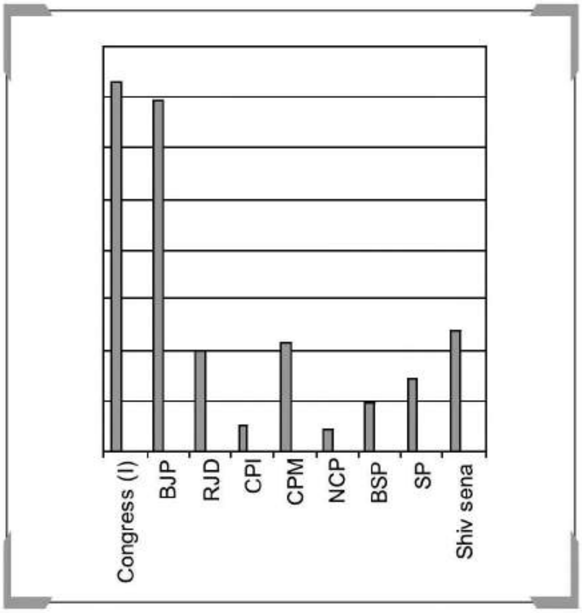
The bar chart, in comparison to the pie chart is more versatile in representing data. It has been proven that representation using lengths as in the case of bar charts is a better indicator of data vis-a-vis pie charts wherein data is categorized in terms of areas.



In the above diagram, the same data has been represented length-wise in the bar chart and area-wise in the pie chart. Obviously, it is easier to see the contribution of the various segments in the bar chart than in the pie chart.

A therefore, is a chart with rectangular bars of lengths proportional to the values which they represent. Usually, the terms 'bar chart' and 'bar graph' are used interchangeably. It should also be noted that in a bar chart, what matters is the length of the bar and not the width of the bar.

The following bar chart represents the number of seats won by different parties in the last general election.

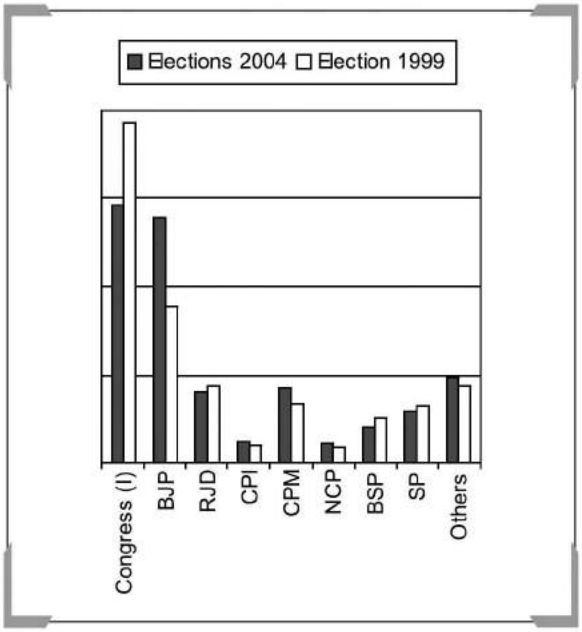


Significance of Bar Charts

Following are the major specialities of bar charts:

Unlike the pie chart, a single bar chart can be used to compare two, or more than two continuous variables.

The following bar chart represents the number of seats won by different parties in two general elections:



Since the length parameter is easier to study than the area parameter, a bar chart gives a quick understanding of the various ranks. Hence the time taken to understand the data becomes condiserably lower in a bar chart as compared to other formats of data presentation.

Types of Bar Charts

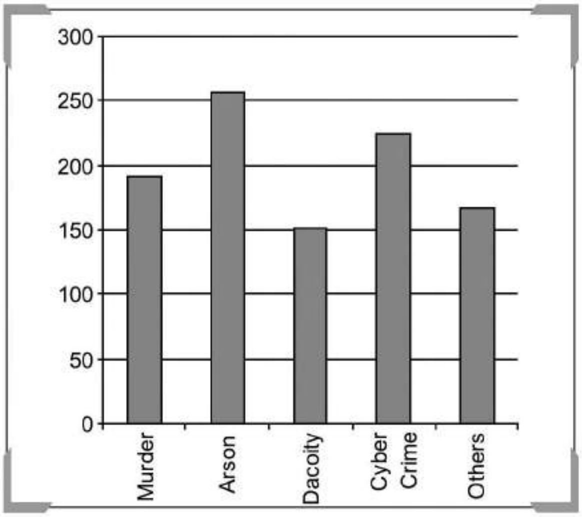
There are three types of bar charts:

(A) Normal Bar Chart

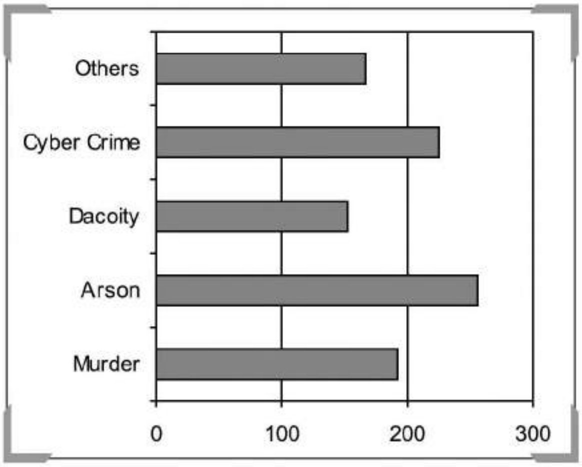
This is a simple bar chart with the values of different segments represented in the form of bars,which could be either horizontal, vertical or both.

The following bar chart represents the crime cases reported in Delhi in 2007:

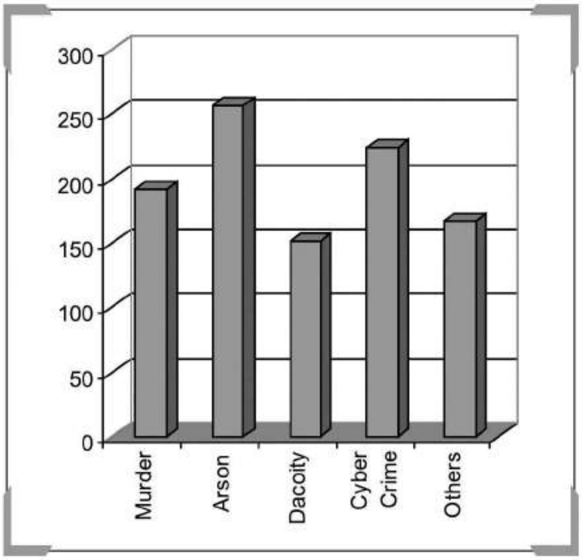
Bar Chart with Vertical Bars



Bar Chart with Horizontal Bars



Bar Charts with 3–D Effects



(B) Stacked Bar Chart Value-wise

When the same variable is to be represented on more than one parameters like; year etc., then we can have a stacked bar chart.

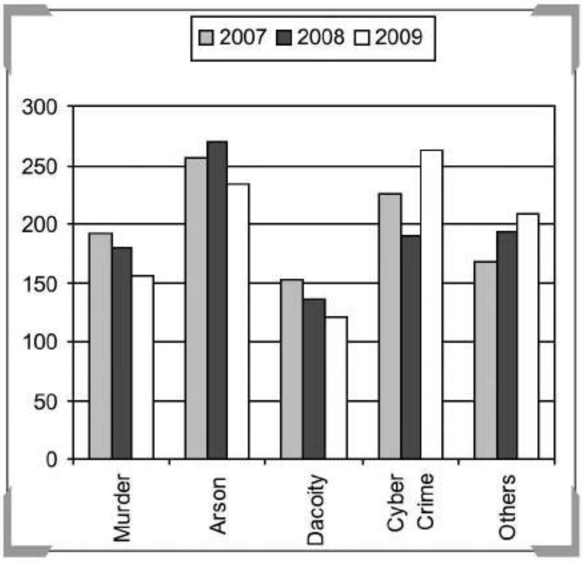
(C) Stacked Bar Chart Percentage-wise

The only difference between value-wise and percentage-wise bar charts is that in the former we apply values to stack the bars and in the latter we apply percentages.

Stacked Bar Charts are also known as Cumulative Bar Charts.

Here, we will see the same data in a normal bar chart, value-wise stacked bar chart and stacked bar chart percentage-wise

Normal Bar Chart



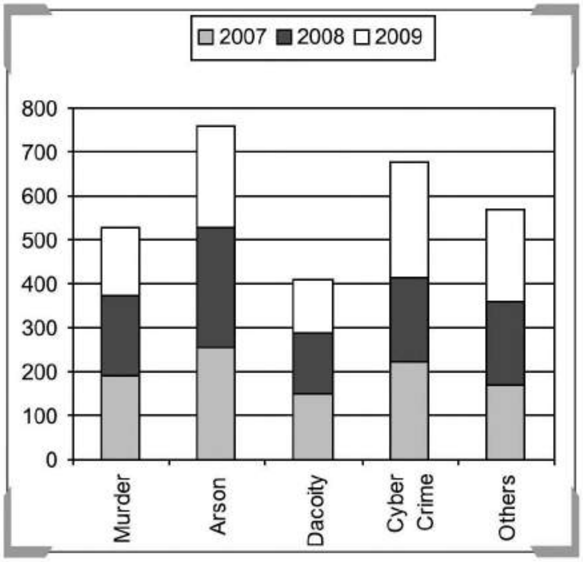
Following things can be observed in the above bar chart:

Bars representing different crimes in different years are proportional to the crimes reported.

The above bar chart takes the minimum value as 100, however the same bar chart could take the minimum value as 0 or 50 or anything else as well. The question here is, what will happen if we take the minimum value as 150?

Since some of the values are less than 150, what will happen to those? Find out yourself without using a computer.

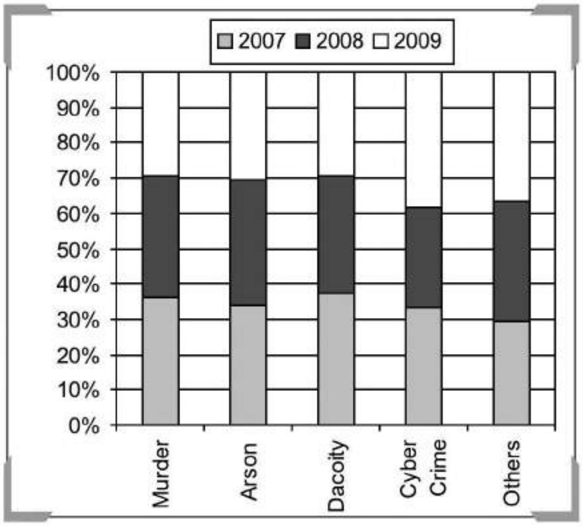
Stacked Bar Chart Value-wise



Following things can be observed in the above stacked bar chart:

- The total number of murders have been added up in one bar and the different years are shown as a part of that total.
- The same data could also have been represented by taking the sum of all the crimes reported in a particular year as a total, and the individual crimes as a part of that total.

Stacked Bar Chart Percentage-wise



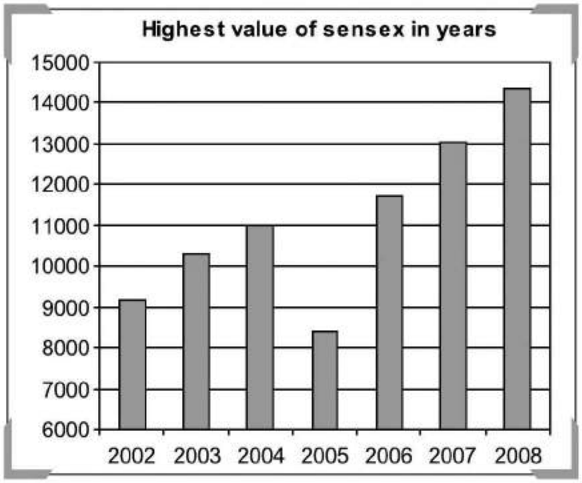
Following things can be observed in the above stacked bar chart:

- The total number of murders have been added up and they equal to 100%; and the murders reported in different years have been taken as a percentage of that total.
- The same data could have been represented by taking the sum of all the crimes reported in a particular year as a total, and the individual crimes as a part of that total.

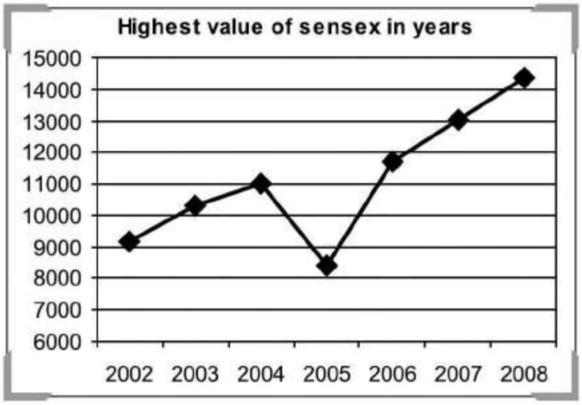
Line Chart/X–Y Chart

Line charts are seen as simplified forms of the normal bar chart.

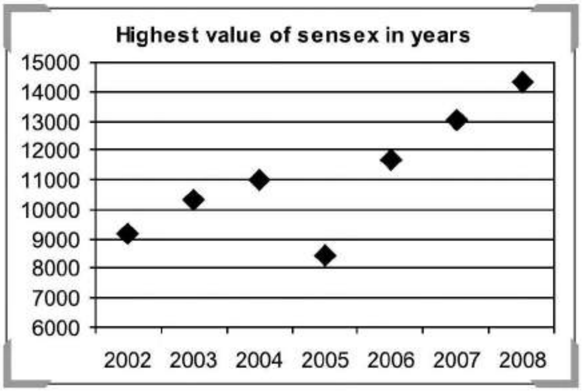
The bar chart given below represents the highest values of the sensex in the given years.



If we convert the same data into a line chart, it will look like this:



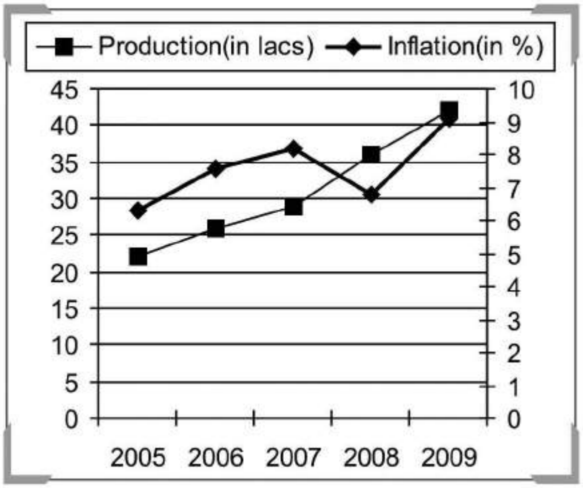
Sometimes in case of line charts, the lines are not given and only the dots are indicated in the graph. Let us see an example with the same data:



Significance of Line charts

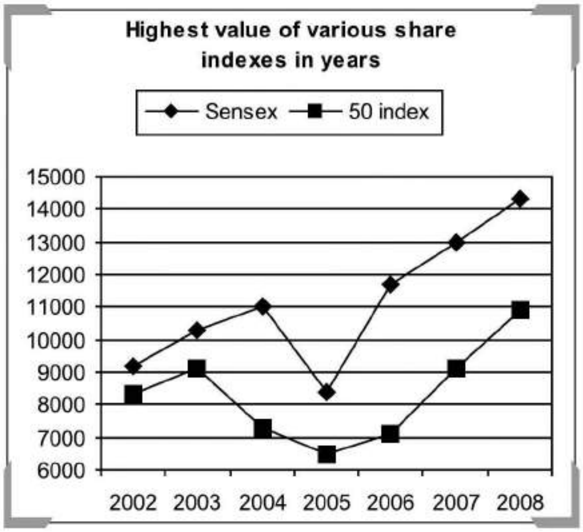
- Generally Line charts are used in case time is one of the variables in the data. The 'time' variable can be in the form of hour, day, months years or anything that represents chronological order of events.
- It is easier to calculate the percentage changes in a line chart and thereby understand the trends of the data in a better way.
- A line chart becomes very handy in case of data with two different scales.

The line chart given below represents the production of soaps by a company over a period of five years and the inflation for the same period.

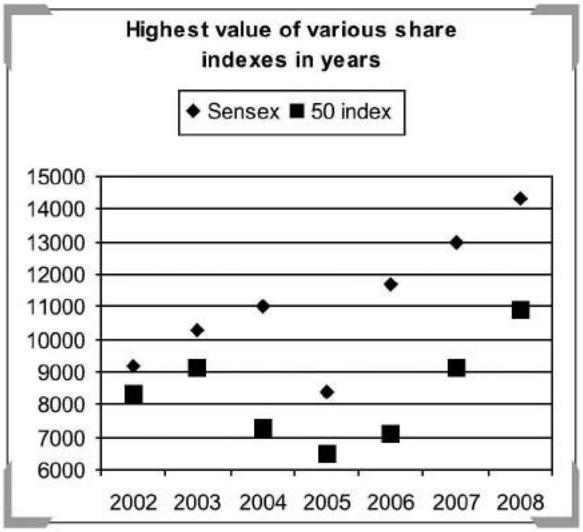


- Two or more than two variables can be represented on a line chart very easily. Besides, we can see the movement of data very easily in case of a line chart.

Below given line chart represents the movements of the highest value of two indices over a given period:



The same data can be represented using dots only also:



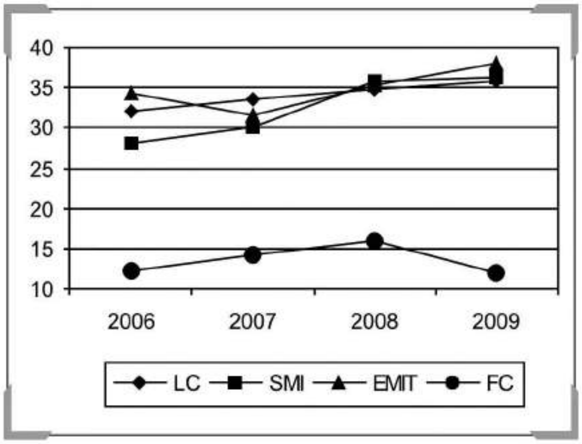
Types of Line Charts

Broadly, line charts are of three types:

Normal Line Chart

This is a simple line chart representing two or more than two variables.

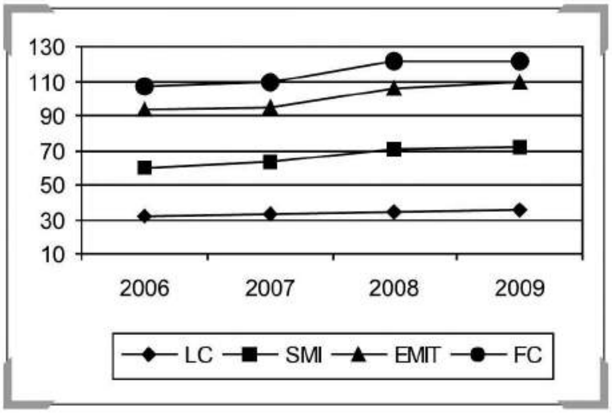
In the chart given below, The total number of enrolments for four different years of for four coaching institutes are represented:



Stacked Line Chart

In a stacked line chart, the values keep on getting added to obtain the next value.

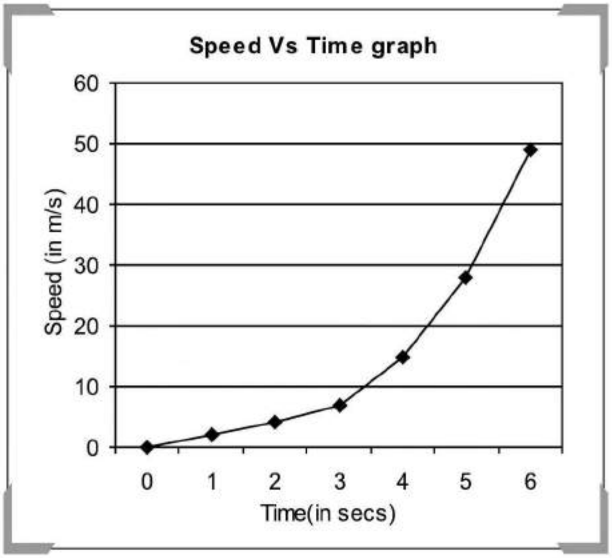
Here, the bottom line (of LC) gets added with the values of SMI to represent the value of SMI. Now this represented value of SMI is added to the actual value of EMIT to give the value of EMIT on the chart. And finally, the same is done with the value of FC.



Hence, in a stacked line chart, to obtain the values of different constituents/segments, either start with the top line or the bottom line and then keep on subtracting the values to obtain the next value.

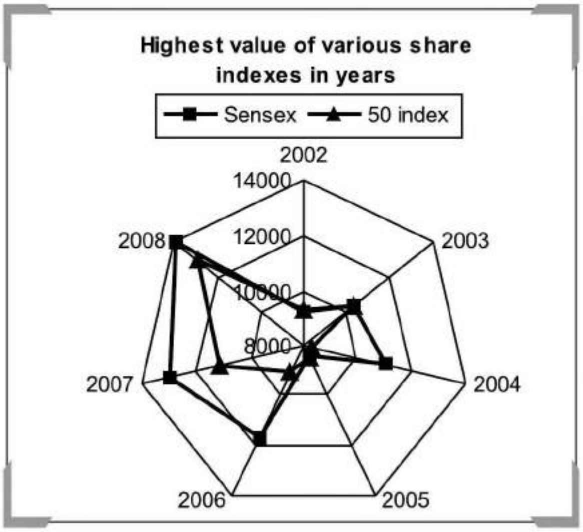
Time-Speed-Distance Line Chart

This line chart is used in tables of data collected from experiments on physical processes.



Radar Diagram

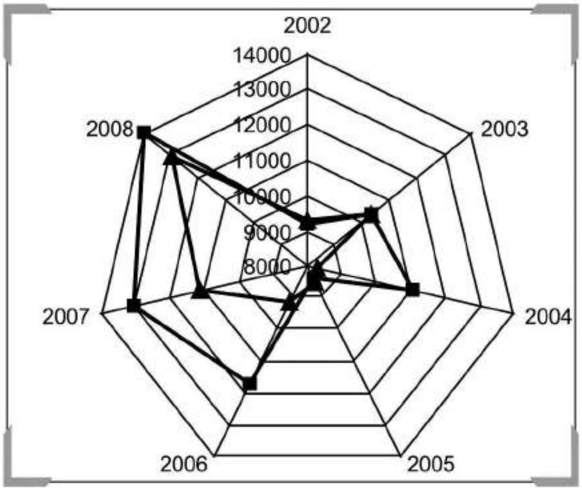
This diagram every value is represented with respect to a central point. All the changes in the values are expressed in the form of distance from this center point.



The radar diagram given below represents the highest value of two indices over a given period.

It can be seen that the centre value = 8000. With every passing circle the value increases by 2000. Since here are seven years, the diagram takes the shape of a heptagon.

Had there been only six years, it would have taken the shape of a hexagon, as shown below:



Similarly, had there been only four years, the diagram would have been in the shape of a square.

Area Diagram

In case of an area diagram, the values are represented in terms of areas.

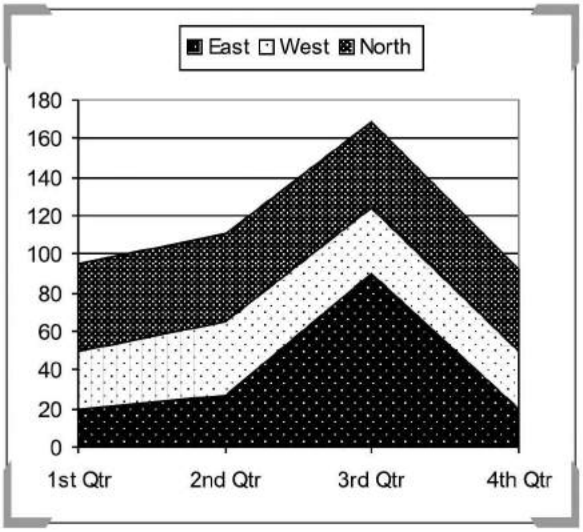
Example 2

Convert the data given below to a stacked area diagram:

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
East	20	27	90	20
West	30	38	34	30
North	45	46	45	42

Solution

The stacked area diagram corresponding to the table given above will be like this:



Floating Diagram

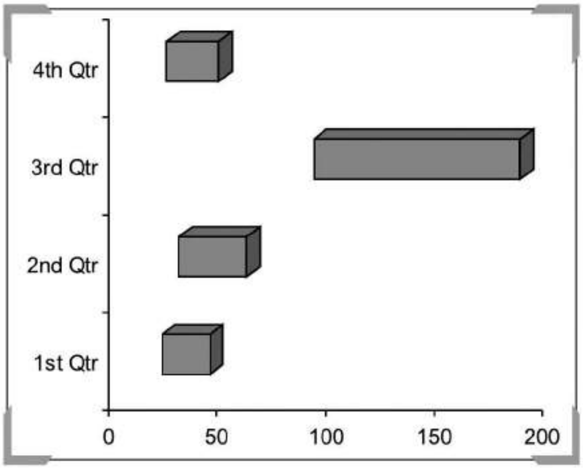
A floating diagram is used to represent the difference in any given variable between two different periods.

Example 3

Convert the following table into a floating diagram:

	1st Quarter	2nd Quarter	3rd Quarter	4th Quarter
2000	20	28	90	22
2001	22	31	95	24

Solution

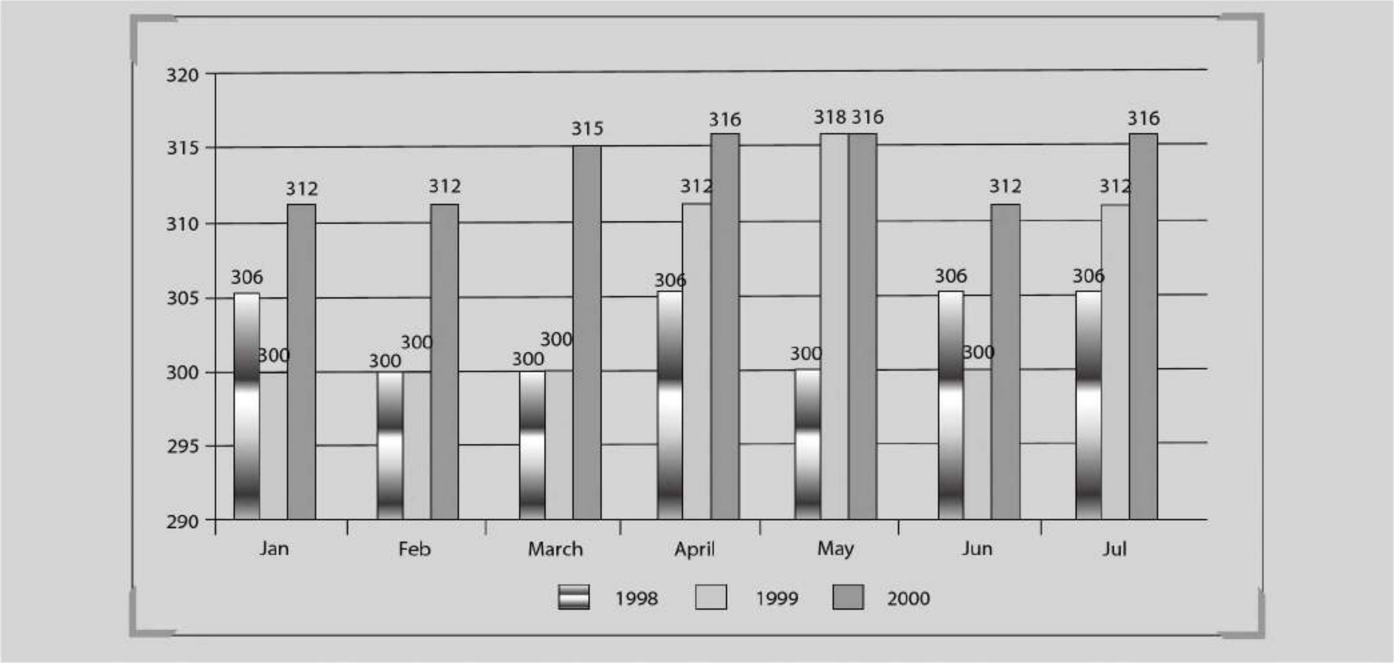


PRACTICE EXERCISE

BAR CHART

[redacted]

The following bar chart shows the monthly expenditure of a family comprising of five persons over a period of seven months during three different years.



Q 1. In any of the given years, which month sees the maximum percentage increase in expenses with respect to the previous month?

(a) Feburary (b) March
(c) April (d) June

Q 2. What is the average monthly expenditure of the whole family in the year 2000?

(a) Rs 314000 (b) Rs 315000
(c) Rs 316000 (d) Cannot be determined

Q 3. In April 2000, what was the percentage increase in expenditure over April 1999?

(a) 1.2% (b) 2%
(c) 6% (d) 9.4%

Q 4. Which period has shown the same change in trend across all the given three years?

(a) February-March
(b) May-June
(c) March-April
(d) January-February

Q 5. Which of the following statements is correct?

(a) In 1998, May-June were the two consecutive months during which the expenditure was the maximum.
(b) During May-June 1999, the expenditure was the maximum for the year.
(c) Expenditure during January-Februrary was the same in 1998 as well as in 1999.
(d) None of these

Q 6. For how many months of the given years the expenditure has been consistently increasing or decreasing?

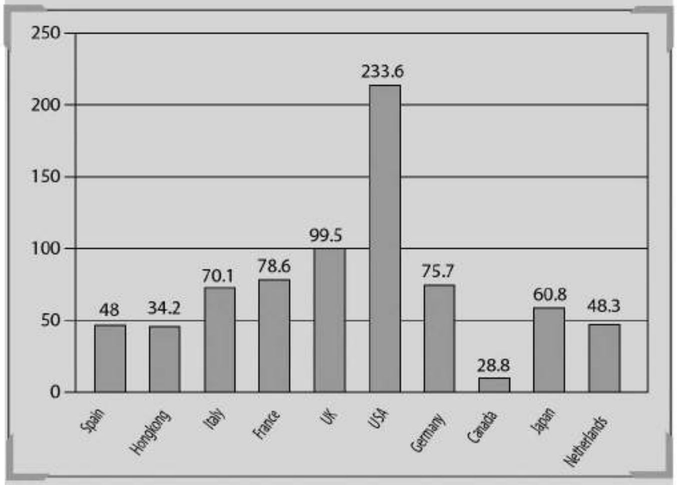
(a) 0 (b) 1
(c) 2 (d) 3

Q 7. Out of the following months in the options, Which month accounts for the maximum combined expenditure for three years?

(a) March (b) May
(c) April (d) Feburary

Direction for questions 8 to 14: Refer to the following bar graph and solve the based on it.

The bar chart given below represents top 10 exporters and their export value (in billion $) in 2008.



Q 8. If India also joins the top ten exporters list by becoming the 11th largest exporter, then what will be the effect on the average exports of these 11 countries; provided the export value of India is $ 10 billion?
(a) The average will increase by 1 billion dollars
(b) The average will decrease by 1 billion dollars
(c) The average will increase by 1.1 billion dollars
(d) None of these

Q 9. The ratio of exports between Spain and Japan is (approximately)
(a) 3 : 4 (b) 19 : 15
(c) 15 : 19 (d) 4 : 5

Q 10. What is the average export of all the countries in billion dollars for the year 2008?
(a) 77.66 (b) 76.76
(c) 76.67 (d) Cannot be determined

Q 11. If the exchange rate per dollar is Rs 43 then find the difference between the values of exports by Hong Kong and UK in Rupee.
(a) 28.179×10^9
(b) 28.179×10^{10}
(c) 28.079×10^{10}
(d) 28.079×10^{11}

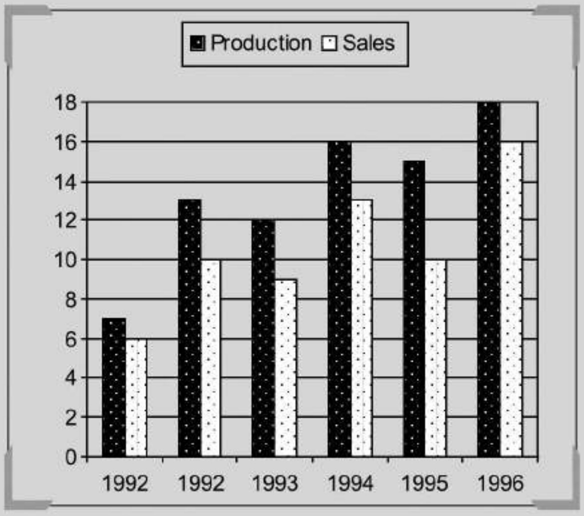
Q 12. If the average export of the top ten exporters is calculated how many of the top ten exporting countries have higher export than this average?
(a) 3 (b) 4
(c) 5 (d) Cannot be determined

Q 13. If the average export of the top ten exporters is calculated, how many countries have higher export than this average?
(a) 3 (b) 4
(c) 5 (d) Cannot be determined

Q 14. To fight the US hegemony, the countries have decided to merge their operations by adding their exports value. What is the minimum number of countries needed to merge their operation so that their combined exports is more than the exports value of the US? (None of these ten countries export to each other.)
(a) 3 (b) 4
(c) 5 (d) None of these

Direction for questions 15 to 20: Refer to the following bar graph and solve the questions based on it.

Amount of production and sales by a company over the years (in lacs tonnes)



Q 15. What is the difference between the sales of 1993 and 1995 (in thousand tonnes)?
(a) 1000 (b) 50
(c) 100 (d) 500

Q 16. Total sales of 1991 and 1992 together are approx-imately what percentage of the sales in 1994?
(a) 120% (b) 100%
(c) 50% (d) 200%

Q 17. What is the approximate percentage increase in production from 1993 to 1994?
(a) 20% (b) 33%
(c) 70% (d) 100%

Q 18. The percentage of sales to production is maximum in which of the following years?
(a) 1992 (b) 1994
(c) 1996 (d) 1991

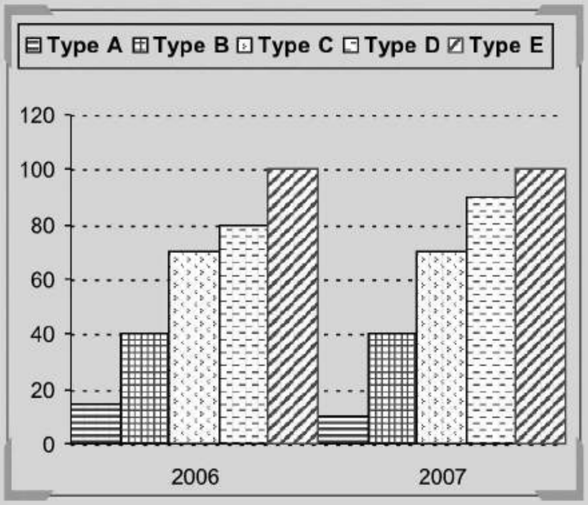
Q 19. What is the percentage drop in sales from 1992 to 1993?
(a) 5% (b) 40%
(c) 10% (d) 60%

Q 20. In which year is the difference between the production and sales the maximum?
(a) 1992 (b) 1993
(c) 1995 (d) 1996

Direction for questions 21 to 25: Refer to the following bar graph and solve the questions based on it.

The following bar chart gives the cumulative percentage of five different types of bikes produced by Kajaj during the given two years:

Total Bikes produced in 2006 is 4,50,000



Total bikes produced in 2007 is 520,000.

Q 21. What is the difference in the production of the C type bikes between 2006 and 2007?
(a) 5,000
(b) 7,500
(c) 10,000
(d) None of these

Q 22. If 85% of the E type bikes produced during 2006 and 2007 are being sold by the company, then how many E type bikes are left unsold?
(a) 1,42,800
(b) 21,825
(c) 29,100
(d) 21,300

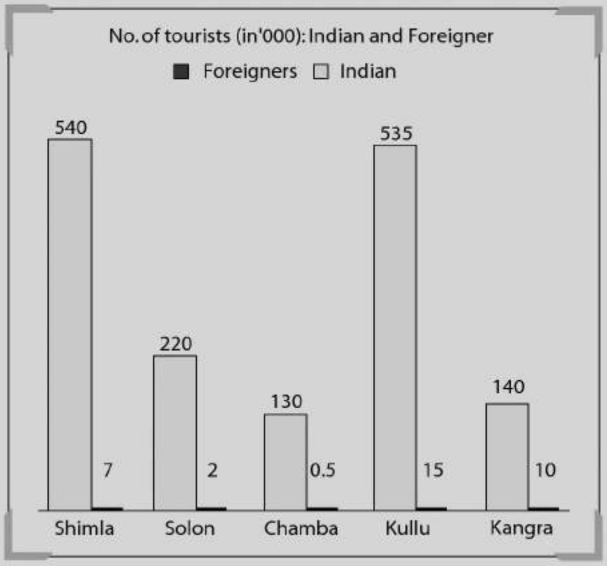
Q 23. What is the approximate percentage share of the A type bikes in the total production of (2007 + 2006)?
(a) 11 (b) 13
(c) 15 (d) 9

Q 24. In the case of which of the following types of bikes is the percentage increase from 2006 to 2007 the maximum?
(a) A (b) E
(c) D (d) B

Q 25. Production of B type bikes in 2007 is what percentage of bikes produced in 2006?
(a) 30% (b) 40%
(c) 35% (d) 25%

Direction for questions 26 to 30: Refer to the following bar graph and solve the based on it.

The bar graph given below shows the five districts of the state of Himachal Pradesh and the number of tourists visiting them. For the given bar chart, total number of Indian tourists is more that the total number of foreign tourists. None of the tourists can be Indian as well as foreigner.



Q 26. By what percentage are the Indian tourists visiting Chamba less than those visiting Shimla?
(a) 30% (b) 50%
(c) 60% (d) 75%

Q 27. Approximately what percentage of the total tourists visiting Himachal Pradesh are foreigners?
(a) 2% (b) 8%
(c) 4% (d) 5%

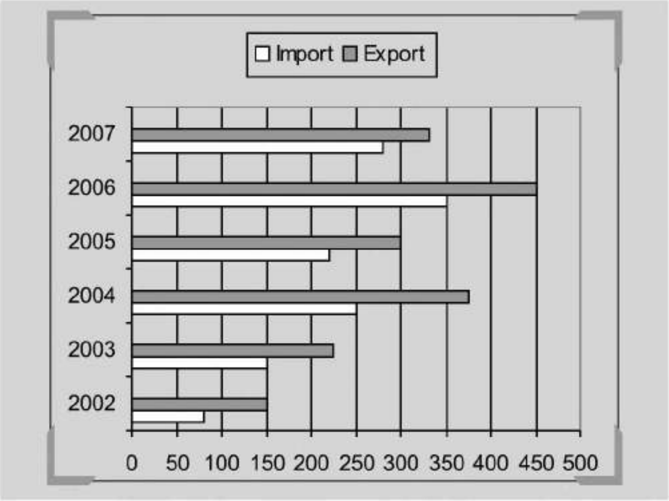
Q 28. What is the ratio between the Indian tourists and the foreign tourists visiting Kullu?
(a) 105 : 3 (b) 70 : 3
(c) 107 : 3 (d) 35 : 1

Q 29. Which of the following districts has the maximum ratio of Indian and foreign tourists?
(a) Shimla (b) Solon
(c) Chamba (d) Kangra

Q 30. Which of the following districts has the minimum ratio of Indian and foreign tourists?
(a) Shimla (b) Solon
(c) Chamba (d) Kangra

Direction for questions 31 to 35: Refer to the following bar graph and solve the questions based on it.

The following bar charts represents the value of exports and imports (in Rs crores) of a country for the given period:



(All the values in the above bar chart are the multiples of 10).

Q 31. The value of exports in 2006 was what percentage of the average value of imports in the years 2004, 2005 and 2007?
(a) 200% (b) 100%
(c) 150% (d) None of these

Q 32. The value of exports in 2004 was exactly what percentage of the value of imports in the same year?
(a) 200%
(b) 100%
(c) 150%
(d) None of these

Q 33. What is the approximate difference (in Rs crores) between the average exports and the average imports for the given years?
(a) 65 (b) 85
(c) 105 (d) 135

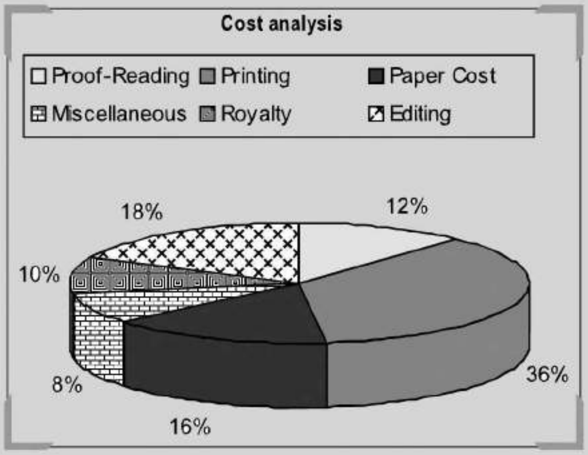
Q 34. In which year is the difference between the exports and the imports closest to Rs 100 crores?
(a) 2005
(b) 2006
(c) 2007
(d) 2008

Q 35. What is the percentage increase in the value of exports in 2006 over 2005?
(a) 30% (b) 50%
(c) 90% (d) 120%

PIE CHART

Direction for questions 1 to 5: Refer to the following pie chart and solve the questions based on it.

Following is the cost analysis of a book "Pearson's Guide to *Quantitative Aptitude* for CAT".



Q 1. What is the central angle showing the cost of paper?
(a) 42.8° (b) 32.6°
(c) 36.8° (d) 57.6°

Q 2. If the cost of printing is Rs 23,400 what would the cost of royalty be?
(a) Rs 6500 (b) Rs 2340
(c) Rs 4680 (d) Rs 7840

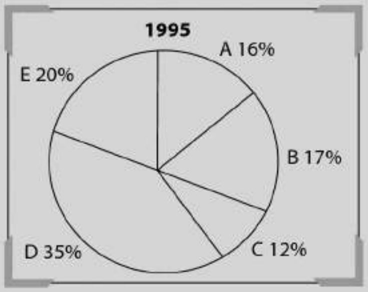
Q 3. If the miscellaneous expenditure amounts to Rs 18,000 then what is the expenditure on editing?
(a) Rs 8000 (b) Rs 14400
(c) Rs 46800 (d) None of these

Q 4. The royalty on the book is less than the editing expenditure by
(a) 8% (b) 80%
(c) 44.44% (d) None of these

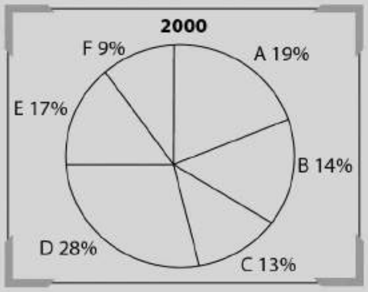
Q 5. If 5500 copies of the book are published and the miscellaneous expenditure amounts to Rs 36,960 and the marked price is 40% above the cost price, then the marked price of each copy is
(a) Rs 122.50
(b) Rs 117.60
(c) Rs 126.40
(d) Rs 92.40

Direction for questions 6 to 10: Refer to the following pie charts and solve the questions based on it.

The pie charts given below represent the market share of different players in the electronics market in the year 1995 and 2000. A new player F has entered the market in the given period.



Market size = $ 2.3 billion



Market size = $ 3.7 billion

Q 6. What is the percentage increase in the sales of A in the year 2000 over 1995?
(a) 90% (b) 60%
(c) 80% (d) 50%

Q 7. Find the CAGR with which the market has grown for the period 1995-2000?
(a) 18%
(b) 10%
(c) 5%
(d) 12%

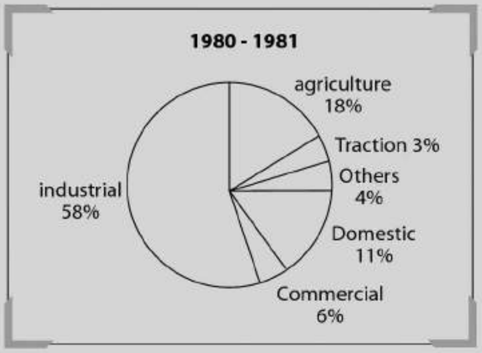
Q 8. If the new entrant F has grown at the expense of B, C, and E, what would be the change in the net value of sales of E in the year 2000, had F not entered the market?
(a) $ 111 million
(b) $ 340 million
(c) $ 247 million
(d) Cannot be determined

Q 9. What is the increase in sales (in $ million) of A, B and C put together, over the 5 years period?
(a) 103
(b) 1202
(c) 668
(d) 854

Q 10. If the market grows at 10% p.a. over the next 2 years and D captures 1/3rd of the new market, what would be its market share in 2002?
(a) 29% (b) 27%
(c) 25% (d) 32%

Direction for questions 11 to 17: Refer to the following pie charts and solve the questions based on it.

Electricity Consumption by Sector



Q 11. By what percentage has the consumption of electricity by agriculture increased in 1993-1994 over 1980-1981?
(a) 66% (b) 33%
(c) 133% (d) Cannot be determined

Q 12. The electricity consumption of how many sectors has definitely increased over the given period?
(a) 2 (b) 3
(c) 4 (d) Cannot be determined

Q 13. If the total electricity consumption in 1993-94 is 1.2 times of the total electricity consumption in 1980- 1981, then how many sectors have definitely increased by more than 50% during the same period?
(a) 1 (b) 2
(c) 3 (d) 4

Q 14. If the total electricity consumption in 1993-1994 is 1.5 times of the total electricity consumption in 1980-1981, then what is the percentage increase in the electricity consumption of the agricultural sector in the given period?
(a) 50% (b) 100%
(c) 150% (d) None of these

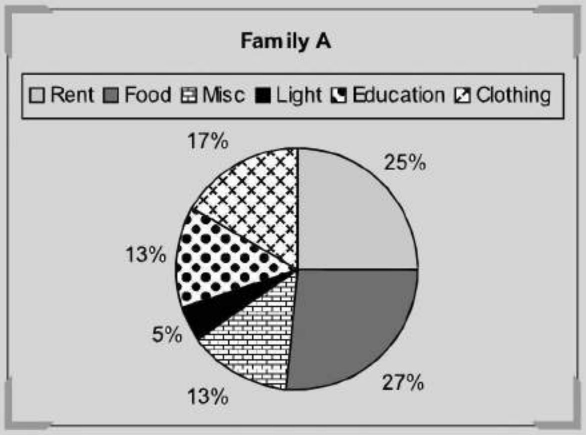
Q 15. What is the minimum number of sectors required to be added up in order to be more than 50% of the consumption for the period in 1993-1994?
(a) 1 (b) 2
(c) 3 (d) 4

Q 16. The agricultural consumption of electricity doubled from 1980-81 to 1993-94. By how much percentage has the total electricity consumption grown from 1980-81 to 1993-94?
(a) 20% (b) 25%
(c) 50% (d) Cannot be determined

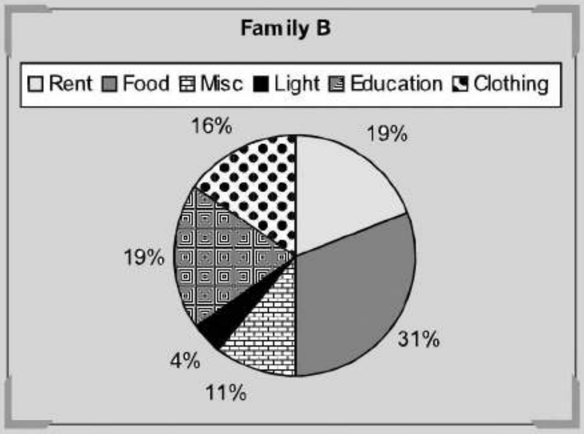
Q 17. If the electricity consumption of the 'others' category has remained constant over the period, then what is the percentage increase in the electricity consumption of the domestic category?
(a) 63% (b) 38%
(c) 58% (d) Cannot be determined

Direction for questions 18 to 23: Refer to the following pie charts and solve the questions based on it.

The two pie charts given below provide the expenses of two families:



Total expenses = Rs 48, 000



Total expenses = Rs 72,000.

Q 18. Both the families decide to double the total expenditure keeping the pattern of spending the same as given above. What will be the new ratio of expenditure on food between family A and family B?
(a) 18:31 (b) 31:27
(c) 2:3 (d) 3:2

Q 19. If the total expenses of family B increase three-fold, keeping the expenses on education the same as given above, what will be the expense on education?
(a) 6.33% (b) 57%
(c) 19% (d) None of these

Q 20. What will be the expenses on light by family A, as a percentage of expense on light by family B?
(a) 120% (b) 83.33%
(c) 62.5% (d) 66.66%

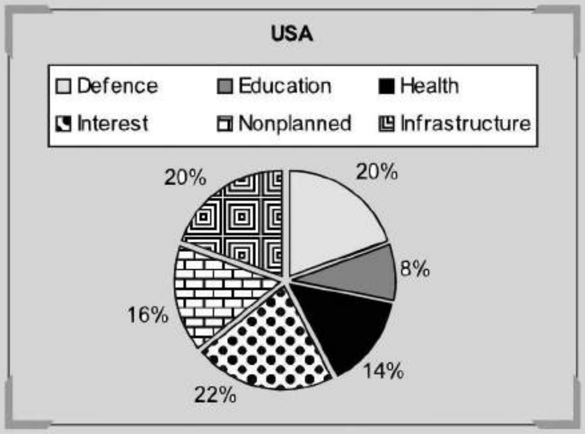
Q 21. If family A and family B decide to combine their expenses, then which one of the following heads will be responsible for the highest expenses?
(a) Rent (b) Misc
(c) Food (d) Education

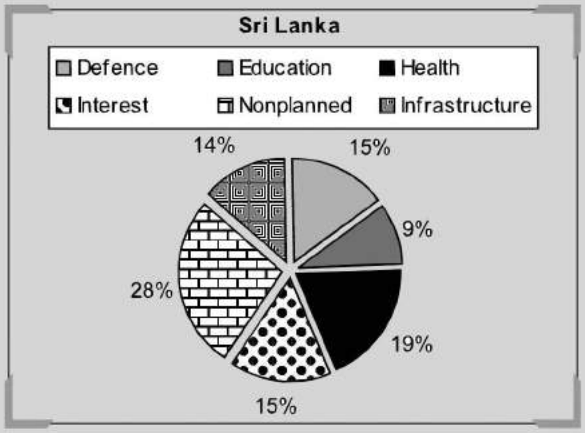
Q 22. In the above question, how many heads will have a lower percentage share in the combined total expenses of both the families than the percentage share of family B under the same head?
(a) 1 (b) 2
(c) 3 (d) 4

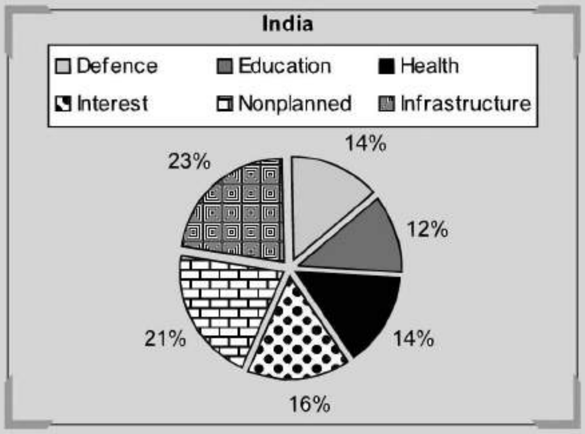
Q 23. Under how many heads are the expenses of family B more than the expenses of family A?
(a) Less than 3
(b) More than 3
(c) Equal to 3
(d) Cannot be determined

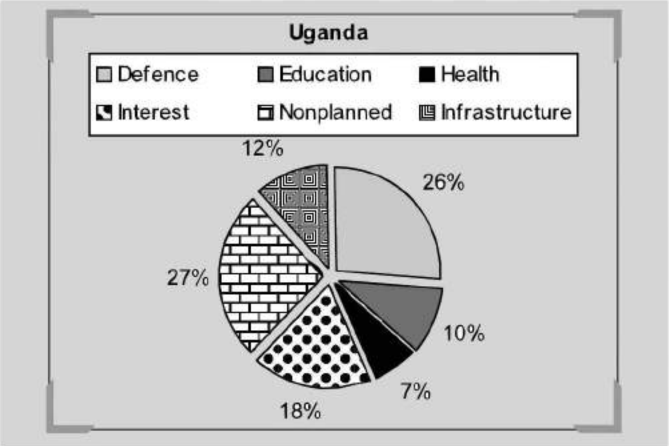
Direction for questions 24 to 30: Refer to the following pie charts and solve the questions based on it.

The following pie charts represent the budget expenditure of certain countries on various sectors for the year in 2007:









Q 24. A country is said to be progressive if its education, health and infrastructure expenditures are in the top four expenditure sectors. How many of the four countries are progressive?
(a) 0 (b) 1
(c) 2 (d) 3

Q 25. A country is said to be developing if its combined expenditure on education, health and infrastructure is atleast 50% of the total expenditure. How many of the four countries are developing nations?
(a) 0 (b) 1
(c) 2 (d) 3

Q 26. If a country is under military rule, it will spend maximum on defence and minimum on either health or education. How many of the above countries are under military rule?
(a) 0 (b) 1
(c) 2 (d) Cannot be determined

Q 27. Which of the following sectors will have same ranking in the countries budget expenditures?
(a) Nonplanned (b) Education
(c) Health (d) None

Q 28. If the budgetary expenditures of all the four countries are combined, which sector will account for the maximum expenditure?
(a) Non-planned (b) Interest
(c) Defence (d) Cannot be determined

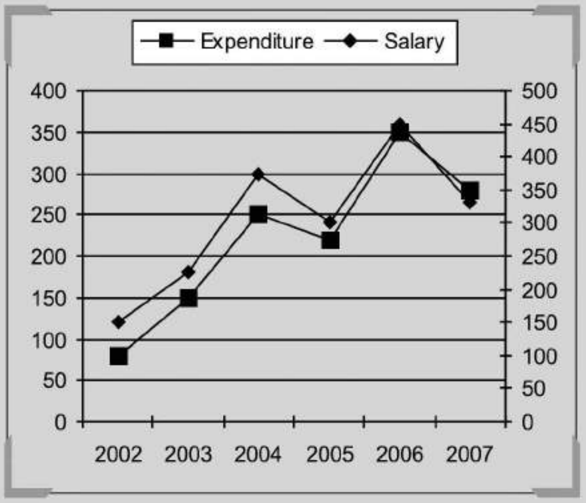
Q 29. The total budgetary expenditure of USA is four times the total budgetary expenditure of India; and the population of India is twice the population of USA. Then the per capita budgetary expenditure of USA is how many ….. times per capita expenditure of India.
(a) 8 (b) 4
(c) 1/8 (d) 1/4

Q 30. The total budgetary expenditure of Sri Lanka is twice the total budgetary expenditure of Uganda. Under how many sectors is the budgetary expenditure in Uganda more than the budgetary expenditure in Sri Lanka?
(a) 0 (b) 1
(c) 2 (d) 3

LINE CHART

Direction for questions 1 to 7: Refer to the following line chart and solve the questions based on it.

The line chart given below represents the salary and expenditure (in Rs) of Guru Gulab Khatri for the given period.



It is given that the salary is greater than the expenditure atleast for four years. All the values are multiples of 5.

Q 1. By what percentage has the salary of Guru Gulab Khatri increased in 2004 over 2002?
(a) 210% (b) 150%
(c) 100% (d) 250%

Q 2. By what percentage has the expenditure of Guru Gulab Khatri increased in 2004 over 2003?
(a) 66.66% (b) 133.33%
(c) 150% (d) None of these

Q 3. For how many years the salary is more than expenditure?
(a) 4 (b) 5
(c) 6 (d) Cannot be determined

Q 4. Which year is the percentage increase in the salary of Guru Gulab Khatri maximum?
(a) 2003 (b) 2004
(c) 2006 (d) 2003 or 2006

Q 5. A year is said to be comfortable, if Guru Gulab Khatri is able to save atleast 30% of his salary. How many years are comfortable during the given period?

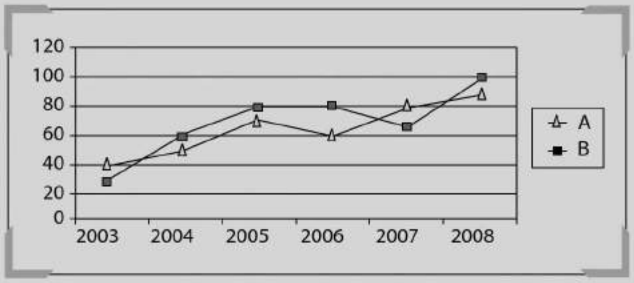
(a) 2 (b) 3
(c) 4 (d) 5

Q 6. What is the maximum difference (in Rs) between the salary and the expenditure for any year?
(a) 70 (b) 75
(c) 125 (d) 100

Q 7. For how many years is the difference between the salary and the expenditure more than Rs 80?
(a) 2 (b) 3
(c) 4 (d) 5

Direction for questions 8 to 12: Refer to the following line chart and solve the questions based on it.

Percentage profit earned by the two companies A and B over the years



Q 8. If the income for company A in the year 2004 was Rs 35 lacs what was the expenditure for company B in the same year?
(a) Rs 123.5 lacs
(b) Rs 128 lacs
(c) Rs 132 lacs
(d) Cannot be determined

Q 9. If the income of company A in 2006 and the income of company B in 2007 are equal, what will be the ratio of expenditure of company A in 2006 to the expenditure of company B in 2007?
(a) 26 : 7 (b) 17 : 16
(c) 15 : 170 (4) None of these

Q 10. During which of the following years is the ratio of percentage profit earned by company A to that of company B, the maximum?
(a) 2003 and 2006
(b) 2005 and 2007
(c) 2003 only
(d) 2008 only

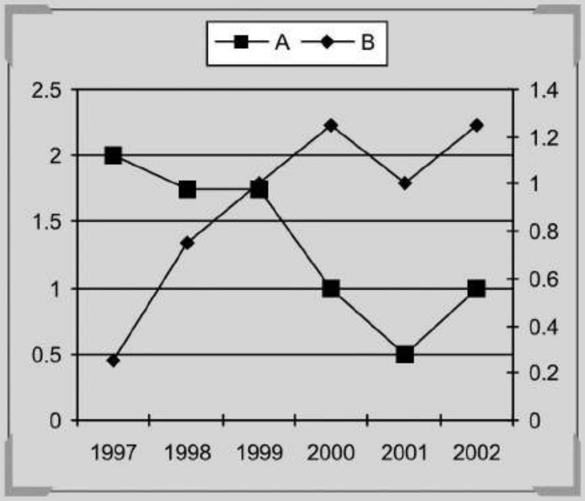
Q 11. If the expenditure of company B increases by 20% from 2005 to 2006, the income in 2006 will be how many times the income in 2005?
(a) 2.16 times
(b) 1.2 times
(c) 1.8 times
(d) None of these

Q 12. If the income of company A in 2006 was Rs 36 lacs, what was the expenditure of company A in 2006?
(a) Rs 12.5 lacs
(b) Rs 18.8 lacs
(c) Rs 20 lacs
(d) None of these

Direction for questions 13 to 17: Refer to the following line chart and solve the questions based on it.

The following line chart represents the ratio of exports (in Rs) to imports (in Rs) of two countries A and B over the given period:

Left Axis = Country A ; Right Axis = Country B



All the values are multiples of 0.25.

Q 13. For how many years do both the countries have their respective exports more than their imports?
(a) 1 (b) 2
(c) 4 (d) None of these

Q 14. Due to some compulsions, one year during the period given these two countries have to do business between themselves only and they are not allowed to export to or import from any other countries. Which out of the following years is possibly that year?
(a) 1997 (b) 1998
(c) 1999 (d) None of these

Q 15. What can be said regarding the imports/exports of the two countries?
(a) The value of exports of country A is constantly on the decline from 1997 to 2001.
(b) The value of imports of the country B is constantly on the rise from 1997 to 2000.
(c) The value of imports of country A is equal to the value of import in 2002 (ratio = 1 for both the years)
(d) none of these

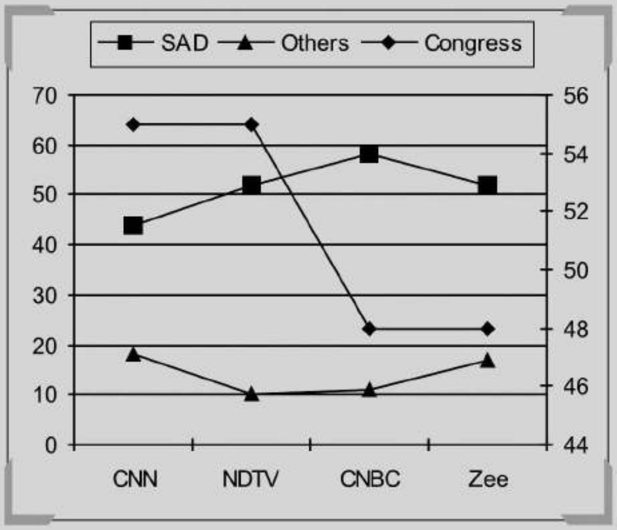
Q 16. For how many years is the value of exports of country B is constantly on the rise over the previous year for country B?

(a) 2 (b) 3
(c) 4 (d) Cannot be determined

Q 17. For how many years is the value of imports of country A is constantly on the rise over the previous year?
(a) 2 (b) 3
(c) 4 (d) Cannot be determined

Direction for questions 18 to 20: Refer to the following line chart and solve the questions based on it.

The line chart given below represents the projections of seats for the coming Punjab assembly elections according to surveys done by four different agencies.



The following observations have also been made regarding the projected seats:

(i) The others have won less than 20 seats according to the projections made by each of the agencies.
(ii) According to NDTV, the Congress' projected seats are more than the projected seats for the SAD.
(iii) The total number of seats have been won by only three parties—Congress, SAD and the others.
(iv) According to CNBC, the difference between projected seast of SAD and Congress is 20.

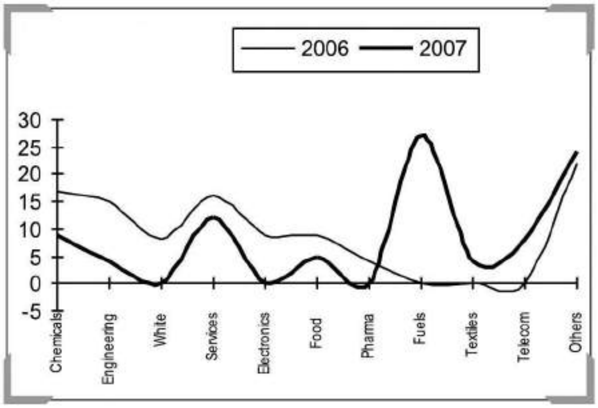
Q 18. If we add the projected seats of each of the parties individually, then which of the following will have the maximum number of projected seats?
(a) Congress
(b) SAD
(c) Congress or SAD
(d) Others

Q 19. According to how many agencies' projections is the Congress getting more seats than the SAD?
(a) 0
(b) 1
(c) 2
(d) 3

Q 20. For how many agencies' projection about seats has the sum of the two bottom ranked parties been more than the top ranking party?
(a) 0
(b) 1
(c) 2
(d) 4

Direction for questions 21 to 25: Refer to the following line chart and solve the questions based on it.

The following line charts give the FDI (in $ billion) in 2006 and 2007 for the different sectors in India:



Q 21. For how many sectors, is there an increase in the FDI in 2007 over 2006?
(a) 2 (b) 3
(c) 4 (d) 5

Q 22. Which of the following sectors has seen the maximum percentage increase in its value in 2007 over 2006 (only for those sectors which have their presence in both the years)?
(a) Fuel (b) Others
(c) Chemical (d) None of these

Q 23. Which of the following sectors has seen the maximum percentage decrease in its value in 2007 over 2006 (only for those sectors which have their presence in both the years)?
(a) Fuel
(b) Others
(c) Chemical
(d) None of these

Q 24. Which of the following sectors will have the maximum FDI added in both the years?
(a) Fuel (b) Others
(c) Chemical (d) None of these

Q 25. Which of the following sectors will have the minimum FDI added in both the years?
(a) Fuel (b) Others
(c) Chemical (d) Textile

Table

Direction for questions 1 to 4: Refer to the following table and solve the questions based on it.

The following table gives the exports of top six industries (in value terms) from India in 2007:

	Exports (Rs crore)	Growth (in %)
Glass industry	1093	11.7
Coal Industry	1080	7
Ruby Industry	913	–8.7
Nickel Industry	663	7.1
Software Industry	605	–12.3
Diamond Industry	505	–1.5

The overall exports from India increased by 20% to Rs 19,500 crore in 2007.

Q 1. What is the share of the Glass industry in the total exports for 2007?
(a) 2.5%
(b) 4%
(c) 5.6%
(d) 7%

Q 2. For how many of the given six industries has the share in exports increased in 2007?
(a) 0
(b) 1
(c) 2
(d) 3

Q 3. Which of the following has the maximum share among the given industries in 2006?
(a) Glass Industry
(b) Ruby Industry
(c) Software Industry
(d) Coal Industry

Q 4. What is the percentage increase in the exports of the given six industries?
(a) 0.1%
(b) 0.5%
(c) 1.05%
(d) 1.75%

Direction for questions 5 to 9: Refer to the following table and solve the questions based on it.

The following table gives the number of vehicles (in '000s) of different models and colours sold in two cities—Patna and Lucknow–in a given year.

Type	Patna					Lucknow				
	Colour					Colour				
	Black	Red	Blue	White	Silver	Black	Red	Blue	White	Silver
A	40	25	55	75	15	45	32	40	60	20
B	20	35	60	80	20	30	37	39	81	35
C	35	30	50	90	35	40	42	41	6	37
D	45	40	45	85	40	35	39	37	90	42
E	50	35	35	60	30	50	44	43	77	22
F	55	42	40	65	52	47	34	45	87	17

Q 5. For which of the following models, difference between white—coloured vehicles sold in the two cities is minimum?
(a) A
(b) C
(c) D
(d) None of these

Q 6. The total number of blue—coloured vehicles of model E and D sold in Lucknow is exactly equal to the number of white-coloured vehicles of which model in Patna?
(a) B
(b) F
(c) C
(d) A

Q 7. What is the difference between the number of blue-coloured vehicles of model C sold in Patna and the number of red coloured vehicles of model F sold in Lucknow?
(a) 8,000
(b) 10,000
(c) 12,000
(d) None of these

Q 8. The total number of silver-coloured vehicles sold in lucknow is approximately what percentage of that in Patna?
(a) 130%
(b) 140%
(c) 90%
(d) 100%

Q 9. In Patna the number of vehicles sold was the maximum for which of the colour–model combinations?
(a) white-C
(b) blue-B
(c) silver-B
(d) white-D

Direction for questions 10 to 14: Refer to the following table and solve the questions based on it.

The table given below gives the marks obtained by six students in six subjects:

Student	History (150)	Science (200)	Geography (150)	English (200)	Hindi (100)	Math (200)
A	75	110	90	140	75	170
B	105	130	75	130	80	140
C	95	105	80	150	90	160
D	85	115	95	125	65	135
E	115	135	110	145	70	125
F	120	160	96	110	55	145

Marks in bracket are Total Marks.

Q 10. What is the difference between the percentage marks obtained by student B in History and the percentage marks obtained by student C in Hindi?
(a) 25 (b) 20
(c) 35 (d) 30

Q 11. The marks obtained by student A in Math are how many times the percentage marks obtained by student F in Science?
(a) 2.5
(b) 4.125
(c) 1.125
(d) 2.125

Q 12. What is the difference between the percentage marks obtained by student C in English and the average percentage marks of all the six subjects?
(a) 82
(b) 38
(c) 7
(d) 14

Q 13. In how many of the given subjects has student D got more than seventy percent marks?
(a) none
(b) one
(c) two
(d) three

Q 14. Approximately, what is the average percentage marks obtained by the six students in English?
(a) 67%
(b) 72%
(c) 80%
(d) Data inadequate

Direction for questions 15 to 18: Refer the following table and solve the questions based on it.

The following table gives the number of students from different locations who appeared and passed in the Xth standard examination conducted by CBSE over the years:

Year	Rural		Semi-urban		State Capitals		Metro-politans	
	Appeared	Passed	Appeared	Passed	Appeared	Passed	Appeared	Passed
1990	1651	209	7897	2511	5057	1464	9537	3212
1991	1832	314	8561	2932	7163	3242	10152	4015
1992	2154	935	8133	2466	8251	3151	9697	3033
1993	5035	1792	9436	3525	8526	3627	11245	5157
1994	4911	1656	9782	4012	9013	4315	12512	6323
1995	5626	2391	9965	4266	1729	4523	13625	6411

Q 15. For the students from which of the following locations was there a continuous increase in both the appeared and passed member of candidates?
(a) semi-urban
(b) state capital
(c) state capital and rural
(d) None of these

Q 16. In which of the following years was the percentage passed to appeared candidates from the semi-urban area the least?
(a) 1991 (b) 1993
(c) 1990 (d) 1992

Q 17. By what approximate value did the percentage drop in the number of semi-urban candidates who appeared from 1991 to 1992?
(a) 5 (b) 10
(c) 15 (d) 8
(e) 12

Q 18. The total number of candidate who passed from the rural locations in 1993 and the semi-urban locations in 1990 was exactly equal to the total number of candidates who passed from the state capitals in which of the following years?
(a) 1990
(b) 1993
(c) 1994
(d) None of these

Direction for questions 19 to 21: Refer to the following table and solve the questions based on it.

The following table gives the average marks obtained by 20 boys and 20 girls in five subjects from five different schools P, Q, R, S and T:

Subject	Maximum Marks	P		Q		R		S		T	
		B	G	B	G	B	G	B	G	B	G
English	200	85	90	80	75	100	110	65	60	105	110
History	100	40	55	45	50	50	55	40	45	65	60
Geography	100	50	40	40	45	60	55	50	55	60	65
Math	200	120	110	95	85	135	130	75	80	130	135
Science	200	105	125	110	120	125	115	85	90	140	135

In the above table, B = Boys and G = Girls

Q 19. What were the total marks obtained by the boys in History from school Q?
(a) 900 (b) 1000
(c) 800 (d) 1300

Q 20. In which of the following subjects did the girls have the highest average percentage marks in all the schools?
(a) Science
(b) Geography
(c) English
(d) History

Q 21. What is the average mark obtained by all the students of school P taking all the subjects into account?
(a) 82
(b) 84
(c) 80
(d) Cannot be determined

Direction for questions 22 to 28: Refer to the following table and solve the questions based on it.

The following table shows the domestic sales of cars of five manufacturers from 1995 to 2000. (All the figures are in thousands)

Manufacturer	1995	1996	1997	1998	1999	2000
A	440	480	470	500	520	510
B	400	410	415	415	420	430
C	380	390	390	400	420	495
D	360	380	400	415	440	500
E	480	440	440	420	425	435

Q 22. Considering the period given, the domestic sales of which manufacturer are the highest?
(a) A (b) B
(c) C (d) D

Q 23. During 1996, what is the approximate share of domestic sales of cars of the manufacturer B?
(a) 10.5%
(b) 25.5%
(c) 15.5%
(d) 19.5%

Q 24. During 2000, the sales of which manufacturer has shown the maximum percentage increase over the previous year?
(a) A (b) B
(c) C (d) D

Q 25. With respect to which of the following combinations, is the sales of cars the highest over the given period?
(a) D, 2000
(b) A, 1998
(c) A, 2000
(d) A, 1999

Q 26. Which of the following manufacturers, has a consistent increase across the years given?
(a) A (b) B
(c) C (d) D

Q 27. How many of the manufacturers have never shown a decline in the number of cars sold in any particular year over the previous year?
(a) 1 (b) 2
(c) 3 (d) 4

Q 28. How many times have the number of cars sold been more than 450 for any manufacturer in any year more than once? (A manufacturer can achieve this feat)
(a) 6 (b) 7
(c) 8 (d) 9

Direction for questions 29 and 30: Refer to the following table.

The following table captures the comparative performance of LIC and HDFC in the housing sector in the last four years. (All figures are in Rs crores).

Attribute	1994-1995		1993-1994		1992-1993		1991-1992	
	LIC	HDFC	LIC	HDFC	LIC	HDFC	LIC	HDFC
Loan Sanctions	618.0	1495.0	564.0	1025	607	859	511	758
Dues	24.2	29.6	20.0	27.8	16.1	25.17	5.32	19.57
Total Loans	1672.0	3747.0	1283.0	3071	852	2561	401	2125
Default Percentage	1.45	0.79	1.56	0.91	1.89	.98	1.32	0.93
Total Revenue	251.0	780.0	175.0	608	109	473	44.4	371
Net Profit	41.4	146.1	21.53	105	13.5	55.55	6.71	45.6

Q 29. For how many years are the LIC loan sanctions 70% or more of HDFC's loan sanctions?

(a) 0 (b) 1
(c) 2 (d) 3

Q 30. The ratio of total revenue to the loan sanctions is the least for LIC in which of the following years?

(a) 1992-1993 (b) 1993-1994
(c) 1994-1995 (d) 1991-1992

ANSWERS

Bar Chart

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(a)	(c)	(d)	(b)	(b)	(d)	(d)	(d)	(d)	(a)	(a)	(a)	(c)	(a)	(b)	(c)	(c)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35					
Answer	(d)	(d)	(b)	(c)	(c)	(d)	(a)	(c)	(c)	(c)	(d)	(c)	(b)	(b)	(b)					

Pie Chart

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(d)	(c)	(b)	(a)	(b)	(d)	(c)	(a)	(d)	(d)	(b)	(c)	(b)	(a)	(a)	(a)	(a)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(c)	(b)	(b)	(a)	(a)	(d)	(d)	(d)	(a)	(a)										

■■■ Line Chart

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(a)	(c)	(b)	(b)	(c)	(a)	(d)	(b)	(c)	(b)	(d)	(d)	(d)	(d)	(d)	(d)	(c)	(c)	(d)
Q. No.	21	22	23	24	25															
Answer	(c)	(b)	(d)	(b)	(d)															

■■■ Table

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(a)	(d)	(c)	(d)	(a)	(d)	(c)	(a)	(b)	(d)	(c)	(a)	(a)	(d)	(d)	(a)	(d)	(a)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(d)	(a)	(d)	(c)	(d)	(d)	(c)	(c)	(b)	(d)										

HINTS AND SOLUTIONS

Bar Chart

[illegible]

1. This question can be solved by visual inspection.
2. To find the average monthly expenditure during the year 2000, the total income for the year has to be known. In the chart, the income of only seven months is known. Therefore it cannot be determined.
3. Value (April 2000) = 316

   Value (April 1999) = 312

   Approximate % age increase = 1.2%
4. The same increasing trend is seen every year during the period March-April.
5. In 1998, the period during which the expenditure was maximum was June-July.

   In 1999, the period of maximum expenditure was April-May.

   So, option (a) and (b) are incorrect. Option (c) is obviously incorrect.

   Hence option (d) has to be true.
6. Only April satisfies these conditions.
7. Visual inspection we can see that option (b) is correct. It should also be noted that we have to find out of the options given.

[illegible]

8. The average will decrease by 6.11 billion dollars. Hence, option (d) is the correct answer.
9. Required ratio = 4 : 5
10. Cannot be determined because the data is given only for the top ten exporters.
11. Required difference = (99.5 – 34.2) × 43 × 10^9 = Rs 28.079 × 10^{11} (1 billion = 10^9)
12. The average is 77.6. Now it can be seen that only three countries have more than this.
13. There is no difference between this and the last question. Understand that the average is calculated for the top ten countries only, and the only countries which can have exports higher than the average exports can be the countries which are in the list of the top ten exporter countries.

[illegible]

15. The difference between sales of 1993 and 1995 = 1 lacs or 100,000
16. Sum of sales of 1991 and 1992 = 6 + 10 = 16 lacs

    Required Answer $= \frac{16}{13} \times 100 = 123\%$
17. Percentage increase in production from 1993 to 1994

    $\frac{16-12}{12} \times 100 = 33.33\%$

18. Percentage of sales to production is maximum in the year when the difference between them is minimum.

19. Percentage drop in sales from 1992 to 1993

$$=\frac{1}{10}\times 100=10\%$$

20. By visual inspection we can see that the correct answer is option (c).

[illegible]

22. Production of E type bikes in 2006

= (100 – 80)% of 4,50,000

= 20% of 4,50,000 = 90,000

And in 2007 = 10% of 5,20,000 = 52,000

Total production = 90,000 + 52,000 = 1,42,000

So, the number of bikes

= 15 % of 1,42,000 = 21,300

23. Production of A type bikes in 2006 = 67,500

Production of A type bikes in 2007 = 52,000

$$\text{Required \%}=\frac{67,500+52,200}{9,70,000}\times 100=12.3\%$$

≅ Closest to option (b)

24. By visual inspection we can say that option (C) is correct answer

25. Percentage production of B type bikes in 2007 = that in 2006 (given) = (40 – 15) = 25% of 5,20,000 = 1,30,000.

[illegible]

26. $$\text{Required \%}=\frac{540\text{-}130}{540}\times 100=75\%$$

27. Percentage share of foreign tourists visiting HP

$$\frac{34.5}{1600}\times 100=2\%$$

28. Ratio = 535 : 15 = 107 : 3

[illegible]

31. $$\text{Required \% age}==\frac{450}{250}\times 100=180\%$$

Pie Chart

[illegible]

1. Required Angle = (16/100 × 360) = 57.6°

4. If the editing charges are Rs 18, royalty is Rs 10. On Rs 18, it is less by 8. On Rs 100, it is less by (8/18 × 100)% = 44.44%

5. Let the total expenditure be Rs x. Then, 8 : 100 = 36960 : x. So x = Rs 462000. Therefore Cost Price of 5500 copies of the book = Rs 4,62,000. Cost Price of each copy = Rs (4,62,000/500) = Rs 84. So, marked price = 140% of Rs 84 = Rs 117.60.

[illegible]

6. There are two successive increases—First is the total increase of 60% approx.($ 3.7 bn in 2000 from $ 2.3 billion in 1995) and second is the percentage share of A by 20% approx. Hence total percentage increase = 92%.

7. Market size in 2000 = $ 3.7 billion

Market size in 1995 = $ 2.3 billion

Therefore, growth rate

$= [(3.7/2.3)^{1/5} – 1] \times 100 = 10\ \%$

Hence, option (b) is the correct answer.

[$3.7/2.3= 1.6$, now to find $(1.6)^{1/5}$, $1.1^3= 1.331$ and $1.1^2 = 1.21$ and $1.3 \times 1.2 = 1.6$

Thus, $1.6^{1/5}= 1.1$]

8. Since we do not know what part of market share of B. C and E has been eaten up by F, so cannot be determined.

9. Sales value for A + B + C in 2000 = (13 + 14 + 19)% of 3.7 billion = 46 % × 3.7 = 1.702 billion

Sales value for A + B + C in 1995 = (16 +17 + 12) % of 2.3 billion = 1.035 billion.

Hence increase in sales value = 1703 – 1035 = $ 668 million.

Hence, option (c) is the correct answer.

[illegible]

11. Since the total consumption for the given years is not known, we cannot determine it.

12. Since the total consumption for the given years is not known, we cannot determine it.

13. Total consumption has increased by 20%. So, any sector that shows an increase of even 25% will give us a net increase of 50%. There are only two sectors–agriculture and domestic. Therefore, option (b) is the correct answer.

14. There are two increases—one is the increase in the share of agriculture (66.66%) and the other is an increase of 50% in total. Net increase = 150% Therefore, option (c) is the correct answer.

15. By visual inspection. We can see that the correctansever is option (b)

16. There is an increase in the share of agriculture (66.66%), to make the agricultural consumption double, total consumption is to be increased by 20%.

17. The electricity consumption of the ‘others’ category has remained constant over the period. It means that the total consumption of 1980-1981 = Total consumption of 1993-1994. So, the only increase = [(18—11)/11] × 100 = 63.64%

[illegible]

18. If the percentage increase in the expenditure of both the families is the same then the ratio will be the same.

19. The total consumption has become 3 times more keeping the expenses on education the same. Hence the percentage consumption on education will become 1/3rd of the present.

21. By visual inspection we can see that option (c) is the correct answer.

22. It should be understood that the final percentage of expenditure will always be in between the percentage of family A and family B (It is true of any mixture that the percentage composition of the mixture will be always in between the percentage compositions of the components).

23. By visual inspection we can see that the correct answer is option (b).

[illegible]

24. By visual inspection we can see that option (a) is the correct answer.

25. By visual inspection we can see that option (a) is the correct answer.

26. Understand the logic that “If a country is under military rule, it will spend maximum on defence and minimum on either health or education” does not mean that if a country is spending maximum on defence and minimum on either health or education, then it will under military rule.
[For more–See Logical Links in Part 1 of this book]

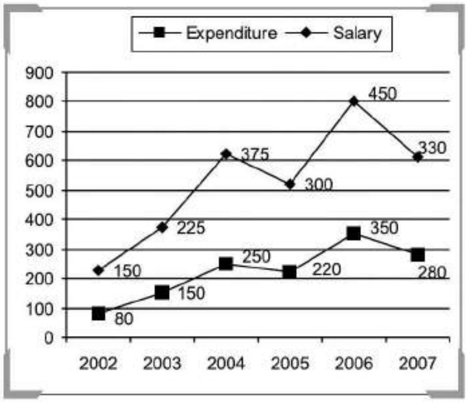
27. By visual inspection. we can see that option (d) is the correct anser.

28. Since individual expenditures of the countries are not given, we cannot determine it.

Line Chart

[illegible]

For the sake of convenience, use the following chart (with the same data) to find out the solution:



Now all the questions can be done with ease.

[illegible]

8. Income-expenditures of company A and B cannot be correlated. Hence cannot be determined.

9. Expenditure of company A in 2006 =

Income in 2006 of (A) × (100/160) = Income in 2006 of (A) × 5/8

Expenditure of Company B in 2007 =

Income in 2007 of (B) (100/170) = 10/17 of Income in 2007 of B

Now, [Expenditure in 2006 of (A)/Income in 2007 of (B)] = (5/8) ÷ (10/17) [Since Income in 2006 of (A) = Income in 2007 of (B)]

$$= \frac{5}{8} \times \frac{17}{10} = \frac{17}{16}$$

10. Ratio A : B is greater than 1 in only 2003 and 2007. It is 1.33 in 2003 and 1.1 in 2007.

11. Percentage profits are the same for two years, hence if expenditure increases by 20% the income should also increase by 20%. Hence, the required ratio = 120/100 = 1.2

[illegible]

13. This never happens. Understand that if export is more than import, then the ratio will be more than 1. If export is equal to import, then the ratio will be equal to 1 and if export is less than import, then the ratio will be less than 1.

14 . The ratio of one to the other should be reciprocal.

15. Despite the ratio being on a decline for country A, we cannot comment about the value of exports. it might be a possibility that imports have increased keeping exports constant. Similarly both the other options (b) and (c) can be refuted on the same grounds.

16. Since only the ratio is given, we cannot comment on the values.

17. Since only the ratio is given, we cannot command on the values.

[illegible]

Following is the table pertaining to the data set given:

	CNN	NDTV	CNBC	Zee
SAD	44	52	58	52
Congress	55	55	48	48
Others	18	10	11	17

[illegible]

Following is the table pertaining to the data given:

	Chemical	Engineering	White Goods	Services	Electronics	Food	Phamacentional	Fuel	Textiles	Telecommunication	Others
2006	17	15	8	16	9	9	4	0	0	0	22
2007	9	4	0	12	0	5	0	27	4	8	24

Table

[illegible]

	Export (Rs Crore)	
	2006	**2007**
Glass Industry	978	1093
Coal Industry	1009	1080
Ruby Industry	1000	913
Nickel Industry	619	663
Softwares Industry	690	605
Diamond Industry	513	505
Total	16250	19500

1. The share of Glass in the total exports for 2007 = 1093/19500 × 100 = 5.6%.

2. Overall exports have increased by 20% while none of the given six industries have grown by 20%. So, we can conclude that the market shares for all the six industries will decrease in 2003. Hence option (a) is the correct answer.

3. Coal has the maximum exports in 2006. Hence, its market share is the maximum.

4. Exports for the given six industries in 2006 = 4809 crore

   Exports for the given six industries in 2007 = 4859 crore

   Therefore, percentage increase in the exports = [(4859 – 4809)/4809] × 100 = 5000/4809 =1.05%.

[illegible]

5. The difference between the white-coloured cars sold is the minimum in the B type model.
6. Blue (E + D) = 43 + 37 = 80 = White (B)
7. Required difference = (50 – 34) × 1000 = 16,000
8. Required Percentage = 173/192 × 100= 90%
9. White-c = 90,000

[illegible]

10. The difference in percentage marks of B (History) and C (Hindi) = 90 – 70 = 20

11. The percentage of marks obtained by F in Science = 160/200 × 100 = 80%

    Required Value = 170/80 = 2.125

12 Average percentage of marks of C = 680/1000 × 100 = 68%

Percentage of marks of C in English = 150/200 × 100= 75%

Required difference = 75 – 68 = 7

14. Average percentage of marks in English

$$=\left(\frac{140+130+150+125+145+110}{6}\right)\times\frac{100}{200}$$

= 67%

[illegible]

15. By visual inspection.

[illegible]

19. Average marks obtained by 20 boys in History from school Q = 45

    Therefore, Total marks = 20 × 45 = 900

20. From visual inspection it is clear that Science is the desired subject.

21. The table gives the data pertaining to only 40 students of the school and it is not given if this is the total number of students in the school. Therefore it cannot be determined

[illegible]

22. It is a simple calculation. Alternatively, it can be observed that A has always been more than the others except but for one year.

23. Required share
= (410/480 + 410 + 390 + 380 + 440) × 100 = 19.5%

24. It can be observed that the difference is the highest in case of manufacturer C. We can further observe that the largest percentage growth would be for C, as the base is the smallest.

26 By visual inspection we can say that option (d) is the correct answer.

27. By visual inspection we can say that option (c) is the correct answer. Manufacturers are B, C, D.

28. By visual inspection we can say that option (d) is the correct answer.

[illegible]

29. It happens only in 1992–1993

2

CHAPTER

Developing the Skills

LEARNING OBJECTIVES

- **Different constituents of Data Interpretation**
- **Various skill-sets required and the methods to acquire the same**
- **Calculations required**
- **How to excel in these calculations and the calculations specific to a particular data-set**
- **How to interpret the data**
- **Connecting diferent data-sets**

CONSTITUENTS OF DATA INTERPRETATION (DI)

Data Interpretation is primarily aimed at assessing the candidate's ability to analyse the given data, arrive at meaningful conclusions and to take appropriate decisions. In many ways, this section truly reflects a business situation; where one is required to take a decision on the basis of available information and future trends.

In any DI question, students will be required to go through the data-set and make necessary relationships between "what is given?" and "what is asked?" These questions simulate the kinds of detailed analyses of relationships that management students must perform in solving problems under strict time restrictions.

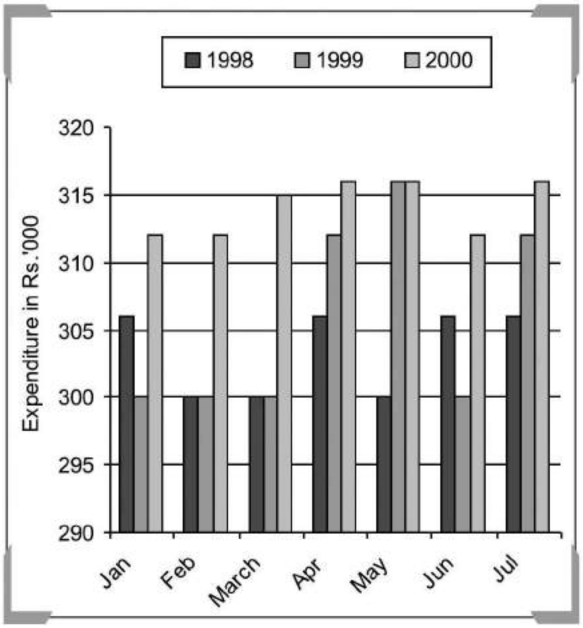
Each DI question contains three separate parts —

(A) Data-set,
(B) Explanations and restrictions given for the data and
(C) Question/s.

Data-Set

The data pertaining to the question can be represented in various formats—pie chart, bar chart, line chart etc. The data-set introduces a sample or a report of some activity, done in the past, present or the future.

For example,



In the above data-set, we see the expenses of a family for three years from January to July.

The same data-set can be represented by other formats like the line chart etc. keeping the essence of the data, the same.

We will learn more ways of presenting the data and how they can be inter-related, ahead in this chapter.

Explanations and Restrictions

These should be seen as the guiding principles or the helping factors associated with the data-set. Explanations help us in knowing and understanding the data better; on the other hand, restrictions narrow down our focus to what is being asked and what is to be extracted.

For example, the following are the final results of standard X. The students were tested in English, Hindi, Science, Math, Social Science, Drawing, Craft, PT and Scout-Guide. For a better presentation of the mark sheet, the scores in the first four subjects are consolidated into A1, the next three into A2 and last three into A3. The final score is the total of these three scores A1, A2 and A3. On the basis of the final score the students get their ranks. The table below gives the scores of the top eleven students of the class.

Name	Grade	Final Score	A1	A2	A3	Drawing	PT	Social service
Anana	A	725	282	217	226	80	81	69
Pankaj	B	864	310	87	267	85	86	92
Vamshi	C	812	300	290	222	70	91	74
Ram	C	805	270	262	273	65	79	97
Shann	D	900	391	292	217	96	86	81
Saket	D	876	301	297	278	65	87	93
Mini	C	825	296	296	233	50	89	91
Rashi Singh	E	905	320	290	295	72	99	99
Parul	B	895	310	290	295	72	99	98
Rakesh Rathi	B	870	276	296	298	63	99	99
Gauri	C	888	361	270	257	82	78	92

Without the given explanation, we would not have any idea about how to arrive at A1/A2/A3 and how the grades are being awarded.

Example 1 And now finally, the tools to check how much you have been able to interpret the data and relate it to the explanations/restrictions given.

Here, are a few questions based upon the data-set given above:

1. How many students have got grade A and more than 250 marks in all the group scores (A1, A2 and A3)?
   (a) 0
   (b) 1
   (c) 2
   (d) None of these
2. In the drawing subject, what is the difference between the total marks of grade A students and grade C students?
   (a) 50
   (b) 52
   (c) 48
   (d) None of these

We will see the solution of the questions given above in the next part. Let us now go through the skill-set required to excel in the Data Interpretation section.

SKILL-SET REQUIRED

Till now, we have understood that DI questions test the following:

- Ability to understand the data and retain that in mind.
- Exposure to various types of data presentation formats
- Understanding of how restrictions limit and order behaviour
- The 'logic' inside a data-set and the algorithm to extract it to arrive at a solution

It should be noted, that even if a student has not gone through a formal training or practice of DI for any aptitude exam including CAT, can have an understanding of data and exposure to the various formats of presenting the data, just by virtue of going through any standard newspaper regularly.

However, while discussing the skill-set required, we will assume that the student is at the basic level and has just started going through DI.

Skill 1—Knowing the Terminology

There can be various economic or non-economic terms like CAGR, inflation etc., involved in the data-set, and it is a prerequisite to have a good understanding of these terms.

Example 2 If the production of wheat in 2004 was 100 million tonnes and in 2007 it was 133 million tonnes, then find the (A) Percentage Growth, (b) SAGR, (c) CAGR.

Solution

(A) Percentage Growth = [Growth/Initial value] × 100 = 33%

(B) SAGR (Simple Annual Growth Rate)–It is the average growth rate over the given period. It is calculated by dividing the growth percentage by the number of years involved.
In the above case, SAGR = 33/3 = 11%

(C) CAGR (Compounded Annual Growth Rate)–It is the year to year growth rate over the given period. To calculate this, we take the help of the fundamentals of compound interest. We take the final value as the amount after the given period and the initial value as the principal.
In the above case,

$$133 = 100\left(1 + \frac{r}{100}\right)^3$$

Hence, r = 10% (approx.)

Skill 2—Knowing the Different Formats of Presenting the Data

Having discussed earlier, the significance of organized data, we can automatically conclude that the same data when presented in an unorganized way makes difficult reading and is not of much use. The same data can be presented differently. In one format, it might make easy reading; in another format, it might make difficult reading.

Example 3

	Location 1	Location 2	Location 3	Location 4	Location 5
2004	112	128	98	132	102
2005	118	147	120	120	108
2006	120	144	126	124	113

A company is selling its products at five different locations and the sales value (Rs thousands) of different locations for three consecutive years is given.

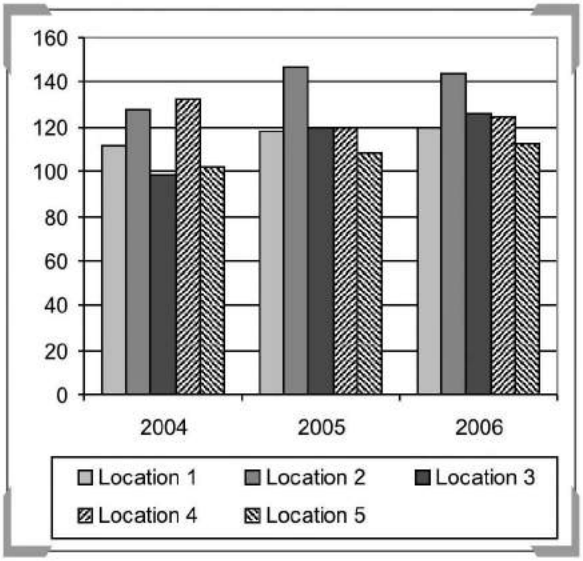
Question

1. Which of the following locations has the maximum sum of sales value for the three years?
   (a) Location 1 (b) Location 2
   (c) Location 3 (d) Location 4

Solution

This question simply requires the sum of the sales value of each of the locations for three years. Obviously, location 2 is the answer.

However, if we convert the same data into a bar chart, then it can be seen by simple observation that location 2 is the answer.



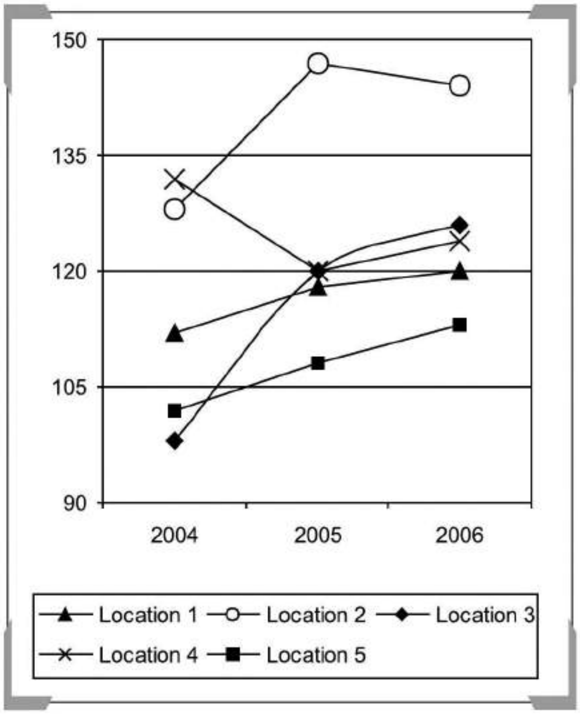
Since the bar of location 2 is above the other two for two years and in 2004, it is just a bit lower than one of the bars; we can simply say that location 2 produces the maximum combined sales value for the three years.

Example 4 Which of the following locations has the minimum sum of sales value for the three years?

(a) Location 1
(b) Location 2
(c) Location 3
(d) Location 5

Solution It can be seen that the bar of location 5 is lower than all the bars for two years and once it is equal to the bar of location 3; hence, location 5 produces the minimum combined sales value for the three years.

If we convert the same data into a line chart, it becomes more obvious than the bar chart that location 2 and location 5 give the maximum and minimum sales value for the given period.



Example 5 Which of the following locations has seen the maximum percentage increase in the sales value in 2005 over 2004?

Solution If we try to solve this question with the help of bar chart, it becomes a bit difficult without having the actual values. So, we are left with either the line chart or the given table.

In case of a table, this question can definitely be solved by calculation and observation. But if we see the same situation in case of a line chart, we can simply rule out location 1/location 4/location 5 as the answer. Now, from the remaining two locations 2 and 3, despite the slope being almost the same, the base in location 3 is very low with respect to the base in location 2. Hence, location 3 will have more percentage increase.

We will learn more about the different formats of presenting data, ahead in this chapter.

Hence, we can see that with different forms of data representation, one can gather different conclusions just by visual inspection. However, the speed with which this conclusion can be drawn depends heavily on the kind of chart given.

Skill 3—Knowing What is to be Found Out

In my experience of dealing with students from varied backgrounds, many a times I have seen that students are not able to solve a question because they have got very impressed with the data. And as a result, they lose track of what is to be fond out.

Remember, never ever get 'emotionally attached' to a question.

As an examinee, you are not supposed to get into the technicalities of the data. Rather, you should focus upon what is being asked in the question, and how to extract information from the data-set relevant to the given question.

However, during practice, one needs to focus not just on solving the problems on hand, but also on building an algorithm to solve similar problems whenever they occur.

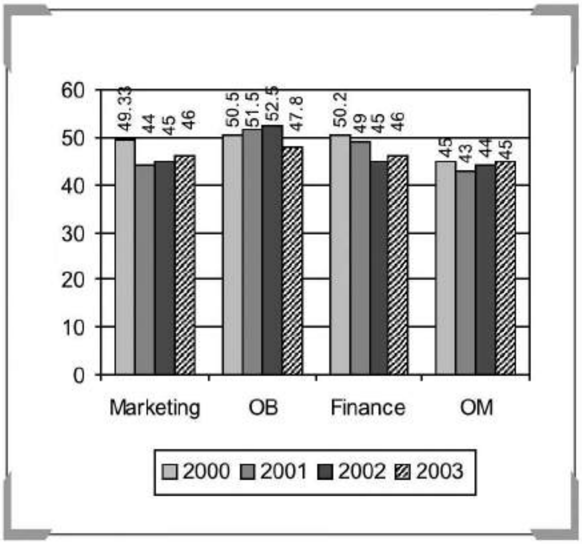
Cracking the questions require a very systematic approach. The primary focus should be on understanding the question. It would be worthwhile to read the question patiently and carefully. At this juncture, efforts towards building your vocabulary and improving your reading skills might help you. One must pay attention to otherwise innocuous words and phrases such as "at most" or "at least", "cannot be less than" , "cannot be more than" etc.

Till now we have discussed data, the significnace of organized data and the skill set required to solve the questions.

Let us see a few questions from an actual CAT DI paper:

CAT 2005

A management institute was established on 1 January 2000 with 3, 4, 5, and 6 faculty members in the Marketing, Organizational Behaviour (OB), Finance, and Operations Management (OM) areas respectively, to start with. No faculty member retired or joined the institute in the first three months of the year 2000. In the next four years, the institute recruited one faculty member in each of the four areas. All these new faculty members, who joined the institute subsequently over the years, were 25 years old at the time of their joining the institute. All of them joined the institute on April 1. During these four years, one of the faculty members retired at the age of 60. The following diagram gives the area-wise average age (in terms of number of completed years) of faculty members as on April 1 of 2000, 2001, 2002, and 2003.



Example 6 From which area did the faculty member retire?

(a) Finance
(b) Marketing
(c) OB
(d) OM

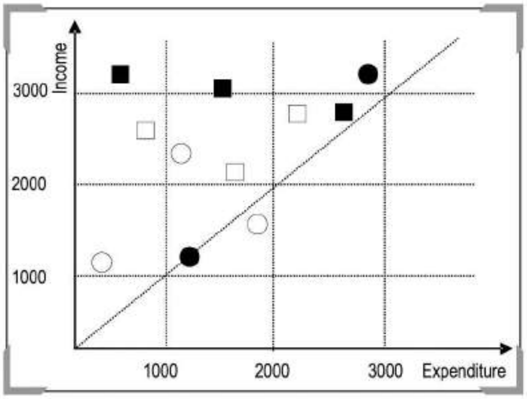
Solution To solve this question, following things were required:

- Understanding the passage given at the beginning of the DI set.
- Understanding the above bar chart.
- Understanding the question and the process of how to inter-relate the data.

Of course we can calculate the sum of ages of all the faculty members in all the areas and for all the years. There were 3 faculty members in Marketing in 2000 (inception of the institute), hence, the sum of completed years of these three faculty members in 2000 = 49.33 × 3 = 148 years. But the main problem lying ahead is, for how many years and areas can we do the same thing? As this will involve a good amount of calculation, it will require a good amount of time and patience.

CAT 2004

The data points in the figure below represent the monthly income and expenditure data of individual members of the Ahuja family (■), the Bose family (□), the Coomer family (○), and the Dubey family (●). For these questions, saving is defined as



Example 7 Which family has the lowest average income?

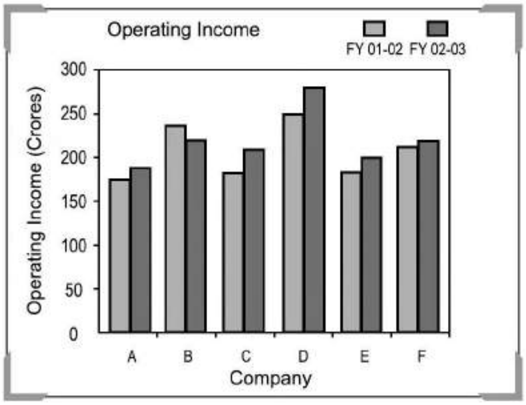
(a) Ahuja (b) Bose
(c) Coomar (d) Dubey

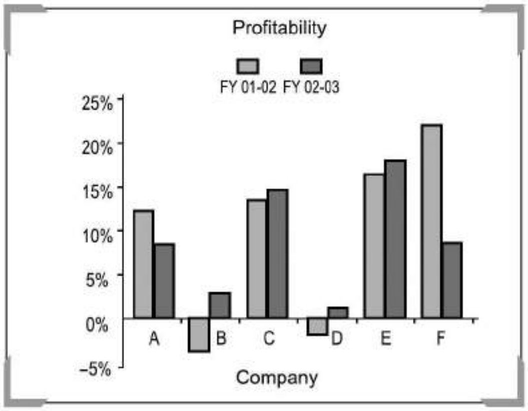
Solution To solve this question, following things are required—

- Understanding the data as given above.
- Understanding the question and the process of how to inter-relate the data.

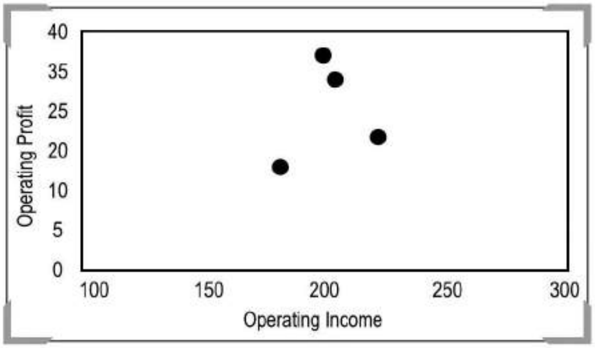
CAT 2003 (Re-Test)

The profitability of a company is defined as the ratio of its operating profit to its operating income, typically expressed in percentage. The following two charts show the operating income as well as the profitability of six companies in the Financial Years (FYs) 2001-2002 and 2002-2003.





The operating profits of four of these companies are plotted against their respective operating income figures for the FY 2002-2003, in the third chart given below.



Example 8 What is the approximate average operating profit, in FY 2001-2002, of the two companies excluded from the third chart?

(a) –7.5 crore	(b) 3.5 crore
(c) 25 crore	(d) cannot be determined

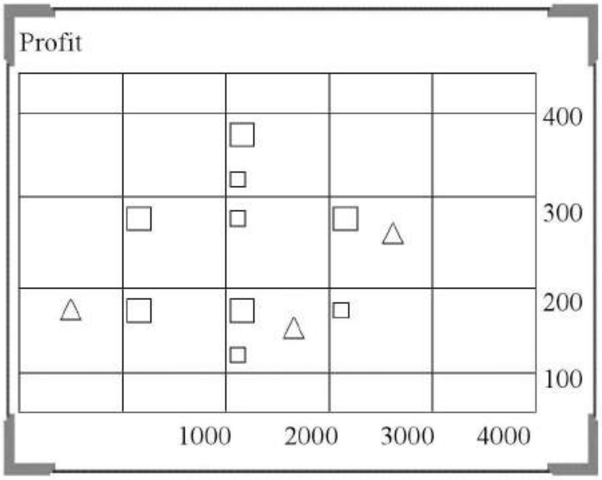
Solution To solve this question, following things are required—

- ■ Understanding of the definitions given above
- ■ Understanding of the above representations of the data-set

The biggest problem one would face while solving this question is how to inter-relate the data given.

CAT 2003 (Leaked)

Each point in the graph below shows the profit and turnover data for a company. Each company belongs to one of the three industries: textile, cement and steel.



Example 9 For how many companies does the profit exceed 10% of the turnover?

(a) 8	(b) 7
(c) 6	(d) 5

Solution To solve this question, following things are required:

- ■ Understanding of the above representations of the data-set.
- ■ Finding out which one is textile, cement and steel.

(For solution of all these questions refer to CAT papers given in this book.)

- ■ The above examples illustrate a few simple facts:
- ■ Data is an important tool for decision-making and drawing inferences.
- ■ However, having only the data does not serve the purpose; it should be presented in a proper format.
- ■ Despite the data being presented in a proper way, the most important thing is having the capability to interpret it.

CALCULATION TECHNIQUES

Despite the trend at CAT having shifted from easy-to-understand-data-with-difficult-calculations to more logical questions with lesser calculations, it is not advisable to sit for CAT without being reasonably good at quick calculations and approximations.

The importance of a good calculation speed can be understood with the following statistics:

We are taking a sample of a student who attempts 15 questions in DI at CAT. All the figures taken ahead are 'on an average' basis.

Number of calculations/questions

= 4–5 calculation

Hence, the total number of calculations done

= 60 – 75 calculations

If a calculation takes 6 seconds on an average, then time invested in doing calculations only

= 360 – 450 secs

= 6 – 7:30 minutes

Now if we double our calculation speed, we would have 3-4 minutes of extra time. What leverage can be taken from this 'generated time' depends totally upon the caliber of the student. Even if a student solves only one question than what he actually would have, and gets it right; the percentile in DI might differ by 1–2 percentile.

Following are the most frequent type of calculations done at CAT—

Addition

This is the most important calculation and takes the lion's share of the whole calculation section. Addition is also treated as the building block for all the other calculations like subtraction, multiplication etc.

Unfortunately, we don't have any scut for additions. The only thing that can be done is, practice.

I would suggest a 4 × 4 table for this practice:

	Column 1	Column 2	Column 3	Column 4
Row 1				
Row 2				
Row 3				
Row 4				

Fill in this table with two digit numbers and add all the columns and rows. To obtain a faster speed, it is advisable that both the digits are added simultaneously. For this, we need to train our mind to retain both the digits of a number simultaneously and then add it in one go with the two digits of the other number.

I would suggest, that rather than adding the unit digit first, add the tens place digit first; and while your brain is adding the tens place digit, your eyes should have captured the unit digit.

Sample table:

	Column 1	Column 2	Column 3	Column 4	
Row 1	36	42	67	98	243
Row 2	46	54	28	72	200
Row 3	58	45	82	89	274
Row 4	49	58	36	92	235
	189	199	213	351	

Rate your calculation speed:

Equal to or less than 15 seconds— Excellent

More than 15 seconds— Less than or equal to 30 seconds— Good

More than 30 seconds— Less than or equal to 60 seconds— Average

More than 60 seconds— A lot work is needed

The above given time includes time for writing the sum and it does not consider the time taken for creating the table and putting values inside it.

The idea is that if, 3 + 4 = 7 can be done without any effort (ask yourself after doing the addition if did you put any effort), then we could be in a position to do 79 + 68 = 147 also without any effort. And with this practice exercise, we want to achieve the same.

Subtraction

Once we attain a good speed in addition, subtraction too will become fast as a natural outcome.

Multiplication

Here, again addition is the key. However, it is advisable to know the basics of Vedic maths for multiplication.

Some of the methods are given below:

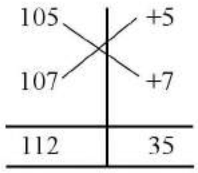
Base Method

In this method of multiplication, we use a number as a base, like 10, 50, 100 etc. We should try to assume the base that is closer to both the numbers.

Examples 10 105 × 107

Solution Both the numbers are close to 100, so let us assume 100 as the base. We will now find the deficit/surplus from the base.

Base = 100, Surplus = 5 and 7



Right part (after slash) is the product of the surplus. Since base = 100 and surplus are 5 and 7, so product would be 5 × 7 = 35.

Left part (before slash) could be either number plus surplus of the other multiplicand. Hence left part would be either (105 + 7) or (107 + 5) = 112 (both will always be the same) i.e., 112.

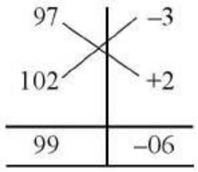
Left part would be the equivalent to Number × 100. In this case, 112 × 100 = 11200

Now we will add both the right part and the left part = 11200 + 35 = 11235

Hence, the result of the multiplication would be 11235.

Example 11 97 × 102

Solution



97 × 102

Base = 100, Deficit = 97 – 100 = – 3,

Surplus = 102 – 100 = 2

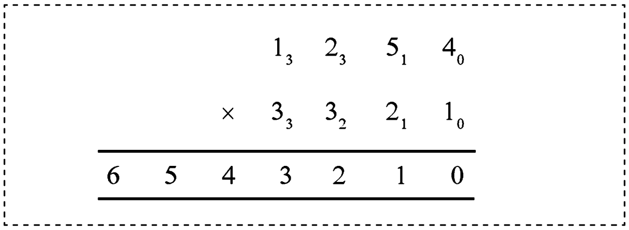
Right part will now be (–3) x 2 i.e., –06. To take care of the negative we will borrow 1 from the left part, which is equivalent to borrowing 100 (because we are borrowing from the hundred digits of the left part). Thus this part will be 100 – 06 = 94.

So, the answer = 9894

Place value Method

In this method of multiplication, every digit is assigned a place value and we do the multiplication by equating the place values of multiplicands with the place value of the product.

Examples 14

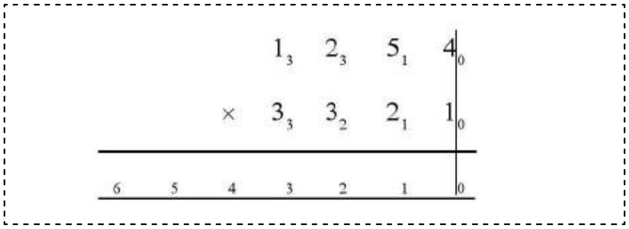

1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

Solution Conventionally, the unit digit is assigned a place value of 0, tens place digit is assigned a place value of 1, hundreds place digit is assigned a place value of 2, thousands place digits is assigned a place value of 4 and so on.

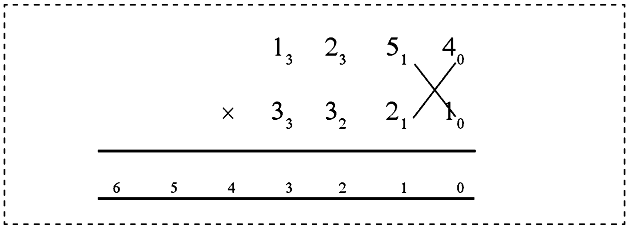
Now, this multiplication is a two-step process:

Step 1–Add the place values of digits of the numbers given (1254 × 3321) to obtain the place value of the digits of the product.

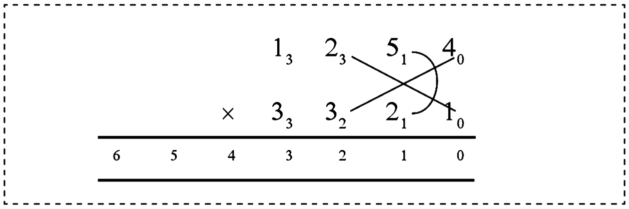
For example, using the place values of the multiplicands i.e., using 0, 1, 2 and 3 of the number 1254 and the same place values 0, 1, 2 and 3 of the other multiplicand 3321, we can get 0 place value in the product in just one way i.e., adding 0 and 0.


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

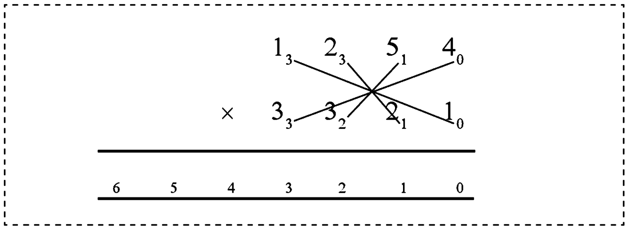
Place value 1 in the product can be obtained in two ways:


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

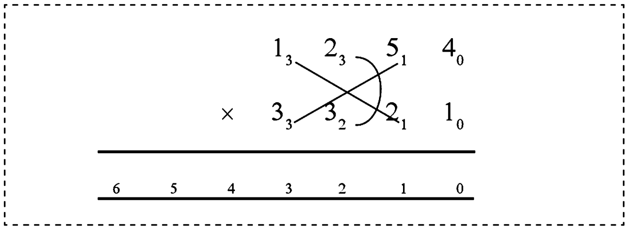
Place value 2 can be obtained in three ways:


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

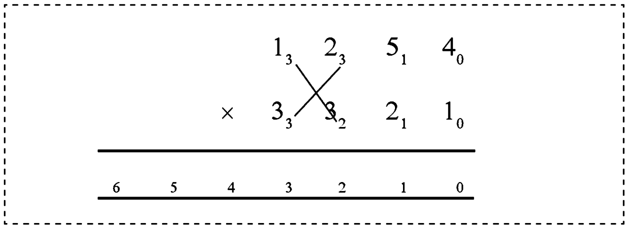
Place value 3 can be obtained in four ways:


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

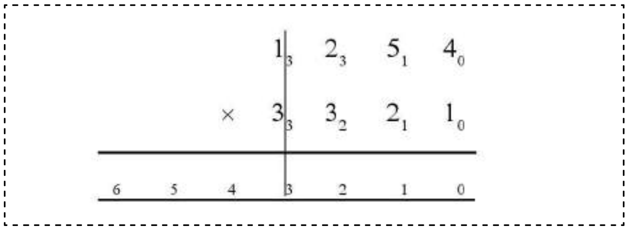
Place value 4 can be obtained in three ways:


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

Place value 5 can be obtained in two ways:

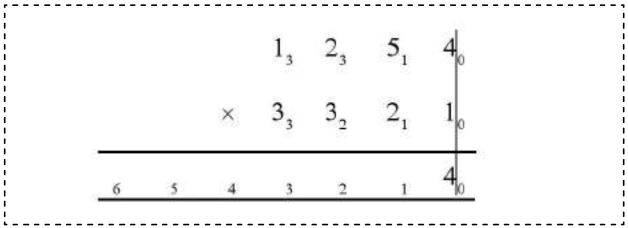

1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

Place value 6 can be obtained in one way:

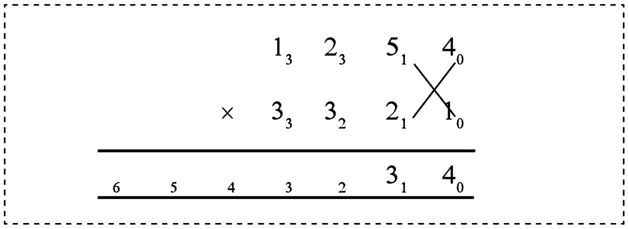

1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 0

And this is the maximum place value that can be obtained.

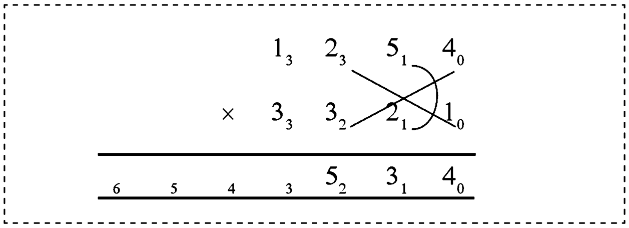
Step 2–Now multiply the corresponding numbers one by one.


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 1 4_0

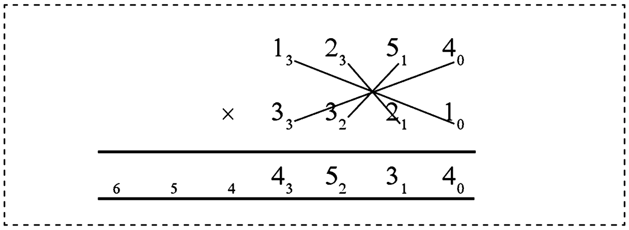
↓


1_3 2_3 5_1 4_0
× 3_3 3_2 2_1 1_0
6 5 4 3 2 3_1 4_0

↓



↓

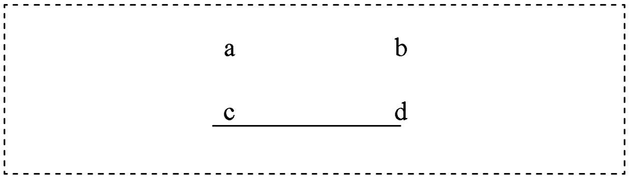


And so on, we can find the product = 4164534

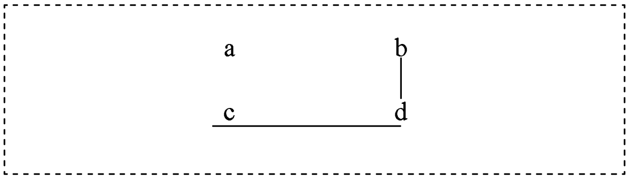
This method is most useful in case of multiplications of 2 digits × 2 digits or 2 digits × 3 digits or 3 digits × 3 digits multiplication.

Example 13 ab × cd

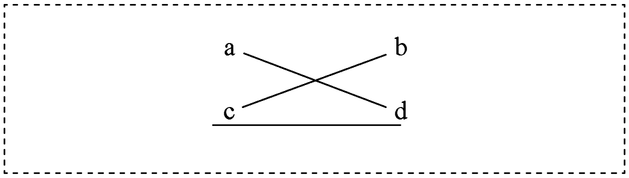
Solution



↓



↓



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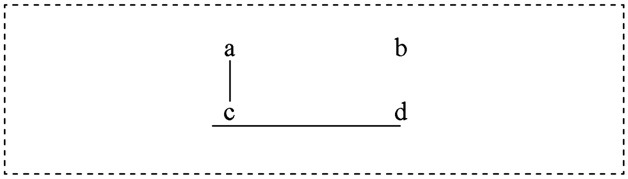


Similarly, we can have a proper mechanism for multiplication of 2 digits × 3 digits or 3 digits × 3 digits using the above given place value method.

Division

Most of the division calculations can be done by approximation techniques without actually finding the original value.

Example 14

$$\frac{3548}{4182}$$

Solution Let us approximate this value.

There are three golden rules of approximation:

1. Range of approximation should be ± 2% with respect to the actual result.
2. Approximation should be done only if the options are at a considerable gap, say ± 5% atleast.
3. One of the key things to be kept in mind while doing approximation is the direction of the approximation i.e., one should know if the actual result will be more or less than the approximated result.

We should know that in any division, if the numerator and the denominator are both either increased or decreased by the same percentage, then the result will be exactly the same. While it is practically not possible to find such situations always, we will use this to know the direction while doing approximation for the above calculation:

Approximation 1

Eliminate the unit digit of both the numerator and the denominator

$$\frac{3548}{4182} \cong \frac{354}{418}$$

Now, the objective is to break the numerator into the parts of the denominator.

354 = 209(50% of 418) + 145

145 = 105(25% of 418 approx.) + 40

40 = 10% of 418

Hence, net value = 85% = 0.85

Is this value more or less than the actual value?

Let us compare it with the actual result = 0.8483, which is a slightly less than the approximated result of ours.

Approximation 2

Eliminate the unit and tens place digit of both the numerator and the denominator

$$\frac{3548}{4182} \cong \frac{35}{41}$$

35 = 20.5(50% 0f 41) + 14.5

14.5 = 12(30% of 40 approx.) +2.5 = 30% of 41 + 6% of 41(approx.)

[Since 0.41 = 1% of 41]

Hence, net value = 86% = 0.86. This is slightly more than the actual value.

Ratio Comparison

This is one of the most important calculations and is very extensively sought after in DI. On an average, while performing 100 odd calculations in DI, atleast 8-10 calculations will be on comparing the ratios. Normally there are two methods for comparing two or more than two ratios:

Cross Multiplication Method

This is one of the conventional methods of comparing two ratios.

Example 15 Let us compare 11/15 and 13/18.

$$\frac{11}{15} \qquad \frac{13}{18}$$

Solution Cross multiplying the numerator of the 1st fraction with the denominator of the 2nd fraction and denominator of the 1st fraction with the numerator of the 2nd fraction,

$$\frac{11\times18}{198} \qquad \frac{13\times15}{195}$$

Since, 198 is greater than 195; so the 1st fraction (11/15) is greater than the 2nd fraction (13/18).

Decimal Calculation Method

$$\frac{11}{15} = 0.7333 \qquad \frac{13}{18} = 0.722$$

Obviously, here the 1st fraction $\left(\frac{11}{15}\right)$ is greater than the 2nd fraction $\left(\frac{13}{18}\right)$.

However, if we have to compare 3156/5438 and 3423/5822, then using any of the above two methods becomes cumbersome and a good amount of time will be required.

Here, we will compare ratios with the help of percentage.

Percentage Comparison Method

Let us first understand this with the help of the following ratios:

1st Case

$$\frac{10}{15}\xrightarrow[100\%]{}\xrightarrow{100\%}\frac{20}{30}$$

2nd Case

$$\frac{10}{15}\xrightarrow[100\%]{}\xrightarrow{200\%}\frac{30}{30}$$

3rd Case

$$\frac{10}{15}\xrightarrow[300\%]{}\xrightarrow{200\%}\frac{30}{60}$$

In the 1st case, percentage change in numerator (100%↑) = percentage change in denominator (100%↑), So ratios are equal.

In the 2nd case, percentage change in the numerator (200%↑) > percentage change in the denominator(100%↑). So the 2nd ratio is greater than the 1st ratio.

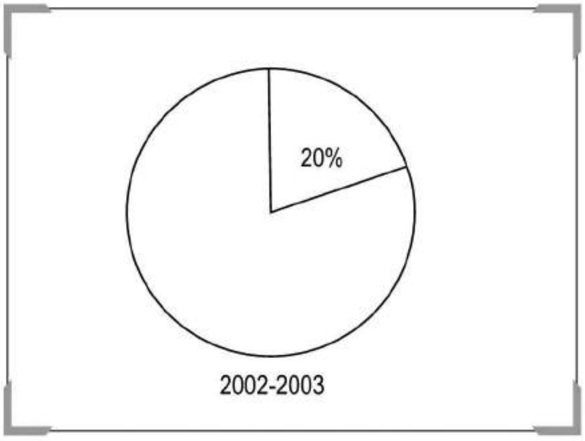
In the 3rd case, percentage change in the numerator (200%↑) < percentage change in the denominator (300%↑). So the 1st ratio is greater than the 2nd ratio.

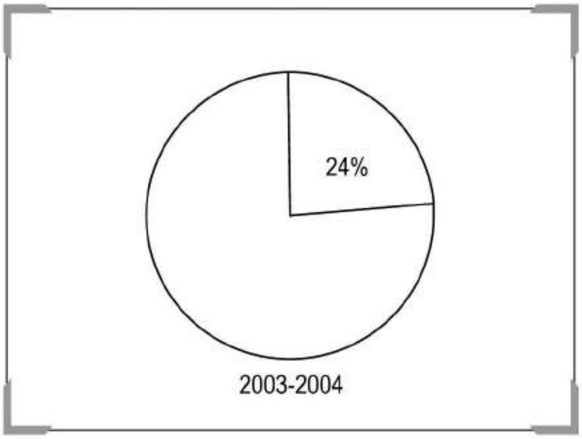
This particular example can also be seen as a general rule for determining the order of ratios.

Besides these general calculation techniques, there are certain techniques specific to particular types of data presentation formats:

Pie Chart Calculation Technique

The following figure shows the sales of *Due North Inc.* for the year 2002-2003 and 2003-2004. It is also given that the share of North India in the total sales figure is 20% and 24% for the two respective years.





The total sales for 2002-2003 is Rs 1202 crores and for 2003-2004, it is Rs 1381 crores.

Example 16 What is the percentage growth in the sales value of North India in 2003-2004 over the sales value of 2002-2003?

Solution The normal way of doing this question is –

Sales value of North India in 2002-2003
= Rs 240.4 crores
Sales value of North India in 2003-2004
= Rs 331.4 crores
So, growth = Rs 91 crore
So, Percentage growth =

$$\frac{Growth}{Initial\,Value} \times 100 = 37.93\%$$

Alternatively, there are percentage increases in two factors of the given pie-charts, i.e., share is increasing from 20% to 24% and then the total sales value is increasing from Rs 1202 crores to Rs 1381 crores.

Rather than calculating the sales value of the individual years, we can directly calculate the net percentage change by calculating the percentage changes independently in the above mentioned two factors.

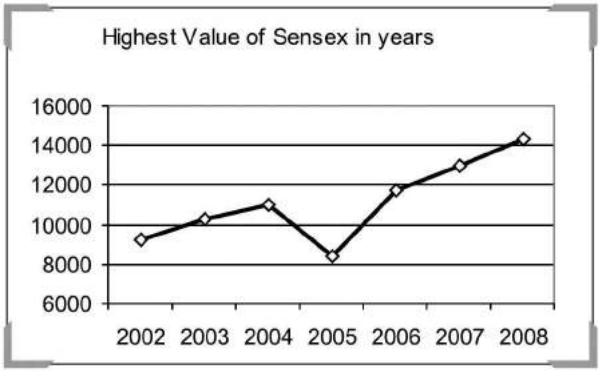
Percentage growth in share = 20%

$$\left(\frac{Growth \times 100}{Initial\,Value}; \text{and remember that it is not 4\%}\right)$$

Percentage growth in total value ≅ 15% approx.
So, 100 → 20%↑ → 120 → 15%↑ → 138, OR
100 → 15%↑ → 115 → 20%↑ → 138
So, net percentage growth = 38%

Line Chart/Bar Chart

In case of line charts bar charts, understanding the process of slope calculation between two consecutive periods/ constituents/segments, can give us the highest or lowest percentage change in the whole chart.



Example 17 When does the highest percentage increase occur between any two years during the whole period?

Solution Let us start by going through the slopes of different lines joining the two values.

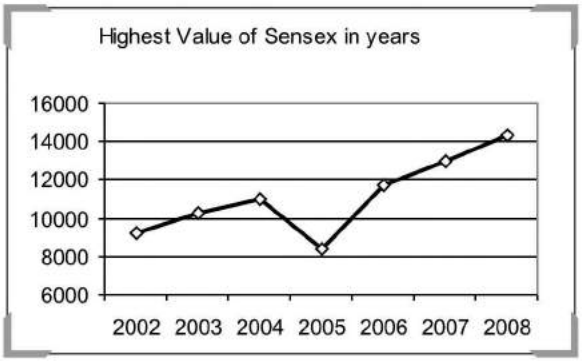
(Slope is defined as the tangent of the angle formed by the line in an anti-clockwise direction with the X-axis.)

If we find out the angle at the place of slope, it will serve the purpose.

It can be seen in the above line chart that the angle between 2005 and 2006 is the highest. Hence, the maximum percentage increase occurs during 2005-2006.

What is to be done if the angle formed between the two different periods is almost the same?

Let us see this through a final example:



In the line chart given above, the line between 2006-2007-2008 is almost straight; hence the slope is almost the same. But, as we can see, the base (or, denominator) is lower in 2005 than in 2006, hence, the percentage increase in 2005-2006 will be higher than 2006-2007.

Whatever we have discussed till now about line charts is true for bar charts as well.

CONNECTING DIFFERENT DATA-SETS

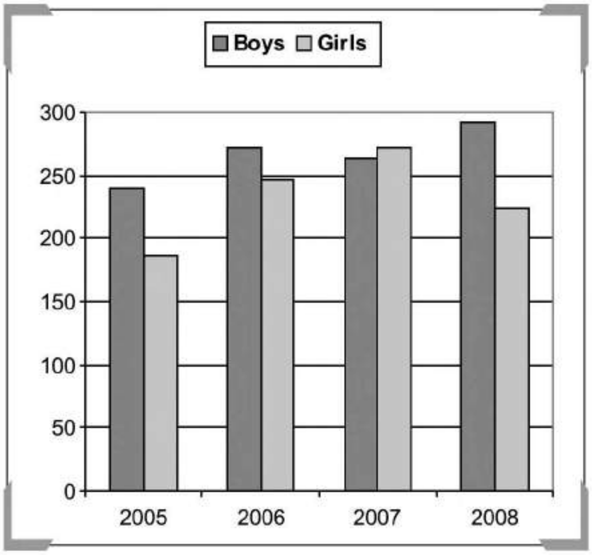
We have seen in Chapter 1, Introduction to Data Interpretation, that there exists a certain relationship among almost all the data presentation formats, it is quite possible to convert a data-set given in one format to a data-set in another format.

A number of boys and girls passed out from a school during 2005-2007:

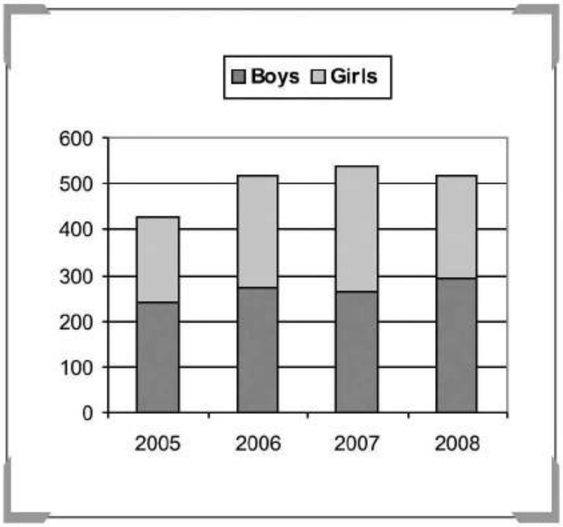
Tabular Presentation

	Boys	Girls
2005	240	186
2006	272	246
2007	264	272
2008	292	224

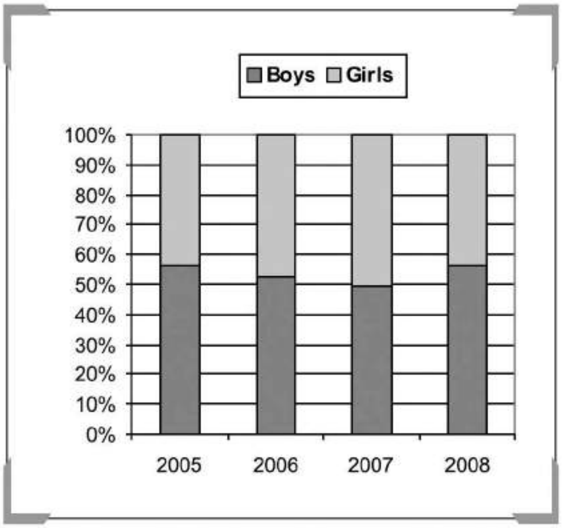
Bar Chart



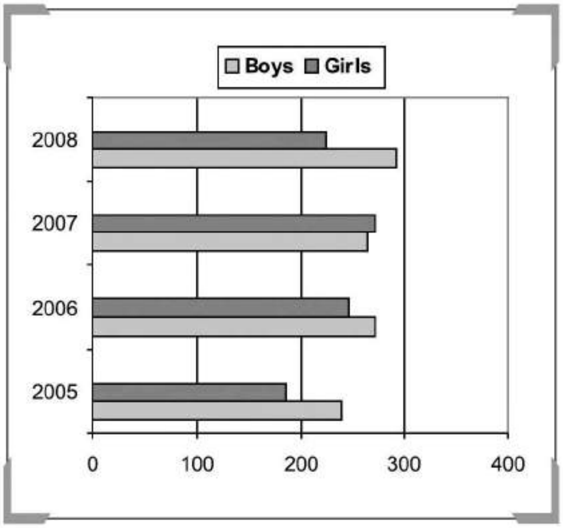
Value-Wise Stacked Bar Chart



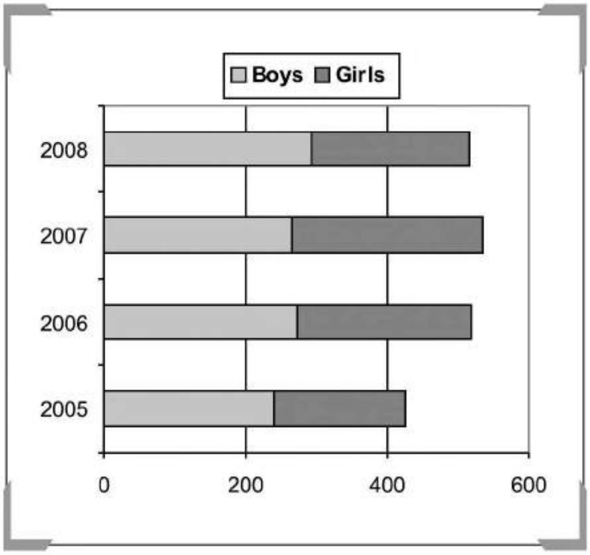
Percentage-wise Stacked Bar Chart



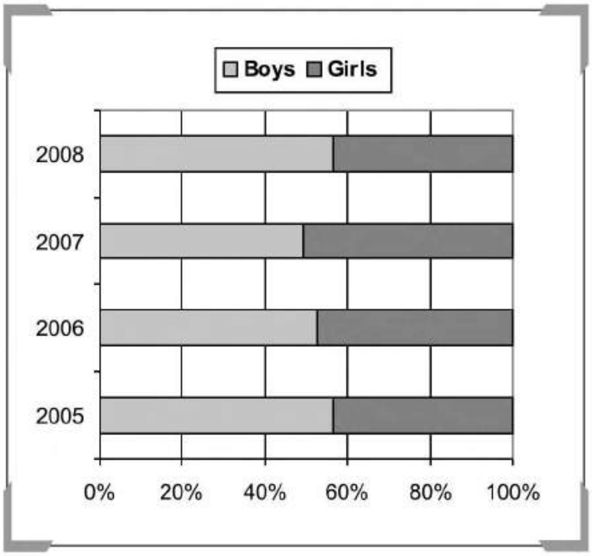
Horizontal Bar Chart



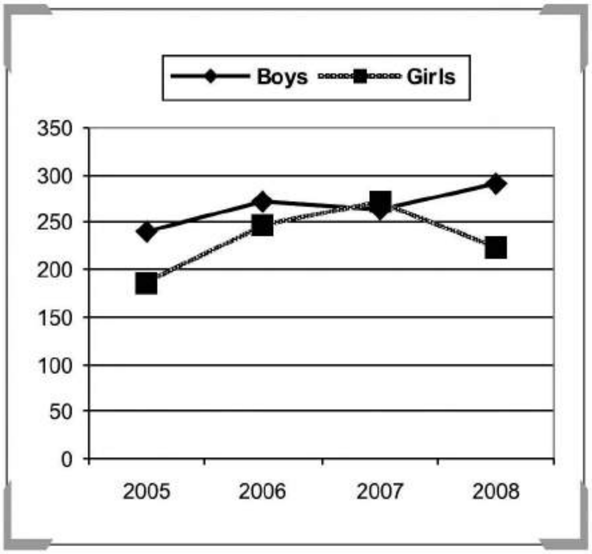
Horizontal Value-wise Stacked Bar Chart



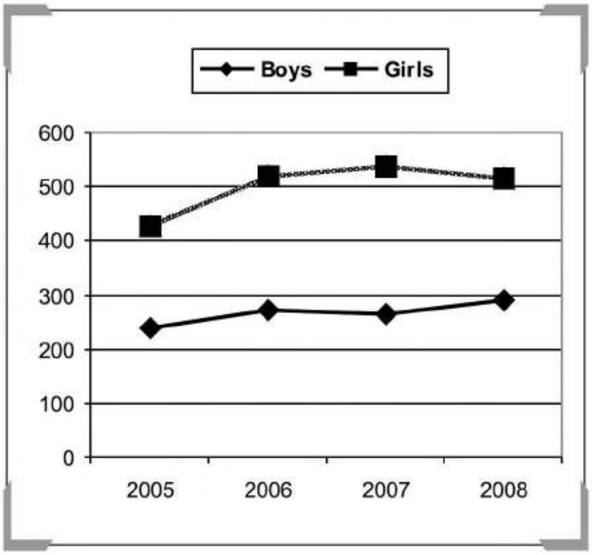
Horizontal Percentage-wise Stacked Bar Chart



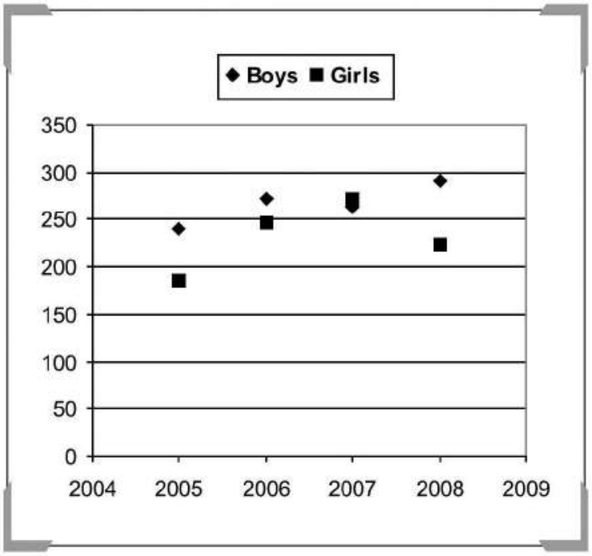
Line Chart



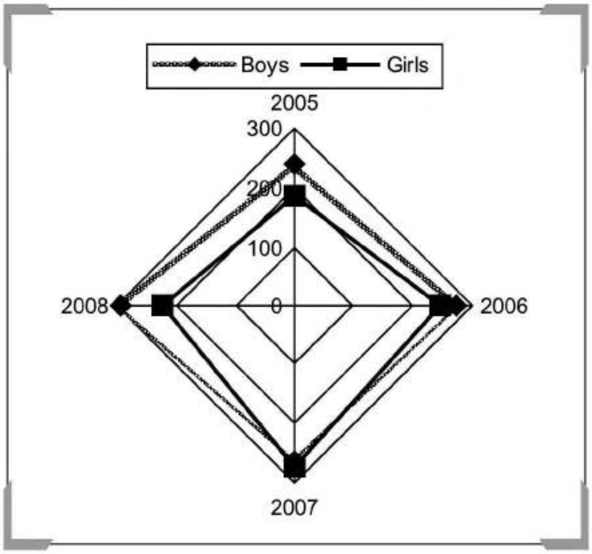
Stacked Line Chart



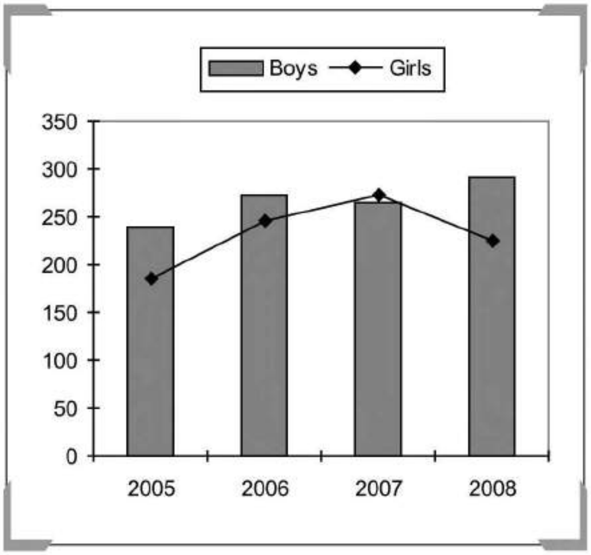
Line chart (with dots)



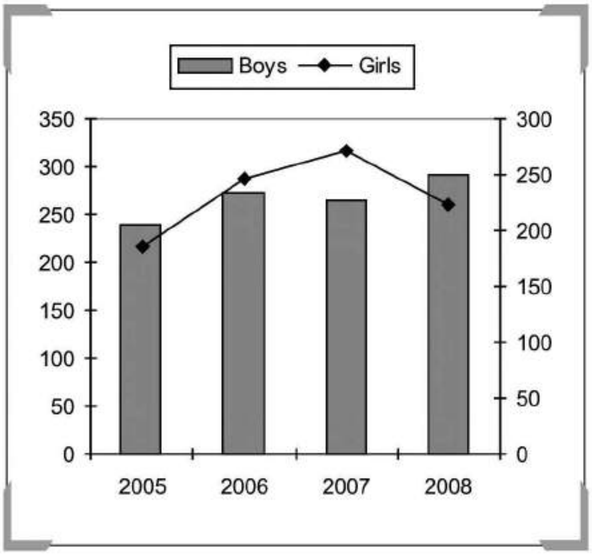
Radar Diagram



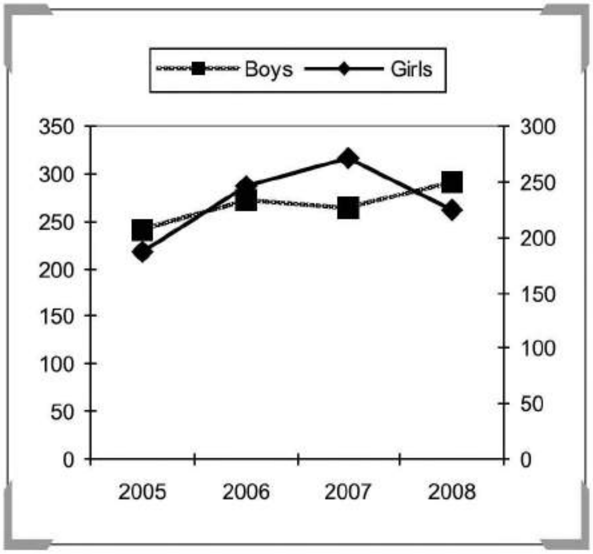
Combination Chart (Line Chart + Bar Chart)



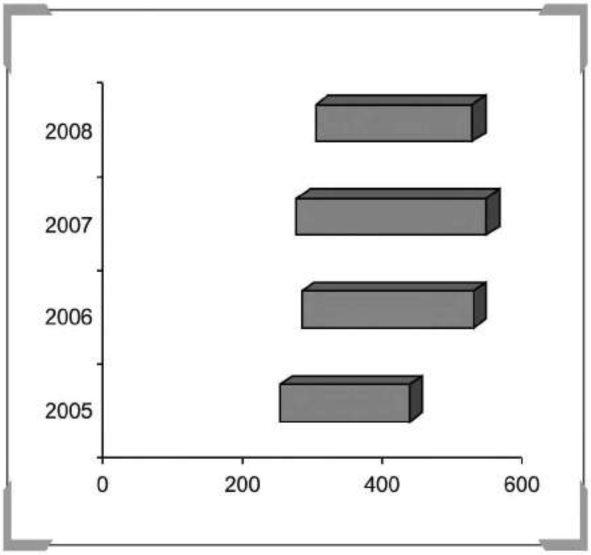
Combination Chart (Line Chart + Bar Chart) with Independent Axes



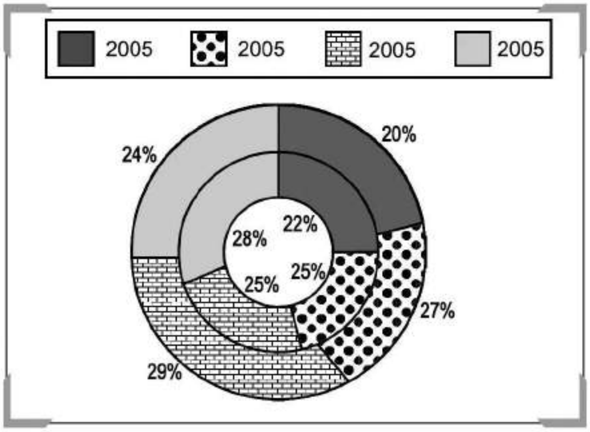
Line Chart with Independent Axes



Floating Diagram



Doughnut Graph



PRACTICE EXERCISE

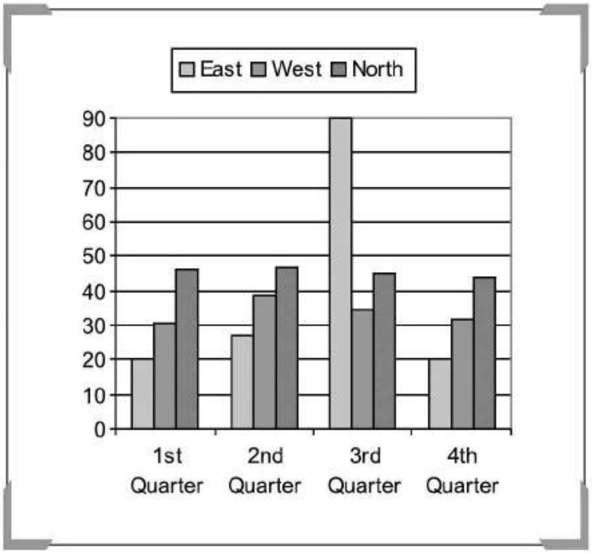
■■■

Q 1. Convert the following table into (i) bar chart (ii) line chart (iii) set of pie charts.

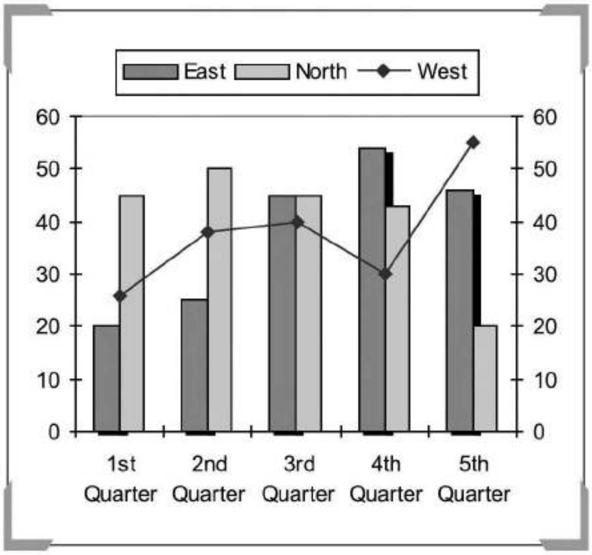
	BJP+	Congress	Communist	RJD+	Others
1996	154	214	38	32	72
1999	198	172	46	46	56
2004	234	146	52	42	34

Q 2. Convert the above given table into a line chart with two different axes—BJP on one axis and all the other parties on the second axis.

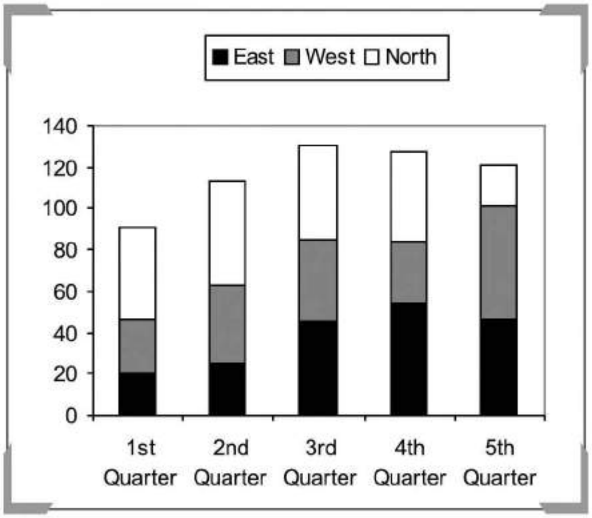
Q 3. Convert the below given bar chart into (i) line chart (ii) table



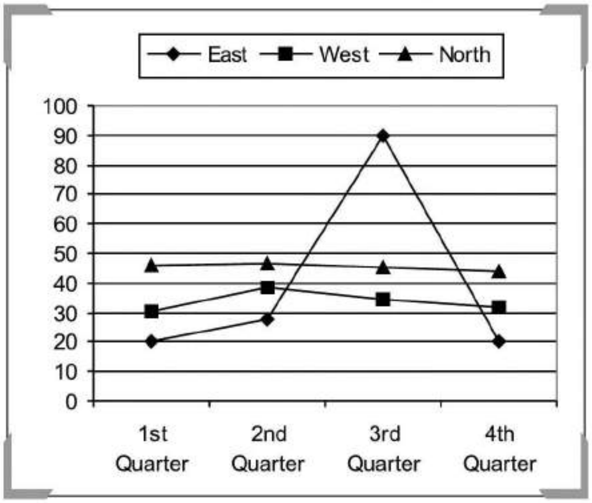
Q 4. Convert the below given line chart on two axes into (i) line chart on a single axis (ii) table (iii) bar chart.



Q 5. Convert the below given stacked bar chart into (i) normal bar chart (ii) line chart (iii) pie charts.



Q 6. Convert the below given line chart into (i) bar chart (ii) stacked bar chart (iii) pie charts.



Q 7. Rank the following ratios in descending order (without actual calculation):

3426/4536
3954/4865
3136/4129
3754/4765
4024/5128

Q 8. Rank the following ratios in ascending order(without actual calculation):

7324/6453
8345/7128
6873/5896
7212/6123
8523/7027

3

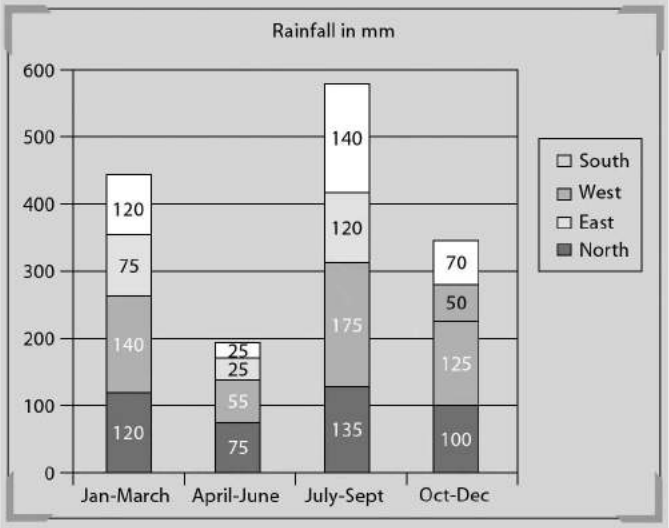
C H A P T E R

Foundation Exercises

PRACTICE EXERCISE 1

Direction for questions 1 to 5: Refer to the bar chart below and answer the question based on it.

Below given bar chart shows the quantity of rainfall (in mm) in the four sectors of India in the four different periods of the year:



Q 1. Which sector received the maximum rainfall in the given year?
(a) North (b) East
(c) South (d) West

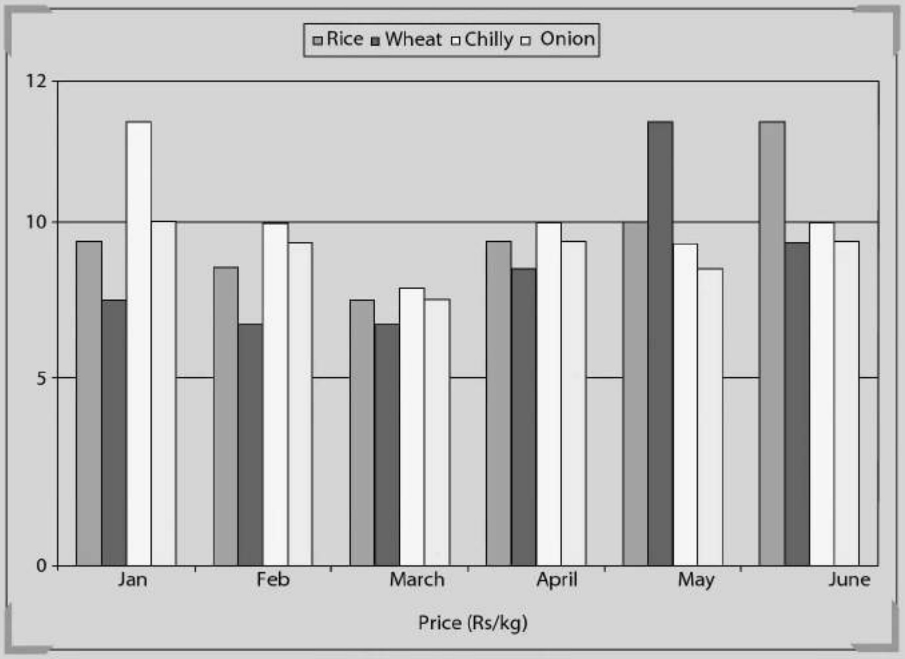
Q 2. During which period there was minimum rainfall?
(a) Jan-March (b) April-June
(c) July-Sep (d) Oct-Dec

Q 3. The average rainfall for the given year (rainfall/month) in the north sector was (approx)
(a) 33.8 (b) 34.6
(c) 35.3 (d) None of these

Q 4. In how many periods did the East receive more rainfall than the South?
(a) 0 (b) 1
(c) 2 (d) 4

Q 5. Which of the following statements is false?
(a) The rainfall followed a similar pattern for all the four sectors.
(b) The western sector received the least rainfall in Jan-March across all the regions.
(c) Rainfall was the least during the months of April-June.
(d) None of these

Direction for questions 6 to 10: Refer to the bar chart below and answer the questions based on it.



Q 6. Which of the following statements is correct?
- (a) Price of rice is maximum in the month of May
- (b) In all the months price of onion is less that that of chilly
- (c) Price of wheat is minimum in January
- (d) None of these

Q 7. Months of May and June differ from the rest of months for
- (a) Price of onions is less than that of chilly
- (b) Price of rice is more than Rs 8 per kg
- (c) Price of wheat is more than Rs 8 per kg
- (d) None of these

Q 8. Which month saw the maximum combined price of all the items?
- (a) January (b) April
- (c) May (d) None of these

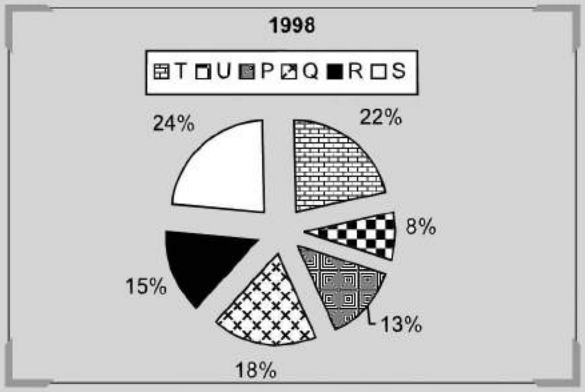
Q 9. Of the four given commodities how many have seen a drop in the price for two consecutive months?
- (a) 0 (b) 1
- (c) 2 (d) 3

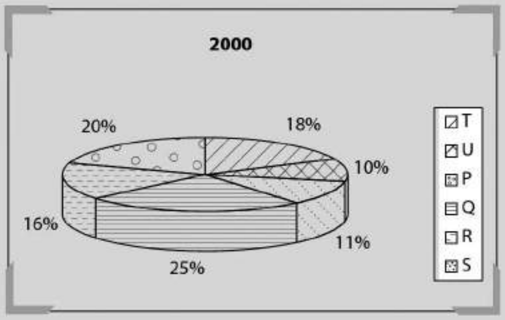
Q 10. A commodity is said to be 'Vary-Most' if its price keeps fluctuating between increase and decrease or vice-versa on alternate months. How many of the four commodities are 'Vary-Most'?
- (a) 0 (b) 1
- (c) 2 (d) 3

Direction for questions 11 to 15: Refer to the pie charts below and answer the questions based on it.

The following pie charts show the number of workers of different categories of a factory for the two different years.

The total number of workers in 1998 is 2000. Number of workers in 2000 grew by 20% over the number of workers in the year 1998.





Q 11. Find the percentage increase in the number of workers in category U in the year 2000.
- (a) 25%
- (b) 50%
- (c) 33.33%
- (d) 75%

Q 12. What is the total number of increased workers for the categories in which the number of workers has increased?

(a) 468 (b) 404

(c) 382 (d) None of these

Q 13. Which of the following categories has shown a decrease in the number of workers from the year 1998 to 2000?

(a) P (b) R

(c) S (d) None of these

Q 14. Find the maximum difference between the number of workers of any two categories taken together for any one year and that of other two categories for the other year.

(a) 660 (b) 636

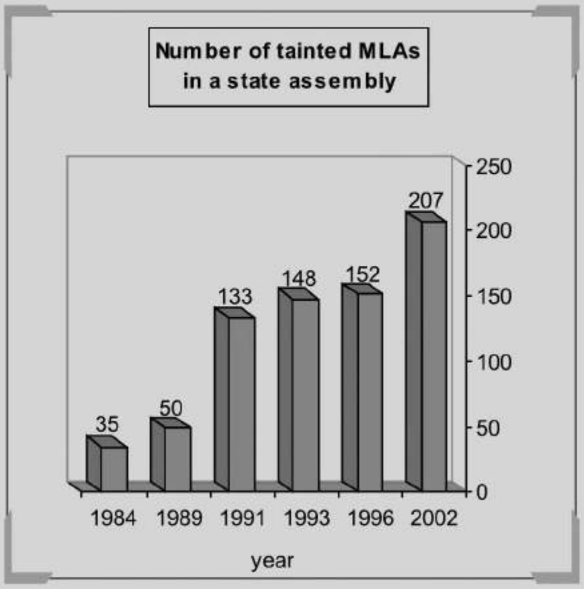
(c) 416 (d) 502

Q 15. For how many categories, number of workers have increased?

(a) 6 (b) 3

(c) 4 (d) 5

Direction for questions 16 to 19: Refer to the bar chart below and answer the questions based on it.



Tainted MLAs of different parties in the 2002 state assembly.

SPA	BP	SBJP	Congred	Lokmat	Others
84	54	41	7	7	14

Total number of MLAs in 2002 state assembly = 403

Q 16. Find the approximate percentage increase in the number of tainted MLAs from 1991 to 1993.

(a) 11.1 (b) 11.3

(c) 12.2 (d) 11.7

Q 17. During which year was the percentage increase in the no. of tainted MLAs from previous recorded year the lowest?

(a) 1989 (b) 1991

(c) 1993 (d) 1996

Q 18. Which of the following statements is not true?

(a) In the 1991 elections, there were 133 tainted MLAs–more than a double over 1989.

(b) Of the 403 legislators in 2002, there were 207 tainted MLAs of which a maximum of 84 were from SPA.

(c) The maximum percentage increase of tainted MLAs from the previous given year was for the year 1991.

(d) The maximum increase in the number of tainted MLAs from the previous given year was for the year 2002.

Q 19. After 2002 assembly elections, the ratio of tainted MLAs of BP to those of SPA is

(a) 14:9 (b) 9:21

(c) 21:9 (d) 9:14

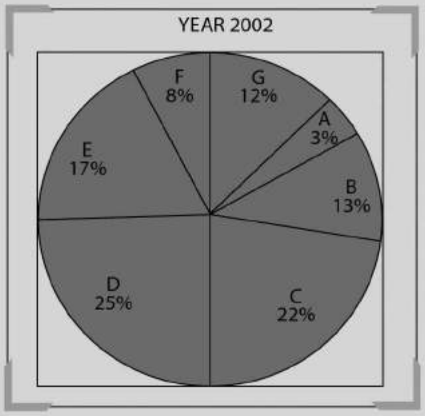
Q 20. Which year exhibits the maximum percentage increase in the tainted MLAs over the previous recorded year?

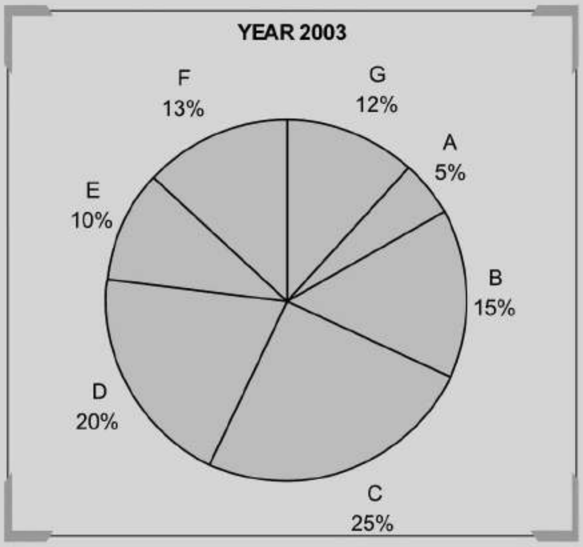
(a) 1989 (b) 1991

(c) 1993 (d) 1996

Direction for questions 21 to 25: Refer to the pie charts below and answer the questions based on it.

There are seven types of employees in an organization doing seven different jobs. Following pie charts show the percentage of these employees in the organization across two years—2002 and 2003.





It is also given that the total number of B type employees in 2002 and 2003 were 1560 and 2250 respectively.

Q 21. C and D types of employees are known as grade IV employees. Find the total no. of grade IV employees in both the given years.

(a) 12390
(b) 11590
(c) 12790
(d) 11290

Q 22. A, E and F type of employees belong to higher income group (HIG). What is the number of HIG employees in 2003?

(a) 3600 (b) 4000
(c) 4200 (d) 3200

Q 23. Approximately what percentage of B type employees in 2002 is the number of B type employees in 2003?

(a) 72 (b) 73
(c) 144 (d) 142

Q 24. Which type of employees have recorded maximum increase in the year 2003?

(a) B (b) F
(c) C (d) None of these

Q 25. For how many types of employees, there is an absolute decrease in the number of employees in 2003 over 2002?

(a) 0 (b) 1
(c) 2 (d) 3

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(d)	(d)	(b)	(b)	(c)	(d)	(d)	(a)	(b)	(d)	(d)	(a)	(c)	(b)	(d)	(d)	(d)	(b)
Q. No.	21	22	23	24	25															
Answer	(a)	(c)	(c)	(c)	(b)															

HINTS AND SOLUTIONS

Let us have a tabular representation:

Sector/ Period	Rainfall in mm				
	North	East	West	South	Total
Jan-March	120	140	80	120	460
April-June	70	50	20	20	160
July-Sept	140	180	120	140	580
Oct-Dec	100	120	50	60	330
Total	430	490	270	340	1530

1. East

2. All four values for April–June season are less than those of other season so, the rainfall was minimum.

3. The answer is none of these

4. East received more rainfall than South in all the four periods.

5. Western sector received the least rainfall in April-June. So, statement b is false.

11. There are two increases–one in total number of workers (20%) and another in the percentage share of the workers(20%).

    Using successive percentage change, the net change = 50%.

12. Visual inspection.

13. Only T has decreased.

14. Case I: Two categories having the highest number of workers in 1998 and simultaneously the two categories having the least number of workers in 2000.

    Therefore, Difference = (480 + 440) – (240 + 264) = 416

    Case II: Two categories having the highest number of workers in 2000 and simultaneously the two categories having the least number of workers in 2000.

    Therefore, Difference = (600 + 480) – (160 – 260) = 660.

15. There are four categories with an increase in number of workers–U, P, Q and R. S remains constant and the T decreases.

21. Total no. of C and D type employees in 2002 = 5640
    Total no. of C and D type employees in 2003 = 6750
    Hence total number of employees = 12390

22. No. of A, E and F type employees in 2003
    $= (2250/15) \times (5 + 10 + 13) = 4200$

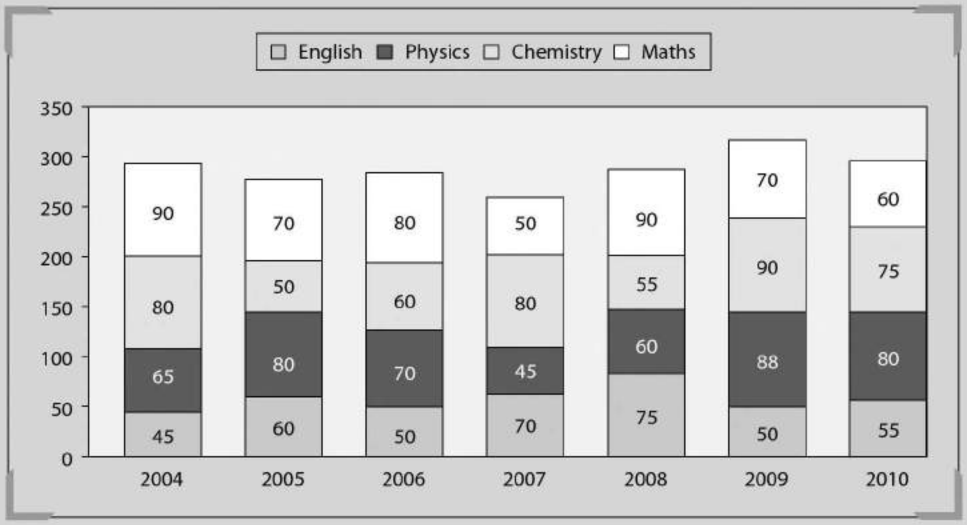
23. Required percentage = $(2250/1560) \times 100 \approx 144\%$

24. Here we are talking about the number of employees only.

25. Only for E.

PRACTICE EXERCISE 2

Direction for questions 1 to 4: Go through the bar chart given below and solve the questions based on it.



The bar chart shows average marks obtained in four different subjects for different years at A R Academy. Marks obtained in any subject in any year can be the multiples of five only.

Q 1. Which of the following subjects shows maximum increase in average marks over any two consecutive years?
(a) Chemistry
(b) Maths
(c) Physics
(d) English

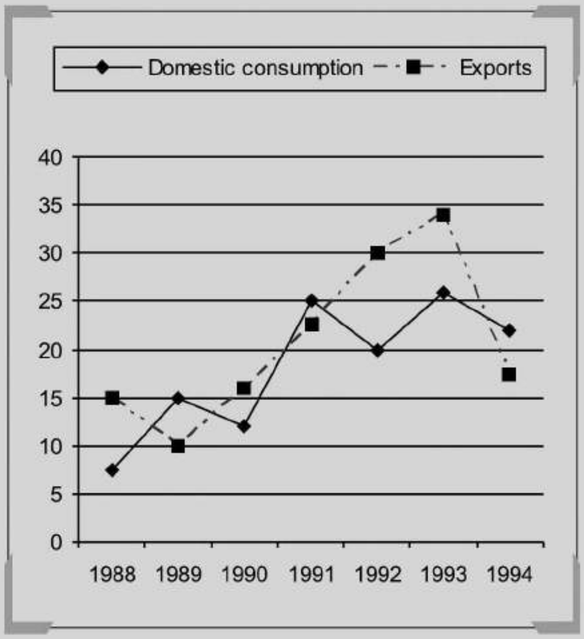
Q 2. The percentage increase in the average marks in Maths in 2006 with respect to the previous year = N% and the percentage increase in the average marks in Chemistry in 2006 over the previous year = M%. Find M – N.
(a) 5.7%
(b) 6.5%
(c) 4.3%
(d) 3.5%

Q 3. Find the year representing the maximum average marks, combining all the subjects.
(a) 2004
(b) 2008
(c) 2009
(d) 2010

Q 4. The total average marks in the year 2010 is decreased with respect to the total average marks in year 2004 by
(a) 2% (b) 3%
(c) 3.7% (d) 4.5%

Direction for questions 5 to 9: Go through the line chart given below and solve the questions based on it.

(In thousand tones)



Q 5. In which of the following years was the total of the exports and domestic consumption highest among the given year?
(a) 1991 (b) 1993
(c) 1992 (d) None of these

Q 6. The difference between the domestic consumption between 1990 and 1991 was exactly equal to the difference in exports between which of the following pairs of year?
(a) 1991 and 1992
(b) 1989 and 1991
(c) 1988 and 1990
(d) 1991 and 1994

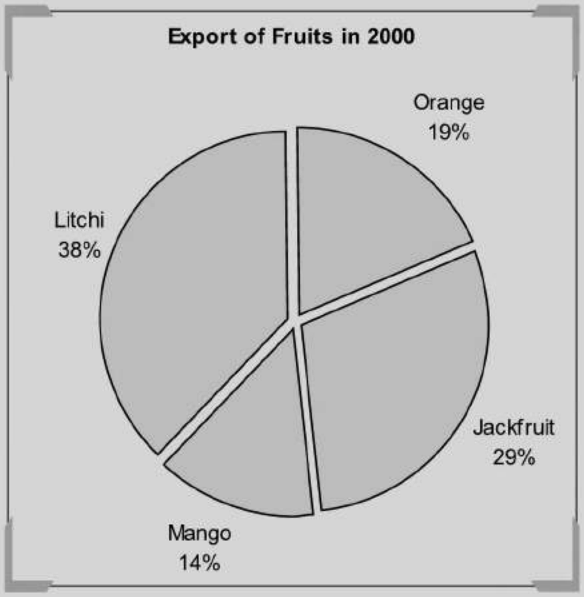
Q 7. If there was a 40 percent increase in the domestic consumption from 1994 to 1995, what would have been the difference between the domestic consumption and exports in 1995 (in tonnes)?
(a) 5,000 (b) 7,000
(c) 8,500 (d) Cannot be determined

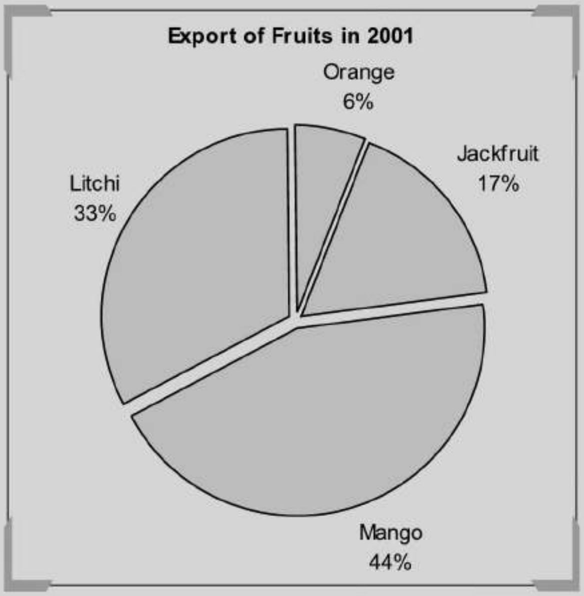
Q 8. In which of the following years was the percentage fluctuation (increase/decrease) in exports the maximum from the previous years?
(a) 1989 (b) 1991
(c) 1994 (d) 1990

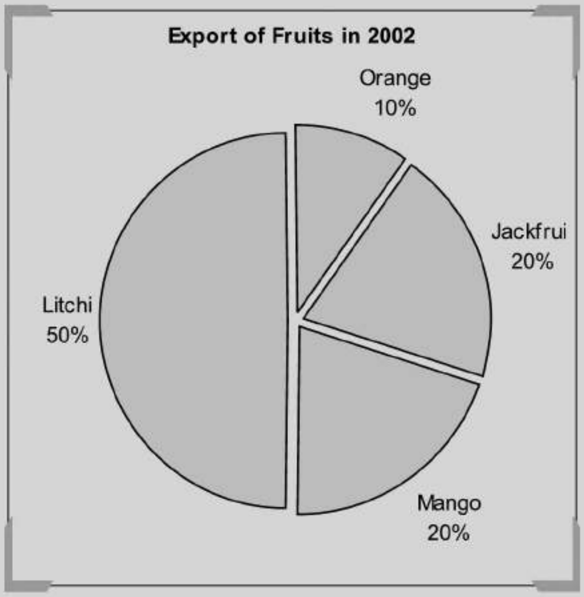
Q 9. What was the difference (in tonnes) between the average domestic consumption and the average exports during the given years?
(a) 2,000 (b) 3,000
(c) 2,500 (d) 4,000

Direction for questions 10 to 14: Refer to the charts below and answer the questions based on it.

The charts given below give the share of export of four fruits from India in three consecutive years. (in volume)







Q 10. In which year were the exports of Jackfruit maximum?
(a) 2000 (b) 2001
(c) 2002 (d) Cannot be determined

Q 11. In which year were the orange exports minimum?
(a) 2000 (b) 2001
(c) 2002 (d) Cannot be determined

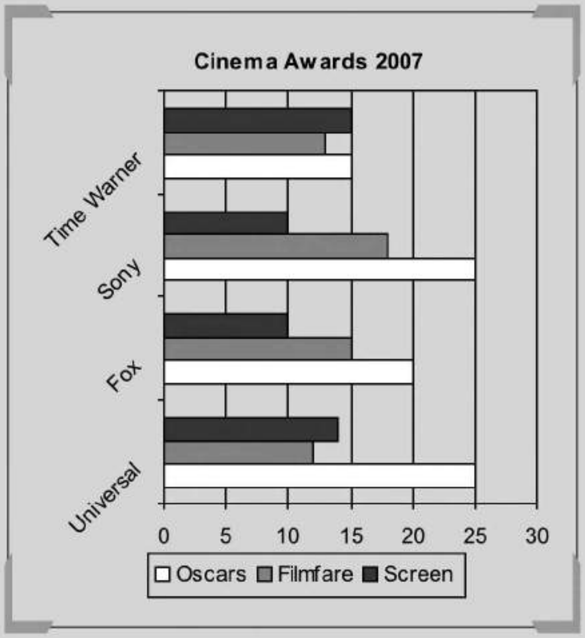
Q 12. In 2001, which fruit has shown maximum percentage increase in export over the last year?
(a) Orange (b) Jackfruit
(c) Mango (d) Litchi

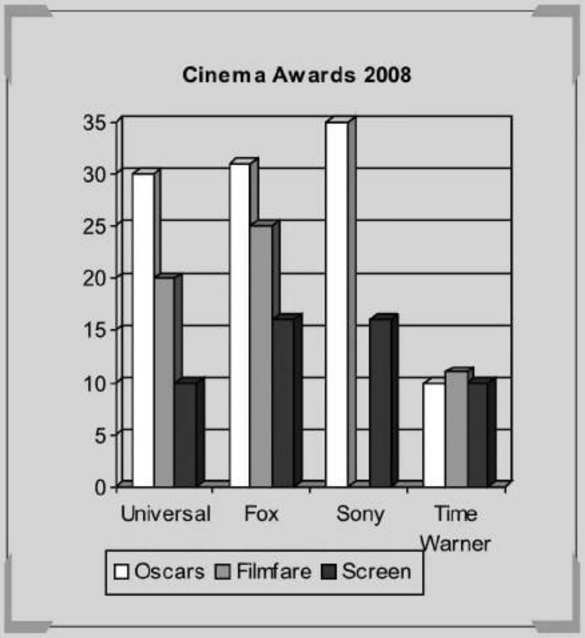
Q 13. Which fruit has the maximum percentage increase in export value in 2002 over the export value in 2000?
(a) Orange
(b) Mango
(c) Jackfruit
(d) Cannot be determined

Q 14. Export of which fruit has constantly been on a rise?
(a) Orange
(b) Mango
(c) Jackfruit
(d) Cannot be determined

Direction for question 15 to 19: Refer to the bar charts below and answer the questions based on the same.

The following bar charts give the details of three different awards won by the movies produced by the four studios:





Q 15. Which studio won the maximum total number of Screen awards in 2007 and 2008?
(a) Universal
(b) Fox
(c) Sony
(d) Time Warner

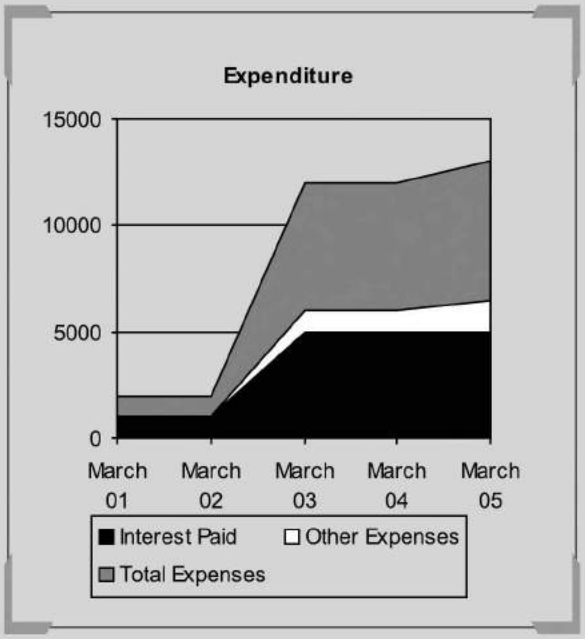
Q 16. Which studio saw the maximum percentage increase in the total number of awards?
(a) Universal
(b) Fox
(c) Sony
(d) Time Warner

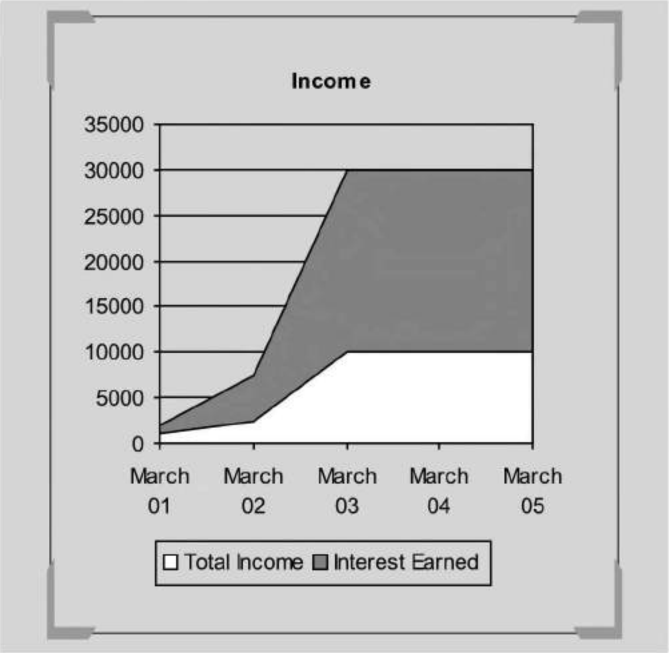
Q 17. Which award had the maximum number of categories in 2008?
(a) Oscars
(b) Filmfare
(c) Screen
(d) Filmfare and Screen

Q 18. Which studio had the maximum drop in the Filmfare awards from 2007 to 2008?
(a) Universal (b) Fox
(c) Sony (d) None of these

Q 19. Which award had the maximum percentage decrease/increase in the number of awards won in 2008 from 2007?
(a) Oscars (b) Filmfare
(c) Screen (d) None of these

Direction for questions 20 to 24: Go through the charts given below and solve the questions based on it.





Q 20. The ratio of interest earned to the total income shows
- (a) Increasing trend
- (b) Decreasing trend
- (c) Erratic trend
- (d) No particular trend

Q 21. Total income and interest earned reported similar decline between the years
- (a) 2001 and 2002
- (b) 2002 and 2003
- (c) 2003 and 2004
- (d) 2004 and 2005

Q 22. The difference between the total income and the total expenditure is maximum in the year
- (a) 2002
- (b) 2003
- (c) 2004
- (d) 2005

Q 23. The ratio of total interest paid to expenditure is minimum in the year
- (a) 2002
- (b) 2003
- (c) 2004
- (d) 2005

Q 24. Which of the following statement is correct?
- (a) Total income and total expenditure follow a similar trend
- (b) Total income and interest paid follow a similar trend
- (c) Total income and total expenditure do not follow a similar trend
- (d) Other expenses and interest paid follow a similar trend

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(a)	(c)	(c)	(b)	(b)	(d)	(d)	(c)	(d)	(d)	(c)	(d)	(d)	(a)	(b)	(a)	(c)	(a)	(a)
Q. No.	21	22	23	24																
Answer	(c)	(b)	(d)	(c)																

HINTS AND SOLUTIONS

1. Maximum increase in average marks is observed for Maths from 2007 to 2008.
2. Percentage increase in average marks in Maths in 1996 over the previous year = 14.28%

   Percentage increase in average marks in Chemistry in 1996 over the previous year = 20%

   Difference = 20% – 14.28% = 5.72%.

3.

Year	Total marks
1994	280
1995	260
1996	260
1997	245
1998	280
1999	295
2000	270

4. Average marks in 2004 = 280
Average marks in 2010 = 270
Percentage decrease = 3.7%.

7. Growth or decline of exports is not given.

8. Percentage change in exports is maximum in 1990.

10. Since the value is not given, we cannot have the numbers. Hence cannot be determined.

11. Since the value is not given, we cannot have the numbers. Hence, cannot be determined.

12. Mango is the only fruit which has shown growth. Understand that total exports in 2001 might be increasing but it will impact the answer to this question because exports of all the fruits will be equally affected by it.

14. Since the value is not given, we cannot have the numbers. Hence, cannot be determined.

19.

Award	2007	2008	% Increase
Oscars	25 + 20 + 3+ 15 = 63	30 + 31 + 35 + 10 = 106	> 70%
Film-fare	12 + 15+18+13 = 58	20 + 25 + 0 + 11 = 56	= 4%
Screen	14 + 10 +10 + 15 = 49	10 + 16 + 16 + 10 = 52	= 6%

20. From the income graph, we see that the interest to the total income is showing an increasing trend.
Hence, option (a), is the answer

21. From the income graph, we can see that the interest earned and the total income reported a similar decline in 2003 and 2004, however, the decline as seen is very imperceptible.

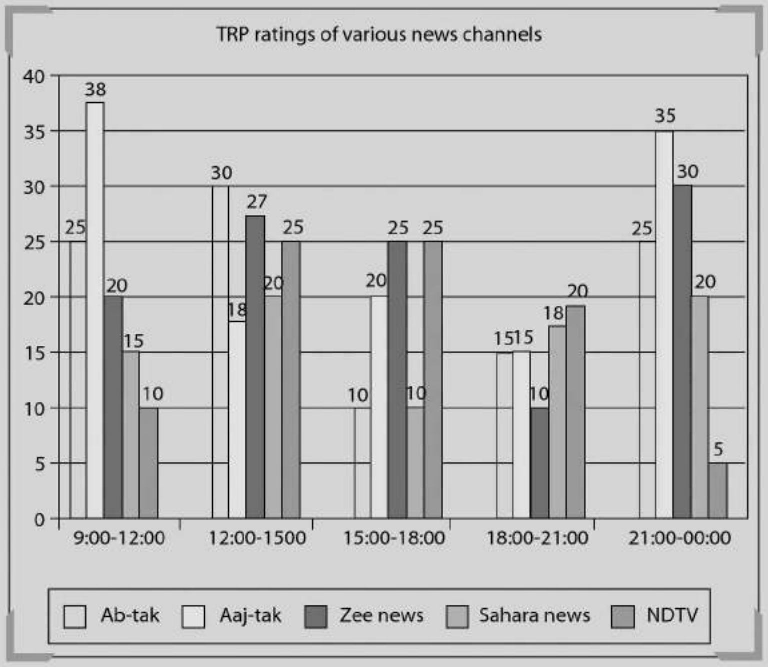
22. From both the graphs we see that the difference between the total income and the expenditure is maximum in 2003, So (b).

23. As seen from the expenditure graph, Ratio for
2002 =1/2
2003 = 8/22 = 0.33
2004 = 7/19 = 0.36
2005 = 5/19 = 0.26

PRACTICE EXERCISE 3

Direction for questions 1 to 5: Go through the bar chart given below and solve the questions based on it.

Following bar chart gives the TRP rating of five news channels in five time slots.



Q 1. Which time-slot is the most popular for watching news?
(a) 9:00-12:00 (b) 12:00-15:00
(c) 15:00-18:00 (d) 18:00-21:00

Q 2. Which TV channel has the maximum difference in the TRP ratings for any two time-slots?
(a) Ab-tak
(b) Aaj-Tak
(c) Sahara News
(d) None of these

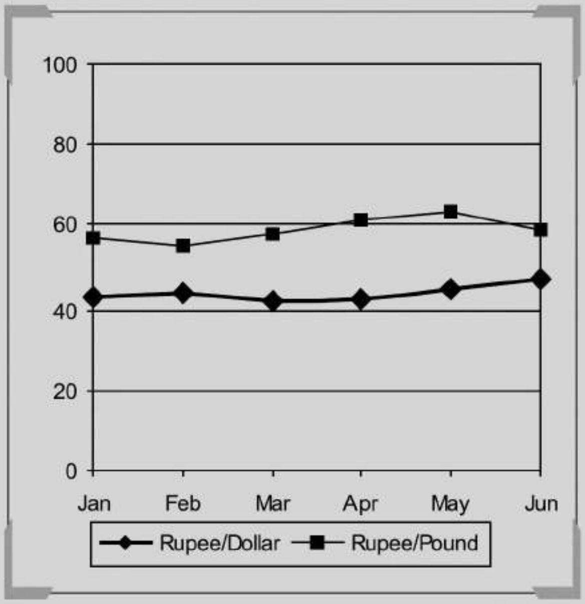
Q 3. For which time-slot is the difference between the maximum and minimum TRP rating the minimum?
(a) 9:00-12:00 (b) 12:00-15:00
(c) 15:00-18:00 (d) 18:00-21:00

Q 4. For how many time-slots does Zee news have a higher TRP rating than at least two more channels?
(a) 2 (b) 3
(c) 4 (d) None of these

Q 5. Under a new points scheme, news channels are given the points equal to their rank in the various time slots. The lower the sum of rank, the more popular the channel. Which channel is the most popular channel in accordance with this points scheme?
(a) Ab-tak (b) Aaj-Tak
(c) Sahara News (d) Zee news

Direction for questions 6 to 10: Refer to the graphs below.

Following line charts present exchange rates of three currencies for 6 months as in the month of January (Jan) 1 Dollar = Rs 43; 1 Pound = Rs 57.



Q 6. In which month can a person get least number of pounds with $ 100?
(a) Feb (b) Mar
(c) Apr (d) May

Q 7. If a person had 65 pounds, then in which month would he have exchanged them to get maximum dollars?
(a) Jan (b) Feb
(c) Jun (d) Apr

Q 8. $1 in Mar is worth 130 yens. How many yens can I purchase, if I sold 100 pounds to purchase rupees and that rupees to purchase the dollars to purchase yens?
(a) 18,000
(b) 16,000
(c) 5800
(d) 17,250

Q 9. If I purchased $ 75 in Jan paying in rupees, then in which month should I sell the dollars to get rupees so that the loss would be highest?
(a) March
(b) April
(c) May
(d) Any month

Q 10. For how many months, the exchange price of pound was more than the exchange price of dollar?
(a) 4
(b) 5
(c) 6
(d) None of these

Direction for questions 11 to 15: Go through the pie charts given below and solve the questions based on it.

There are six types of employees working in an organization. Following is the percentage breakup of different types of employees in a company in two successive years 2007 and 2008. It is also given that the total no. of employees in 2007 and 2008 is 42,980 and 48,640 respectively.
(All the percentage value are rounded off).

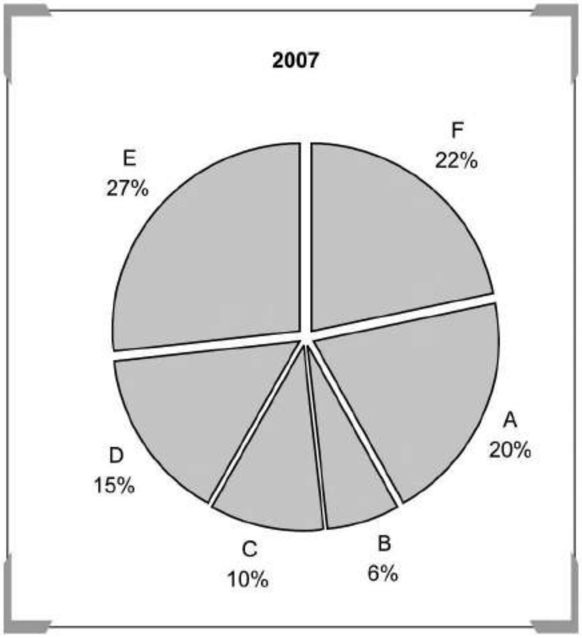
Q 11. The number of A type employees in 2008 was approximately what percentage of A type employees in 2007?
(a) 115 (b) 140
(c) 125 (d) 130

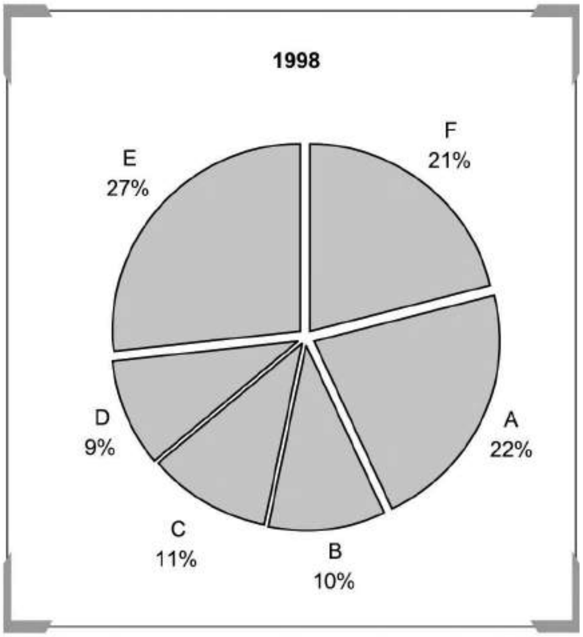
Q 12. Number of which of the following types of employees have increased maximum in 2008 over 2007?
(a) B (b) D
(c) C (d) None of these

Q 13. Approximately, what was the difference in the number of B type employee between 2007 and 2008?
(a) 2285 (b) 2325
(c) 2620 (d) 1825

Q 14. The total number of which of the following pairs of types of employees in 2007 was approximately equal to A type employees in 2008?
(a) B and C
(b) A and C
(c) D and E
(d) C and D

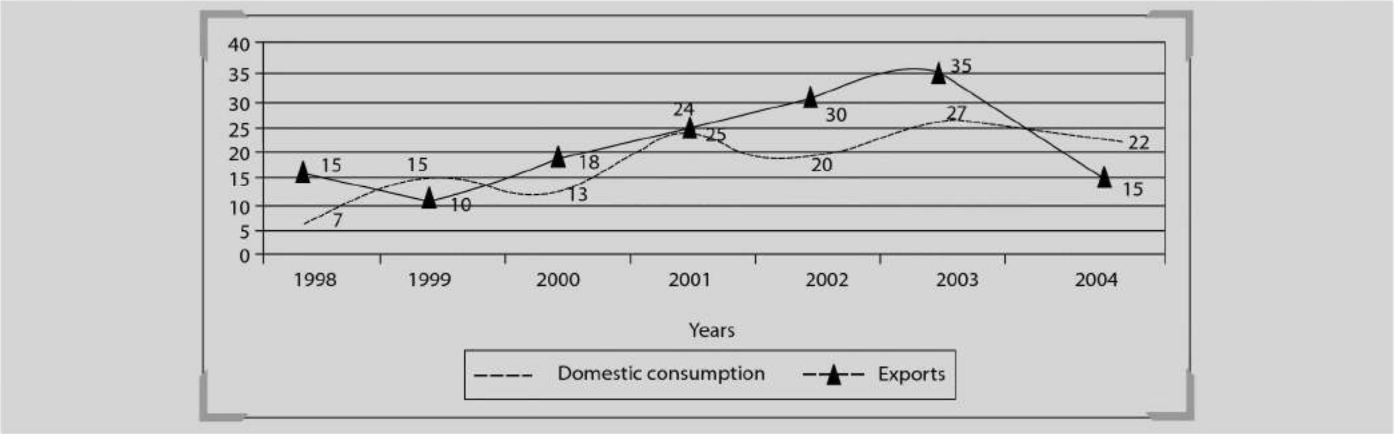
Q 15. Which type of employees have recorded maximum percentage growth in 2008 over 2007?
(a) B (b) D
(c) C (d) A





Direction for questions 16 to 20: Go through the line chart given below and solve the questions based on it.

Below given line chart represents the domestic consumption and export (In thousand tones) of rice from India in seven consecutive years.



Q 16. In which of the following years was the total of the exports and domestic consumption highest among the given years?

(a) 2001 (b) 2003
(c) 2004 (d) 2002

Q 17. The difference between the domestic consumption between 2000 and 2001 was approximately equal to the difference in exports between which of the following pairs of years?

(a) 2001 and 2002
(b) 1999 and 2001
(c) 1998 and 2000
(d) 2001 and 2003

Q 18. If there was 40% increase in the domestic consum-ption from 2004 to 2005, what would have been the difference between the domestic consumption and exports in 2005 (in tonnes)?

(a) 5,000
(b) 7,000
(c) 8,500
(d) Cannot be determined

Q 19. In which of the following years was the percentage fluctuation (increase/decrease) in exports the maximum from the previous years?

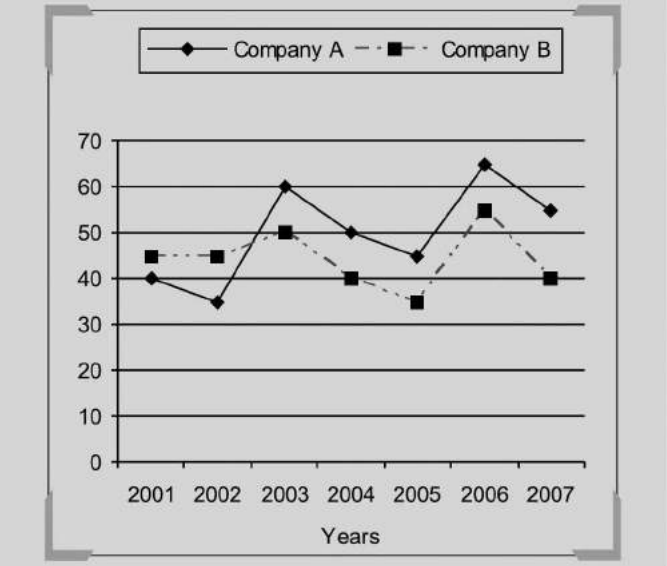
(a) 1999 (b) 2000
(c) 2004 (d) 2002

Q 20. What was the difference between the average domestic consumption and the average exports during the given years?

(a) 2,000 tonnes
(b) 3,000 tonnes
(c) 4,000 tonnes
(d) None of these

Direction for questions 21 to 25: Go through the line chart given below and solve the questions based on it.

Following line chart represents percentage net profit of two companies over the years



$$\text{Percentage Profit} = \frac{\text{Income - Expenditure}}{\text{Expenditure}} \times 100$$

Q 21. If the total income in 2002 for company B was 140 crores, what was the total expenditure in that year?

(a) 10 cr
(b) 110 cr
(c) 98 cr
(d) None of these

Q 22. If the total expenditure of 2003 and 2004 together of company B was Rs 279 crores, what was the total income in these years?

(a) Rs 121.5 Cr
(b) Rs 135 Cr
(c) Rs 140 Cr
(d) Cannot be determined

Q 23. In how many of the given years the percentage of expenditure to the income of Company A was less than fifty?

(a) One
(b) Two
(c) Three
(d) None of these

Q 24. If the total expenditure of Company B in 2004 was Rs 200Cr, what was the total income?
(a) Rs 160 Cr
(b) Rs 200 Cr
(c) Rs 260 Cr
(d) None of these

Q 25. In which of the following years was the total income more than double the total expenditure in that year for Company B?
(a) 2005
(b) 2003
(c) 2007
(d) None of these

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(c)	(b)	(a)	(d)	(b)	(c)	(c)	(c)	(a)	(a)	(d)	(a)	(b)	(d)	(d)	(b)	(d)
Q. No.	21	22	23	24	25															
Answer	(d)	(d)	(d)	(d)	(d)															

HINTS AND SOLUTIONS

[illegible]

Let us tabulate the given info in the bar chart:

TRP	Slots						
Channels	**9:00 - 12:00**	**12:00 - 15:00**	**15:00 - 18:00**	**18:00 - 21:00**	**21:00 - 00:00**	**Total**	**Max. Difference in TRP any two time slots**
Ab-tak	25	30	10	15	25	105	20
Aaj-tak	38	18	20	15	35	126	23
Zee News	20	27	25	10	30	112	20
Sahara News	15	20	10	18	20	83	10
NDTV	10	25	25	20	5	85	20
Total	108	120	90	78	115	511	-
Max.Diff. in TRP for any 2 channels	28	12	15	10	30	-	-

[illegible]

Let us first establish the relation between dollar and pound:
Dollar-pound rate = Rupee-pound rate/Rupee-dollar rate

Jan	$\frac{57}{43} = 1.32$
Feb	$\frac{55}{44} = 1.25$
Mar	$\frac{58}{42} = 1.38$
Apr	$\frac{61}{42.5} = 1.44$
May	$\frac{63}{45} = 1.4$
Jun	$\frac{59}{47} = 1.26$

11. Required percentage

$$= 48{,}640 \times 22 \times 100 \approx 125\%\ 42{,}980 \times 20$$

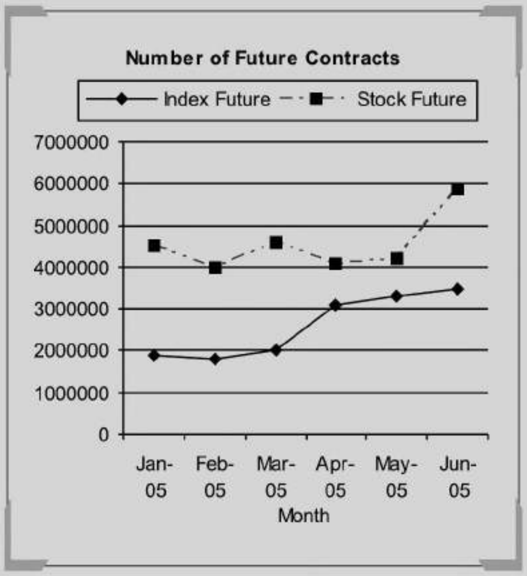
13. Required difference = 4864 – 2579 = 2285.

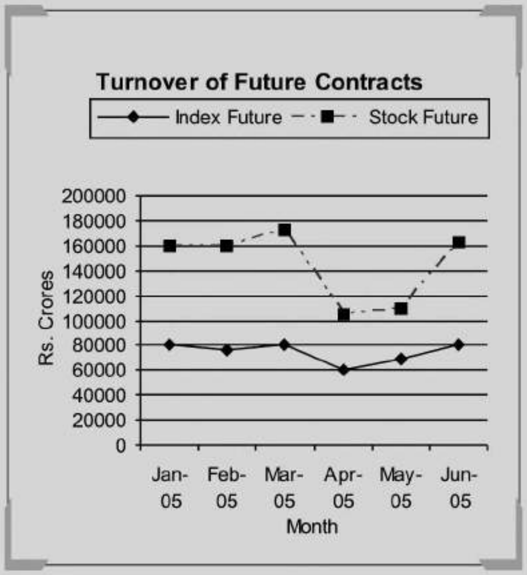
24. $I = E\left[\frac{100+\%\text{Profit}}{100}\right] = 200 \times 1.4 = 280$

25. $I > 2E$, so we can conclude that profit percentage is more than 100%. This is not correct for any of the given years.

PRACTICE EXERCISE 4

Direction for questions 1 to 7: Go through the charts given below and solve the questions based on it.





Q 1. The trend of stock and index futures contracts diverged in the month of
I. March to April II. April to May
III. May to June
(a) I and II (b) II and III
(c) I and III (d) Cannot be determined

Q 2. The difference in the growth or decline rates between number and turnover of futures contract in any given month is the highest for
I. Stock futures II. Index Futures
(a) I only (b) II only
(c) I and II (d) Insufficient Data

Q 3. The highest monthly percentage growth or decline occurred in the case of
(a) Number of stock futures contracts
(b) Number of Index futures contracts
(c) Turnover of stock futures contracts
(d) Turnover of index futures contracts

Q 4. The value of a contract is defined as the ratio of turnover to the number of contracts. The value of the contract was highest for both stock and index futures in
(a) January (b) February and March
(c) March (d) Insufficient data

Q 5. The highest growth rate between January and June occurred in the case of
(a) No. of stock futures contracts
(b) No. of index futures contracts
(c) Turnover of stock futures contracts
(d) Turnover of index futures contract

Q 6. Consistent growth pattern is observed for
(a) No. of stock futures contracts
(b) No. of index futures contracts
(c) Both stock and index futures contracts
(d) None of the above

Q 7. Which of the following is true?
(a) No. and turnover of stock futures follow a similar pattern
(b) No. of turnover of index futures follow a similar pattern
(c) No. of stock futures and turnover of index futures follow a similar pattern
(d) No. of index futures and turnover of stock futures follow a similar pattern

Direction for questions 8 to 11: Go through the table given below and solve the questions based on it.

All India Installed capacity (in MW) of power

Region	Sector	Total	Hydro	Thermal	Nuclear
Northern Region	State	17898	6401	11491	0
	Central	13642	3810	8652	1180
Western Region	State	19611	4180	15405	0
	Central	7387	875	5752	760
Southern Region	State	18744	10395	8318	0
	Central	7120	0	6340	780

All India Installed capacity (in MW) of power					
Region	**Sector**	**Total**	**Hydro**	**Thermal**	**Nuclear**
Eastern Region	State	7915	2256	5656	0
	Central	8552	204	8348	0
North Eastern Region	State	1098	274	824	0
	Central	1235	860	375	0
All India	State	65265	23505	41692	0
	Central	37936	5749	29467	2720

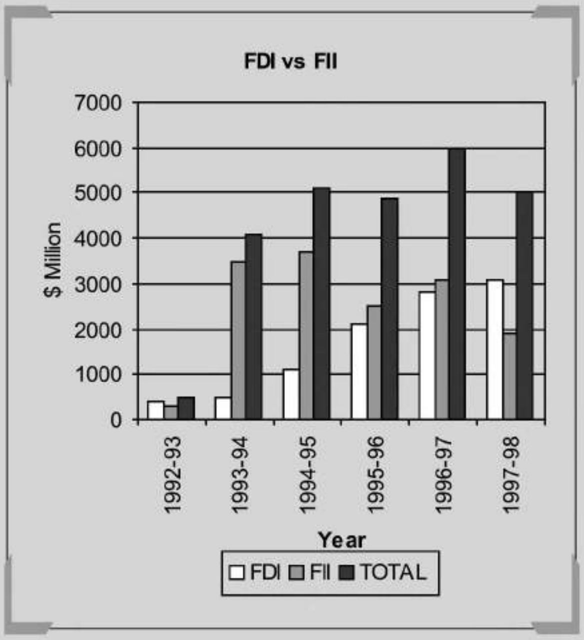
Q 8. Which region has the highest installed capacity by the central sector as compared to the state sector?
(a) Northern region (b) Southern region
(c) Eastern region (d) North Eastern region

Q 9. What is the percentage of nuclear power in total installed capacity in the country?
(a) 2.74 (b) 2.81
(c) 2.51 (d) 2.63

Q 10. In any sector, across the regions, the highest contribution from anyone of the three sources to the total installed capacity is
(a) 79% (b) 78%
(c) 89% (d) 90%

Q 11. Hydro power is the major source of energy in
(a) Northern and Southern regions
(b) Eastern and Western regions
(c) Southern and North Eastern regions
(d) None of these

Direction for questions 12 to 16: Go through the charts given below and solve the questions based on it.



Q 12. FII flows recorded negative growth rates in
(a) One Year
(b) Two Years
(c) Three Years
(d) Four Years

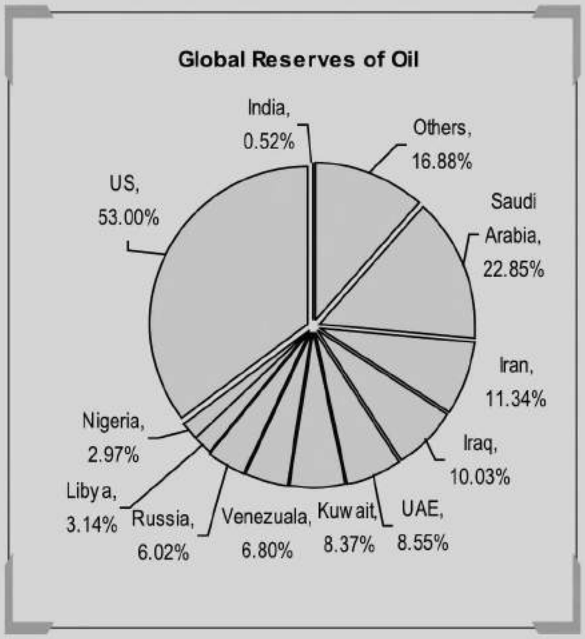
Q 13. FDI flows recorded highest growth between
(a) 1992–93 and 1993–94
(b) 1993–94 and 1994–95
(c) 1994–95 and 1995–96
(d) 1995–96 and 1996–97

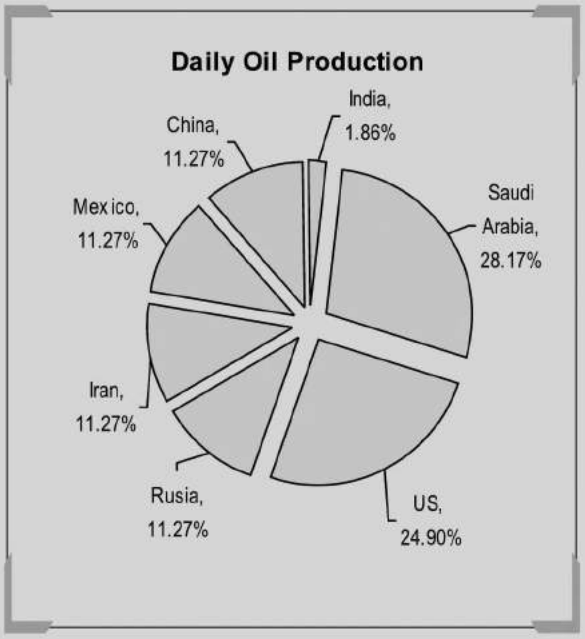
Q 14. FDI flows were greater than FII flows in
(a) 1992–93 and 1993–94
(b) 1996–97 and 1997–98
(c) 1993–94 and 1997–98
(d) 1992–93 and 1997–98

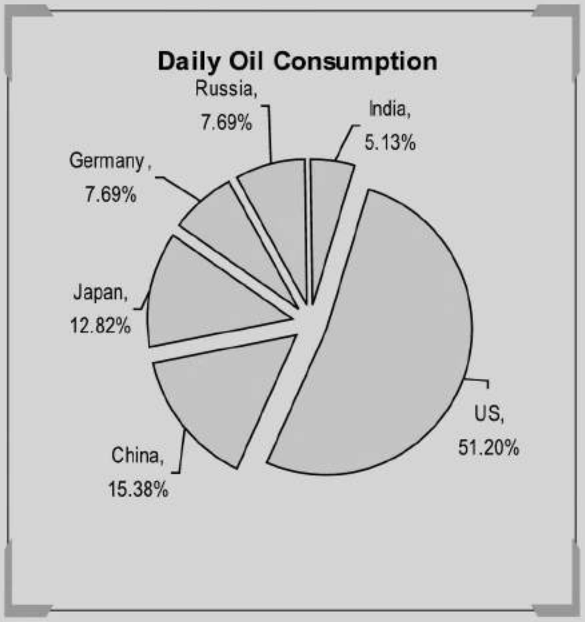
Q 15. Over the years, which of the following flows are highly volatile?
(a) FII flows
(b) FDI flows
(c) Total flows
(d) Both FII and total

Q 16. Difference between FII and FDI flows is maximum in the year
(a) 1992–93
(b) 1993–94
(c) 1994–95
(d) 1995–96

Direction for questions 17 to 25: Go through the charts given below and solve the questions based on it.







Q 17. If estimated global reserves of oil is 1146.49 billion barrels, the level of reserves located in the US (in billion barrels) would be approximately?

(a) 29	(b) 34
(c) 36	(d) 98

Q 18. The daily production and consumption of the US is 8.84 and 20 million barrels respectively. Assuming that this level remain constant, how long would the oil reserves in the US last? (Year = 360 days, Global reserves = 1146.49 billion barrels)

(a) 4 years	(b) 6 years
(c) 8 years	(d) 9 years

Q 19. The daily production and consumption figures of India are 0.66 and 2.2 million barrels. For which of the following countries the difference between consumption and production is the lowest?

(a) Iran	(b) India
(c) Saudi Arabia	(d) Cannot be determined

Q 20. How many days of India's oil consumption is equivalent to daily consumption of US?

(a) 8	(b) 10
(c) 11	(d) Insufficient data

Q 21. Which of the charts truly represent the global position?

(a) All the charts
(b) Two of the charts
(c) One of the charts
(d) Cannot be determined

Q 22. The reserves and annual production figures for India are 5.58 and 0.24 billion barrels respectively. Which country has the lowest ratio of annual production to reserves?

(a) USA
(b) Iran
(c) Saudi Arabia
(d) India

Q 23. The current reserves and the annual consumption of India is 5.58 and 0.72 billion barrels. If the expected annual growth rate of consumption for India is 2%, the oil reserves of India (in billion barrels) after 5 years would be?

(a) 1.74
(b) 1.76
(c) 1.78
(d) Cannot be determined

Q 24. The reserves located in Libya as percentage of reserves located in Russia is

(a) 50
(b) 52
(c) 54
(d) 56

Q 25. Which of the following is true?

(a) Saudi Arabia and US are the highest consumer and producer respectively
(b) Saudi Arabia and US are the highest producer and consumer respectively
(c) Russia is the lowest producer and consumer both
(d) None of these

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(b)	(c)	(b)	(d)	(a)	(d)	(d)	(d)	(d)	(b)	(c)	(d)	(d)	(b)	(a)	(c)	(d)	(b)
Q. No.	21	22	23	24	25															
Answer	(d)	(c)	(d)	(b)	(d)															

HINTS AND SOLUTIONS

1. As it is not given whether we need to scan the graph with respect to number or turnover of future contracts, so the answer cannot be determined.

2 We need to find out the individual growth rate or declined rate for number of future contracts (both stock and index) and turnover of future contracts (both stocks and index). After we have calculated this, we need to find out the difference in their growth/ declined rates.

Difference for index futures in between March–April is highest. So, (b) is the answer.

8. In this question, we have to simply find the value of (Central/State). And North-Eastern region is more than 1.5, which is highest.

9 Simple calculation involved. Percentage of nuclear power in total installed capacity = (Nuclear power/ total installed capacity) = (2720)/(103201) = 2.63%

10. It can be seen that highest contribution comes from thermal in eastern region, central sector.

12. 1995–1996 and 1997–98 are the two years.

15. Visual inspection. Bar of FII is sometimes getting up and down. Similar is the case with bar of total flows.

17. Estimated reserved = $1146.49 \times \frac{2.53}{100} = 29$

18. As calculated, US estimated reserve

= 29 billion barrels.

Daily net consumption of USA

= 8.84 – 20 = 11.16 million barrels.

So no. of years that the oil will cost

$$= \frac{29 \times 1000}{11.16 \times 360} = 8 \text{ years}$$

19. Difference between production and consumption figures for India = 2.2 – 0.66

= 1.54 billion barrels.

Russia = –0.7

Consumption for Saudi Arabia and Iran is not given.

So answer cannot be determined (d).

20. No. of days equivalent to one day of consumption of US

$$= \frac{51.28}{5.13} = 10$$

21. No information is given about the total reserves, daily oil production, daily oil consumption. So, we cannot infer the given.

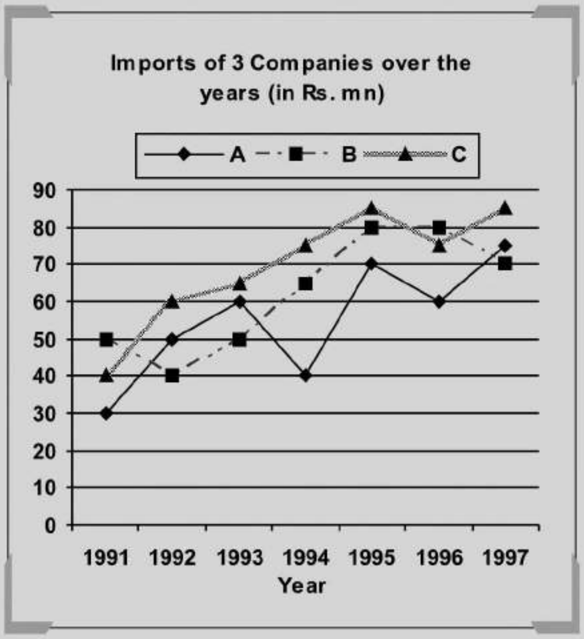
23. Since the production figures five years from now is not given, we cannot say any thing about the oil reserves.

24. Required Percentage = $\frac{3.14}{6.02} = 52.15$

25. Looking at the charts, we see that none the first three options is true.

PRACTICE EXERCISE 5

Direction for questions 1 to 5: Go through the line chart given below and solve the questions based on it.



In the above line chart, all the figures are in the multiples of five.

Q 1. In which of the following years, the imports made by company A was exactly equal to average imports made by it over the given years?
(a) 1992 (b) 1993
(c) 1994 (d) None of these

Q 2. In which of the following years was the difference between the import made by the company B and C the maximum?
(a) 1995 (b) 1994
(c) 1991 (d) 1992

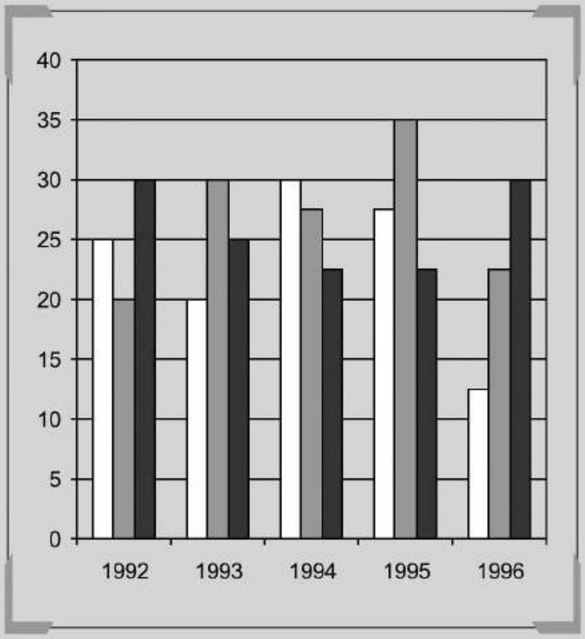
Q 3. In which of the following years was the import made by company A exactly half of the total imports made by company B and C together in that year?
(a) 1992 only (b) 1993 only
(c) 1992 and 1993 (d) 1995 only

Q 4. What was the percentage increase in imports by company B from 1992 to 1993?
(a) 10 (b) 25
(c) 40 (d) 50

Q 5. In which of the following years, the total exports made by all the three companies the maximum?
(a) 1994 (b) 1995
(c) 1996 (d) 1997

Direction for questions 6 to 10: Go through the bar chart given below and solve the questions based on it.

Production of three types of tea leaves over the five years (in mn kg)



All the values in the above chart are in the multiples of 2.5.

Q 6. What was the percentage drop in the C type tea leaves produced from 1993 to 1994?
(a) 25 (b) 10
(c) 20 (d) 25

Q 7. What was the difference between the B type tea leaves produced (in kg) in 1994 and 1995?
(a) 10,00,000 (b) 20,000,000
(c) 10,000,000 (d) None of these

Q 8. The total production of all the three types of tea leaves was the least in which of the following year?
(a) 1995 (b) 1996
(c) 1992 (d) 1994

Q 9. In which of the following years was the percentage production of B type tea leaves over the production of C type tea leaves the maximum?
(a) 1994 (b) 1992
(c) 1996 (d) 1995

Q 10. Which year the production of tea leaves of all the types the maximum?
(a) 1994 (b) 1992
(c) 1996 (d) 1995

Direction for questions 11 to 15: Go through the table given below and solve the questions based on it.

Following table gives the details of five leading opener's performance in 20 one-day cricket international matches:

Openers	Total runs	Highest Runs	No. of matches with runs		
			100 or more	50-99	0's
A	994	141	5	3	1
B	751	130	1	8	2
C	414	52	-	2	2
D	653	94	-	4	1
E	772	85	-	7	-

Q 11. What is the difference between the average runs of top two openers, if matches in which 0's are scored are not to be counted?
(a) 4.7
(b) 13.7
(c) 11.1
(d) None of these

Q 12. If matches having zero runs and the one with highest runs are ignored, what will be the average runs for opener C?
(a) 21.29
(b) 21.79
(c) 20.17
(d) 20.19

Q 13. What is the maximum number of 25 runs inning that could have been played by B?
(a) 6
(b) 8
(c) 9
(d) Cannot be determined

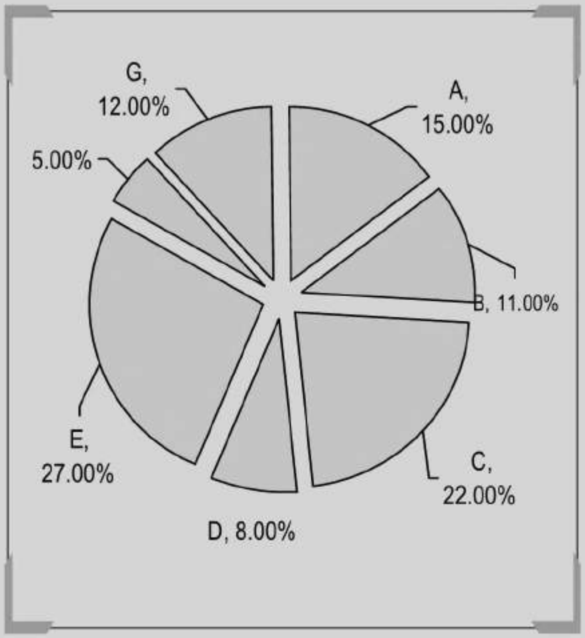
Q 14. Which of the given pairs of openers have ratio 3:2 in their highest runs?
(a) B and D (b) B and C
(c) A and D (d) D and C

Q 15. If each of the above given five openers have scored their highest run in a single match, then what is the average score per member of the team (there are eleven playing members in a cricket team)?
(a) 25 (b) 15
(c) 30 (d) Cannot be determined

Direction for questions 16 to 19: Go through the charts given below and solve the questions based on it.

There are seven companies—A, B, C, D, E, F and G, engaged in the production of naphtha of two types I and II. The comparatives data about production of these types by the seven companies is given in the following chart and table.

Percentage of the total production produced by the seven companies.



Cost of the total production (Both types together) by seven companies = Rs 25 crores and the production cost of each of the companies is same. Cost of individual company can be calculated using above pie-chart.

Ratio of production between types I and II and percentage profit earned for the two types:

Company	Ratio of Production		Percentage profit earned	
	Type I	Type II	Type I	Type II
A	2	3	25	20
B	3	2	32	35
C	4	1	20	22
D	3	5	15	25
E	5	3	28	30
F	1	4	35	25
G	1	2	30	24

Q 16. What is the total cost of the production of type I by companies A and C together in Rs Crore?
(a) 9.25 (b) 5.9
(c) 4.1625 (d) 4.9

Q 17. What is the amount of profit earned by company D on type II?
(a) Rs 3.125 cr
(b) Rs 31.25 cr
(c) Rs 3.125 lakh
(c) Rs 31.25 lakhs

Q 18. Cost of production of type I by company F is what percentage of the cost of production of type II by company D?
(a) 16%
(b) 33.33%
(c) 66.67%
(d) None of these

Q 19. What is the total profit (in Rs. lacs) earned by company G for types I and II together?
(a) 78 (b) 162
(c) 780 (d) 16.2

Direction for questions 19 to 22: Go through the table given below and solve the questions based on it.

The percentage change in the sales of six departmental stores from 1993 to 1995 is shown in the following table:

Store	Percentage change	
	From 1993 to 1994	From 1994 to 1995
UCB	+20	–20
VAN	–20	+9
Wrangler	+5	+12
X-tra	–7	–15
Yo	+17	–8
Zodiac	+21	+9

Q 19. In 1995 which of the stores had greater sales than any of the other stores?
(a) Van (b) Wrangler
(c) Zodiac (d) Cannot be determined

Q 20. Sales in store UCB amounted to Rs 80 lacs in 1993. What was the sales amount of the same store in 1995?
(a) Rs 96 lacs (b) Rs 74 lacs
(c) Rs 80 lacs (d) None of these

Q 21. In store Yo the sales for 1994 amounted to about what percentage of the sales for 1995?
(a) 117 (b) 109
(c) 101 (d) 113

Q 22. If in 1993 the sales in store Wrangler amounted to Rs X, the sales for that store in 1995 amounted to:
(a) 1.5X (b) 1.17X
(c) 1.32X (d) 1. 61X

Direction for question 23 to 26: Refer to the table below and answer the questions based on it.

Maruti has recently slashed the price of its car Alto. The following is the information on its previous and present price (ex. show room, Delhi)

	Old (Rs in Lakh)	New (Rs in Lakh)
Showroom price	6.30	4.90
(NAV)	3.98	3.12
Excise	1.59	1.25
Sales Tax	0.44	0.34
Dealer Commission	0.12	0.12
Company Margin	0.17	0.07

Q 23. What has been the fall in the showroom price of cars?
(a) 15% (b) 16.67%
(c) 22.2% (d) 25%

Q 24. Excise is levied as percentage of NAV. If NAV of a car is Rs 6.4 lakh, then what is its excise?
(a) Rs 1.4 lakh
(b) Rs 2.8 lakh
(c) Rs 2.2 lakh
(d) Rs 2.6 lakh

Q 25. As a percentage of the showroom price, the dealer commission has changed by how many percentage point?
(a) –0.35% (b) +0.55%
(c) –0.75% (d) +1.00%

Q 26. What is the drop in the sales tax collections per car?
(a) 16% (b) 18%
(c) 20% (d) 23%

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(d)	(a)	(b)	(d)	(b)	(d)	(b)	(d)	(d)	(d)	(a)	(b)	(c)	(d)	(b)	(c)	(d)	(d)	(d)
Q. No.	21	22	23	24	25	26														
Answer	(b)	(b)	(c)	(d)	(b)	(d)														

HINTS AND SOLUTIONS

1. Average imports made by company A = (30 + 50 + 60 + 40 + 70 + 60 + 75)/7 = 55

   In none of the given years the imports is equal to Rs 55 mn.

3. 1992 only is the desired year. What we are required to see here is the fact that A should lie exactly midway B and C.

4 . Required percentage increase $= \frac{50-40}{40} \times 100 = 25\%$

6. Required percentage Drop = $\frac{25-22.5}{25} \times 100 = 100\%$

7. Required difference = (35–22.5) lakh

   = 12,50,000

9. Production of B type leaves is more than the production of C type leaves only in the years 1993, 1994 and 1995. We see the largest difference exists in the year 1995. So, the answer is 1995.

11. Average runs of A $= \frac{994}{19} = 52.31$

    Average runs of B $= \frac{751}{18} = 41.72$

    Difference = 52.31 – 41.72 = 10.59

12. Average runs of C = $\frac{414-52}{17}$

    $= \frac{362}{17} = 21.29$

14. Ratio of A and D = 141 : 94 = 3:2

16. Total cost for A $= \frac{2}{5} \times 15\% \times 25$ crores

    Total cost for C $= \frac{2}{5} \times 15\% \times 25$ crores

    So, total cost of A & C = 5.9 crores

17. Amount of profit earned by company D on type II

    $= \frac{5}{8} \times \frac{8}{100} \times 25 \times \frac{25}{100} = 31.25$ lakhs

18. Cost of production of type I by company F

    $= \frac{1}{5} \times \frac{5}{100} \times 25 =$ Rs. 0.25 cr

    Cost of production of type II by company D

    $= \frac{5}{8} \times \frac{8}{100} \times 25 =$ Rs.1.25 cr.

    Therefore, required % $= \frac{0.25}{1.25} \times 100 = 20\%$

20. In 1993 the sales in store UCB amounted to Rs 80 lacs. In 1994 the sales in store UCB amounted to 120% of Rs 80 lacs = 96 lacs. In 1995 the sales in store U amounted to 80% of Rs 96 lacs = Rs 76.8 lacs.

21. If x is the amount of sales for store Yo in 1994, then 0.92x is the amount of sales for 1995. So, the answer is: 1/0.92 = 1.086 = 1.09

23. Percentage decrease in showroom price = 22.22%

24. Excise as the % of NAV = 1.59/ 3.98 × 100 = 40%

    Therefore, Excise is 40% of NAV.

    When NAV = Rs 6.4 lakhs,

    Excise = 40/100 × 6.4 = 2.56 lakhs.

25. Initial dealer commission as percentage of show-room price = 0.12/6.30 × 100 = 1.90%,

    And similarly final dealer commission on showroom price = 2.45%

    Therefore, the dealer commission has risen by

    2.45 – 1.90 = 0.55%

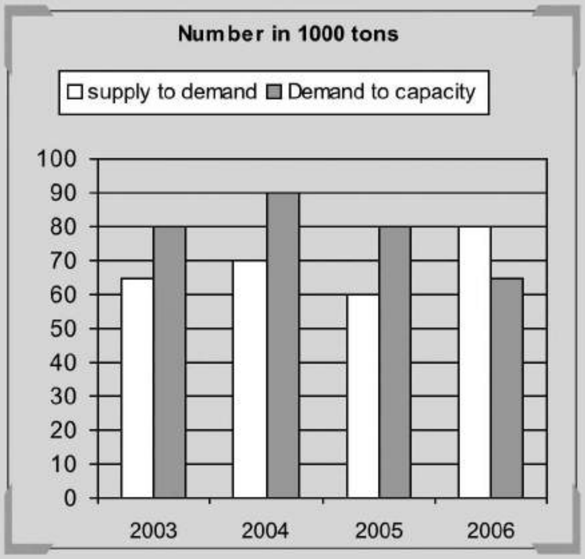
26. Percentage decrease in sales tax

    = (0.44 – 0.34)/0.44 = 23% (Approx.)

PRACTICE EXERCISE 6

Direction for questions 1 to 6: Go through the chart given below and solve the questions based on it.

The following bar chart represents supply as a percentage of demand of a product as well as the demand as a percentage of capacity of the industry to produce the same product.



All the values are in the multiple of 5.

Q 1. If the product supply for the year 2005 is 6000 kgs, then the total capacity of the product is (in kg)
(a) 12500
(b) 16500
(c) 10000
(d) None of these

Q 2. If the capacity of the industry to produce the product remains constant at 10000 kgs, what is the approximate average supply (in kgs) for the given period?
(a) 5100 (b) 5800
(c) 4400 (d) 6200

Q 3. If the product supply remains unchanged for a given period then what does the trend of the industry capacity signify?
(a) The industry capacity steadily increases over the given period.
(b) The industry capacity steadily decreases over the given period
(c) The industry capacity steadily fluctuates over the given period.
(d) None of these

Q 4. Which year shows the highest increase in the difference between the industry capacity and supply over the previous year, if demand for the product is 10,000 kgs for each of the given years?
(a) 2003
(b) 2004
(c) 2005
(d) 2006

Q 5. If we interchange the bars of 'supply to demand' and 'demand to capacity' with each other, then which of the bars will represent inconsistency in data?
(a) 2003
(b) 2004
(c) 2005
(d) None of these

Q 6. If the supply is constant for each of the years given above, then for which year is the capacity highest?
(a) 2003
(b) 2004
(c) 2005
(d) 2006

Direction for questions 7 to 9: Given below is a table showing percentage out of a total of 700 employees ranking six attributes that help in promotion. Rank I is the highest. Study the table carefully and answer the questions that follow:

No employee can give the same rank to two or more than two attributes.
Rank 1 = 6 points. rank 2 = 5 points, and so on rank 6 = 1 point.
Higher the points, more is the chance of promotion.

Attribute	Percentage of Employees Giving Different Ranks					
	I	II	III	IV	V	VI
Seniority	32	17	22	19	5	5
Perservance	14	19	17	9	27	14
Efficiency	15	19	21	14	14	17
Intelligence	10	14	10	14	17	35
Honesty	24	17	7	9	27	16
Sociability	5	14	23	35	10	13

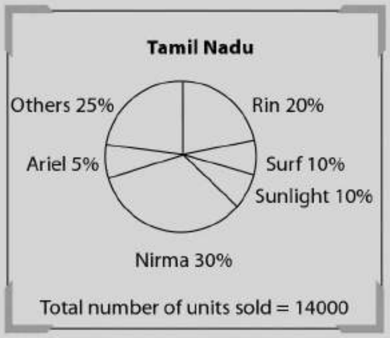
Q 7. Which attribute for promotion has received the highest rank?
(a) Perservance (b) Seniority
(c) Honesty (d) Sociability

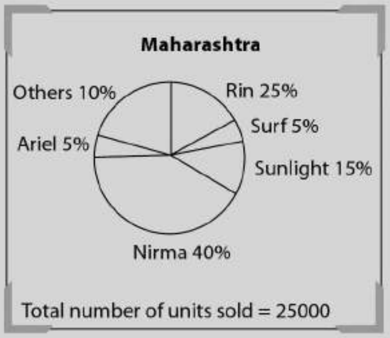
Q 8. How many employees gave rank III to intelligence?
(a) 119 (b) 98
(c) 77 (d) 70
(e) 10

Q 9. Which attribute is considered the least importance for promotion?
(a) Honesty (b) Intelligence
(c) Perservance (d) Efficiency
(e) Sociability

Direction for questions 10 to 12: Go through the pie charts given below and solve the questions based on it.

The market shares of various detergent brands in the year 2008 in the states of Tamil Nadu and Maharashtra are given below:





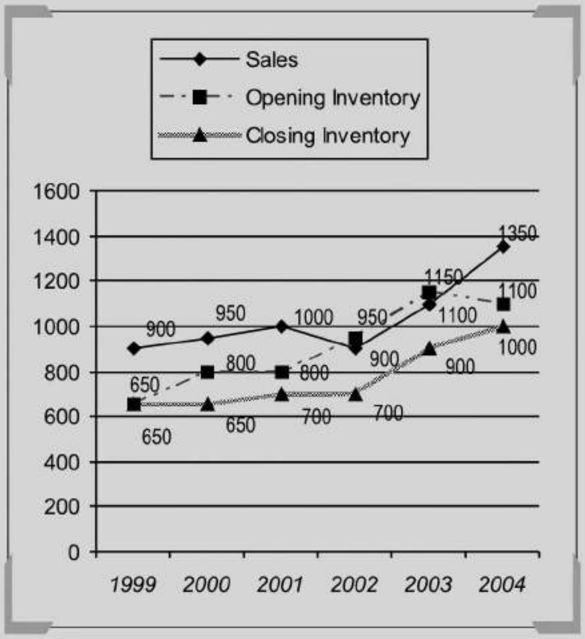
These are the top brands in these 2 states.

Q 10. Which brand has the largest difference in the units sold in the two given states?
(a) Rin
(b) Surf
(c) Sunlight
(d) None of these

Q 11. Total sale of Sunlight in the two states is:
(a) 4,850 units
(b) 5,000 units
(c) 5,150 units
(d) 5,300 units

Q 12. Which detergent is having the maximum sales in both the states taken together?
(a) Ariel
(b) Nirma
(c) Sunlight
(d) Cannot be determined

Direction for questions 13 to 15: Go through the chart given below and solve the questions based on it.



Sales = Opening Inventory + Production – Closing Inventory

Q 13. What is the percentage growth in average annual sales of the last 3 years versus average sales of first 4 years?
(a) 21% (b) 19%
(c) 24% (d) 23%
(e) None of the above

Q 14. What is the difference between the average closing inventory and the average opening inventory over the given period?
(a) 57
(b) 54
(c) 59
(d) 53
(e) None of the above

Q 15. If production figures are plotted on the graph, then the shape of the production line
(a) forms a linear
(b) resembles more like the sales curve
(c) resembles more like the opening inventory curve
(d) resembles more like the closing inventory curve
(e) None of the above

Direction for questions 16 to 20: Percentage of marks obtained by seven students in six subjects

	History (75)	Geography (75)	Maths (150)	Science (200)	English (100)	Hindi (50)
A	84	72	81	67	62	68
B	76	58	87	73	65	72
C	59	64	75	61	71	56
D	69	78	64	58	61	66
E	88	68	93	83	74	78
F	82	66	89	91	77	68
G	78	72	66	71	75	80

Note: The figures in the brackets are the maximum marks for each subject.

Q 16. What is the total marks obtained by student 'A' in History, Geography and Science together?

(a) 261 (b) 245
(c) 342 (d) 251
(e) None of these

Q 17. What is the percentage of total marks obtained by student 'B' in all subjects together?

(a) 71.83 (b) 73.54
(c) 76.84 (d) 72.75
(e) None of these

Q 18. What is the average marks obtained by the seven students in Maths?

(a) 126
(b) 119
(c) 115
(d) 111
(e) Cannot be determined

Q 19. What is the percentage of marks obtained by student 'E' in History Geography, Maths and Hindi together?

(a) 93.2
(b) 89.6
(c) 81.4
(d) 84.4
(e) Cannot be determined

Q 20. What is the difference of the marks obtained by C & G is all subjects?

(a) 42.75
(b) 96.25
(c) 112.75
(d) No difference
(e) None of these

Direction for question 21 to 25: Study the following table carefully and answer the questions given below:

Number of students appeared and qualified in an examination from five states over the years 1993 to 1998

	A		B		C		D		E	
	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
1993	950	230	880	200	1075	315	975	260	1120	325
1994	1090	320	960	250	1125	290	990	305	1208	308
1995	860	240	1100	330	950	275	1400	375	1375	380
1996	1200	350	1250	290	1240	325	1175	310	1060	265
1997	1150	300	1300	325	1220	340	1080	295	995	235
1998	980	280	1080	336	1140	305	1150	280	1075	290
Total	6230	1720	6570	1731	6750	1850	6770	1825	6833	1803

Q 21. Approximately, what is the percentage of students that qualified from the states B and C together in the year 1996?

(a) 32% (b) 20%
(c) 25% (d) 30%
(e) 35%

Q 22. The total number of students that appeared from state 'A' from 1996 to 1998, is approximately what percentage of the total number of students appeared from that state from 1993 to 1995 i.e.,

(a) 98% (b) 105%
(c) 85% (d) 87%
(e) 115%

Q 23. The total number of students the qualified from all the states in 1998 is approximately what percentage of the number of students appeared from all the states in that year i.e.,

(a) 25%
(b) 28%
(c) 32%
(d) 30%
(e) 35%

Q 24. The number of students that qualified from state 'D' from 1993 to 1995 together is approximately what percentage of the students appeared from the state 'D' in these three years i.e.,

(a) 27% (b) 32%
(c) 30% (d) 28%
(e) 25%

Q 25. The percentage of students qualified is the highest for which of the following states for all the six years together?

(a) A (b) B
(c) C (d) D
(e) E

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(a)	(c)	(c)	(d)	(c)	(b)	(d)	(e)	(d)	(c)	(b)	(b)	(e)	(e)	(d)	(b)	(b)	(d)	(e)
Q. No.	21	22	23	24	25															
Answer	(c)	(e)	(b)	(d)	(a)															

HINTS AND SOLUTIONS

[illegible]

1. Supply is 60% of the demand and demand is 80% of the production. Supply = 6000 kgs, Hence demand = 10,000 kgs, Hence capacity = 12,500 kgs

5. Data becomes inconsistent if the supply exceeds the production. This will not be observed for any of the years.

[illegible]

13. Average annual sales of the last three years
= 3350/3 = 1116.67
Average annual sales of the first four years
= 3750/4 = 937.5
Percentage growth = 19.11%

14. Average closing inventory = 766.66
Average opening inventory = 908.33
Difference = 141.66

15. The corresponding figures for the production is – 900, 800, 900, 650, 850, 1350 over the corresponding years. Obviously graph pertaining to these values will not resemble to any of the given graphs.

[illegible]

17. Marks obtained by B in history = 57
Marks obtained by B in Geography = 43.5
Marks obtained by B in Maths = 130.5
Marks obtained by B in Science = 146
Marks obtained by B in English = 65
Marks obtained by B in Hindi = 36
Total marks obtained by B = 478
Maximum marks = 650
Percentage marks obtained = 73.54%

19. Marks obtained by E in history = 66
Marks obtained by E in Geography = 51
Marks obtained by E in Maths = 139.5
Marks obtained by E in Hindi = 39
Total marks obtained by E in the given four subjects
= 295.5
Maximum marks in these four papers = 350
Percentage marks obtained = 84.4 %

Explanations

21. Total number of students appeared from states B and C together in 1996 = 2490
    Total number of students passed from states B and C together in 1996 = 615

22. Total no. of students appeared from state A from 1996 to 1998 = 3330. Total no. of students appeared from state A from 1996 to 1998 = 2900

23. Total number of students qualified from all the states in 1998 = 1491. Total number of students appeared from all the states in 1998 = 5425. Percentage required = 28% (approx.)

24. Total number of students qualified from state D from 1993 to 1995 = 940. Total number of students appeared from state D from 1993 to 1995 = 3395

PRACTICE EXERCISE 7

Direction for questions 1 to 5: Read the passage below and solve the questions based on it.

Following is the points scored by all the teams participated in Benson and Hedges World Cup cricket. Total number of matches played by any team is equal to the total number of wins, total number of matches lost, total tied matches and no result matches. No result matches are those matches which could not be completed due to rains.

Team	Points	Played	Win	Loss	No result	Tied
New Zealand	14	8	7	1	0	0
England	11	8	5	2	1	0
South Africa	10	8	5	3	0	0
Pakistan	9	8	4	3	1	0
Australia	8	8	4	4	0	0
West Indies	8	8	4	4	0	0
India	5	8	2	5	1	0
Sri Lanka	5	8	2	5	1	0
Zimbabwe	2	8	1	7	0	0

Q 1. In how many matches, there was no result?
(a) 2 (b) 4
(c) 0 (4) Cannot be determined

Q 2. Top six teams will automatically qualify for the next world cup cricket to be played in India in 1996. To find out the top teams, following is the criteria to select the top six teams based upon the priorities:

Priority 1 – More Points

Priority 2 – More Win

Priority 3 – Less number of losses

Which of the following teams will not be selected?
(a) Australia (b) Sri Lanka
(c) West Indies (d) None of these

Q 3. What is the number of points scored by a win?
(a) 1 (b) 2
(c) 3 (d) Cannot be determined

Q 4. New Zealand must have won its matches against all the teams except:
(a) England (b) South Africa
(c) Australia (d) Cannot be determined

Q 5. In how many matches, was it possible to find the winner of the match?
(a) 36 (b) 32
(c) 34 (d) None of these

Direction for questions 6 to 10: Read the passage below and solve the questions based on it.

Following is the medals obtained by different countries participating in the 2004 summer Olympics.

There is only one gold medal, one silver medal and one bronze medal associated with any event.

Additional information for questions 6–8: Use the information given below to answer the given questions.

Ranking of the countries is done on the basis of gold medals won by the country. If the tally of the gold medals is same, then number of silver medals won decides the rank. If both the number of gold medals and silver medals won are same, then the ranking is decided on the basis of bronze medal. If all these three medals won by any two or more than two countries are same, then they are awarded the same rank.

Country	Gold	Silver	Bronze	Total
United States	12	9	7	28
Australia	7	5	3	15
Japan	3	1	4	8
Holland	2	3	2	7
Ukraine	2	0	1	3
France	1	2	3	6
Iceland	1	2	0	3
South Africa	1	1	1	3
Zimbabwe	1	1	1	3
China	1	1	0	2
Romania	1	0	1	2
Austria	0	2	0	2
Germany	0	1	4	5
Hungary	0	1	1	2

Country	Gold	Silver	Bronze	Total
Italy	0	1	1	2
Croatia	0	1	0	1
Russia	0	1	0	1

Q 6. Which of the following ranks will be obtained by more than one country?

(a) 7 (b) 8

(3) 10 (4) 11

Q 7. How many countries will be ranked 17?

(a) 0 (b) 1

(c) 2 (d) Cannot be determined

Q 8. At how many ranks, more than one country will stand at?

(a) 0 (b) 1

(c) 2 (d) None of these

Q 9. Ranking of the countries is done on the basis of total medals won by the country.

Which country will be ranked 5 according to this scheme of ranking?

(a) Japan

(b) Ukraine

(c) France

(d) None of these

Q 10. In the above question, what will be the rank of Germany?

(a) 6

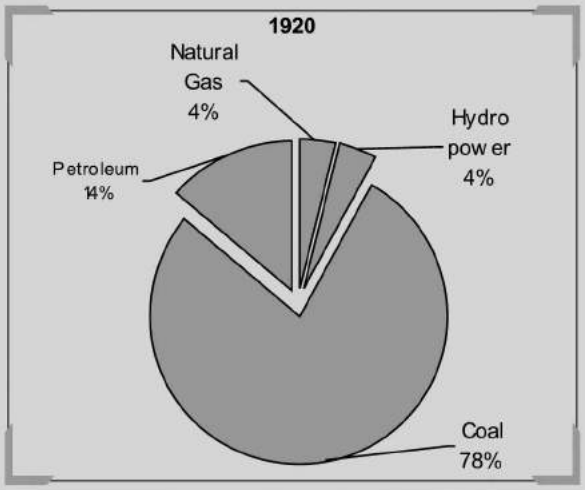
(b) 7

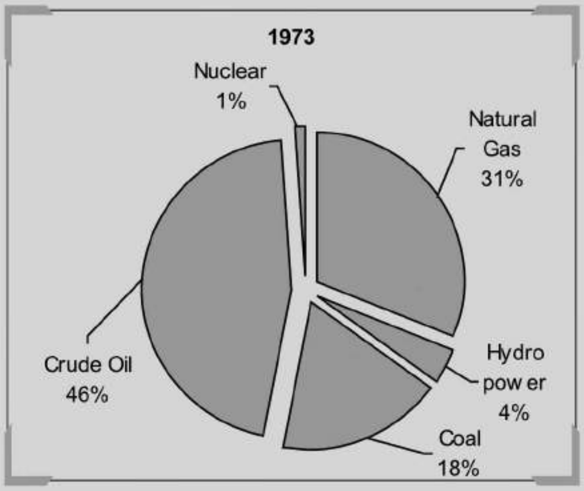
(c) 8

(d) None of these

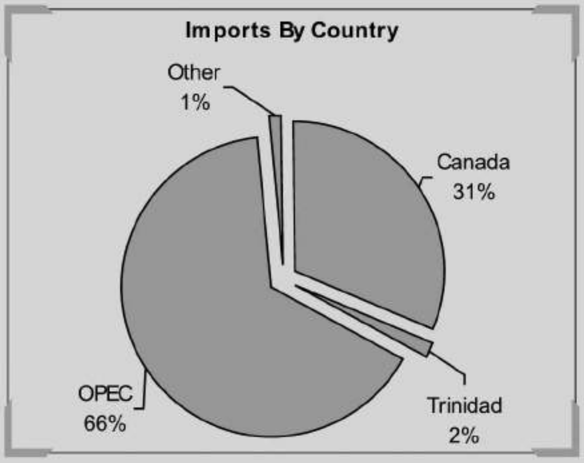
Direction for questions 11 to 14: Go through the charts given below and solve the questions based on it.

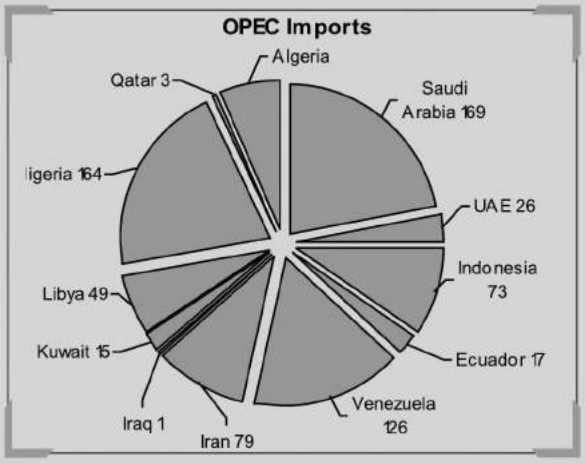
Sources of US Energy Supply: 1920 and 1973





1973 Crude Oil Imports to US (millions of barrels)





Q 11. What percentage of US energy supply in 1973 is fulfilled through the crude oil imports from OPEC?

(a) 46% (b) 65.5%

(c) 30.13% (d) cannot be determined

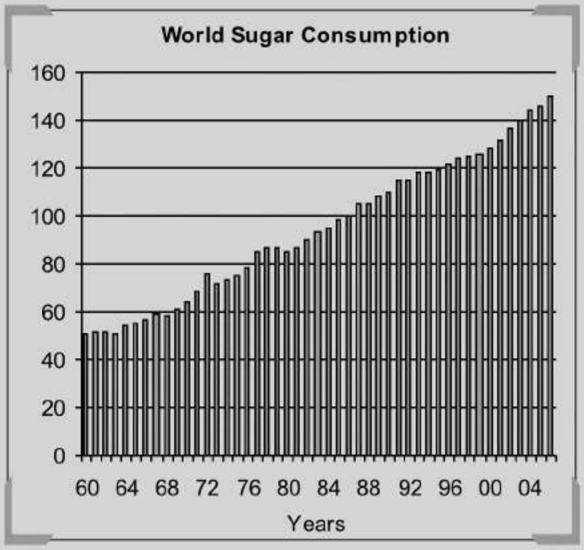
Q 12. Which of the following countries has a bigger share in crude oil imports to US in 1973 than the share of Canada same year?

(a) Saudi Arabia (b) Venezuela
(c) None (c) Cannot be determined

Q 13. Saudi Arabia supplies what percentage of total crude oil imports in 1973 to US?
(a) 28.7% (b) 34.4%
(c) 20.6% (d) Cannot be determined

Q 14. Which of the following can definitely be concluded after looking at the US energy supply in 1920 and 1973?
(a) No new source to generate hydropower has been identified.
(b) Total usage of coal has decreased over the period.
(c) At least one new source to generate nuclear power has been identified.
(d) None of these

Direction for questions 15 to 18: Go through the chart below and solve the questions based on it.



Q 15. For how many years during the given period consumption has increased over the previous year?
(a) 24
(b) 25
(c) 26
(d) None of these

Q 16. How many years the world sugar consumption has been more than 125 million tonnes per year?
(a) 7
(b) 8
(c) 9
(d) None of these

Q 17. Average sugar consumption per year is to be calculated for the given years. This value is most likely to be approximately equal the value of a year in between
(a) 75-80
(b) 80-90
(c) 90-95
(d) None of these

Q 18. For how many years the world sugar consumption has been more than 75 million tonnes and less than 100 tons per year?
(a) 9
(b) 10
(c) 11
(d) 12

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Answer	(a)	(b)	(b)	(d)	(c)	(b)	(a)	(c)	(c)	(a)	(d)	(c)	(d)	(c)	(d)	(d)	(b)	(c)

HINTS AND SOLUTIONS

Following is the medal table:

Rank	Nation	Gold	Silver	Bronze	Total
1.	United States	12	9	7	28
2.	Australia	7	5	3	15
3.	Japan	3	1	4	8
4.	Holland	2	3	2	7
5.	Ukraine	2	0	1	3
6.	France	1	2	3	6
7.	Iceland	1	2	0	3
8.	South Africa	1	1	1	3
9.	Zimbabwe	1	1	1	3
10.	China	1	1	0	2
11.	Romania	1	0	1	2
12.	Austria	0	2	0	2
13.	Germany	0	1	4	5
14.	Hungary	0	1	1	2
15.	Italy	0	1	1	2
16.	Croatia	0	1	0	1
17.	Russia	0	1	0	1

11. It is not given that what percentage of US energy supply is imported and what percentage is indigenous. Hence, cannot be determined.

12. Despite the value of Algeria is not given, it can be calculated. For the value to be more than that of Canada's value, it should be more than 31.3%, i.e., approximately a bit less than half of the OPEC countries share. Obviously it is not the case. Hence no such country is there.

13. Since the value for Algeria is not given, we do not have the data for total OPEC. Hence, cannot be determined.

14. Statement 1 and statement 2 need not be true always because it is percentage share of total.
    Statement 3 has to be true because nuclear power was not present in 1920, and it is present in 1973.

15. After looking at the options, one is not expected to find the actual value in this question. And it can be seen very easily that the years have to be more than 26 years out of the given 60-06.

17. It is difficult to calculate the actual average without having the values of individual years. But looking at the options, it has to be between one of the years in the block of 80-90, as the values are constantly increasing and the mid-value will be obtained in between 80-90.

PRACTICE EXERCISE 8

Direction for questions 1 to 4: Go through the table given below and solve the questions based on it.

Table given below presents the different wars fought by US with total cost in dollars and cost per person per year (CPP). Cost per person per year is calculated by dividing total cost by average US population per annum during that period.

Today's Price of US wars

	Total Cost (Billions in $)	CPP	
Revolutionary War (1775-83)	2.4	481	
			Period 1
War of 1812 (1812-15)	1.2	129	
			Period 2
Mexican War (1846-48)	1.7	73	
			Period 3
Civil War (1861-65)	6.7	1,814	
			Period 4
Spanish-Americal war (1998)	10.3	118	
			Period 5
World War I (1917-18)	205.1	2,678	
			Period 6
World War II (1941-45)	3,116.4	21,937	
			Period 7
Korean War (1950-53)	361.4	2,438	
			Period 8
Vietnam War (1964-72)	531.9	2,372	
			Period 9
Gulf War (1990-91)	81.9	329	
			Period 10
Irag/Afganistan War (2001-)	549.0	1,829	

Q 1. Which period saw the maximum percentage increase in the population of US?

(a) Period 3 (b) Period 5

(c) Period 6 (d) Cannot be determined

Q 2. Price of dollar remains constant during this whole period. Assuming that any war started on 1st January of the given year and ended on 31st December of the given year, then which war resulted into maximum US cost per annum?

(a) World war 1

(b) World war 2

(c) Iraq/Afghanistan

(d) Cannot be determined

Q 3. The price of dollar remains constant during this whole period. Which period of war corresponds to the maximum US population per annum?

(a) Revolutionary war

(b) World war 1

(c) World war 2

(d) None of these

Q 4. Price of dollar remains constant during this whole period. Which period of war corresponds to the minimum US population per annum?

(a) Revolutionary war

(b) World war 1

(c) World war 2

(d) None of these

Direction for questions 5 to 9: Go through the table given below and solve the questions based on it.

Year		Export	% Growth over pervious year
2000-2001	Quantity (In Tonnes)		28.41
	Value (In Rs Crore)		25.94
	Value (In Millions US$)		19.11
2001-2002	Quantity (In Tonnes)	424470	(–) 3.63
	Value (In Rs Crore)	5957.05	(–) 7.56
	Value (In Millions US$)	1253.35	(–) 11.51

Year		Export	% Growth over pervious year
2002-2003	Quantity (In Tonnes)		10.09
	Value (In Rs Crore)		15.52
	Value (In Millions US$)		13.69
2003-2004	Quantity (In Tonnes)	267546	
	Value (In Rs Crore)	3911.00	
	Value (In Millions US$)	847.00	

Q 5. What was the approximate value of exports (in million US $) in 2000-2001?
(a) 1360
(b) 1415
(c) 1520
(d) 1484

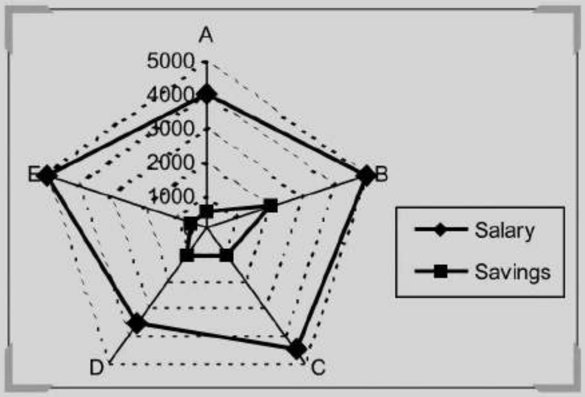
Q 6. What is difference between the quantity (in tonnes) exported in 2000-2001 and 2003-2004?
(a) 133000
(b) 153000
(c) 173000
(d) 193000

Q 7. For which of the following years, the value (in Rs) per unit weight is the lowest?
(a) 2000-2001
(b) 2001-2002
(c) 2002-2003
(d) 2003-2004

Q 8. For which of the following years, the price of dollar is maximum?
(a) 2000-2001
(b) 2001-2002
(c) 2002-2003
(d) 2003-2004

Q 9. What is the percentage decrease in the export value (in million US $) in 2003-04 over than that of 2000-2001?
(a) 20%
(b) 30%
(c) 40%
(d) 50%

Direction for questions 10 to 14: Go through the radar diagram given below and solve the questions based on it.



The above given radar diagram gives the salary and savings of five persons A, B, C, D and E for the month of July'06. All the values are in the multiples of Rs 500.

Salary – Savings = Expenditure

Q 10. Whose expenditure is the maximum for the month of July'06?
(a) B (b) C
(c) D (d) E

Q 11. Whose expenditure is minimum for the month of July'06?
(a) B (b) C
(c) D (d) E

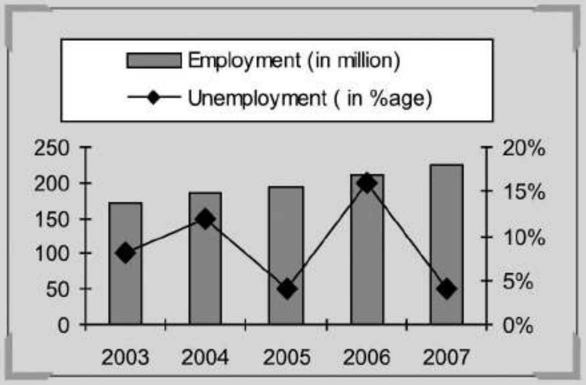
Q 12. Salary of B increases by 20% in the month for the month of August 06 than the month of July 06. However expenditure of B increases only by 10% in the month of August 06 than the month of July 06. By what percentage the savings of B has increased in the month of August 06 than the month of July 06?
(a) 33.33% (b) 23.66%
(c) 26.66% (d) None of these

Q 13. For who among the following, expenditure expressed as a percentage of salary for the given period the maximum?
(a) B (b) C
(c) D (d) E

Q 14. For how many of the given persons, the savings is more than 20% of the salary for the given period?
(a) 1.2 (b) 2.3
(c) 4 (d) 5

Direction for questions 15 to 20: Go through the chart given below and solve the questions based on it.

Following chart presents the employment scenario in India in different years:



Employment is in the multiples of 5 million and unemployment percentage is in the multiples of 4%. People can be either employed or unemployed.

Q 15. For which of the given years the number of persons unemployed is maximum?
(a) 2003 (b) 2005
(c) 2006 (d) 2007

Q 16. What is the total population of India in 2005 (in million)?
(a) 190 (b) 203
(c) 215 (d) 223

Q 17. For which year, the population of India is maximum?
(a) 2003 (b) 2005
(c) 2006 (d) 2007

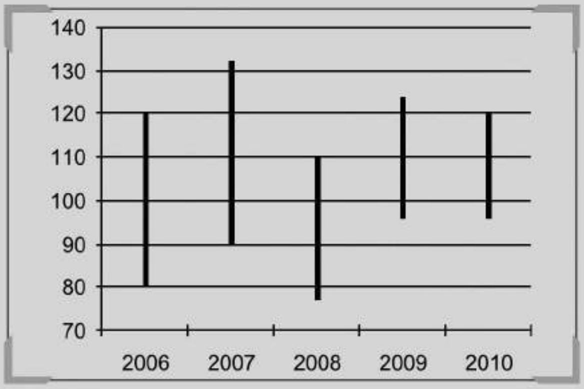
Q 18. If the percentage change in the population of India in 2008 is same as that of 2007 and the rate of unemployment in 2008 is 10%, then how many people in 2008 are employed in India (in million)?
(a) 185 (b) 191
(c) 197 (d) 207

Q 19. In the above question, total number of persons employed in the government sector to the non-government sector is in the ratio 2 : 3. What is the number of persons employed in government sector in 2007?
(a) 93 mn (b) 139 mn
(c) 162 mn (d) Cannot be determined

Q 20. How many of the following statement/s is/are correct?
I. The population of India is constantly on a rise during the given period every year.
II. The number of unemployed persons is constantly on a rise during the given period every year.
III. The total number of unemployed persons is decreasing every year by atleast 10% over the unemployed persons in the previous year.
(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 21 to 23: Go through the chart given below and solve the questions based on it.

In the chart given below, bottom of the bar represents the productivity of the company and the top of the bar presents the production (in million units) of the company for that particular year. Productivity is production as a percentage of production capacity.

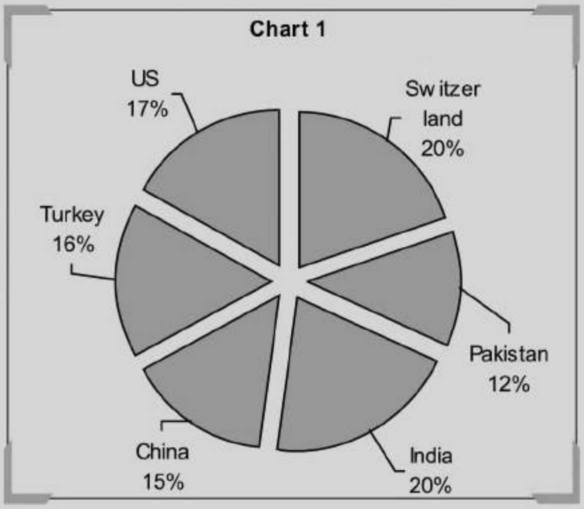


Q 21. For which year is the production capacity of the company the maximum?
(a) 2006
(b) 2007
(c) 2008
(d) None of these

Q 22. For which year is the production capacity of the company the minimum?
(a) 2007 (b) 2008
(c) 2009 (d) 2010

Q 23. Which of the following statement is true?
(a) The production capacity of the company is constantly decreasing year by year.
(b) The purnover of the company is constantly on a rise.
(c) Production capacity of the company is decreasing first and then increasing.
(d) None of these

Direction for questions 24 to 25: Go through the chart given below and solve the questions based on it.



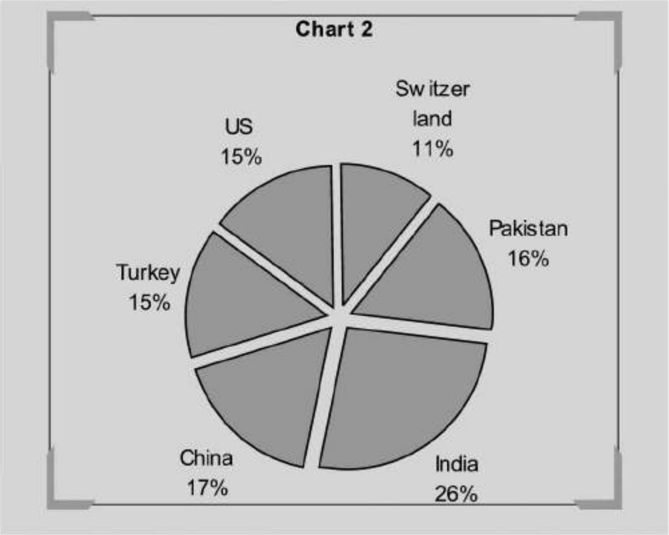


Chart 1 shows the distribution by value of the top 6 suppliers of MFA textiles in 1995. Chart 2 show the distribution by quantity of top 6 suppliers of MFA textiles in 1995. The total value is Rs 5760 million. The total is 1.055 million tonnes.

Q 24. The country which has the lowest average price is
(a) US
(b) India
(c) Pakistan
(d) None of these

Q 25. The average price in Rs per kilogram for Turkey is approximately
(a) 3.20
(b) 4.60
(c) 5.60
(d) 7.20

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(d)	(a)	(b)	(c)	(a)	(c)	(c)	(d)	(c)	(c)	(d)	(b)	(c)	(b)	(c)	(c)	(d)	(a)
Q. No.	21	22	23	24	25															
Answer	(a)	(d)	(a)	(c)	(c)															

HINTS AND SOLUTIONS

1. Since it is not given that price of dollar changes at what rate, so we cannot determine the population of US during different periods. Hence, cannot be determined.
2. It can be simply calculated by dividing total cost by number of years involved in war. Value for world war 2 is more than even Iraq/Afghanistan war(assuming it lasts only for one year).
3. To solve this question, we are supposed to divide total cost by CPP. It is maximum for Iraq/Afghanistan.
4. Using the process given in Q 3. Minimum is obtained for Revolutionary war.
   This is the table having all the data:

Year		Export	% Growth over pervious year
2000-2001	Quantity (In Tonnes)	440473	28.41
	Value (In Rs Crore)	443.89	25.94
	Value (In Millions US$)	1416.32	19.11

Year		Export	% Growth over pervious year
2001-2002	Quantity (In Tonnes)	424470	(–) 3.63
	Value (In Rs Crore)	5957.05	(–) 7.56
	Value (In Millions US$)	1253.35	(–) 11.51
2002-2003	Quantity (In Tonnes)	467297	10.09
	Value (In Rs Crore)	6881.31	15.52
	Value (In Millions US$)	1424.90	13.69
2003-2004	Quantity (In Tonnes)	267546	
	Value (In Rs Crore)	3911.00	
	Value (In Millions US$)	847.00	

8. We will divide the value (in Rs crores) by value (in million US $) of the corresponding year. It comes maximum for 2002—2003.

   For

   2001 – 2002 = Rs 47.5

   For

   2002 – 2003 = Rs 48.3

   For

   2003 – 2004 = Rs 46.1

9. Using the above table, it can be calculated that

   $$\text{percentage decrease } = \frac{1416-847}{1416}\times 100 = 40\%$$

[illegible]

This is the table pertaining to the data given above:

	A	B	C	D	E
Salary	4000	5000	4500	3500	5000
Savings	500	2000	1000	1000	500
Expenditure	3500	3000	3500	2500	4500

Now all the questions can be answered.

[illegible]

This is the table pertaining to the data given.

	2003	2004	2005	2006	2007
Employment (in million)	170	185	195	210	225
Unemployment (in %age)	8%	12%	4%	16%	4%
Population (in million)	184	210	203	250	234

Now all the questions can be answered.

18. The percentage change in the population in 2007 over 2006 = 6.4%. Hence, the population in 2008 = 219 mn

    10% unemployment means 90% people are employed.

    Hence, number of persons employed = 0.9 × 219 = 197 mn

19. The data given in Q,18 is about 2008, whereas Q, 19 is about 2007. Hence, cannot be determined.

20. None of the statements is correct.

[illegible]

	2006	2007	2008	2009	2010
Production (In million units)	120	132	110	124	120
Productivity	80	90	77	96	96
Production Capacity (In million units)	150	146.66	142.85	129	125

PRACTICE EXERCISE 9

Direction for questions 1 to 5: Go through the data set given below and solve the questions based on it.

S. No.	Station Name	Arrival Time	Dep. Time	Distnace	Day
1.	New Delhi	Source	16:25	0	1
2.	Aligarh	19:05	19:10	132	1
3.	Tundla	20:15	20:25	210	1
4.	Etawah	21:28	21:30	302	1
5.	Kanpur Central	23:15	23:25	440	1
6.	Allahabad	02:00	02:10	634	2
7.	Mughal Sarai	04:25	04:40	787	2
8.	Buxar	05:52	05:54	881	2
9.	Ara	06:40	06:42	949	2
10.	Danapur	07:14	07:16	989	2
11.	Patna	07:45	07:55	999	2
12.	Bakhtiyarpur	08:33	08:35	1044	2
13.	Mokameh	09:18	09:20	1088	2
14.	Kiul	09:55	09:57	1122	2
15.	Jamui	10:18	10:20	1150	2
16.	Jhajha	10:57	11:02	1176	2
17.	Jaisidih	11:38		1220	2

Following is the time table of Poorva express which leaves New delhi station at 4.25 am. and reaches Jasidih station on next day at 11.38 pm. Below given table gives the arrival time and the departure time of Poorva express at different stations with the distance of that station from the source.

Q 1. According to the time-table given above, what is the maximum time for which Poorva express runs non-stop?
(a) 155 minutes (b) 160 minutes
(c) 135 minutes (d) None of these

Q 2. For how many of the stations from the list given above, the stoppage time is more than 5 minutes (stoppage time is the difference between the departure time and the arrival time)?
(a) 4 (b) 5
(c) 6 (d) None of these

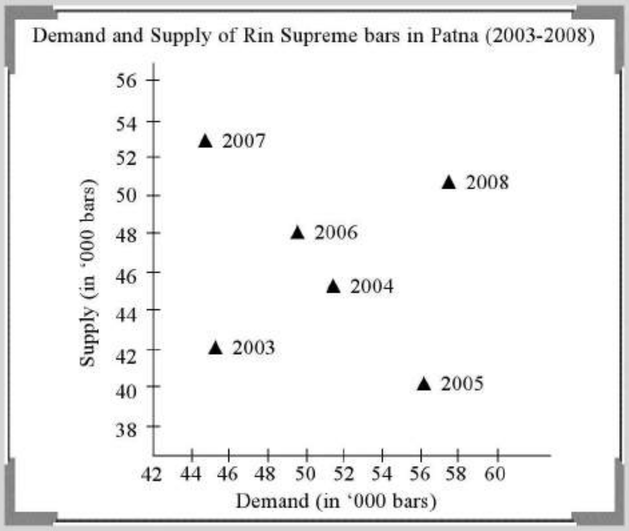
Q 3. In between which stretch the train obtains its maximum speed?
(a) Ara-Danapur (b) Buxar-Ara
(c) Kiul-Jamui (d) None of these

Q 4. If the speed of the train is increased by 20% in all the stretches, then at what time (approximate) the train will reach at Jasidih station keeping all the other things constant?
(a) 8.17 am (b) 8.39 am
(c) 9.12 am (d) 9.29 am

Q 5. What is the approximate average speed (kmph) of the train during the whole journey?
(a) 60 (b) 63
(c) 66 (d) 69

Direction for questions 6 to 10: Go through the data set given below and solve the questions based on it.

The following graph captures the demand and supply situation of Rin supreme bars in a town Patna over the years 2003-2008.



Q 6. For who many years, the supply was not less than the demand?
(a) 0 (b) 1
(c) 3 (d) 7

Q 7. If the price of one bar of Rin supreme is directly proportional to the ratio of demand to supply, in which year, in the time period 2003-2008, was the price of a bar the highest?

(a) 2007 (b) 2005

(c) 2006 (d) 2008

Q 8. Over the time period of 2003-2008, in how many years did both demand and supply show a similar trend (i.e., either both increased or both decreased) over the previous year?

(a) 1 (b) 2

(c) 3 (d) 4

Q 9. Patna had an inventory of 20,000 bars at the start of 2003. In which year was the deficit of bars the highest? [Deficit is defined as demand minus available quantity and available quantity is supply of year plus the inventory. In a year where availability is less than the demand, the demand goes unsatisfied in that year and is not carried forward. Excess availability over demand goes into inventory]

(a) 2008 (b) 2007

(c) 2006 (d) 2005

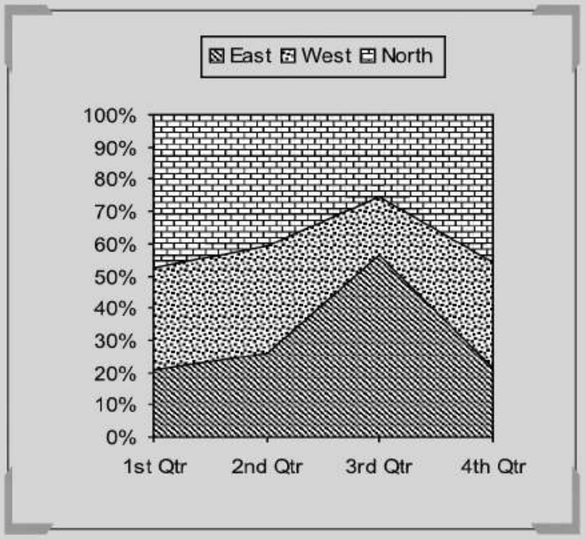
Q 10. The demand and supply given above is to be re-drawn using the line chart with the year along the X axis and the demand and supply along the Y axis (same scale being used for both demand and supply). At how many distinct points would the two lines cut?

(a) 0 (b) 1

(c) 2 (d) 3

Direction for questions 11 to 15: Go through the data set given below and solve the questions based on it.

Following is the area diagram given for the sales value of ABC Limited for the four quarters of the year 2007. ABC Limited has its operations in three zones—North, West and East. Below given data set represents the stacked contribution of these three zones in every quarter of the year 2007.



Q 11. If we rank the contribution of zones in each of the quarters, then how many zones will have the same rank across all the quarters?

(a) 0 (b) 1

(c) 2 (d) 3

Q 12. Which zone represents maximum sales value in the year 2007?

(a) North (b) West

(c) East (d) Cannot be determined

Q 13. If we convert the above given data set into a line chart (not stacked), then total how many times the lines will intersect?

(a) 0 (b) 2

(c) 4 (d) None of these

Q 14. There is an increase of 10% every quarter over the last quarter for the given period. Which of the following represents the maximum sales value?

(a) North zone Qtr 1

(b) East zone Qtr 3

(c) West zone Qtr 2

(d) North zone Qtr 4

Q 15. Data set given above is arranged in bars (in percentage) in the following way—First North zone, then West zone, then East zone of the 1st quarter and in the similar order for the 2nd, 3rd and 4th quarter. How many times there will be an increase in the height of bar with respect to the height of the previous bar?

(a) 1 (b) 2

(c) 3 (d) 4

Direction for questions 16 to 18: Go through the data set given below and solve the questions based on it.

In a B-School, the students have to specialize in at least one of the areas—Finance, Marketing or Systems. The table given below represents the distribution of students in different specializations according to their subjects chosen. There were a total of 80 girls and 120 boys from arts background and 40 girls and 70 boys from science background comprising the entire batch.

	Arts		Science	
	Girls	**Boys**	**Girls**	**Boys**
Finance	25	45	25	65
Marketing	23	50	20	15
System	32	100	25	15

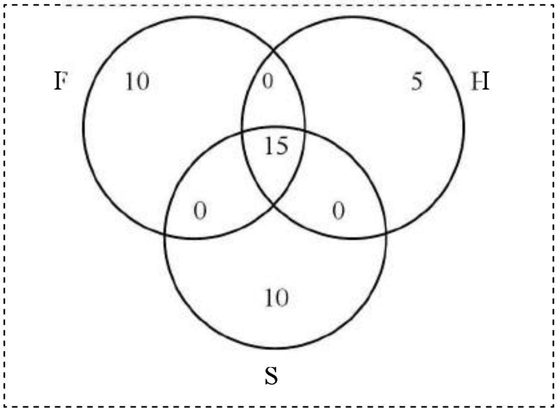
Q 16. What is the percentage of girls from arts specializing in all three areas?

(a) 16.5% (b) 16%

(c) 100% (d) 0%

Q 17. If 60% of boys and 70% of girls are successful in at least one of the specializations taken by them, what is the percentage of students who have not passed in any subject?
(a) 33.9% (b) 36.1%
(c) 38.4% (d) Cannot be determined

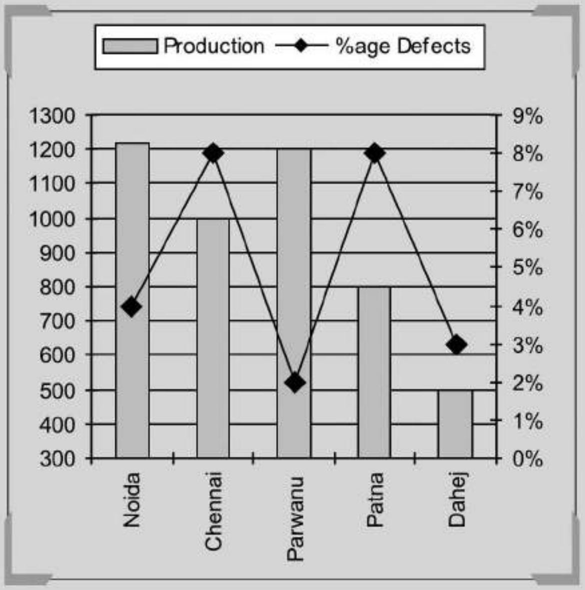
Q 18. Find the maximum number of girls from science who could have specialized in all 3 subjects.
(a) 23 (b) 20
(c) 15 (d) Cannot be determined



Direction for questions 19 to 25: Go through the data set given below and solve the questions based on it.

Following bar chart represents the number of units produced (in millions) and percentage defective units produced out of the total units produced by the individual plants of an electronics company. These plants – Noida, Chennai, Parwanu, Patna and Dahej are the only plants of this company.

All the defective units are rejected and total units supplied from any of the plants
= Total units produced – total defective units.



All %age defects are in integers

Q 19. Which of the following plants can supply maximum units?
(a) Noida
(b) Chennai
(c) Parwanu
(d) Dahej

Q 20. Which of the following plants can supply minimum units?
(a) Noida
(b) Chennai
(c) Patna
(d) Dahej

Q 21. It has been observed that if the employees working at two plants are frequently given an opportunity to interact, then the combined percentage defect at these two plants becomes the harmonic mean of the individual percentage defect of these two plants. What is the total number of units (in lacs) that can be supplied from taking Chennai and Patna together?
(a) 1656
(b) 16560
(c) 1728
(d) 17280

Q 22. In the above question, which of the following two plants should be merged so that the number of units supplied from these two units combined is maximum?
(a) Noida + Chennai
(b) Parwanu + Patna
(c) Noida + Parwanu
(d) None of these

Q 23. The average of total number of units supplied by all the plants taken together will be closest to the number of units supplied by
(a) Noida
(b) Chennai
(c) Parwanu
(d) None of these

Q 24. What is the approximate total number of units (in millions) supplied by all the plants taken together?
(a) 4500
(b) 5000
(c) 4000
(d) Cannot be determined

Q 25. The ratio of production of Parwanu plant and Patna plant is same as the ratio of production of Chennai plant and ……. Plant.
(a) Noida
(b) Dahej
(c) Either Noida or Dahej
(d) None of these

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(d)	(b)	(b)	(b)	(b)	(a)	(d)	(c)	(a)	(d)	(c)	(b)	(b)	(d)	(b)	(c)	(c)	(d)
Q. No.	21	22	23	24	25															
Answer	(b)	(c)	(b)	(a)	(b)															

HINTS AND SOLUTIONS

1. This is obtained between New Delhi – Aligarh.
2. Visual inspection.
4. If speed of the train is increase by 20%, then the time taken (in which train is moving) will reduce by 16.66%. However there will not be any impact on the stoppage of the train (= 81 minutes).

   In new case, total time taken by train = 974 minutes.

   Hence 8:39 a.m.
5. Total time taken = 19: 13 mins and

   total distance = 1120 km

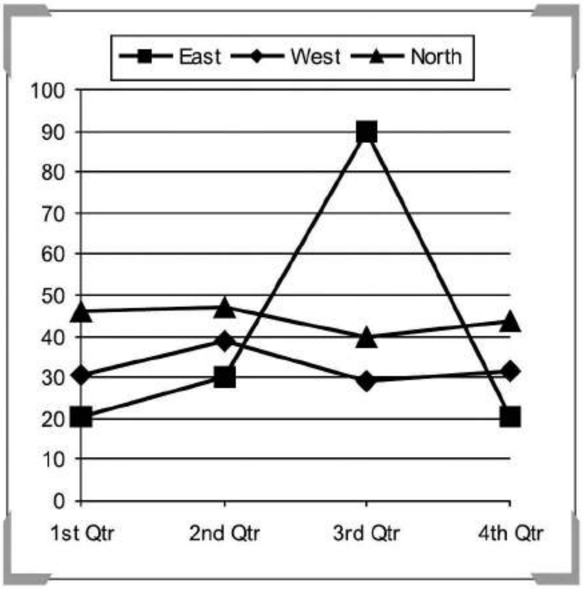
   Hence, average speed = 1120/19:13 = 63.3 kmph
6. Only 2007 is the year when the given condition in the question is fulfilled.
7. By visual inspection, it can be seen that the highest ratio of, it is for year 2005.
8. Only once in year 2004.
9. Highest deficit is in 2005 i.e., 6000 bars.

   In 2006, it is 2000 bars.

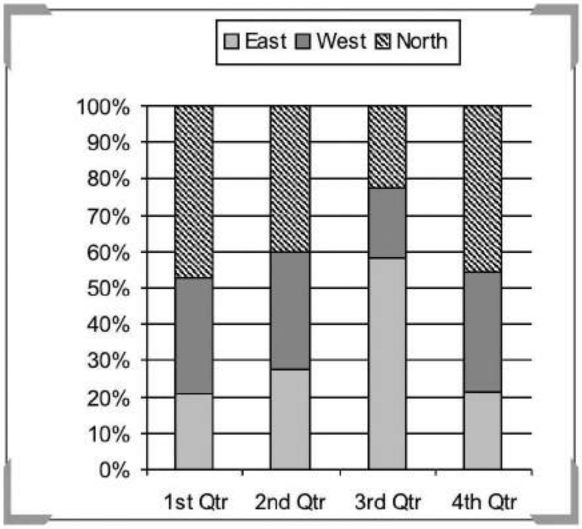
   In 2007, there is an inventory of 7000 bars.

   In 2008, the deficit is 1000 bars.
10. Between 2006 and 2007 and 2007 and 2008.
11. Visual inspection.
12. Since the value for each of the quarters is not given, hence we cannot determine it.
13. It will appear like this:

    We can see now that there are four points of intersection.



14. Start by assuming the value in 1st qtr = 100 and then for the subsequent quarters, the value will increase by 10% over the previous quarter.
15. Following bar chart gives the values of the quarters and zones. Now we will arrange the individual bars to see that the increase is appearing twice—2nd quarter North zone and 3rd quarter East zone.



16. Total number of girls in finance, Marketing and systems = 80.

    Therefore, there are 0% girls who specialize in all three of areas.

17. 40% of boys and 30% of girls are not successful.

    Hence, 40% of 190+30% of 120 = 76+36 = 112

    Hence, percentage of students who are not

    successful = 36%

18. The maximum number is 15.

    Following is the data-sheet pertaining to the data set given above:

	Noida	**Chennai**	**Parwanu**	**Patna**	**Dahej**
Production (in mn units)	1220	1000	1200	800	500
%age Defects	4%	8%	2%	8%	3%
Supplied Units (in mn units)	1171.2	920	176	736	485

21. Use the data sheet given above. Values are required in lacs.

22. Noida and Parwanu are the two highest producing plants with the percentage defects being in the lowest three. Hence, this combination can supply the maximum units.

23. Chennai is the median of all the plants and closely looking upon the values, value for Chennai is closest to the average of all the plants.

24. Use the data-sheet.

25. Visual inspection.

4

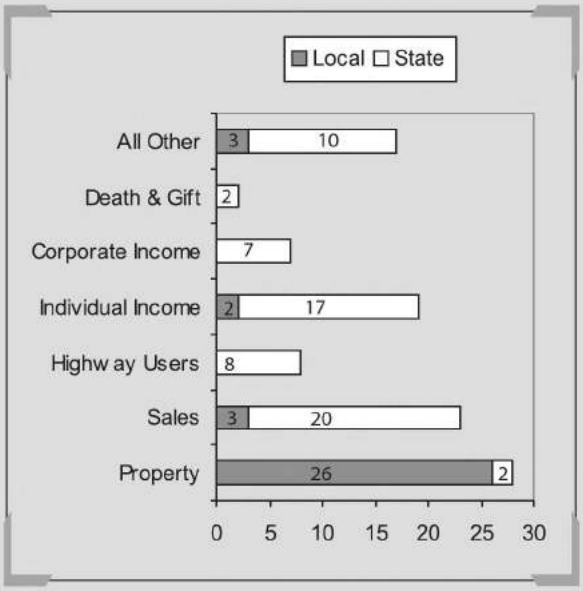
CHAPTER

Moderate Exercises

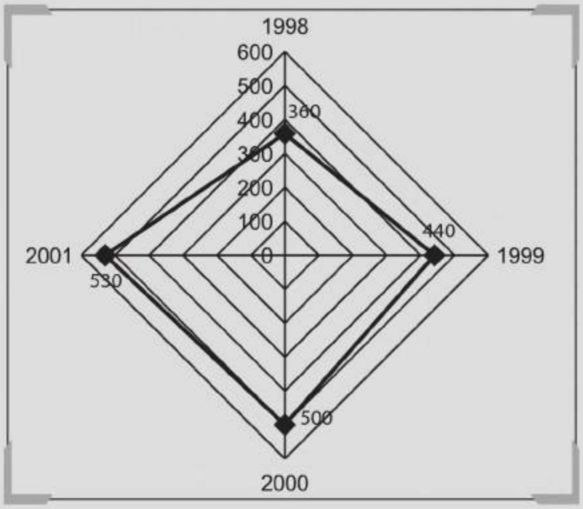
PRACTICE EXERCISE 1

Direction for questions 1 to 5: Refer to the charts below answer the questions that follow.

Chart given below presents the sources of local and state tax revenues for the year 2001 as a percentage of the total tax revenues.



The diagram shown below represents the combined state and local tax revenue (in $ mn).



Q 1. Find the difference between the tax revenue generated from individual income at local and state levels in 2001.

(a) $ 72 mn

(b) $ 78.75 mn

(c) $ 86.25 mn

(d) $ 97.25 mn

Q 2. What is the total tax revenue generated by the state in 1999?

(a) $ 149.6 mn (b) $ 210.3 mn

(c) $ 290.4 mn (d) Cannot be determined

Q 3. If revenue distribution in 2000 was same as in 2001 and tax from highway users was collected for the first time, then what was the percentage increase in tax revenue from source other than highway users in 2000 over the previous year?

(a) 4.5% (b) 7.3%

(c) 9.09% (d) 14.28%

Q 4. Find the compounded annual rate of growth during 1998-2001.

(a) 10.9% (b) 12.3%

(c) 13.4% (d) 14.7%

Q 5. What percentage of the total local tax revenue is generated from property tax in 2001?

(a) 52.4% (b) 76.4%

(c) 86.1% (d) 90.9%

Direction for questions 6 to 10: Refer to the table below and answer the questions that follow.

Length of service (years)	Knowing Shorthand	Knowing Typing	Knowing Both	Total staff
0 – 5	40/35	35/40	35/30	80/120
6 –10	60/15	60/20	40/10	150/75
More than 10	40/40	40/30	15/20	90/110

The chart below represents the statistics pertaining to the employees of Due North Inc.

Following is the representation:

40/35 – Graduate/Non-graduate

Q 6. What percentage of the employees with 10 years or less service know only one of shorthand or typing?

(a) 32 (b) 27

(c) 22 (d) 18

Q 7. What percentage of employees know shorthand only?

(a) 33 (b) 28

(c) 23 (d) 18

Q 8. How many graduate employees know neither shorthand nor typing?

(a) 115 (b) 120

(c) 135 (d) 130

Q 9. Out of those employees knowing shorthand only, how many are graduates?

(a) 63 (b) 58

(c) 53 (d) 48

Q 10. If 10% of those employees knowing shorthand only learn typing and 12% of those knowing typing only learn shorthand, by what % would the number of persons knowing both increases:

(a) 11 (b) 16

(c) 21 (d) 26.

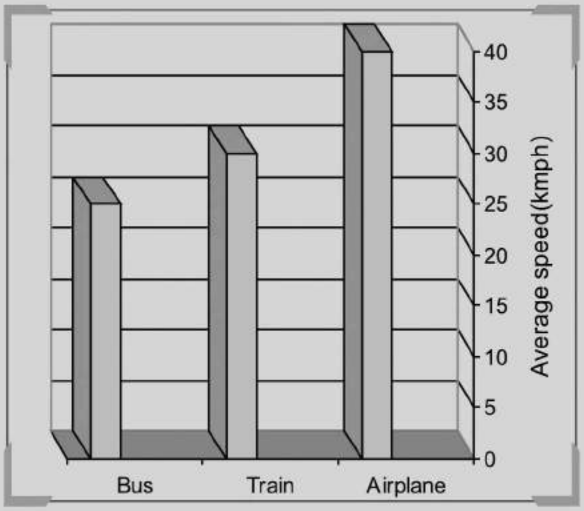
Direction for questions 11 to 14: Refer to the table given below and answer the questions based on it.

Table 1 – Mode of travel between two cities

	Patna	Mumbai	Calcutta	Chennai	Bangalore
Patna	-	Bus	Train	Air plane	Train
Mumbai	Bus	-	Train	Train	Air plane
Calcutta	Train	Train	-	Air plane	Bus
Chennai	Air plane	Train	Air plane	-	Bus
Banga-lore	Train	Air plane	Bus	Bus	-

Table 2 – Distance between two cities (in km)

	Patna	Mumbai	Calcutta	Chennai	Bangalore
Patna	-	100	150	150	300
Mumbai	100	-	200	300	500
Calcutta	150	200	-	500	300
Chennai	150	300	500	-	100
Banga-lore	300	500	300	100	-



All the values in above bar-chart are the multiples of 5.

Q 11. Which city can be reached from Patna in the least time?

(a) Mumbai (b) Calcutta

(c) Chennai (d) Bangalore

Q 12. For which of the following options, travel time is the least?

(a) Mumbai-Chennai

(b) Patna-Bangalore

(c) Bangalore-Chennai

(d) Patna-Bangalore

Q 13. If one can travel by bus or train only, then which two cities will have the least travel time?
(a) Patna-Mumbai
(b) Bangalore-Calcutta
(c) Bangalore-Mumbai
(d) None of these

Q 14. What is the maximum distance one can travel (approx. km) between Patna and Chennai in a day if the Stoppage time between any two consecutive trips is 2 h?
(a) 625 (b) 700
(c) 725 (d) 640

Direction for questions 15 to 19: Go through the table given below and solve the questions based on it.

Following is the final results of standard X. The subjects students tested in are—English, Hindi, Science, Maths, Social science, Drawing, Craft, PT, scout-guide and social service. To have a better presentation of the mark sheet, the scores in first four subjects are consolidated into A1, the next three into A2 and last three into A3. The final score is the total of these three scores A1, A2 and A3. On the basis of the final score students get their ranks. The table below gives the scores of top eighteen students of the class.

Name	Grade	Final Score	A1	A2	A3	Drawing	PT	Social service
Anana	A	725	282	217	226	80	81	69
Pankaj	B	864	310	87	267	85	86	92
Vamshi	C	812	300	290	222	70	91	74
Ram	C	805	270	262	273	65	79	97
Shann	D	900	391	292	217	96	86	81
Saket	D	876	301	297	278	65	87	93
Mini	C	825	296	296	233	50	89	91
Rashi Singh	E	905	320	290	295	72	99	99
Parul	B	895	310	290	295	72	99	98
Rakesh Rathi	B	870	276	296	298	63	99	99
Gauri	C	888	361	270	257	82	78	92
Kush Verma	A	-	315	-	296	62	99	99
Pragya	A	897	812	299	286	69	97	92
Shipra Gupta	D	907	319	289	299	71	100	100
Sachin	E	910	387	290	233	86	69	79
Sarita	A	901	389	270	242	87	79	86
Mahavir Sherawat	C	897	388	297	212	83	69	89
Ritu Sehgal	D	898	299	300	299	73	100	99

Q 15. How many students have got grade A and more than 250 marks in all the group scores A1, A2 and A3?
(a) 0 (b) 1
(c) 2 (d) None of these

Q 16. What is the difference between total marks in drawing of grade A students and grade C students?
(a) 50 (b) 52
(c) 48 (d) None of these

Q 17. What is the least that Kush Verma must get in A 2 that would ensure him 5th rank?
(a) 289 (b) 288
(c) 290 (d) Cannot be determined

Q 18. Who has scored maximum marks in Scout-guide provide the maximum marks in any subject = 100?
(a) Rakesh Rathi (b) Ritu Sehgal
(c) Parul (d) Rakesh Rathi and Ritu

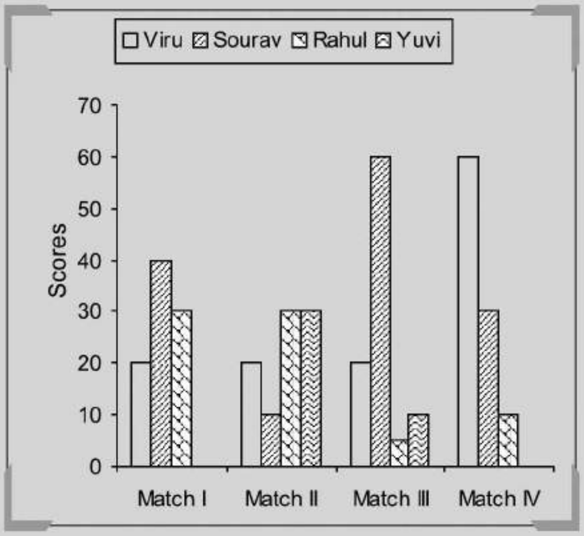
Q 19. What is the difference between the sum of final scores of top five scorers and sum of final scores of bottom five scorers?

(a) 214 (b) 208

(c) 174 (d) None of these

Direction for questions 20 to 24: Go through the diagram below and answer the questions based on it.

Following diagram shows the percentage share of runs scored by four Indian Batsmen in total runs made by India in 4 matches:



Total runs scored by India in any match is in between 250 and 300.

Q 20. In which match is the share of runs scored by the four batsmen out of the total runs scored by India the maximum?

(a) Match I

(b) Match II

(c) Match III

(d) Match IV

Q 21. What is the maximum possible difference between the runs scored by Sourav and Yuvi in any match?

(a) 25

(b) 30

(c) 50

(d) None of these

Q 22. Who is the lowest scorer in all the four matches taken together?

(a) Viru

(b) Sourav

(c) Rahul

(d) Yuvi

Q 23. Find the maximum possible difference in total runs by the four batsmen in any two matches?

(a) 25 (b) 50

(c) 75 (d) 100

Q 24. Among Viru, Sourav and Rahul, who has shown maximum percentage increase in the runs scored in a match over the previous match?

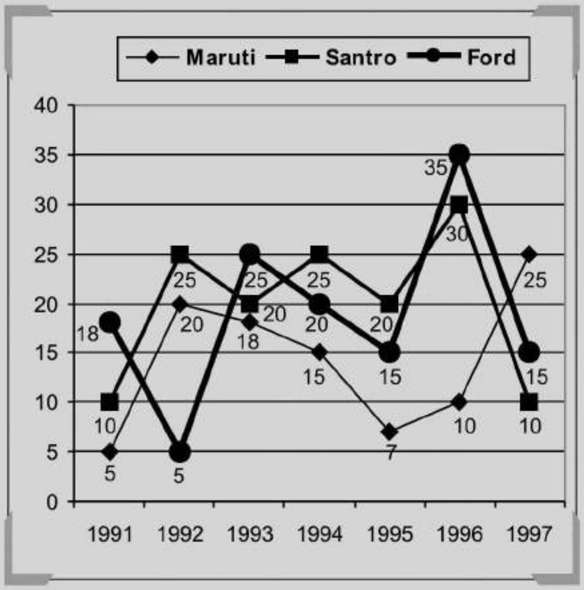
(a) Viru

(b) Sourav

(c) Rahul

(d) Cannot be determined

Direction for questions 25 and 26: Go through the graph below and answer the questions based on it.



The above graph gives the production of cars (in thousands) over the given years.

Q 25. In how many years is the number of cars manufactured by Maruti less than the average number of cars manufactured by Santro over the given years?

(a) 3 (b) 4

(c) 6 (d) None of these

Q 26. If the total average production of all cars increase at the rate of 10% every year from 1997 onwards, what will be the average approx. production of cars in the year 2000?

(a) 19000

(b) 21000

(c) 22500

(d) 24000

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(d)	(a)	(c)	(b)	(d)	(d)	(c)	(a)	(a)	(c)	(b)	(a)	(d)	(b)	(b)	(a)	(d)	(d)	(d)
Q. No.	21	22	23	24	25	26														
Answer	(d)	(d)	(c)	(b)	(c)	(c)														

HINTS AND SOLUTIONS

[illegible]

1. State tax revenue from individual income
   = 17 × 525/100 = 89.25 mn
   Local tax revenue from individual income
   = 2 × 525/100 = 10.5 mn
   Hence, Difference = 89.25 – 10.5 = $ 78.75 mn.
2. Percentage of state tax revenue in total tax revenue
   = 2 + 20 + 8 + 17 + 7 + 2 + 10 = 66%
   Therefore, state tax revenue in 1999
   $= \frac{6\times 440}{100} =$ $26.40 mn
3. Tax revenue from highway users in 2000
   $= \frac{8\times 500}{100} =$ $40.0 mn
   Therefore, tax revenue from other sources in 2000
   = 500 – 40 = $ 460 mn
   Therefore, % increase in tax revenue from other sources
   $= \frac{460-440}{440}\times 100 = 4.5\%$
5. Percentage of local tax revenue in total revenue
   = 26 + 3 + 2 + 3 = 34
   Percentage of local tax revenue from property source in total tax revenue = 26.
   Therefore, percentage of property tax in local tax revenue = (26/34) × 100 = 76.4%

[illegible]

6. Number of people knowing shorthand only with experience 10 year or less
   = (40+35+60+15) – (35+30+40+10) = 35.
   Numbers knowing typing only with experience 10 years or less
   = (35+40+60+20) – (35+30+40+10) = 40.
   No. of members knowing shorthand or typing and having experience of 10 years or less
   = 35 + 40 = 75.
   Percent required = 18.
7. No. of people knowing shorthand only
   = (40 + 35 + 60 + 15 + 40 + 40)
   – (35 + 30 + 40 + 10 + 15 + 20) = 80
   Percentage required= 80×100/625 = 13.
8. No. of graduates knowing neither shorthand nor typing
   = 320 – (40 + 60 + 40 – 35 – 40 – 15) –
   (35 + 60 + 40 – 35 – 40 – 15)–(35+40+15) = 135.
9. People knowing shorthand only
   = (40 + 60 + 40 + 35 + 15 + 40) –
   (35 + 30 + 40 + 10 + 15 + 20) = 80.
   Required percentage = (50/80) x 100 = 63.
10. No. of persons knowing shorthand only
    = 80 ……..(A)
    No. of persons knowing typing only
    = 75 ……..(B)
    If 10% of (A) learn typing and 12% of (B) learn shorthand, the total no. of those knowing both increases by 8 + 9 = 17; 17 × 100/150 = 11%.

[illegible]

Let us make a table for the time taken:

	Patna	Mumbai	Calcutta	Chennai	Bangalore
Patna		4	5	3.75	10
Mumbai	4	-	6.666667	10	12.5
Calcutta	5	6.666667		12.5	12
Chennai	3.75	10	12.5		4
Banga-lore	10	12.5	12	4	-

14. Total time for Patna-Chennai trip
= 3.75 + 2 = 5.75 h
Hence, min. total stoppage = 2 × 4 = 8h
So, in 24 h, 4 trips get complete while 5th trip will be on.
Therefore, distance covered by airplane in 16 h
= 16 × 40 = 640 km.
Hence, option (d) is the answer.

[illegible]

15. Only one such person is there.

16. Total marks in drawing of division A students
= 298
Total marks in drawing of division C students
= 350.
Required difference = 350 – 298 = 52.

17. To stand 5th total score required = 900 marks. Hence marks required to be scored in A2 = 289 marks.

18. Marks in scout-guide can be calculated by subtracting marks of PT and social service from A3. Rakesh Rathi and Ritu Sehgal scored 100 marks in scout guide.

[illegible]

	Percentage runs scored in			
Batsmen	**Match I**	**Match II**	**Match III**	**Match IV**
Viru	20	20	20	60
Sourav	40	10	60	30
Rahul	30	30	5	10
Yuvi	0	30	10	0
Total	90	90	95	100

20. It can be seen from the above chart that in match IV, the share of runs scored by these four batsmen = 100%, which is the maximum.

21. For maximum difference to occur, total runs scored by India and the contribution of these four batsmen, both should be maximum. and in the second case, runs scored by India and the score by individual batsmen should be minimum.

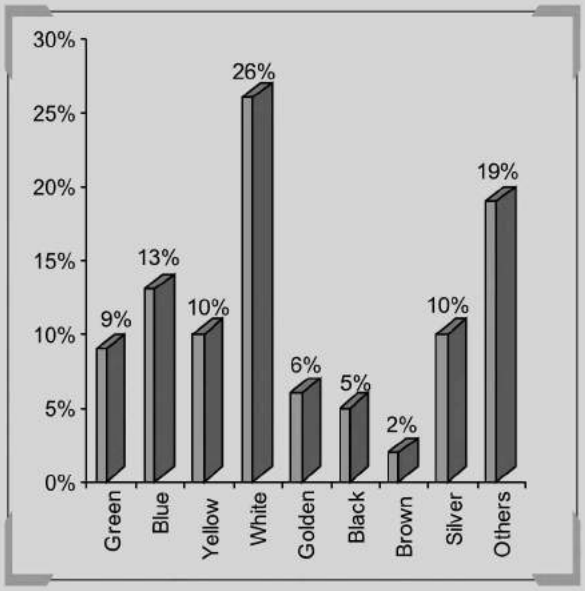
22. It is given that team score lies in between 250 and 300. We can thus see that the total score is more or less same in all the matches. So percentage share in total runs scored by the team is enough to answer who scored the minimum runs.
Yuvi has the lower percentage share.

24. Observe the data. Percentage runs scored by Sourav increased by 500% and the total runs scored by the team can vary by a maximum of 20%. And in no matches other batsmen showed such a high increase.

PRACTICE EXERCISE 2

Direction for questions 1 to 5: Study the following chart carefully and answer the questions based on it.

Following bar chart shows the sales of Cars in USA in 2008 in preference to different colors of the car:



Q 1. Which of the following colour of cars combined consist 50% of the total cars sold?
(a) Black, Golden, Blue, Silver
(b) Blue, Black, Green, Silver
(c) White, Golden, Blue, Black
(d) White, Blue, Green, Black

Q 2. Number of cars sold is directly proportional to the colour of car. Cars of how many colours are atleast 20% more popular than green coloured cars?
(a) 0 (b) 3
(c) 3 (d) Cannot be determined

Q 3. Brown coloured car is the least number of cars sold, and no other colour car's percentage share in total sale of cars is less than 4%. Now the sales figure of different coloured cars are arranged in ascending order. For how many colours' sales figure, we can definitely determine its rank?
(a) 0 (b) 1
(c) 6 (d) Cannot be determined

Q 4. A car can be painted multi-coloured too. However in that case, no colour used in making a car multi-coloured can be used individually. Total percentage share of a multi-coloured car = Sum of percentage share of all the individual colours used.

If a car is made multi-coloured by painting it by atleast two colours and at max by four colours, for how many different combinations it is possible to have 32% market share of total sales of the cars?
(a) 4 (b) 6
(c) 9 (d) None of these

Q 5. To increase the percentage share of all colour cars under 10% of total sales, a new exchange policy has been brought under practice. Following are the terms and conditions of this exchange policy:
i. Only those colour cars can be exchanged (to obtain a new colour car under 10% market share) which have a percentage share of more than 15%.
ii. No colour car whose market share is in between 10%–15% can be used in exchange.

How many different combinations are possible under this exchange scheme?

(A combination is defined as a set of a new car, which a person obtains after exchanging, and an old car defined as a car with the person before this exchange.)
(a) 6 (b) 8
(c) 12 (d) Cannot be determined

Direction for questions 6 to 10: Study the following chart carefully and answer the questions based on it.

Following table shows the percentage population of six states below poverty line and the proportion of male and female.

State	Percentage Population Below poverty Line	Proportion of male and female	
		Below Poverty line	Above poverty line
		M : F	M : F
A	12	3 : 2	4 : 3
B	15	5 : 7	3 : 4
C	25	4 : 5	2 : 3
D	26	1 : 2	5 : 6
E	10	6 : 5	3 : 2
F	32	2 : 3	4 : 5

Q 6. The total population of State A is 3000, then what is the approximate no. of females above poverty line in state A?
(a) 1200 (b) 2112
(c) 1800 (d) 1950

Q 7. If the total population of C and D together is 18000, then what is the total no. of females below poverty line in these two states?
(a) 5000 (b) 5500
(c) 4800 (d) Cannot be determined

Q 8. If the population of males below poverty line in state A is 3000 and that in state E is 6000, then what is the ratio of the total population of state A and E?
(a) 3 : 4 (b) 4 : 5
(c) 1 : 2 (d) None of these

Q 9. If the population of males below poverty line in state B is 500 then what is the total population of B?
(a) 14400 (b) 6000
(c) 8000 (d) 7600

Q 10. If in state E population of females above poverty line is 19800 then what is the population of males below poverty line in that state?
(a) 5500 (b) 3000
(c) 2970 (d) None of these

Direction for questions 11 to 15: Go through the table below and answer the questions based on it.

The table given below represents the Supply to demand ratio per 100 units of 15 items from the period Jan to Oct. For example, supply of item A is Jan was only 98 units where as the demand is of 100 units.

Month/ Items	Jan	Feb	March	April	May	June	July	August	Sep	Oct
A	98	99	98	98	96	99	97	97	96	95
B	91	91	89	87	88	87	86	90	91	92
C	106	105	102	99	102	100	105	96	92	90
D	109	110	112	108	108	112	85	101	86	99
E	95	94	94	96	101	102	96	97	96	92
F	86	83	84	96	94	87	93	99	101	105
G	88	88	89	94	86	97	93	101	92	89
H	98	89	89	96	94	91	86	102	94	84
I	100	98	96	101	104	98	104	99	111	100
J	97	100	101	99	96	94	100	97	101	92
K	98	96	95	97	98	94	93	98	96	100
L	103	97	96	96	95	92	91	99	101	104
M	83	105	100	103	100	99	98	104	107	102
N	104	78	80	82	86	92	91	100	98	87
O	94	91	95	96	99	92	93	92	97	100

Q 11. Which of the following items show the maximum increase in the supply to demand ratio from Feb to August?
(a) F (b) N
(c) G (d) H

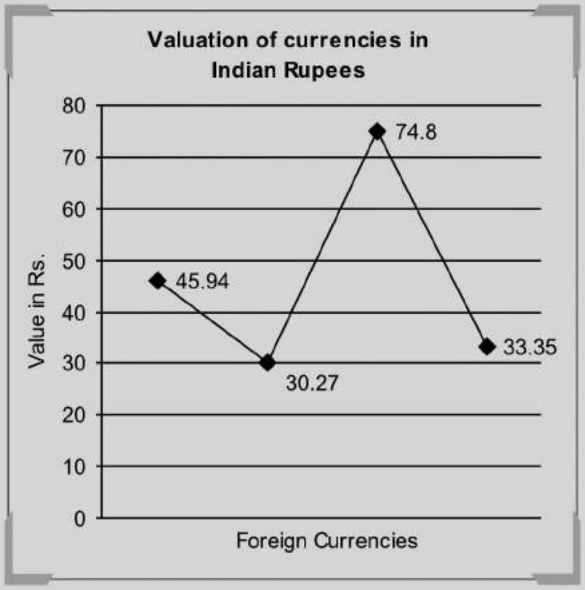
Q 12. For which item does supply exceed demand the maximum number of times?
(a) C (b) M
(c) D (d) J

Q 13. For the item with the least supply to demand ratio, what is the percentage increase in that ratio in the next month?
(a) 2.5% (b) 3%
(c) 4% (d) 2%

Q 14. How many items are constantly under supply through out the given period (under supply means demand is more than the supply)?
(a) 1 (b) 2
(c) 3 (d) 4

Q 15. How many items are constantly under demand through out the given period (An item is under demand if demand exceeds the supply in all the months during the given period)?
(a) 1 (b) 2
(c) 3 (d) 4

Direction for questions 16 to 20: Go through the chart below and answer the questions based on it.



Above given four currencies represents the following: US Dollar, Singapore Dollar, UK Pound and Euro. Following is the order of their valuations: UK pound > US Dollar > Euro > Singapore Dollar.

Q 16. A person wants to buy an article worth 65 Singapore Dollars. If Singapore Dollar loses 2% with respect to the above given exchange rates, then how many US dollars must be spend?

(a) 42.1 (b) 41.9
(c) 44.9 (d) 43.6

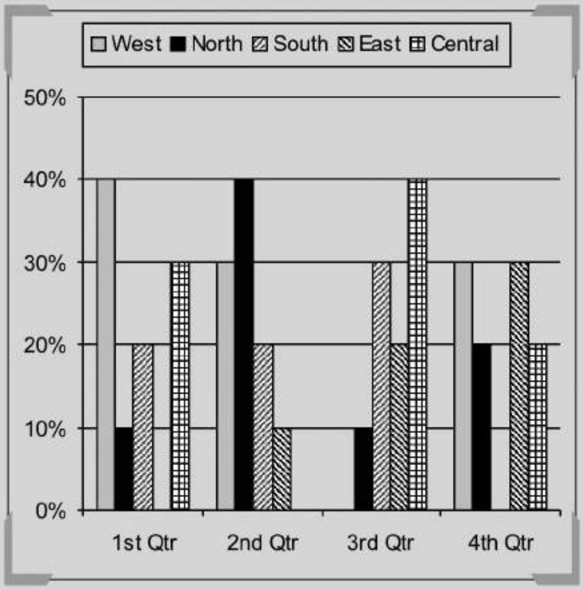
Q 17. A person wants to buy 800 barrels of oil. The payment can be made either in US dollars or in Euros. If the payment is made in Euros, then it would be 47 Euros per barrel while if one pays in US dollars, the rate would be 29 US dollars per barrel. How much money will he save if he goes for the cheaper deal?
(a) 29200 US dollars
(b) 34100 US dollars
(b) 37600 Euros
(d) None of these

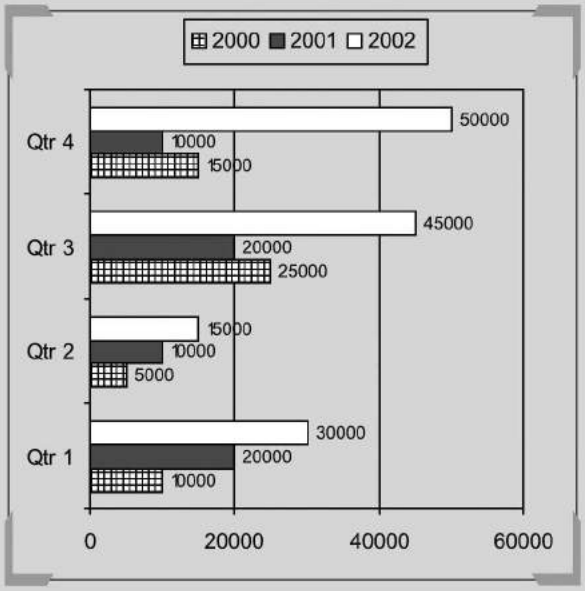
Q 18. In the above question, if there is a discount of16% on paying in Euros and if the person goes for paying in Euros, how much does a person stand to gain or lose?
(a) Rs 12482 gain (b) Rs 12482 loss
(c) Rs 3987 loss (d) Rs 3987 gain

Q 19. US bonds give 9% return on investment. Which of the following when invested in US bonds gives maximum returns?
(a) 4000 US dollars
(b) 2500 UK Pounds
(c) 5300 Euros
(d) 7000 Singapore Dollars

Q 20. Value of rupee depreciates against US Dollar by 10%. What will be the new value of US Dollar in Rs?
(a) Rs 41.34
(b) Rs 51.04
(c) Rs 50.53
(d) Rs 40.83

[redacted]





The bar charts shown here represents the earnings of a Sales executive Mohit. Bar-chart (1) gives us the details of his earnings in the four quarters of the year across the five regions—West, North, South, East and Central. Bar chart (2) gives the details of his total earning in the four quarters in three different years. Bar-chart (1) is applicable for bar chart (2)and vice-versa.

Q 21. Across the given years, which quarter has given the maximum earning to Mohit?
(a) Qtr 1
(b) Qtr 2
(c) Qtr 3
(d) Qtr 4

Q 22. Which Region has brought maximum earning for Mohit in the year 2000?
(a) South (b) West
(c) East (d) Central

Q 23. During which year and quarter the earnings from the north Region have shown maximum change over the previous quarter in that year?
(a) Quarter III, year 2000
(b) Quarter II, year 2002
(c) Quarter II, year 2001
(d) Quarter IV, year 2002

Q 24. What is the maximum difference between any two regional earnings in any quarter?
(a) Rs 15500
(b) Rs 13000
(c) Rs 14500
(d) None of these

Q 25. Find the percentage change in Mohit's earning due to the south zone during 2001 over the previous year.
(a) 14% (b) 20%
(c) 25% (d) 33%

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(d)	(b)	(d)	(d)	(a)	(d)	(d)	(d)	(b)	(b)	(d)	(a)	(b)	(d)	(b)	(d)	(a)	(d)	(b)
Q. No.	21	22	23	24	25															
Answer	(c)	(d)	(d)	(d)	(a)															

HINTS AND SOLUTIONS

[illegible]

1. This question can be done by simple calculation using the options.
2. Since we do not know the colours of cars contained in 'Others' category, we cannot answer this question.
3. We can determine the ranks of only those colour cars which are having a share less than 4%. Reason being unavailability of the data pertaining to the colours contained in 'others' category.
5. We do not know the colours in 'Others' category. It might be a possibility that 'Others' category is containing 4-5 different colours and each one being around 4% -5%.

   Hence, cannot be determined.

[illegible]

7. From the given information, we cannot find the population of states C and D individually; so we can't find the required value.
8. Population of State A below poverty line

   $$= \frac{3000 \times 5}{3} = 5000$$

   Therefore, total population of state A

   $$= \frac{5000 \times 100}{12}$$

   And the population of state E below poverty line
   = 11000

   Therefore, total population of state E

   $$= \frac{11000 \times 100}{10}$$

   $$\text{Therefore, required ratio} = \frac{5}{12} \times \frac{10}{11} = \frac{25}{66}$$

[illegible]

11. By observation we can say that 'N' shows maximum increase in supply to demand ratio.
12. For D, supply exceeds demand the maximum number of times.
13. N has least supply to demand ratio in February

    $$\% \text{ increase} = \frac{2 \times 100}{78} = 2.5\%$$

14. Only items A and B are in under supply.
15. Consider those items also which have a 100 supply viz., O.

[illegible]

16. Taking into consideration above rates.
    1 US \$ = Rs 45.94
    1 Singapore \$ = Rs 30.27
    Hence, 1 US \$ = 1.52 Singapore \$

    The Singapore \$ has lost 2% with respect to the above given exchange rates
    So, 1 US \$ = 1.52 × 1.02 = 1.55 Singapore \$
    Hence, 65 Singapore \$ = 41.9 US

    Thus, the person should give 41.9 US \$ to get 65 Singapore Dollars.

17. Price for 800 barrels = 37600
    Euros = Rs 1253960.
    However, if we are paying in US \$,
    we will be paying \$ 23200 = Rs 1065808.
    Thus, we are paying 188152 Rupees less
    = 4096 US dollars.

18. If there is a discount of 16% on paying in Euros, then a person will have to pay 31584 Euros instead of earlier 37,600

dollars. To buy this much Euros, we will have to spend Rs 12482 less than the earlier case.

So, if we are paying in Euros we will be gaining Rs 12482.

19. Larger the amount larger the returns as rate is constant.

We are just supposed to find out that which amount given in the options is largest.

It can be seen that 7000 Singapore Dollars (4666.7 US dollars) is the largest investment and so it would give the maximum returns.

20. It should be understood that if rupee is depreciating by 10%, then dollar is appreciating by 11.11%.

Questions 21 to 25

21. Use the second bar chart. It is a visual inspection solution.

22

Region	Annual earnings for the region (in thousands)
West	0.4 × 10 + 0.3 × 5 + 0.3 × 15 = 10
North	0.1 × 10 + 0.4 × 5 +0.1 × 25 + 0.2 × 15 = 8.5
South	0.2 × 10 + 0.2 × 5 + 0.3 × 25 =10.5
East	0.1 × 5 + 0.2 × 25 + 0.3 × 15 =10
Central	0.3 × 10 + 0.4 × 25 + 0.2 × 15 = 16

However, we could have done this question without using the actual calculation method too. Just see the major contributing quarters and major contributing Region in it. Major contributing quarters are quarter III and IV while central Region is main contributor in those quarters.

23. It can be seen that whenever percentage share of north Region in any quarter decreases, the total earning during that quarter increases and vice-versa with the only exception being quarter IV, year 2002.

So, the maximum change over the previous quarter must be during quarter IV, year 2002 when it's percentage share and total earnings both have increased.

24. Maximum quarterly zonal earnings are for central Region during quarter III, 2002 = Rs. 18000

Minimum quarterly zonal earnings are East Region during quarter II, 2000 = Rs 500

Hence, Maximum difference

= 18000 – 500 = Rs 17500

25. Let us tabulate the information given:

Region	Annual earning for the region
West	0.4 × Q1 + 0.3 × Q11 + 0.3 × QIV
North	0.1 × Q1 + 0.4 × Q11 +0.1 × Q111 + 0.2 × QIV
South	0.2 × Q1 + 0.2 × Q11 + 0.3 × Q111
East	0.1 × Q11 + 0.2 × Q111 + 0.3 × QIV
Central	0.3 × Q1 + 0.4 × QIII + 0.2 × QIV

Earning in south Region in 2001 = 20000 × 0.2 + 10000 × 0.2 + 20000 × 0.3 = 12000

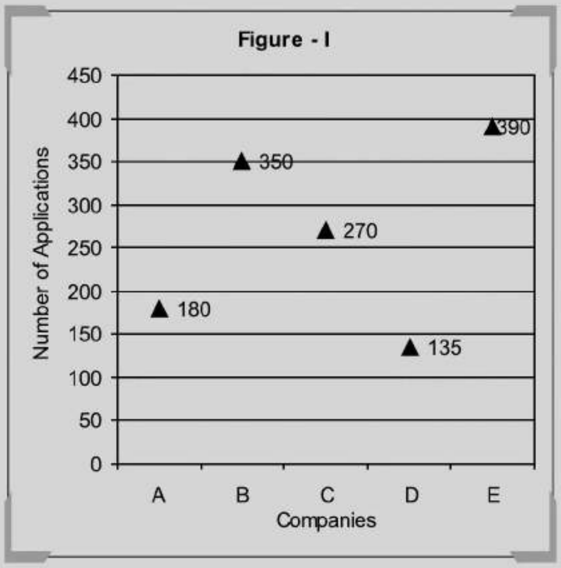
Earning in south Region in 2000 = 10000 × 0.2 + 5000 × 0.2 + 25000 × 0.3 = Rs 10500

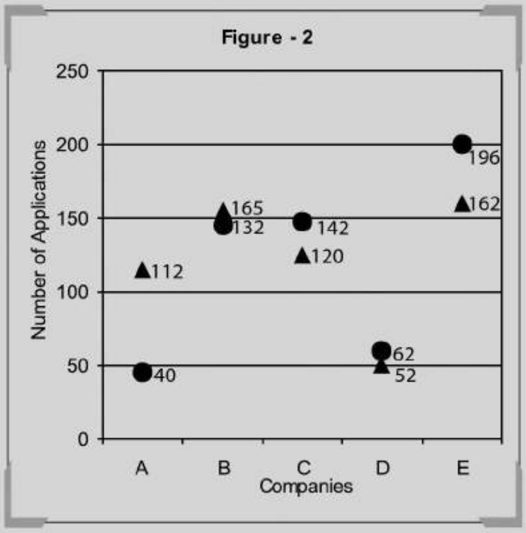
Therefore, percentage change = 14.28%

PRACTICE EXERCISE 3

Direction for questions 1 to 4: Go through the figures below and answer the questions based on it.

Fig. (1) represents the total number of applications received by five companies A, B, C, D and E in the year 2007. Fig (2) represents the total number of applications selected and total number of applications on waiting list. Total number of applications received = Number of applications selected + number of applications on rejected + number of applications on waiting list.





For companies A and B, no. of applicants selected is more than no. of applicants on waiting list.

Q 1. Which company has the minimum ratio of number of rejected applicants to the total number of applicants received?

(a) A (b) C
(c) B (d) E

Q 2. In company E, 29% of selected applicants are rejected due to health reasons and 36% of the waiting list applicants are selected. Then the percentage of selected applicants has increased/decreased by:

(a) 0.5% increase (b) 2% increase
(c) 0.5% decrease (d) 2% decrease

Q 3. Which company has maximum ratio of selected to rejected applicants, if the number of applicants selected and the number of applicants on waiting list are interchanged?

(a) E (b) A
(c) C (d) B

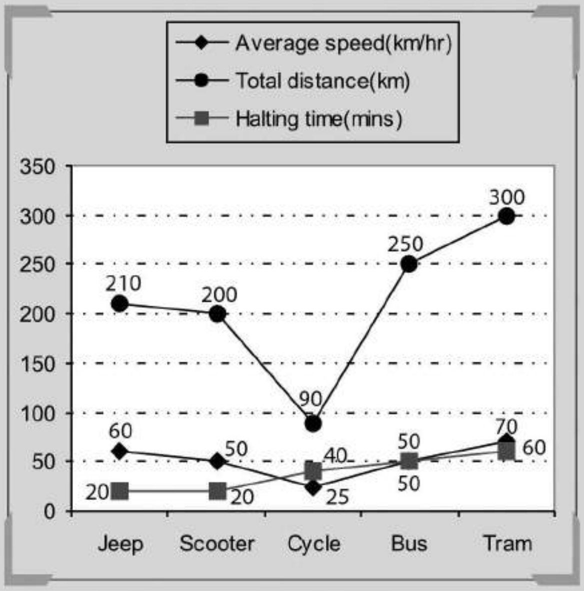
Q 4. For company A, 12.5% of the waiting list applicants are finally selected and 25% of the rejected candidates are added to the waiting list. Now, what percentage of applicants are on the waiting list?

(a) 60% (b) 45%
(c) 58% (d) 54%

Direction for questions 5 to 9: Refer to the line chart below and answer the questions based on it.

The below given line chart shows the total distance from Patna to Delhi, average speed and halting time for five different modes of travel.

It can be seen that distance from Patna to Delhi may change with the different mode of transport as different routes are taken. To calculate the average speed, we do not take halting time into consideration.



Q 5. Which mode of travel takes the least time after taking halting time into consideration?
(a) Jeep (b) Scooter
(c) Cycle (d) Tram

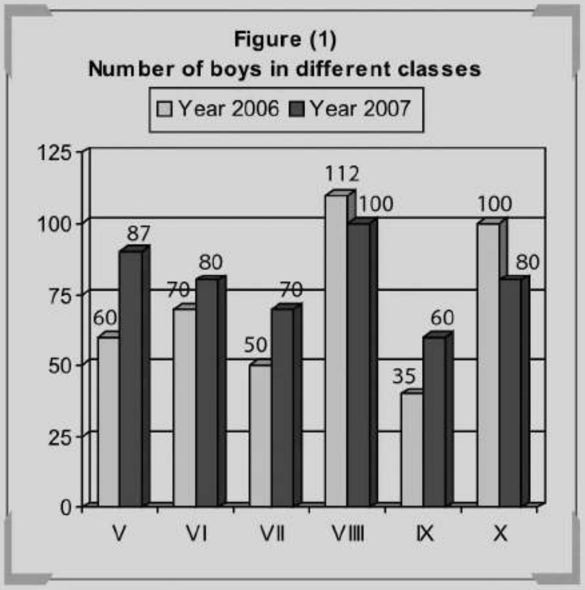
Q 6. What is the percentage change in the time taken to travel from Patna to Delhi by cycle due to halts taken during journey?
(a) 6.66%
(b) 11.1%
(c) 12.5%
(d) 16.66%

Q 7. What is the average speed of all the modes of transport during the journey without taking halting time into consideration?
(a) 46 kmph
(b) 54 kmph
(c) 50 kmph
(d) 58 kmph

Q 8. Find the maximum time taken to travel from Patna to Delhi taking halting time into consideration.
(a) 4.5 hours
(b) 5 hours
(c) 6.25 hours
(d) None of these

Q 9. If one can change the mode of transport at the halfway, then which of the following combinations takes the least time to reach B from A without taking halting time into consideration?
(a) Jeep and Scooter
(b) Cycle and Jeep
(c) Jeep and Tram
(d) Tram and Scooter

Direction for questions 10 to 13: Go through the bar charts given below and solve the questions based on it.



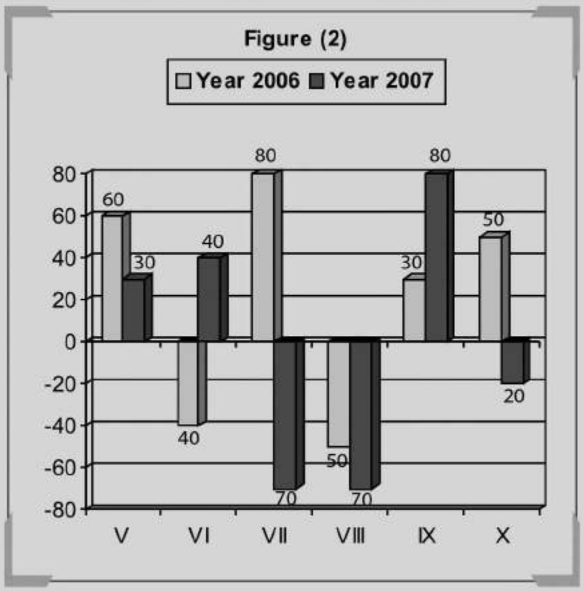


Fig. (1) gives the number of boys in different classes in year 2006 and year 2007 and fig. (2) gives the number of boys as a percentage of number of girls. If the value indicated in the chart is negative, then it is the number of girls as a percentage of number of boys.

Q 10. Find the difference in the total number of students in class VI in the year 2006 and 2007.
(a) 150 (b) 182
(c) 130 (d) 198

Q 11. Find the average number of girls studying in each standard in the year 2007.
(a) 92 (b) 103
(c) 118 (d) 136

Q 12. If 29% of the girls in class X passed in the year 2006 and 37.5% of girls passed in the year 2007, then find the approx. ratio of the no. of girls failing in the year 2006 to the no. of girls failing in the year 2007.
(a) 14 (b) 16
(c) 11 (d) 18

Q 13. Which of the following is false?
(1) Number of girls in class V is maximum in the year 2007.
(2) Number of girls in year 2006 is greater than in year 2007 in class VII.
(3) Number of girls in class VII is minimum in the year 2007.
(4) Number of girls in class X has decreased in 2007 as compared to that in 2006.

Direction for questions 14 to 17: Go through the table below and solve the questions based on it.

The chart gives details of fights operated by Indian airline in a week from Delhi to various destination and back.

From	To	Days	Departure	Arrival
DELHI	Coimbatore	Daily	6:20	7:35
DELHI	Coimbatore	2,4,6	19:15	21:15
DELHI	Cochin	Daily	9:45	11:20
DELHI	Madurai	1,3,5	20:00	21:40
DELHI	Mangalore	3,7	9:45	13:20
DELHI	Mangalore	1,3,5	7:00	9:25
DELHI	Bangalore	1,3,5	9:30	10:25
DELHI	Bangalore	2,4,6	9:30	10:25
DELHI	Bangalore	1,3,5	7:00	7:55
DELHI	Goa	2,4,6	9:30	14:00
DELHI	Hubli	1,3,5	9:30	12:05
DELHI	Huderabad	2,7	7:00	11:40
DELHI	Bombay	1,3,5	9:30	14:00
DELHI	Agatti	1,5	9:45	13:20
DELHI	Pune	2,4,6	9:30	15:20
DELHI	Trichy	2,4,6	19:15	20:10
DELHI	Trichy	1,3,5	20:00	20:50
DELHI	Vijaywada	2,6	7:00	8:20
DELHI	Vizag	2,6	7:00	9:40
DELHI	Bhuneshwar	4,6	7:00	11:20
DELHI	Trivandrrum	1,3,5	14:00	15:45
DELHI	Calicut	1,3,5	14:00	17:10

1. Monday
2. Tuesday
3. Wednesday
4. Thursday
5. Friday
6. Saturday
7. Sunday

Additional information:

In a week:

No. of departures delayed by less than 1 h = 12

No. of departures delayed by more then 1 h = 2

No of arrivals delayed by less than 1 h = 11

No. of arrivals delayed by more than 1 h = 3.

Q 14. An airlines is said to operate at 100% efficiency if 90% of its flights are on time. The efficiency of IA
(a) 80.9% (b) 88.23%
(c) 79.4% (d) 75.4%

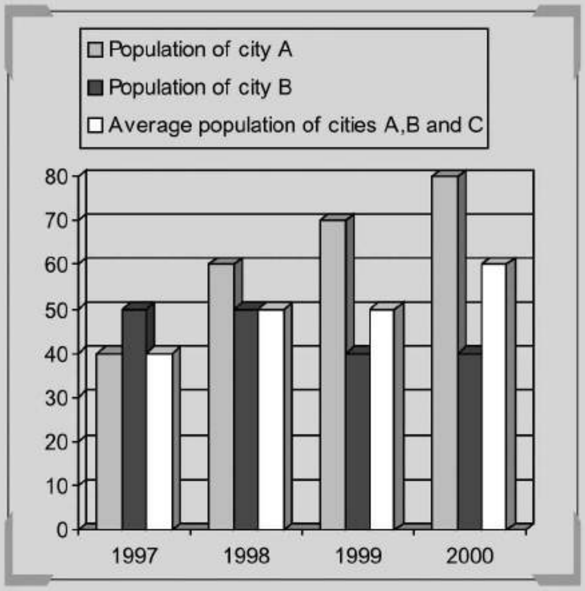
Q 15. If the airlines aims to achieve 90% efficiency on arrivals, then the no. of flights arriving on time must increase by (use data of the previous question)
(a) 3 (b) 2
(c) 4 (d) 5

Q 16. If the airlines suffers a loss of Rs 50000 on every flight delayed for more than one hour, the loss suffered by the airlines in a week is
(a) Rs 2 lakh (b) Rs 2.5 lakh
(c) Rs 3 lakh (d) Rs 3.5 lakh

Q 17. Average delay in departure is measured by delay occurring in each flight, adding up all delays and dividing by the total no. of delayed flights. The average delay in departure for flights operated by IA is
(a) 35 min (b) 25 min
(c) 40 min (d) Cannot be determined

Direction for questions 18 to 22: Go through the bar charts below and solve the questions that follow.

There are only three cities—A, B and C, in a state. Following bar chart gives us the population figures of these cities (Figure in '000):



Q 18. In which year did city C show the highest percentage growth in population?
(a) 1998 (b) 1999
(c) 2000 (d) cannot be determined

Q 19. In which year was the population of city A the highest in state's total population?
(a) 1997 (b) 1998
(c) 1999 (d) 2000

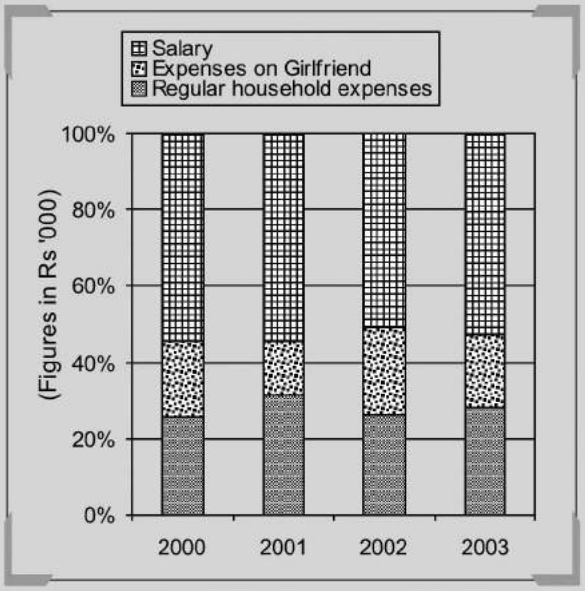
Q 20. In 1998, the ratio of men to women in the given state is 1:1 and in 1999, the same ratio is 2 : 3. What is the percentage increase in the women population from 1998 to 1999?

(a) 30% (b) 60%
(c) 20% (d) None of these

Q 21. If increase in the population of city can only be due to new births and decrease in population of city can only due to deaths, then number of new births in city A in year 2000 is (in '000):
(a) 20 (b) 40
(c) 38 (d) Cannot be determined

Q 22. For which year is the ratio of population of city A to the population of city B the highest?
(a) 1997 (b) 1998
(c) 1999 (d) 2000

Direction for questions 23 to 27: Go through the bar chart below and answer the questions based on it.



Mayank spends his yearly salary on two types of expenses only (given in the chart above) and saves the remaining amount. Given above is the graphical representation of his yearly salary and expenses over the four-year period.

Salary = Total Expenses + Savings

Q 23. If regular household expenses as a percentage of salary exceed 50%, then the living style in that year is said to be aristocratic. How many years Mayank has lived aristocratically in all the years given above?
(a) 0 (b) 1
(c) 2 (d) 3

Q 24. The ratio of expenses on girlfriend to salary has been the highest in which one of the following years?
(a) 2000
(b) 2001
(c) 2002
(d) 2003

Q 25. What is average monthly salary of Mayank over the four-year period (in Rs '000)?
(a) 10.8
(b) 12.7
(c) 14.8
(d) Cannot be determined

Q 26. In which year was the savings rate the highest [Saving rate = savings/salary × 100]?
(a) 2000
(b) 2001
(c) 2002
(d) 2003

Q 27. If his salary is same for the given four years, which year is the percentage increase in total expenses the highest?
(a) 2000 (b) 2001
(c) 2002 (d) 2003

ANSWERS

■■■

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(a)	(c)	(c)	(a)	(b)	(a)	(d)	(c)	(b)	(c)	(a)	(c)	(b)	(b)	(b)	(d)	(a)	(c)	(c)
Q. No.	21	22	23	24	25	26	27													
Answer	(d)	(c)	(d)	(c)	(b)	(a)	(d)													

HINTS AND SOLUTIONS

[illegible]

1.

Company	Number of applicants Rejected/Number of applicants Received	Number of applicants Rejected
A	28/180 = 0.155	180 – (40 + 112) = 28
B	53/350 = 0.151	350 – (132 + 165) = 53
C	8/270 = 0.029	270 – (120 + 142) = 8
D	21/135 = 1.55	135 – (52 + 62) = 21
E	32/390 = 0.08	390 – (162 + 196) = 32

So, the ratio is minimum for company C.

2. For company E, Total applicants received = 390
Number of applicants selected = 196
Number of applicants on waiting list = 162
Number of applicants rejected = 32
Now, 29% of selected applicants are rejected. So, number of selected applicants retained
= 0.71 × 196 = 139
And 36% of waiting list applicants are selected
Therefore, total selected applicants
= 139 + 0.36 x 162 = 197
Hence, increase = [1/196] × 100 = 0.5%.

3.

Company	Selected Applicants/Rejected Applicants
A	112/28 = 4
B	165/53 = 3.11
C	120/8 = 15
D	52/21 = 2.4
E	162/32 = 5.1

Obviously the ratio is maximum for C.

4. For company A, no. of applicants received = 180
Number of selected applicants = 40
Number of waiting list applicants = 112
Number of rejected applicants = 28
Now, 12.5% of waiting list applicants are selected.
Therefore, waiting list applicants
= 0.875 × 112 = 98
And 25% of rejected candidates list
= 0.25 × 28 = 7
Therefore, number of applicants on waiting list
= 7 + 98 = 105
Required percentage = 58%

[illegible]

Let us tabulate the data given:

Mode of transport	Average speed (Km/h)	Total distance (km)	Time taken for actual Journey (h)	Halting Time (h)	Total Time taken (h)	Actual average speed (km/h)
Jeep	60	210	3.5	0.25	3.75	56
Scooter	45	195	4.33	0.25	4.58	42.6
Cycle	20	90	4.5	0.5	5	18
Bus	40	250	6.25	0.75	7	35.7
Tram	75	300	4	1	5	60
Total	-	1045	22.58	2.75	-	-

5. Jeep takes the least time after taking halting time into consideration.

6. Time taken cycle without taking halt = 4.5 h
Change in time due to halt = 0.5h
Hence, percentage change in time = 11%

7. Total distance covered = 1045 km.
Time taken for actual journey = 22.58h
Hence, average speed = 46 km/h

8. Maximum time taken to travel from Patna to Delhi is for Bus and it is equal to 7 h

9. Mode of transport Total time taken (h)
Jeep + scooter = 1.75 + 2.165 = 3.915
Jeep + cycle = 1.75 + 2.25 = 4
Jeep + Tram = 1.75 + 2 = 3.75
Scooter + Tram = 2.165 + 2 = 4.165
Hence Jeep and Tram take minimum time for journey together.

[illegible]

10. In year 2000 number of boys in standard VI = 70
Number of girls = 0.4
Number of boys
Therefore, number of girls = 0.4 x 70 = 28
Total number of students = 98
In year 2001 number of boys = 80

$$= \frac{100}{0.5} = 200$$

Total number of students = 280
Difference = 280 – 98 = 182.

11.

Standard	Number of girls in year 2001
V	300
VI	200
VII	49
VIII	70
IX	75
X	16

12. In class X
Total number of girls in year 2000

$$\text{Ratio} = \frac{142}{10}$$

Total number of failed girls = 0.71 × 200 = 142
Total number of girls in year 2001 = 80 × 0.2 = 16
Total number of failed girls = 0.625 z 16 = 10

$$\text{Ratio} = \frac{142}{10} = 14.2$$

13. Only Statement [c] is wrong.

[illegible]

14. Total departure from Delhi = 68,
Total arrivals from Delhi = 68.
Total no. of delayed arrivals and departure = 28
10% of Total no. of flights that can be delayed = 136/10 = 13.6

No. of flights on time = 136 – 28 = 108 = 79.41% of flights operated. Efficiency
= 100 × 79.41/90 = 88.23%

Alternatively, for 100% efficiency, no. of flights to be on time = 90% of 136

Since only 108 flights are on time, so % age efficiency = 108/(90% of 136)

15. No. of arrivals = 68. For 100% efficiency no. of arrivals to be on time = 68 × 0.9 = 61.2.
For 90% efficiency no. of arrivals to be on time
= 62.2 × 0.9 = 55.08 (nearly)
No. of present arrivals on time = 68–14 = 54.
Hence 2 more flights are to be on time.

16. In a week no. of departures and arrival involving a delay of more than one hour = 5.
Loss suffered = 5 × 50000 = 2.5 lakhs.

17. We don't have the exact time for which the flights are delayed. So, average delay cannot be determined.

[illegible]

18. The first step would be to approximate the lengths of various bars then one can prepare the table as follow:

Population	1997	1998	1999	2000
Average	40	50	50	60
Total (3 × average)	120	150	150	180
City A	40	60	70	80
City B	50	50	30	40
City C Total − (A+B)	30	40	50	60

The population of city C increases by 10000 every year.

Hence, highest percentage increase is in 1998 over 1997 as it has the lowest base.
So, [a].

19. We are supposed to find here the ratio of 1st bar and 3rd bar.

21. Population (2000) = Population (1999) +
No. of new births (2000) –
No. of deaths (2000).
The no. of births figure cannot be determined as the no. of deaths is not known.
Hence, [d]

22.

	2000	1997	1998	1999
City A	40	60	70	80
City B	50	50	30	40

The ratio is the highest for the year 1999.

[illegible]

23. Total income has never been the 50% of the total income. Hence, for none of the years this has happened.

24. Ratio of expenses on girlfriend to salary is tabulated below:

2000	55/150
2001	40/155
2002	60/135
2003	65/170

It is clear now that the ratio is the highest for year 2002. Hence, [c].

25. Average yearly salary

$$= \frac{150+155+135+170}{4} = \frac{610}{4} = 152.5$$

Therefore, average monthly salary = 12.7

PRACTICE EXERCISE 4

Direction for questions 1 to 4: Go through the table below and solve the questions based on it.

Given below are the win loss histories of four tennis players in their career.

Result/Players	P	Q	R	S
Won	23	18	10	16
Won	13	16	13	14
Lost	8	13	16	13
Lost	6	8	20	17

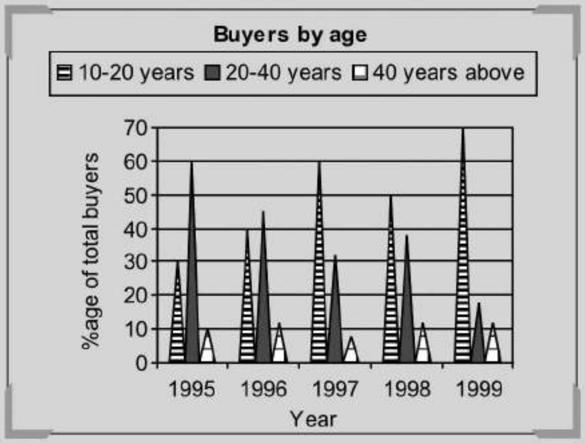
Q 1. If a win carries an award of Rs 1500 and a loss carries of Rs 1000, what is the largest difference in prize money earned among the four?
(a) 7000 (b) 14000
(c) 5000 (d) 9000

Q 2. As in the case above what is the lowest average prize money won per match by any player?
(a) 1135 (b) 1300
(c) 1195 (d) 1150

Q 3. As in the case above what is the highest average prize money won per match by any player?
(a) 1495 (b) 1360
(c) 1460 (d) 1410

Q 4. Who has lost the maximum percentage of matches played?
(a) P (b) Q
(c) R (d) S

Direction for questions 5 to 9: Go through the bar chart given below and solve the questions based on it.



Q 5. By how many percentage has the number of buyers in the age 20–40 years age group has declined from year 1995 to 1999?
(a) 26.6 (b) 73.4
(c) 44 (d) 20

Q 6. If the total number of people surveyed is 2000 in each of the five years given, then find the number of buyers changed from year 1995 to 1999 in the age group 10–20 years?
(a) 800 (b) 600
(c) 1200 (d) 1400

Q 7. Which of the following age group shows a declining trend in the purchases in the given five years?
(a) 10–20 years
(b) 20–40 years
(c) 40 years and above
(d) No particular trend in any group

Q 8. By how many has the percentage number of buyers in the age 10-20 years group has increased from year 1997?
(a) 13.6 (b) 15
(c) 16.7 (d) 10

Q 9. Which of the following age groups show no particular trend?
(a) 10-20 years
(b) 20-40 years
(c) 40 years and above
(d) no particular age group

Direction for questions 10 to 17: Go through the table given below and solve the questions based on it.

There are eight major locations in Sanjay Gandhi Botanical Garden in Patna. A survey was conducted to find out the movement of the people within the garden. Following results were obtained:

		To							
		A	B	C	D	E	F	G	H
From	A		48	46	66	24	4	68	64
	B	20		64	152	64	90	108	86
	C	28	108		88	86	46	130	130
From	D	54	92	66		42	82	20	4
	E	70	48	50	110		26	46	22
	F	24	74	92	68	112		120	60
	G	28	32	134	152	6	86		26
	H	50	70	90	108	68	110	86	

Q 10. The least number of people go to which location?
(a) G (b) D
(c) B (d) A

Q 11. What is the total number of people visiting Zoo?
(a) 3868 (b) 5000
(c) 4000 (d) Cannot be determined

Q 12. What percentage of total visiting E comes from D?
(a) 10 (b) 20
(c) 40 (d) 30

Q 13. What is the difference between the number of people going from B to F and G and D to F and G?
(a) 48 (b) 96
(c) 72 (d) 84

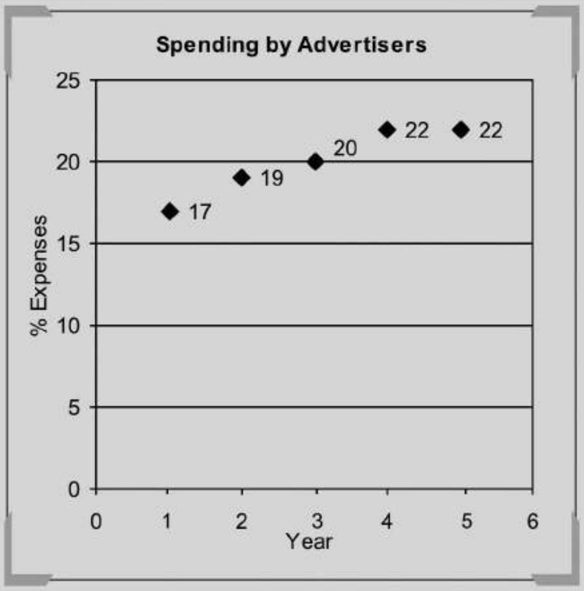
Q 14. The least number of people leave from which place?
(a) H (b) G
(c) A (d) E

Q 15. The number of people leaving F and C is …… the number of people leaving C and F.
(a) Greater than
(b) Less than
(c) Equal to
(d) Cannot say with certainty

Q 16. The number of people going to A from B and C is equal to the number of people going to ……from ……and ……..
(a) E, A, C (b) E, D, C
(c) F, A, C (d) H, E, G

Q 17. The highest number of people leave from which place?
(a) C (b) D
(c) H (d) F

Direction for questions 18 to 24: Go through the chart given below and solve the questions based on it.



Total for all the five years = Rs 100 crores.

Q 18. In year 3, out of the total money spent by the advertisers, 12% had been spent on contingent expenses. What was the amount spent on contingent expenses (in Rs crore)?
(a) 2.4
(b) 2.1
(c) 2.5
(d) 2.6

Q 19. Expenses of year A added to one fourth the expenses of another year B equals the expenses of another year C. Identify A, B and C.
(a) 1-3-4
(b) 2-3-4
(c) 1-2-3
(d) 3-1-4

Q 20. In how many years, has the expenditure has got increased by the atleast 15% with respect to year 2?
(a) 1
(b) 3
(c) 2
(d) Insufficient data

Q 21. The difference in the expenditure between which two years equals the difference between the expenditure in year 2 to year 1?
(a) Year 5 and Year 3
(b) Year 4 and Year 3
(c) Both (1) and (2)
(d) Neither (1) and (2)

Q 22. If in year 6 the expenses increase at the same rate as they did from year 3 to year 5. what will be the expenditure in year 6 (Rs crore)?
(a) 28.2
(b) 24.2
(c) 25.2
(d) 23.2

Q 23. If in year 4, out of the money spent by the advertisers, 10% is spent on disaster recovery expenses, what was the amount spent on disaster recovery expenses?
(a) 2.2
(b) 2.6
(c) 2.3
(d) 2.4

Q 24. How many years, the spending was more than the average spending per annum for the given period?
(a) 0
(b) 1
(c) 2
(d) 3

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(d)	(b)	(c)	(b)	(a)	(b)	(d)	(c)	(d)	(d)	(d)	(a)	(b)	(c)	(a)	(a)	(a)	(a)	(c)
Q. No.	21	22	23	24																
Answer	(c)	(d)	(a)	(c)																

HINTS AND SOLUTIONS

1. Ee get the following table of earnings:

Persons	Win Total	Average
P	36 × 1500 + 14 × 1000 = 68000	1360
Q	34 × 1500 + 21 × 1000 = 72000	1310
R	23 × 1500 + 36 × 1000 = 68500	1160
S	30 × 1500 + 30 × 1000 = 60000	1000

Now we can see that the largest difference
= Max – Min = Rs 75000 – Rs 68000 = Rs 7000

2. Using the data from above question, lowest average is Rs 1160.
3. Highest average is Rs 1360
5. Required percentage = (60–16)/60 x 100 = 73%
6. Total no. of persons = 2000.
Change has been 40% (70–30).
So required answer = 2000 × 0.4 = 800
9. Required percentage = 16.7%
12. The information given is for 8 major locations in the Zoo. There can be many more minor locations in the Zoo for which information about movement of people is not given.

13 Total people visiting
E = 24 + 64 + 86 + 42 + 112 + 6 + 68 = 402.
So, required percentage = 42/402 × 100 = 10%.

14. Number of people going from
B to F and G = 90 + 108 = 198.
Number of people going from
D to F and G = 82 + 20 =102.
Since difference = 96.
17. Number of people going to A from B and C
= 20 + 28 = 48.
Go through the options now.
19. Required amount = 0.12 × 20 = 2.4 crore.
20. Use the options.
21. The difference between the expenditure in year 2 and year 1 is 2%. Use the options now.
22. The rate of increase in expenses from year 3 to year 5 is 10%. So expenditure in year 6
= 22 × 1.1 = 24.2 crore.
23. Required amount = 22 × 0.1 = 2.2 crore.

PRACTICE EXERCISE 5

Direction for questions 1 to 7: Go through the table given below and solve the questions based on it.

The XYZ manufacturing company produces a product which has a 6 month demand cycle as given below:

Month	Demand in Units	Working days/month
January	300	22
February	500	19
March	400	21
April	100	21
May	200	22
June	300	20

Company's policy is to retain safety stock of 20% of monthly demand and each month's safety stock becomes the opening stock for the next month, and opening stock for January is 50 units. Each unit requires 10 labour-hours and company works 8 hour/day.

Q 1. What is the opening stock for June?
(a) 20 (b) 40
(c) 60 (d) 100

Q 2. What is production requirement for March?
(a) 220 (b) 310
(c) 380 (d) 320

Q 3. In which month production requirement is highest?
(a) February (b) March
(c) January (d) June

Q 4. In which month Opening stock is lowest?
(a) February
(b) March
(c) January
(d) May

Q 5. If there are currently 20 workers employed, and how many workers are needed to be laid off in January to accommodate demand?
(a) 6 (b) 2
(c) 3 (d) 5

Q 6. If constant work force of 20 is retained, how many hours workers sit idle in January month?
(a) 300 h (b) 420 h
(c) 240 h (d) 200 h

Q 7. If constant work force of 20 is retained, how many total hours workers do overtime in February month?
(a) 2000 h (b) 3000 h
(c) 2360 h (d) 3360 h

Direction for questions 8 to 11: Go through the table given below and solve the questions based on it.

The following table gives the tariff (in paisa per kilowatt–hour) levied by the NTPC in 2004–05, in the four sectors and the regions within them. The table also gives the percentage change in the tariff as compared to 2001–02.

	Region 1		**Region 2**		**Region 3**		**Region 4**		**Region 5**	
Sector 1	425	+15	472	+5	420	–4	415	+8	440	+10
Sector 2	430	+12	468	+8	448	+7	423	–3	427	+11
Sector 3	428	+8	478	–4	432	+6	441	+10	439	+8
Sector 4	434	–5	470	+15	456	+10	451	+12	446	–12

Q 8. If the amount of power consumed by the various Regions in Sector 1 is the same, then, as compared to 2001–02, the net tariff in 2004-05…
(a) Increased by 9.5%
(b) Decrease by 1.5%
(c) Decrease by 13.3%
(d) None of these

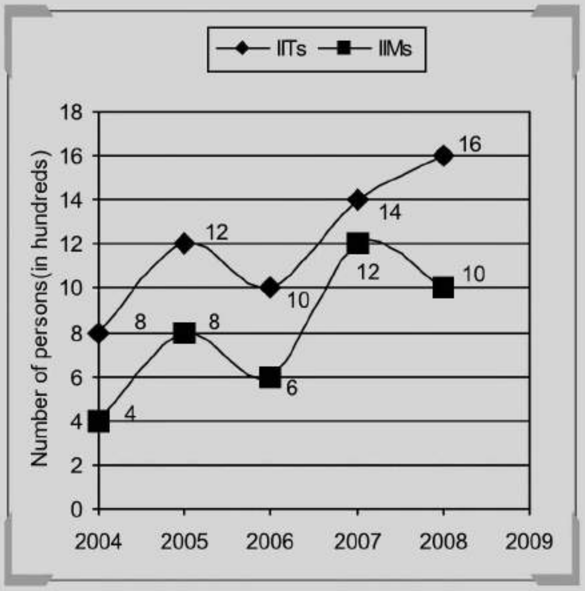
Q 9. What approximately was the average tariff per sector in Region 3 in 2001–02?
(a) 402 (b) 424
(c) 433 (d) None of these

Q 10. Which of the following has got a maximum percentage increase for the given period?
(a) Region 1 – Sector 1
(b) Region 1 – Sector 2
(c) Region 5 – Sector 2
(d) None of these

Q 11. There are a total of 20 units of NTPC divided into regions and then subdivided into sectors, as given in the above table. For how many units of NTPC in 04–05, there is a net increase in the tariff?
(a) 15 (b) 5
(c) 8 (d) None of these

Direction for questions 12 to 17: Go through the chart given below and solve the questions based on it.

Chart given below represents total number of persons (in hundreds)who have passed out from IIMs and IITs in each year from 2004 to 2008. The IITs and IIMs are among the best known professional institutes in India and offer courses in the areas of engineering and management respectively. Both the IIMs and IITs have six campuses located in various cities of India, none of the cities being common.



Q 12. The percentage increase in the number of IIMs pass outs in 2007 over 2004 is
(a) 300% (b) 100%
(c) 200% (d) 75%

Q 13. Compared to 2007, the number of management graduates in 2008 has
(a) Increased
(b) Decreased
(c) Remained same
(d) Cannot be determined

Q 14. How many times during the given period has the growth in pass outs from IITs exceeded 25% over previous year?
(a) Once (b) Twice
(c) Thrice (d) Never

Q 15. During the given period, the total pass outs from IIMs as a ratio of total pass outs from IIT is
(a) 41 : 60 (b) 10 : 13
(c) 6 : 9 (d) 4 : 5

Q 16. What is the ratio of IIM graduates in 1996 to that in 1998?
(a) 0.37 (b) 2.66
(c) 1.04 (d) 0.96

Q 17. On an average, every year 20% of the students who pass IITs go to IIMs the same year. What is the total number (in '000) of IITs pass outs who went to IIM in 2006?
(a) 2 (b) 2.5
(c) 1 (d) Cannot be determined

Direction for questions 18 to 21: Go through the table given below and solve the questions based on it.

The table given below gives the unit cost of transportation of material of three products A, B and C produced by ABC Ltd to five depots D1, D2, D3, D4 and D5. The production figures of the three products A, B and C are 240 units, 1,200 units and 840 units respectively.

Product	Transportation cost per unit (Rs)				
	D1	D2	D3	D4	D5
A	1.5	1.6	2.4	2	1
B	3	3.2	3	2.8	4
C	1.8	1.5	2	1.6	1.1

Q 18. What is the total cost of transportation of all units of all the products to D1?
(a) Rs 5,272
(b) Rs 5,472
(c) Rs 5,282
(d) Rs 5,742

Q 19. If 48 units of A are transported to each of the depots, what is the average cost of transportation?
(a) Rs 1.70
(b) Rs 1.60
(c) Rs 1.65
(d) Rs 1.75

Q 20. Which of the following would involve the least cost?
(a) Transporting 300 units of B to D4
(b) Transporting 200 units of B to D1
(c) Transporting 300 units of C to D2
(d) Transporting 400 units of C to D5

Q 21. By how much is the average transportation cost of B more than that of A?
(a) Rs 1.80 (b) Rs 1.50
(c) Rs 1.60 (d) Rs 1.40

Direction for questions 22 to 25: Read the passage below and solve the questions based on it.

ABC is a firm that deals with furniture. Manufacturing of table requires three levels of assembly. The finished table is at first level. The leg assembly and tabletop are second level. The pieces that go into the leg assembly are at the third level which consist of shortrails, longrails and legs. One unit of table requires one unit of tabletop and one unit of leg assembly. One unit of leg assembly requires 2 units of shortrails, 2 units of longrails and 4 units of legs. Orders are placed just in time to minimize storage.

The lead time for activities are (Lead time is waiting time required to complete one activity)

Parts	Weeks
Assemble table	1
Finished leg assembly	1
Purchase legs	1
Purchase shortrails	1
Purchase longrails	1
Purchase table top	2

The availability of parts at present time:

Parts	Units
Table	50
Leg assembly	100
Legs	150
Shortrails	50
Longrails	0
Table top	50

Demand of finished tables

Details	Week 4	Week 5	Week 6
Demand (units)	200	150	100

Q 22. For meeting the demand of 200 units of finished table of week 4, when would the first order of tabletops be placed?
(a) Week 1
(b) Week 3
(c) Week 5
(d) None of these

Q 23. What is the net requirement of legs for meeting the demand of week 4 finished table?
(a) 200
(b) 50
(c) 400
(d) 800

Q 24. When and how many units of shortrail would be placed for meeting the demand of finished table of week 6?
(a) 100 units in week 1
(b) 200 units in week 3
(c) 300 units in week 6
(d) Cannot be determined

Q 25. Due North Inc. needs a new kind of table in which 8 legs and 4 short rails would be needed for manufacturing a single table. What would be the net requirement of legs and shortrails for meeting the total demand of finished table (including the additional demand of 200 tables for Due North Inc.) in week 5?
(a) 1800, 900
(b) 2200, 1100
(c) 1600, 800
(d) None of these

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(a)	(d)	(c)	(b)	(c)	(d)	(d)	(a)	(a)	(c)	(d)	(b)	(c)	(d)	(d)	(b)	(a)	(d)
Q. No.	21	22	23	24	25															
Answer	(d)	(a)	(b)	(b)	(d)															

HINTS AND SOLUTIONS

[illegible]

We get the following table:

Month	O.S.	Demand	Reserve 20%	Prod. required	Working days
Jan	50	300	60	310	22
Feb	60	500	100	540	19
Mar	100	400	80	380	21
Apr	80	100	20	40	21
May	20	200	40	220	22
Jun	40	300	60	320	20

O.S. = Opening Stock

1. 20% of May demand (200) = 20% of 200 = 40

2. Production requirement for
Mar = Demand + Reserve – opening stock
= 400 + 80 – 1000 = 380

5. One worker in January (22 working day);
Gives = 22 × 8 =1 76 man Hours;
Total man hours required = 310 x 10 = 3100.
No. of workers required to be laid off = 20 – 17 =3.

6. Required No. of man hours Available – No. of Man hours utilized = $20 \times 22 \times 8 - 310 = 3520 - 3100 = 420$.

7. No. of man hours required – No. of man hours Available = 540 × 10 – 20 × 19 × 8 = 2360 h

[illegible]

13. It is not given if IIMs are the only institutes providing management degrees, there can be more institutes providing management degree.

15. IITs = 60; IIMs = 40

Ratio of IIM to IIT pass out $= \frac{4}{6} = \frac{6}{9}$

16.
$$\frac{\left(\frac{6}{10}\right)}{\left(\frac{10}{16}\right)} = 0.96$$

17. The answer is cannot be determined due to various reasons:
Reason 1 – Data is given as ‘on an average’ and not the exact data.
Reason 2 – There can be IITs pass outs from yesteryears making it to IIMs in a particular year.
Hence, cannot be determined.

18. Required cost
= (240 × 1.5) + (1200 × 3) + (840 × 1.8)
= Rs 5472.

19. Required average $= \frac{1.5+1.6+2.4+2+1}{5} =$ Rs 1.7

20. The cost of transportation for each options are:
(a) 300 × 2.8 = Rs 840
(b) 200 × 3 = Rs 600
(c) 300 × 1.5 = Rs 450
(d) 400 × 1.1 = Rs 440
Obviously, the cost of transportation of option (d) is the least.

21. We don’t know how many units of which product are shipped to which depot.

5

CHAPTER

Advanced Exercises

PRACTICE EXERCISE 1

Direction for questions 1 and 2: Read the following passage and solve the questions based on it.

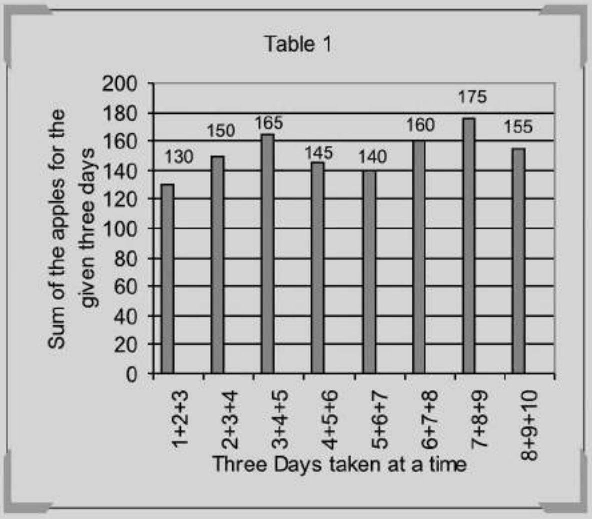
You are the head of a mutual fund, responsible for investing the available money in different funds/securities. From the market trends you find that blue-chip companies and government stocks will generally not do as well as corporate bonds, in the coming year. However, government regulations require that at least one-third of your mutual fund's capital must be in blue-chip stocks and another third in corporate bonds.

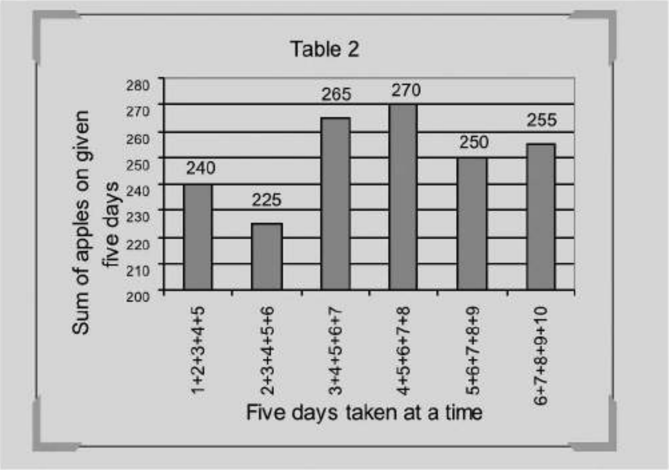
Q 1. Under current regulations, what seems to be the best way for you to invest your funds so as to maximize profit?
(a) two third government stocks, one-third blue-chip stocks
(b) two third government. stocks, one-third corporate bonds
(c) one third government. stocks, two-third corporate bonds
(d) one third each of government stocks, blue-chip stocks and corporate bonds

Q 2. If the return on government stocks suddenly goes up by 5%, you would probably:
(a) sell blue-chip stocks to buy government stocks.
(b) sell corporate bonds to buy government stocks.
(c) sell both corporate bonds and blue-chip stocks to buy government stocks
(d) no conclusive action can be taken

Direction for questions 3 to 8: Read the following passage and solve the questions based on it.

Munu, an intelligent baby girl, plucks apples from her father's orchard on ten consecutive days. When asked by her mother as to how many apples has she plucked from the orchard daily, she gives the following sheet to her mother:





Q 3. What is the number of apples plucked on day 6?

(a) 45 (b) 35
(c) 25 (d) Cannot be determined

Q 4. For how many days, is it possible to determine the exact number of apples plucked by Munu?

(a) 0 (b) 5
(c) 3 (d) None of these

Q 5. Disregarding table 2, how many of the following statements are definitely true?

I. Number of apples plucked on day 2 is more than the number of apples plucked on day 5.
II. Number of apples plucked on day 3 is more than the number of apples plucked on day 5.
III. Sum of the apples plucked on day 1 and day 2 is less than the sum of the apples plucked on day 4 and day 5.
IV. Sum of the apples plucked on day 2 and day 3 is less than the sum of the apples plucked on day 3 and day 4.

(a) 0 (b) 1
(c) 2 (d) 3

Q 6. In the above question, how many of the statements can be verified for being right or wrong (using table 1 only)?

(a) 0 (b) 1
(c) 2 (d) 3

Q 7. Disregarding table 2; using how many of the following statements individually can we find the number of apples plucked each day by Munu?

I. Total number of apples plucked from day 1 to day 10.
II. Sum of the number of apples plucked on all the possible pairs of days.
III. Difference in the number of apples plucked in between all the two consecutive days.
IV. Sum of the number of apples plucked in all the four consecutive day periods.

(a) 0 (b) 1
(c) 2 (d) 3

Q 8. Disregarding table 1; using how many of the following statements individually can we find the number of apples plucked each day by Munu?

I. Total number of apples plucked from day 1 to day 10.
II. Sum of the number of apples plucked on all the possible pairs of days.
III. Difference in the number of apples plucked in between all the two consecutive days.
IV. Sum of the number of apples plucked in all the four consecutive day periods.

(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 9 to 14: Read the following passage and solve the questions based on it.

A company 'Y' is engaged in the manufacturing and selling of instant breakfast food. It has a long standing in the consumer market and has captured a large share of the instant food market. As the company enjoys a long standing, and is quality oriented, it has over the years increased its overhead expenses substantially. However, as it has a large market share, it is surviving with good monetary results. The factory is situated in Jagroaonbad which is very far away from the main consumer market (which are the country's capital town and other bigger towns in the vicinity). This distance from the main consumer market has further attributed to an increased cost of the product, being marketed by company 'Y'.

Another company 'X' has come up a few years back, with its factory situated near Ambalpur, a few kilometers from the main capital town and other bigger towns in the area. The product being manufactured by this company is more or less the same in taste, favour, etc. as that of company 'Y'. In view of its proximity with the consumer market, the transportation cost being incurred by company 'X' is comparatively less. Furthermore, being a new enterprise, it has controlled overhead expenses. With this advantage to its credit, it has priced its product a little less than that of company 'Y' and has it slowly penetrated into the market share of company, 'Y'.

The company 'Y' is now faced with the problem of competing with this new company 'X' without, of course decreasing the quality of its product. Company 'Y' however, cannot reduce the price of its product that as will lower the image of the company in the market and it will also reduce its profits. In the present state the company cannot afford to do this. The chief executive of the company is now faced with the problem of competing with company 'X' without lowering the market reputation of his company and influencing its profits.

The chief executive has decided to undertake a detailed market survey and study all types of instant breakfast foods being marketed in the country. By examining the competitor's products it was revealed that the product being marketed by company 'Y' was a quick consumption item of the highest quality, having a much longer shelf life as compared to other products available in the market. This product was packed in tin containers which after consumption of the contents became scrap without any resale value. The chief executive therefore, decided upon a pre-project study to go in for some different type of packing. The study revealed that they can either go in for glass jars or flexi-packs, or both. As the product is of a high quality and has a longer shelf life, it can withstand climatic conditions in flexi-packs. This, thereby drastically reduces the expenditure on packaging costs (in tin containers this is nearly 38 percent of the total cost of the product and by going in for flexi-packs, the same can be reduced to nearly 15 percent). The saving made in packaging costs can then be released to the consumers by way of reducing the price of the product. This reduction in price will not lower the image of the company as the price reduction will be given along with the background of regarding the change in packaging.

Q 9 Company ‘Y’, however, cannot outrighthy reduce the price of its product because:

I. of the company’s policy
II. it lowers its image in the market
III. it will reduce its profit

(a) I only (b) III only
(c) I and II (d) II and III

Q 10. The selling price of the product manufactured by company ‘X’ was low because :

I. of less transportation costs
II. they had less overheads as compared to company ‘Y’
III. the quantity per unit was less than that of their competitors

(a) I only (b) III only
(c) I and II (d) II and III

Q 11. Why did company ‘Y’ go for flexi-packs?

I. to be in tune with marketing trends
II. easy to handle on filling lines
III. to save on packaging costs

(a) I only (b) III only
(c) I and II (d) II and III

Q 12. Savings made in cost of packaging

I. helped reduce cost of production
II. helped reduce selling price
III. allow more production

(a) I only (b) III only
(c) I and II (d) II and III

Q 13. Why was company ‘Y’ selling the same type of product at a rate higher than that of company ‘X’?

(I) Before company ‘X’ entered the market, company ‘Y’ enjoyed a monopoly in the market and took advantage of the same.
(II) As company ‘Y’ had a long standing, its overheads were comparatively higher.
(III) Their total cost of production was higher.

(a) I only (b) III only
(c) I and II (d) I, II and III

Q 14. The market survey conducted by company ‘Y’ revealed that their product was a quick consumption item of a higher quality with a longer shelf-life. What was the conclusion drawn from the survey?

I. Being a quick consumption item, it can have a cheaper packaging, as the problem of aging will not be there.
II. Longer shelf-life shows that the product has durability and therefore can stand for a longer period in the market without changes in the quality of the product.
III. Their product enjoyed better reputation than that of company ‘X’.

(a) I only (b) III only
(c) I and II (d) II and III

Direction for questions 15 to 17: Read the following passage and solve the questions based on it.

The table below shows the total number of students who enrolled for five marketing courses over the last four years in a B-school. Answer the following questions based on the table.

Courses	1999	2000	2001	2002
Brand Management (BM)	24	29	30	33
Sales Management (SM)	25	30	26	29
Marketing Research (MR)	43	39	38	40
International Marketing (IM)	29	33	34	32
Consumer Behaviour (CB)	21	19	23	24

Q 15. If it is mandated that a student who wishes to enroll for BM or CB has to take MR, then which of the followings statements is necessarily true?

(a) In 1999, only one student must have enrolled for both BM and CB.
(b) In 2001, not more than 7 students could have enrolled both for BM and CB.
(c) In 2000, at least 9 students must have enrolled for both BM and CB.
(d) In 2002, not more than 5 students could have enrolled both for BM and CB.

Q 16. It is mandated that those who enroll for IM should take SM or CB. Then, which of the statements is necessarily true?

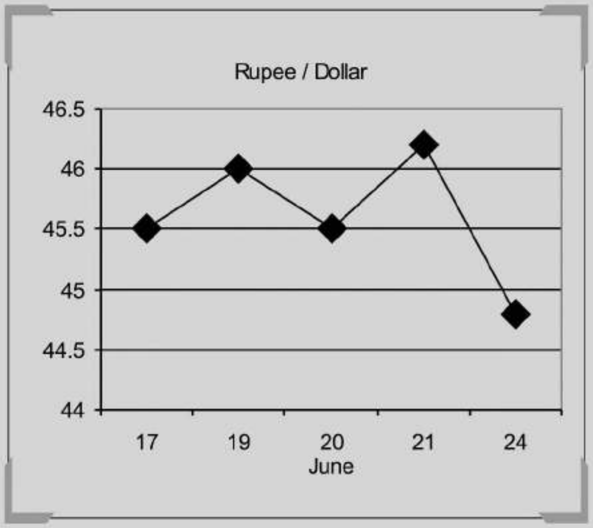
(a) In 1999, the number of students who enrolled for SM, CB and IM is at least 4.
(b) In 2000, the number of students who enrolled for IM and CB and not SN is not more than 10.
(c) If at least four students who took IM and SM also took CB, then the maximum number of students who took CB but not IM is 17.

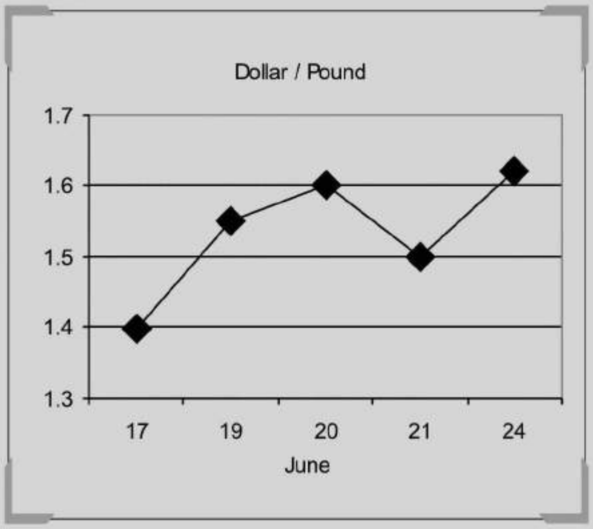
(d) In 2002, the number of students who enrolled for SM, CB and IM is at least 3.

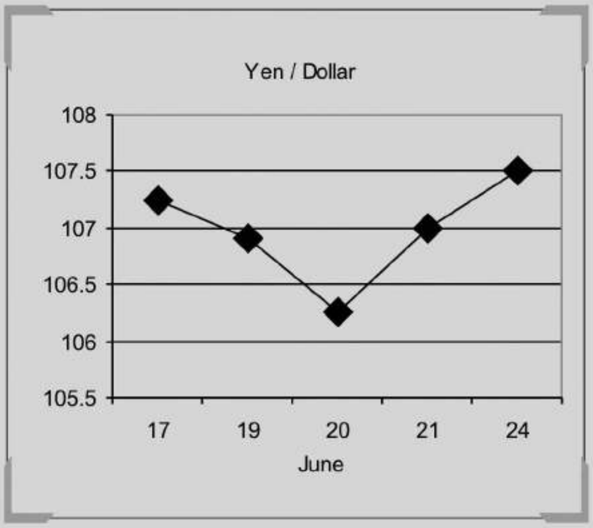
Q 17. If the rules of this school prescribe that a student should take 3 and not more than 3 marketing area courses to specialize in the marketing stream, which of the following statements cannot be true?
(a) In 1999, all those who took CB also took BM and MR.
(b) In 2000, all those who took BM also took SM and IM.
(c) In 2001, all those who took BM also took MR and IM.
(d) all the above

Direction for questions 18 to 21: Refer to the following line charts and solve the questions based on it:

The following line charts show the currency ratio from 17th June to 24th June:







Q 18. On 19th June how much Yen would Rs 8000 amount to?
(a) 18600 (b) 19970
(c) 20200 (d) 17200

Q 19. If the Yen/Dollar ratio follows the same trend from 24th to 27th as from 21st to 24th, then what will be the value of $ 50 on 27th June?
(a) Rs 2000 (b) Rs 5400 Yen
(c) 6000 Yen (d) Rs 3000

Q 20. A jacket costs $ 400. Mohan has Rs 18,200, £ 225 and ¥ 35000 with him. If he can use only one currency to buy the jacket, he would be able to buy the stereo set on 20th June using:
(a) Rs 18,200 (b) £225
(c) ¥ 35000 (d) Cannot be determined

Q 21. On 15th June the Rupee/Dollar ratio was 46.5. On that day Rs 5000 would fetch_________ Pounds.
(a) 80.06 (b) 71.68
(c) 74.15 (d) Cannot be determined

Direction for questions 22 to 25: Read refer to the following table and solve the questions based on it.

The following table shows the highest and the average marks obtained by students in subjects over the years.
The maximum mark in each subject is 100.

Q 22. What were the grand average marks of the five subjects in 1996?
(a) 63 (b) 64
(c) 65 (d) 68

Q 23. The difference in the average marks in History between 1994 and 1995 was exactly equal to the difference in the highest marks in Hindi between which of the following pair of years?
(a) 1992 and 1995 (b) 1993 and 1995
(c) 1992 and 1996 (d) none of these

Q 24. What was the approximate percentage increase in average marks in history from 1992 and 1993?
(a) 20%
(b) 25%
(c) 24%
(d) 16%

Q 25. The average highest marks in English in 1992, 1993 and 1996 were exactly equal to the highest marks in Hindi in which of the following years?
(a) 1996
(b) 1997
(c) 1994
(d) 1993

	SUBJECTS									
	English		Hindi		Maths		Sciences		History	
	High	Avg.	High	Avg.	High	Avg.	High	Avg.	High	Avg.
1992	85	62	75	52	98	65	88	72	72	46
1993	80	70	80	53	94	60	89	70	65	55
1994	82	65	77	54	85	62	95	64	66	58
1995	71	56	84	64	92	68	97	68	68	49
1996	75	52	82	66	91	64	92	75	70	58
1997	82	66	81	57	89	66	98	72	74	62

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(a)	(b)	(d)	(b)	(c)	(b)	(c)	(d)	(c)	(b)	(c)	(b)	(c)	(c)	(a)	(c)	(a)	(b)	(a)
Q. No.	21	22	23	24	25															
Answer	(d)	(c)	(a)	(a)	(d)															

HINTS AND SOLUTIONS

3. Do (1 + 2 + 3) + (4 + 5 + 6) in table 1 and then subtract (1 + 2 + 3 + 4 + 5) from it. Therefore, the correct answer is option (b) 35 apples.

4. We can find it out for all the days starting from day 1. See the solution of Q 3.

5. Only statement III is correct. To find the veracity of statement III, compare (1 + 2 + 3) with (3 + 4 +5)

6. Only statement I and statement III can be verified.

7. Except I, all give us the solution.

9. It is clearly given that company ‘Y’ cannot reduce the price as it lowers the image and reduces its profit also. Therefore, option (d) is the correct answer.

10. Transportation cost of company ‘X’ is comparatively less. Besides being a new entrant, it has kept its overhead expenses low. Therefore, option (c) is the correct answer.

11. Flexi-packs reduce the cost of packaging. Therefore, option (b) is the correct answer.

12. The savings made can be trickled down to the customers by way of reducing the price of the product. Therefore, option (c) is the correct answer.

13. Neither statement I nor II can be verified. However, statement III is mentioned very clearly. Therefore, option (b) is the correct answer.

14. Only statement I and II can be verified. It is given that quality of product ‘Y’ was better than all others, but it cannot be concluded from here that it enjoyed better reputation than company ‘X’. Therefore, option (c) is the correct answer.

15. (29 + 19) – 39 = 9. Hence in 2000, at least 9 students must have enrolled for both BM and CM courses.

16. In 1999, difference of students in IM and
SM = 29 – 25

In SM and CB = 25 – 21 = 4

Hence, atleast 4 students enrolled for SM, CB and IM.

17. Since, in 2001, number of students in BM > number of students in SM

22 Average = $\frac{52+66+64+75+58}{5} = \frac{315}{5} = 63$

23. The difference is 9. Therefore the correct answer is option (a)

24. Percentage Increase = $\frac{55-46}{46} \times 100 = 20\%$

25. Average highest marks

$$\frac{85+80+75}{3} = \frac{240}{3} = 80$$

Therefore the correct answer is option (d)

PRACTICE EXERCISE 2

Direction for questions 1 to 4: Refer to the following table and solve the questions based on it.

Table A
Manpower Costs for a Building Project in Narhi
(All costs in Rs)

Project Phase	Managers	Engineers	Skilled Workers	Unskilled Workers	Supervisors
Design	1	3	--	--	--
Foundation	1	2	5	23	2
Construction	1	2	12	43	4
Finishing (Internal)	1	3	15	31	6
Painting	1	0	5	23	3

Table B
Cost per month for Select Category of Workers

Manpower	Cost (in Rs)
Managers	10,000
Engineers	8,000
Supervisors	5,000
Skilled Workers	3,000
Unskilled Workers	1,800

Table C
Estimated Time and the Actual Time Required for Each Phase of the Project in Months

	Design	Foundation	Construction	Finishing	Painting
Actual	5	3	8	4	2
Estimated	3	2	6	3	2

Q 1. In which phase there was the highest absolute increase between the estimated and the actual cost?
(a) Foundation (b) Design
(c) Construction (d) Cannot be determined

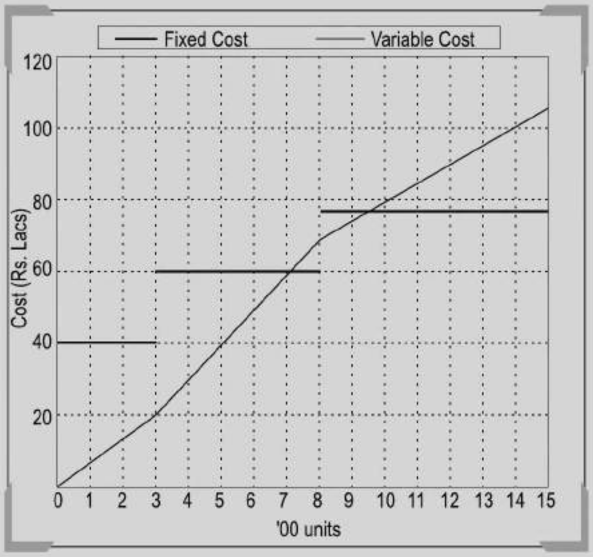
Q 2. For which phase of the project was the highest percentage increase in cost (as compared to the estimated cost)?
(a) Foundation (b) Design
(c) Construction (d) Cannot be determined

Q 3. What is the approximate value of the percentage overflow in costs during construction?
(a) 50% (b) 33%
(c) 66% (d) None of these

Q 4. If internal finishing and painting can be started simultaneously at the end of construction, then what is the overflow of the project as a percentage of the estimated total time for completion of the project?
(a) 1. 27% (b) 33%
(c) 42% (d) Cannot be determined

Direction for questions 5 to 9: Read the following passage and solve the questions based on it.

The following graph shows the variation in the variable and fixed cost of production of a TV. The fixed cost remains fixed in that particular slab and the variable cost varies according to the units produced. It can be seen that the fixed cost remains fixed for the first 300 units, then for the next 500 units and then, for any higher level of units produced. The total production cost is equivalent to the sum of the variable and the fixed cost. It is to be assumed that there is no other cost involved in producing the TVs.



Q 5. At a production level of 500 TVs, the selling price per unit is such that there is a net profit of 20%. What will be the impact on the net profit if the selling price per unit is reduced by 16.66% and the total number of units produced is increased by 40%?
(a) it will increase by less than 20% but more than 10%
(b) it will increase by less than 30% but more than 20%
(c) it will decrease by less than 15% but more than 5%
(d) none of these

Q 6. For production levels of less than 1500 units, the variation of the average cost of production per unit has the following relationship with the volume of production:
(a) decreases uniformly
(b) increases uniformly
(c) decreases, then increases and then increases again
(d) None of these

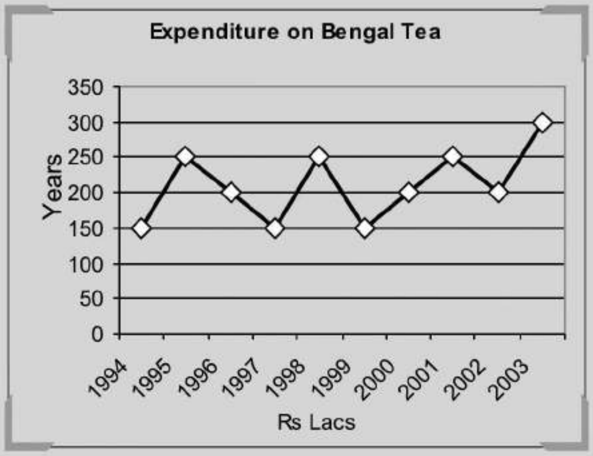
Q 7. The highest production cost per unit will be observed at the following level of production:
(a) 0 –100 (b) 300 – 400
(c) 800 – 900 (d) 1400 – 1500

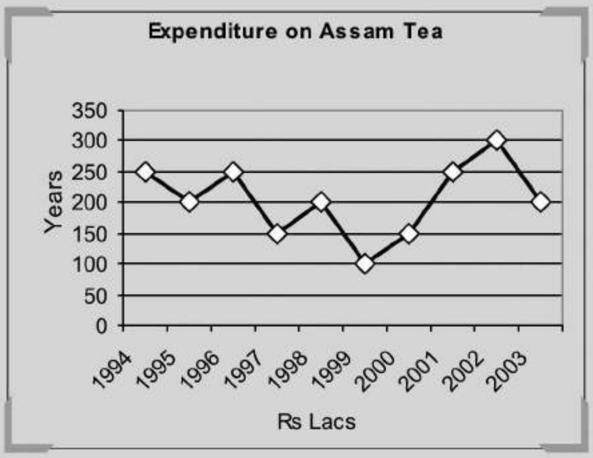
Q 8. What should be the selling price per unit (in Rs) To achieve a break-even if the total number of units produced is 1000?
(a) 12500 (b) 15500
(c) 17500 (d) cannot be determined

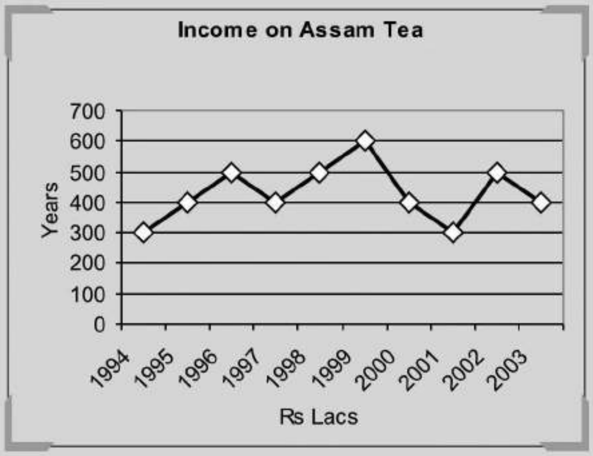
Q 9. What is the approximate additional cost for producing a TV after 920 units have been produced?
(a) 4000 (b) 5000
(c) 6000 (d) 7500

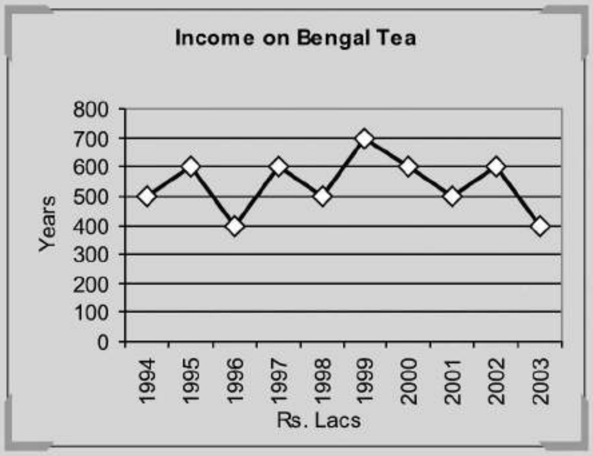
Direction for questions 10 to 14: Refer to the following line charts and solve the questions based on than.

The line charts given below give the expenditure and income of Tata Tea which produces and sells only two variants of its tea—Assam tea and Bengal tea.









Profit = Income – Expenditure

Q 10. In which of the following years, is the total profit of Tata Tea the highest?
(a) 1997 (b) 1999
(c) 2002 (d) 1994

Q 11. For how many years during the period 1995 to 2003, for at least one of the two teas did the income and expenditure not follow the same trend, i.e., both of them did not decrease or did not increase together over the previous year ?
(a) 5 (b) 7
(c) 8 (d) 9

Q 12. In which of the following years was the percentage increase in the income of Bengal Tea over the previous year, the highest?

(a) 1995 (b) 1997

(c) 1999 (d) 2002

Q 13. For how many years during from 1995 to 2003, was there a growth in income compared to that of the previous year but there is a decrease in the income from the year to the next year for each of the two teas?

(a) 1 (b) 2

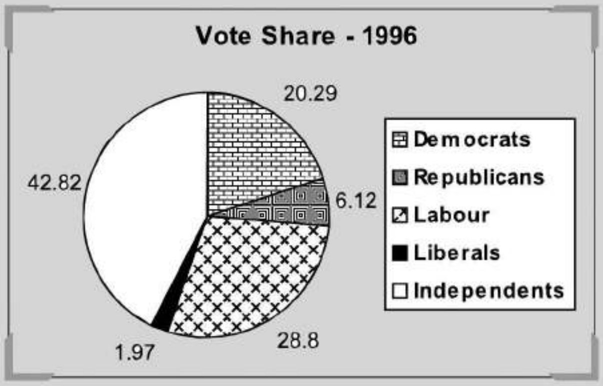
(c) 3 (d) 4

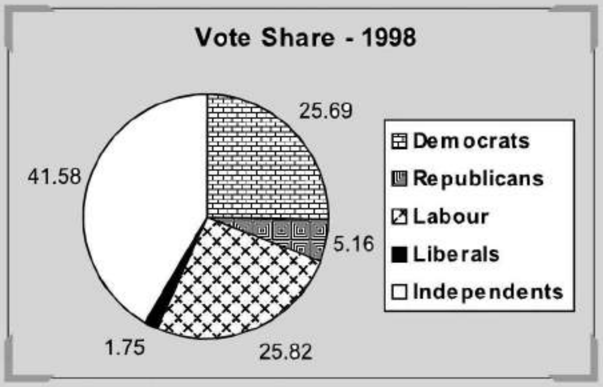
Q 14. In which of the following years during 1995 to 2003, did the profit of Assam Tea decrease by the maximum amount when compared to that of the previous year?

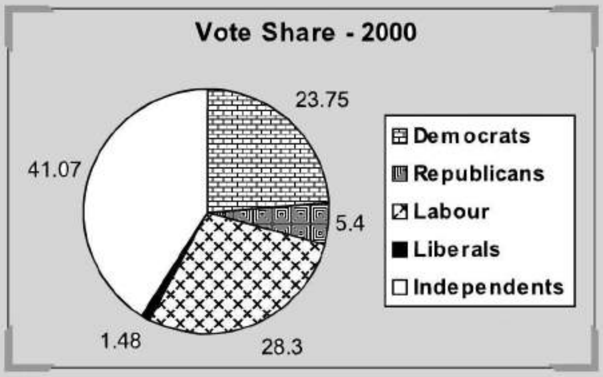
(a) 1999 (b) 2000

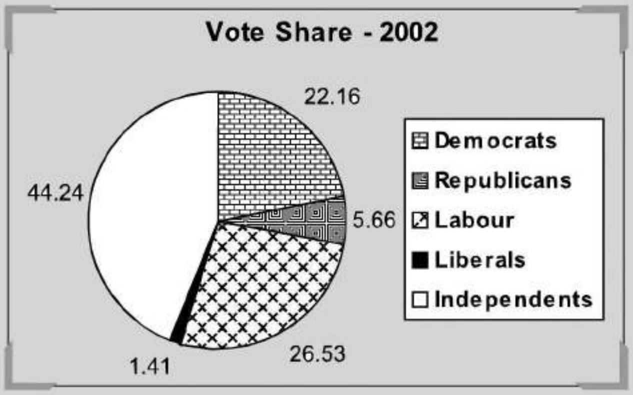
(c) 2001 (d) 2003

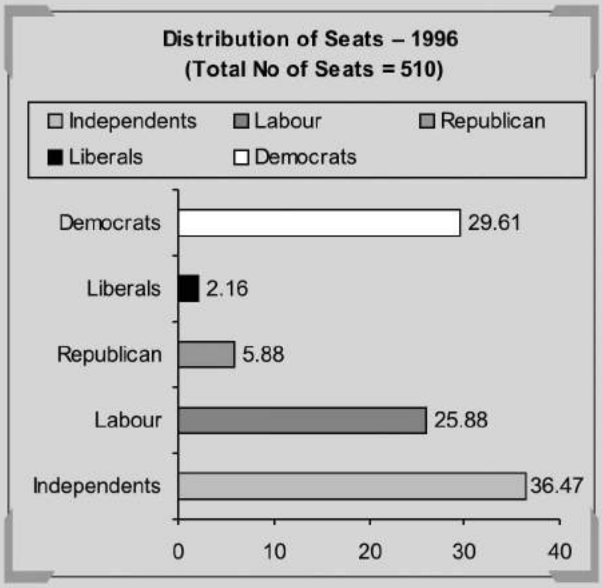
Direction for questions 15 to 18: Refer to the following graphs and solve the questions based on them.

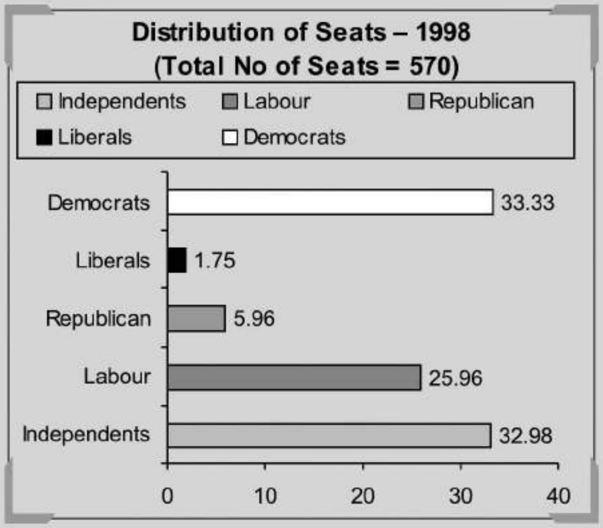

Vote Share - 1996


Vote Share - 1998


Vote Share - 2000

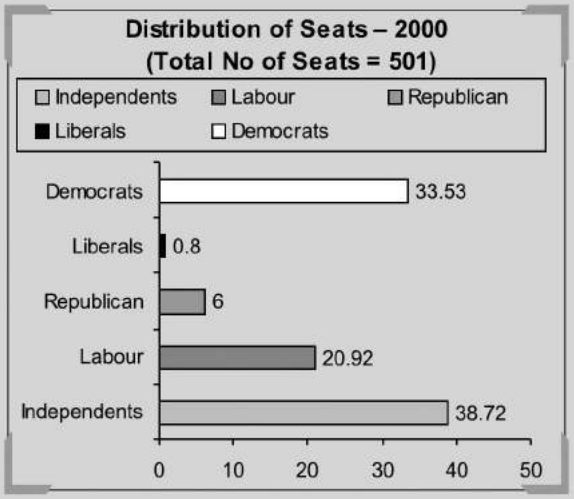

Vote Share - 2002

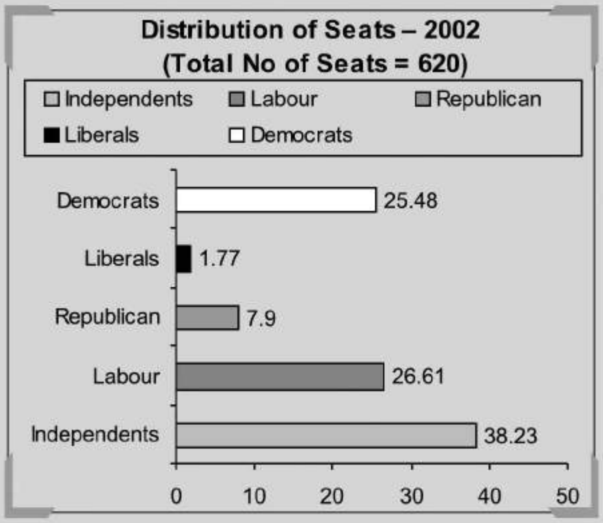

Distribution of Seats – 1996
(Total No of Seats = 510)


Distribution of Seats – 1998
(Total No of Seats = 570)

The given graphs show the vote share and distribution of seats of different parties in the election years during the period 1996-2002. Study the graphs and answer the questions, taking into account the following:

(i) Independents may be considered as a separate party





Q 15. How many of the following statements are correct?

I. In terms of vote share, Labour and Liberal parties taken together lost 3.2 percent in 1998 but gained a total of 16 seats in the same period.

II. Democrats, Republicans and 35% of the Independents could have formed the government only in two election years.

III. No party increased its vote share in every succeeding election.

IV. In the 2000 elections, as compared to the 1996 elections, Republicans and Democrats taken together, gained more in terms of vote share than in terms of percentage of seats.

(a) 0 (b) 1
(c) 2 (d) 3

Q 16. How many of the following statements are incorrect?

I. In the year 2002, in terms of percentage of seats obtained, Democrats and Labour together lost the maximum.

II. Liberals and Republicans taken together lost both in terms of vote share and the number of seats obtained in the year 2000.

III. In the 2000 elections, all the parties lost in terms of number of seats.

IV. The highest gain in the number of seats for Labour was in the year 2002.

(a) 0 (b) 1
(c) 2 (d) 3

Q 17. How many of the following statements are correct?

I. Democrats and Labour together improved their vote share by a larger margin in 1998 over 1996 as compared to the one in 2002 over 1996.

II. The number of seats lost by the Democrats in the 2002 elections was less than the number of seats gained by the Republicans in the same year.

III. In 2002, the Independents gained the most, both in terms of vote share and the number of seats obtained.

IV. If 70% of the Independents had joined Labour, they could have formed a government in 2002 but not in 1996.

(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 18 to 22: Refer to the following table and answer the questions based on it.

Unless otherwise stated, all changes (increases, decreases) indicated in the questions must be calculated over the immediately preceding year.

Q 18. Which of the following statements is not correct?

(a) Wipro's share in the total sales of the electronic sector was lower than the share of Tata Steel in the total sales of the iron and steel sector in each of the given years.

(b) From the group consisting of the textiles and the iron and steel sector, there were five companies whose salaries and wages as a percentage of the total salaries and wages of their respective sectors increased in 2007 as compared to 2005.

(c) Tata Steel's R & D expenditure In each of the given years as a percentage of the sales was lower than that of the iron and steel sector as a whole.

(d) From the group consisting of the pharmaceutical and the electronic sectors, there were five companies which experienced a decline in the growth rate of sales in 2007.

Q 19. In this question, a few statements given is/are right and a few statements is/are wrong. What is the ratio of the right to the wrong among the following statements?

(Rs crores)

Sector/Companies	Sales			Salaries and Wages			R & D			Profit / Loss		
	2005	2006	2007	2005	2006	2007	2005	2006	2007	2005	2006	2007
Textiles (Total)	53145	50184	52616	4296	4360	4315	26	36	39	–3770	–3437	–2441
Indo Rama Synthetics Ltd	2000	1953	2224	48	50	33	0	0	0	19	41	138
Arvind Mills Ltd	1970	740	1552	136	46	102	0	0	0	–499	20	129
Raymond Ltd	1477	992	1035	160	167	168	0	0	1	332	88	90
Century Enka Ltd	960	892	946	32	34	35	0	0	1	70	35	58
Pharmaceuticals (Total)	25245	30273	34731	2009	2291	2716	538	820	993	1758	2586	3069
Ranboxy Laboratory Ltd	2363	3461	4243	167	197	255	77	192	276	262	478	795
Dr. Reddy's Laboratories Ltd	991	1712	1705	83	121	137	42	102	163	144	460	392
Cipla Ltd	1064	1401	1573	50	63	73	41	47	0	179	208	248
Glaxosmithkline Ltd	1143	1197	1242	139	139	139	4	4	4	49	98	172
Electronic (Total)	26944	29323	30022	2164	2218	2323	200	263	247	830	692	–287
Infosys Technologies Ltd	1901	2604	3623	718	1119	1678	17	15	14	629	808	958
Videocon International Ltd	3244	4974	3602	43	63	53	0	0	0	155	156	–100
Bharat Electronic Ltd	1722	1947	2517	394	364	368	89	90	109	155	200	261
Iron and Steel (Total)	54396	55582	70341	5367	5643	6377	73	64	74	–2983	–4604	310
Steel Authority of India Ltd	17320	16624	20665	3135	3255	3728	52	50	55	–729	–1707	-304
Tata Steel Ltd	8491	8277	10517	922	1099	1218	10	8	16	553	205	1012
Rashtriyas Ispat Nigam Ltd	3585	4200	5187	408	376	407	0	3	0	–291	–75	521
Ispat Industries Ltd	3217	2812	4658	45	40	9	9	0	0	–312	–443	83

I. Total salaries wages for 2005-2007 as a percentage of the total sales of the same period had been the highest for the iron and steel sector.

II. In the year 2007, if all the companies were ranked in a descending order in terms of salaries and wages as a percentage of sales; Videocon International would have been ranked the lowest.

III. In the year 2006, if all the companies were ranked in a descending order in terms of R& D as a percentage of sales; Bharat Electronics Limited would have been ranked third.

IV. Of all the companies that made profits every year during 2005-2007, Wipro registered the highest growth in profits for the given period.

(a) 3:1
(b) 1:1
(c) 1:3
(d) All the statements are correct

Q 20 How many of the statements given below is/are correct?

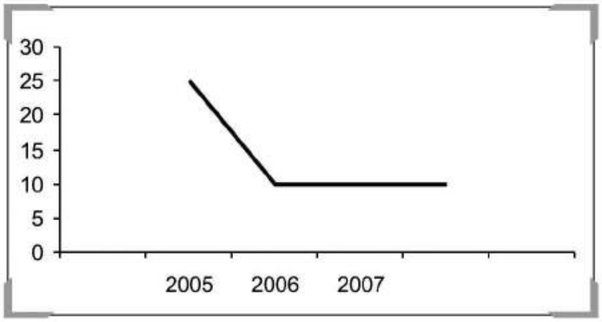
I. Salaries and wages of each company in the iron and steel sector were up to 13 times its losses reported in each of the loss-making years.

II. Total salaries and wages during 2005-2007 as a percentage of the total sales in the same period was minimum for the pharmaceutical sector.
III. Total salaries and wages during 2005-2007 as a percentage of the total sales in the same period for only two companies exceeded 20 percent.
IV. Total profits/Total sales for all the four sectors taken together was higher in 2006 than in 2005.

(a) 0 (b) 1
(c) 2 (d) 3

Q 21. Which of the following statements is incorrect?
(a) Cipla experienced the highest percentage decline in its R&D expenditure in any single year during the given period.
(b) Of all companies which incurred R&D expenditure every year during 2005-2007, the total R&D expenditure/total sales was the highest for Dr Reddy's Laboratories.
(c) During 2005-2007, in terms of sales growth, the best performer in the pharmaceutical sector fared better than the best performer in the iron and steel sector.
(d) Videocon International Ltd experienced the second largest percentage decline in salaries and wages in any single year during the given period.

Q 22. How many of the situations described in the options below, when plotted closely resemble the figure?



I. Raymond Limited 's share in the total sales of the textile sector
II. Tata Steel's share in the total sales of the iron and steel sector
III. Glaxosmithkline Ltd's salaries and wages as a percentage of sales
IV. Raymond Limited's profit as a percentage of sales

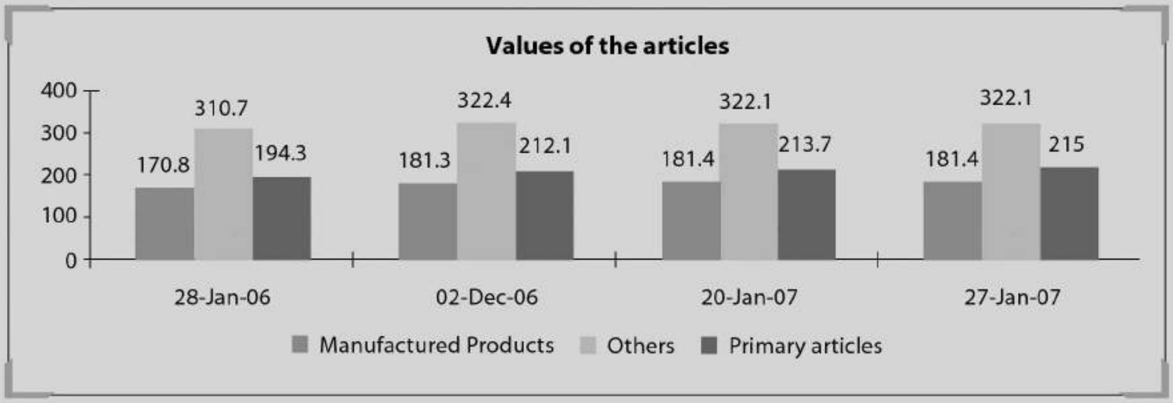
(a) 1 (b) 2
(c) 3 (d) 4

Direction for questions 23- 25: Go through the graphs below and solve the questions based on it.

Inflation is calculated on the basis of the wholesale price index. This index contains three groups of articles– Manufactured products, Primary articles and Others.

Following table represents the weightage of these three groups while calculating the inflation:

	Manufactured Products	Primary Articles	Others
Weightage	63.75%	22.02%	14.23%



Q 23 What is the rate of inflation on 20th Jan 07 over the previous given day?
(a) 5.21% (b) 6.11%
(c) 6.91% (d) 7.81%

Q 24 What is the rate of inflation on 27th Jan 07 over the previous given day?
(a) 5.12% (b) 2. 6%
(c) 6.58% (d) 7.91%

Q 25 What is the rate of inflation on 28th Jan 06 over the previous given day?
(a) 5.12%
(b) 6%
(c) 6.58%
(d) cannot be determined

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(c)	(d)	(d)	(d)	(d)	(d)	(b)	(b)	(c)	(b)	(b)	(b)	(b)	(b)	(c)	(c)	(b)	(a)
Q. No.	21	22	23	24	25															
Answer	(d)	(b)	(b)	(c)	(d)															

HINTS AND SOLUTIONS

5. For n = 500, CP/unit = Rs 20,000

   SP/unit = Rs 24,000

   Profit/unit = Rs 4000

   Hence total profit = Rs 20 lacs

   Now, if the production is increased by 40%, then the total number of units produced = 7000 units.

   CP/unit = Rs 120000/7

   SP/unit = Rs 24000

   Total profit = Rs 20 lacs

   So, the net profit will be the same.

   Therefore, the correct answer is option (d)

6. It can be calculated that the average cost decreases from 0 to 300 units, then increases for the the 301st unit, then decreases till the 800th unit and then increases till the 801st unit and then decreases again.

   Therefore, the correct answer is option (d)

7. From the solution of Q 6. We can see that option (a) is the correct answer.

8. Total cost of production for 1000 units = 155 × 105. Hence, to obtain the break even, the SP/unit = Rs 15,500. Therefore option (b) is the correct answer

PRACTICE EXERCISE 3

Direction for questions 1 to 4: Refer to the following tables and solve the questions based on them.

Table 1 gives the transportation cost of a product from six factories F1, F2, F3, F4, F5 and F6 to five warehouses W1, W2, W3, W4 and W5.

	W1	W2	W3	W4	W5
F1	60	25	45	40	80
F2	85	40	45	60	75
F3	80	20	45	50	90
F4	90	40	55	25	70
F5	95	20	40	45	80
F6	85	35	50	50	75

Table 2 gives the transportation cost of the same product from five warehouses W1, W2, W3, W4 and W5 to six retailers R1, R2, R3, R4, R5 and R6.

	R1	R2	R3	R4	R5	R6
W1	40	20	40	30	85	25
W2	50	50	45	60	70	30
W3	20	10	50	50	80	35
W4	40	30	45	40	90	40
W5	50	30	50	40	70	30

Q 1. What is the minimum transportation cost incurred in transferring the product from any factory to any retailer?
(a) Rs 45 (b) Rs 50
(c) Rs 55 (d) Rs 60

Q 2. What is the maximum transportation cost incurred in transferring the product from any factory to any retailer?
(a) Rs 175 (b) Rs 180
(c) Rs 170 (d) None of these

Q 3. What is the minimum transportation cost at which the product can be supplied to R4 from any factory?
(a) Rs 45 (b) Rs 50
(c) Rs 65 (d) Rs 70

Q 4. What is the maximum transportation cost at which the product can be supplied to R3 from any factory?
(a) Rs 145 (b) Rs 150
(c) Rs 140 (d) None of these

Direction for questions 5 to 10: Answer the questions based on the following information:

In addition to the above tables given, the following table gives the transportation cost incurred for transferring products from different retailers to the five different colonies C1, C2, C3, C4 and C5.

	C1	C2	C3	C4	C5
R1	10	25	10	10	10
R2	15	10	15	10	15
R3	10	20	10	15	10
R4	20	10	15	15	10
R5	15	10	20	20	20
R6	10	15	15	15	15

Q 5. What is the minimum transportation cost incurred in transferring the product from any factory to any colony?
(a) Rs 45 (b) Rs 65
(c) Rs 55 (d) none of these

Q 6. What is the minimum transportation cost incurred in transferring the product from any factory to the colony C2?
(a) Rs 45 (b) Rs 60
(c) Rs 55 (d) None of these

Q 7. What is the maximum transportation cost incurred in transferring the product from any factory to the colony C5?
(a) Rs 175 (b) Rs180
(c) Rs185 (d) None of these

Q 8. If one unit produced in each of the factories is to be sent to each of the colonies, then what will be the total transportation cost incurred?
(a) Rs 1705 (b) Rs 1980
(c) Rs 2185 (d) None of these

Q 9. $F_iW_jR_kC_l$ represents that the product has moved in this cycle – Factory i to Warehouse j to Retailer k to Colony l. If i = 2 and k = 2, then what is the sum of (j + l) for the lowest transportation cost incurred in one product transferred from i = 2 to any of the colonies?
(a) 4 (b) 5
(c) 7 (d) Cannot be determined

Q 10. In the above question, if j = 2, then how many different value/s of (i + k + l) is/are possible for the lowest transportation cost incurred in one product transferred from any of the factories to any of the colonies?

(a) 1 (b) 2
(c) 3 (d) Cannot be determined

Direction for questions 11 to 15: Refer to the following table and solve the questions based on it.

Given below are the forecasts of the world and Asian energy demand for the years 1991, 2001 and 2011. The demand is given in million barrels per day, crude oil equivalent.

	1991		2001		2011	
	World	Asia	World	Asia	world	Asia
Petrol	50.0	4.0	70.0	10.0	80.0	15.0
Naphtha	30.0	0.5	40.0	2.5	50.0	5.0
Solid Fuels	50.0	4.0	60.0	5.0	75.0	10.0
Nuclear Fuel	10.0	0.5	20.0	1.0	25.0	1.3
Hydro Energy	10.0	1.0	10.0	1.5	20.0	2.0
Total	150.0	10.0	200.0	20.0	250.0	33.3

Q 11. What can be inferred from the given table?

(a) Percentage increase in the number of petrol using vehicles/agents has increased in Asia rather than the world, in the decade 1991-2001.
(b) Exploration of Naphtha is low in Asia as compared to the exploration of Naphtha in the world throughout the period.
(c) Both (a) and (b)
(d) None of these

Q 12. During 1991-2011, which two fuels meet more than 60% of the total energy demand of both the world and Asia?

(a) Petrol and Naphtha
(b) Petrol and Solid fuels
(c) Naphtha and Solid fuels
(d) None of these

Q 13. Which fuel's proportion in the total energy demand increases over the decade 1991-2001 and decreases over the decade 2001-2011 for both the world and Asia?

(a) Petrol (b) Naphtha
(c) Solid fuels (d) Nuclear fuel

Q 14. In Asia for which fuel does the proportion in the total energy demand decrease continuously over the period 1991-2011?

(a) Naphtha (b) Solids fuels
(c) Nuclear fuel (d) Hydro energy

Q 15. For which fuel does the proportion in the total energy demand of the world remain constant over the period 1991-2011; but its proportion increases in the total energy demand in Asia?

(a) Solid fuels (b) Nuclear fuel
(c) Hydro energy (d) Naphtha

Direction for questions 16 to 20: Refer to the following table and solve the questions based on it.

The following table gives some incomplete information on the marks obtained by four persons Amit, Binit, Charu and Dilip in four different subjects.

	Physics	Chemistry	Maths	English	Total
Amit	80	70	-	80	-
Binit	60	-	60	-	-
Charu	-	-	60	-	220
Dilip	90	-	70	60	-

(i) Amit's average marks were 72.5, in all the four subjects.
(ii) The average marks in Physics for all four persons were 67.5.
(iii) Binit's average marks for all four subjects were 1.5 times of Charu's marks in Physics.
(iv) Binit and Dilip scored the same marks in English. The average English marks were the same as the average Physics marks.
(v) The total marks in Chemistry were 10 less than the total marks in Physics for all four persons.

Q 16. Who scored the lowest total?

(a) Amit (b) Binit
(c) Charu (d) Dilip

Q 17. The approximate average marks for all four persons (for all the four subjects) are

(a) 65.625 (b) 81.225
(c) 99.5 (d) 105.6

Q 18. Who is the highest scorer in Chemistry?

(a) Amit (b) Binit
(c) Charu (d) Dilip

Q 19. Who is the lowest scorer in Physics?

(a) Amit (b) Binit
(c) Charu (d) Dilip

Q 20. Who is the highest scorer in English?

(a) Amit (b) Binit
(c) Charu (d) Dilip

Direction for questions 21 to 23: ***Refer to the following data-set and solve the questions based on it.***

The following table gives the price of petrol and diesel in India for the given period.

Date	Petrol Change (+/-)	Diesel Change (+/-)
May 2004	33.70	21.74
16 June 2004	35.71	22.74
1 August 2004	36.81	24.16
5 November 2004	39.00	26.28
16 November 2004	37.84	
1 April 2005 (VAT)	37.99	28.22
5 May 2005 (VAT on diesel)		26.45
21 June 2005	40.49	28.45
7 September 2005	43.49	30.45
1 April 2006	43.51	30.39
5 June 2006	47.51	32.47
June 16, 2006	46.85	32.25
November 30, 2006	44.85	31.25
February 15, 2007	42.85	30.25

Q 21. How many times did the price of petrol increase during the given period?

(a) 7 (b) 8
(c) 9 (d) 10

Q 22. How many times did the price of diesel increase during the given period?

(a) 7 (b) 8
(c) 9 (d) 10

Q 23. For how many times is the percentage increase in the price of petrol more than the percentage increase in the price of diesel over their respective previous date?

(a) 4 (b) 5
(c) 6 (d) 7

Direction for questions 24 to 25: Refer to the following data-set and solve the questions based on it.

REDUCTION IN FUEL PRICES
(Rs Per Litre)

Feb 16, 2007	Petrol		Diesel	
	Old	New	Old	New
Delhi	44.85	**42.85**	31.25	**30.25**
Mumbai	50.85	**48.85**	36.11	**34.97**
Kolkata	48.99	**46.91**	33.92	**32.88**
Chennai	49.67	**47.51**	34.41	**33.22**

Q 24. In which city out of the given four cities has the price of petrol decreased by the maximum percentage?

(a) Delhi (b) Mumbai
(c) Kolkata (d) Chennai

Q 25. In which city out of the given four cities has the price of diesel decreased by the maximum percentage?

(a) Delhi (b) Mumbai
(c) Kolkata (d) Chennai

ANSWERS

■■■

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(c)	(c)	(b)	(b)	(d)	(d)	(d)	(d)	(d)	(b)	(a)	(d)	(d)	(c)	(a)	(d)	(c)	(a)
Q. No.	21	22	23	24	25															
Answer	(b)	(b)	(b)	(a)	(c)															

HINTS AND SOLUTIONS

1. F3 to W2 to R6 is the lowest. Therefore, option (b) is the correct answer.
2. F5 to W1 to R5 is the maximum. Therefore, option (b)is the correct answer.
3. F4 to W4 to R4 is the lowest. Therefore, option (c) is the correct answer.
4. F3 to W5 to R3 is the maximum. Therefore, option (c) is the correct answer
5. F3 to W2 to R2 to C1 is the lowest. Therefore, option (b) is the correct answer
6. F3 to W2 to R6 to C2 is the minimum. Therefore, option (c) is the correct answer
9. F2 – W3 – R2 – either C2/ C4. Hence two different values are possible. Therefore, option (d) is the correct answer.
10. There can be two values of i = 3/5, one value of j = 2, one value of k = 6 and one value of l = 1. Hence, two different sums are possible. Therefore, option (d) is the correct answer.
11. Option (b) is obviously cannot be inferred. Option (a) seems to be probable, but number of vehicles/agents need not be increasing. There can be other reasons also like reduction in the efficiency of the vehicle etc. Therefore option (d) is the correct answer.
12.

		1991		**200**		**2011**	
		World	**Asia**	**World**	**Asia**	**World**	**Asia**
Total energy		150	10	200	20	250	33.3
Naphtha	Value	30	0.5	40	2.5	50	5
	Proportion	20%	5%	20%	12.50%	20%	15.01
Solid fuels	Value	50	4	60	5	75	10
	Proportion	**33.30%**	**40%**	**30%**	**25%**	**30%**	**30.03**
Petrol	Value	50	4	70	10	80	15
	Proportion	**33.30%**	**40%**	**35%**	**50%**	**32%**	**45.04**

Hence, solid fuels and petrol combined constitute more than 60 per cent of the total energy in both the world and Asia for the given period.

13. It can be seen from the table that Petrol is the fuel whose proportion in the total energy demand increases during 1991-2001 and decreases during 2001-2011 for both the world and Asia. Therefore option (a) is the correct answer
14. For the answer choices given and for Asia we can make the following table.

		1991	**2001**	**2011**
	Total energy	**10**	**20**	**33**
Naphtha	Value	0.5	2.5	5
	Proportion	**5%**	**12.50%**	**15.15%**

		1991	2001	2011
	Total energy	10	20	33
Solid Fuels	Value	4	5	10
	Proportion	**40%**	**25%**	**30.30%**
Nuclear Fuel	Value	0.5	1	1.3
	Proportion	**5%**	**5%**	**3.90%**
Hydro Energy	Value	1	1.5	2
	Proportion	**10%**	**7.50%**	**6.06%**

Hence, we can see that the proportion of Hydro Energy goes on decreasing over the given period.

15. For the answer choices given and for the world, we can make the following table.

		1991	2001	2011
	Total energy	**150**	**200**	**250**
Naphtha	**Value**	30	40	50
	Proportion	20%	20%	20%
Solid Fuels	**Value**	50	60	75
	Proportion	33%	30%	30%
Nuclear Fuel	**Value**	10	20	25
	Proportion	6.66%	10%	10%
Hydro Energy	**Value**	10	10	20
	Proportion	6.66%	5%	8%

Hence, we can see that the proportion of Naphtha remains constant over the given period.

From the given information the following table can be prepared.

	Physics	Chemistry	Maths	English	Total
Amit	80	70	60	80	290
Binit	60	60	60	60	240
Charu	40	50	60	70	220
Dilip	90	80	70	60	300

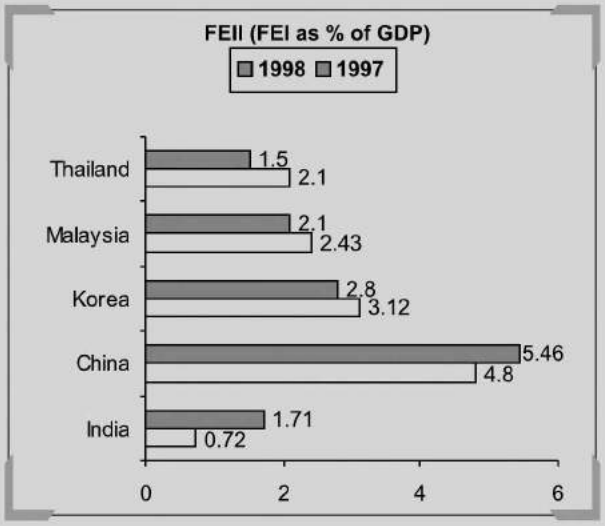
Following is the complete list:

Date	Petrol Change (+/–)	Diesel Change (+/–)
May 2004	33.70	21.74
16 June 2004	35.71 +2.01	22.74 (+1.00)
1 August 2004	36.81 +1.10	24.16 (+1.42)
5 November 2004	39.00 +2.19	26.28 +2.12
16 November 2004	37.84 –1.16	
1 April 2005 (VAT)	37.99 + 0.15	28.22 + 1.94
5 May 2005 (VAT on diesel)		26.45 (–1.77)
21 June 2005	40.49 + 2.50	28.45 (+2.00)
7 September 2005	43.49 +3.00	30.45 (+2.00)
1 April 2006	43.51 +0.02	30.47 (+0.02)
5 June 2006	47.51 +4.00	32.47 (+2.00)
16 June 2006	46.85 –0.66	32.25 (–0.22)
30 November 2006	44.85 –2.00	31.25 (–1.00)
15 February 2007	42.85 –2.00	30.25 (–1.00)

PRACTICE EXERCISE 4

Direction for questions 1 to 5: Refer to the following data set and solve the questions based on it.

The following graph gives the data on the foreign equity inflow index (FEII) for five countries during 1997 and 1998. FEII is taken as the ratio of the foreign equity inflow (FEI) to the country's GDP. FEII is expressed as percentage in the graph.



Q 1. Which country has shown the maximum percentage change in FEII in 1998 relative to 1997?
(a) India
(b) China
(c) Korea
(d) Malaysia

Q 2. If China's FEI is 10 times that of India in the year 1998, which of the following is true for 1998?
(a) China's GDP is 70% more than India's GDP
(b) China's GDP is 40% more than India's GDP
(c) China's GDP is 50% more than India's GDP
(d) Nothing can be inferred

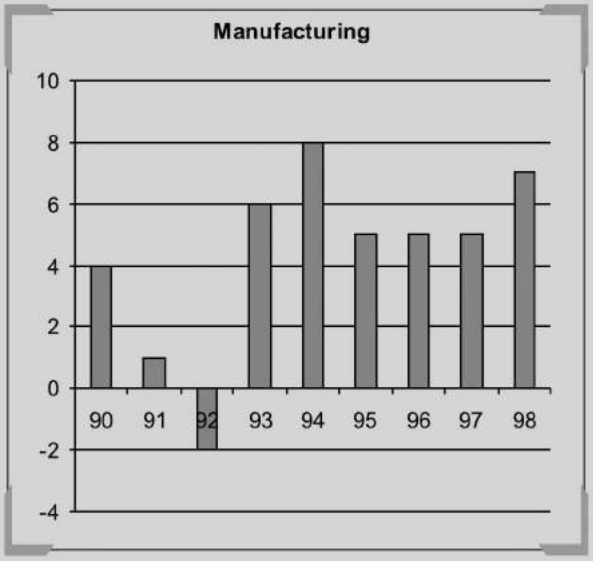
Q 3. If the GDP of India increased by 5%, GDP of China increased by 7% and that of Korea decreased by 2% 1997 to 1998, which of the following is/are true?
I FEI to China was higher in 1998 than in 1997
II. FEI to China was lower in 1998 than in 1997
III. India's FEI has increased from 1997 to 1998
IV. FEI to Korea was lower in 1998 than in 1997
V. Korea's FEI has increased in 1998 from 1997
(a) I and III
(b) II and V
(c) II, III and V
(d) All of the above

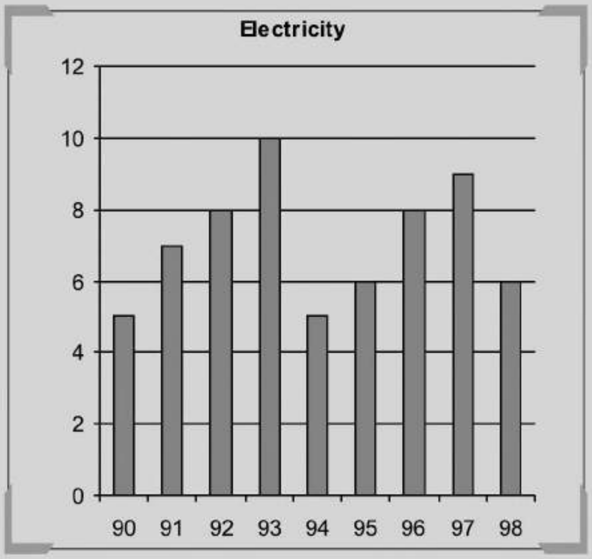
Q 4. Which of the folowing can be inferred from the given data?
(a) China's GDP is more than India's GDP
(b) China's GDP is less than India's GDP
(c) India's GDP is less than Malaysia's GDP
(d) Nothing can be deduced

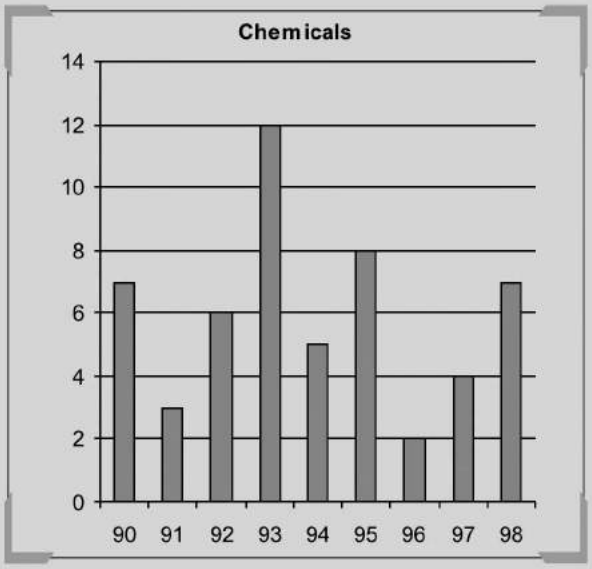
Q 5. For how many countries has the FEI increased from 1997 to 1998?
(a) 1
(b) 2
(c) 3
(d) Indeterminate

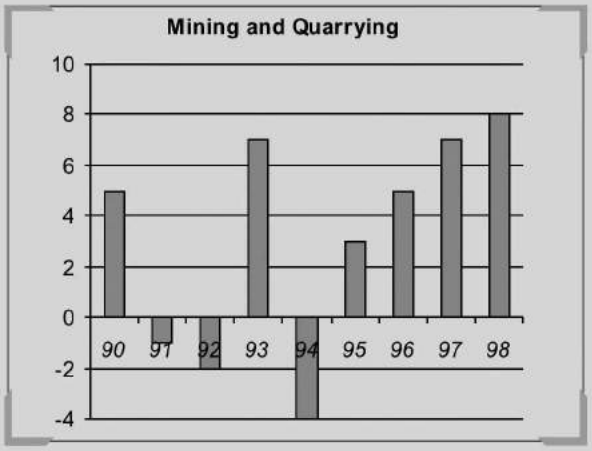
Direction for questions 6 to 12: Refer to the following data-set and solve the questions based on it.

The following graphs provide the data on four industrial sectors in the Republic of Reposia. Manufacturing constitutes 20%, Mining and Quarrying 15%, Electricity 15% and Chemicals 10% of the Republic's total industrial production in 1989. For the period 1990-1998, the graphs give the percentage change in production over the previous year's production. 1989 production values have been assigned an index of 100 for each of the four sectors.









Q 6. What was the average annual growth rate in production for all four sectors combined between 1989 and 1998?
(a) 1.2% (b) 6%
(c) 56.6% (d) 104%

Q 7. During the given period, when was the maximum production level in the electricity sector achieved?
(a) 1989 (b) 1993
(c) 1995 (d) 1998

Q 8. During the given period, when was the minimum production level in the manufacturing sector achieved?
(a) 1989 (b) 1990
(c) 1993 (d) 1998

Q 9. For the given period, which sector showed the highest growth rate?
(a) Electricity (b) Manufacturing
(c) Chemical (d) Mining and Quarrying

Q 10. In 1992, the index for the Chemical sector as compared to the index for the Electricity sector was
(a) greater by 6 (b) less by 5
(c) greater by 20 (d) less by 20

Q 11. By what percentage was the output of the Manufacturing sector more than that of the Chemical sector in 1991?
(a) 84% (b) 91%
(c) 98% (d) Indeterminate

Q 12. When did the maximum percentage change in the production levels of the Mining and Quarrying sector take place?
(a) 1993-1994 (b) 1992-1993
(c) 1996-1997 (d) 1997-1998

Direction for questions 13 to 17: Refer to the following data-set and solve the questions based on it.

The following table gives the break-up of the revenue earned by Phistream Private Limited over a period of five years. All figures in the table are in Rscrores.

	1994-95	1995-96	1996-97	1997-98	1998-99
Hardware					
(a) Domestic	68	54	52	66	35
(b) Exports	540	600	730	1770	1827
Software					
(a) Domestic	142	127	150	320	422
(b) Exports	1100	1200	1339	2500	2215
Peripherals	25	10	25	20	37
Training	140	106	160	240	161
Maintenance	21	19	25	92	126
Others	12	10	19	40	24
Total	2048	2126	2500	5048	4847

Q 13. In which years did the hardware exports lie between 35%-40% of the total revenues for Phistream?
(a) 1997-1998, 1998-1999
(b) 1998-1999, 1995-1996
(c) 1997-1998, 1994-1995
(d) 1995-1996, 1996-1997

Q 14. Which of the following is true?
(a) The revenues from training consistently increased over the given period
(b) Hardware exports consistently increased over the given period
(c) Software exports consistently increased over the given period
(d) none of the above

Q 15. For how many years during the given period , was the total revenue from Training and maintenance less than 10% of the total revenue for the corresponding years?
(a) 2 (b) 3
(c) 4 (d) 5

Q 16. In which of the following years was the revenue from the hardware business more than 50% of that from the software business?
(a) 1994-1995
(b) 1995-1996, 1996-1997
(c) 1997-1998, 1994-1995
(d) None of these

Q 17. If in 1999-2000, the total revenue earned by Phistream decreases by 5% from the previous year but the revenue from software increases to Rs 2437 crores; then what is the approximate percentage decrease in the combined revenue from the rest of the heads(excepting software) in 1999-2000?
(a) 1.0% (b) 1.8%
(c) 2.1% (d) 3.3%

Direction for questions 18 to 23: Refer to the following data-set and solve the questions based on it.

Data Pertains to Marti Udyog Ltd.

Sector	Factories	Employment	Fixed Capital	Variable Cost	Value Added
Government	18	15	14	22	25
Central	8	6	5	7	10
State	4	6	4	8	3
Central/ State	6	3	5	7	12
Public	12	8	6	19	8
Private	55	65	72	54	62
Joint	15	12	8	5	5
Total	100	100	100	100	100

All data is percentage.

Q 18. If the total work force was 76 million, whereas the total values added were 225 million, then which of the following had the maximum value addition per worker?
(a) Central (b) State
(c) Central/State (d) Public

Q 19. Which of the following sectors has the maximum fixed capital invested per factory?
(a) Central (b) State
(c) Central/State (d) Public

Q 20. If the variable is proportional to the number of employees and the production per employee, then for which of the following is the production highest?
(a) Government (b) Private
(c) Joint (d) Public

Q 21. If the government has a fixed capital of $ 200 million in the Iron and Steel industry, which corresponds to 20.012% of its total investment as fixed capital, then how much did the government invest(in Rs million) in Maruti Udyog Ltd which forms 25% of the investment in the joint sector ? (1US $ = Rs 45)
(a) 6500 (b) 2500
(c) 143 (d) 145

Q 22. Maruti Udyog Ltd is a joint project of the Indian Government and Suzuki Motors, Japan; each having equal stake. One fine day, the Indian government decides to disinvest from the venture due to losses occurring from labour problems. How much money will be disinvested?
(a) Rs 246 million (b) Rs 6500 million
(c) $ 246 million (d) $ 6500 million

Q 23. Which of the following statements is true?
(a) The number of government employees is more than the number of factories in the joint sector
(b) The number of employees in the public sector is the same as the fixed capital of the joint sector
(c) Both (a) and (b)
(d) Cannot say

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(b)	(d)	(d)	(b)	(d)	(a)	(a)	(b)	(b)	(d)	(a)	(b)	(d)	(d)	(c)	(c)	(b)	(b)
Q. No.	21	22	23																	
Answer	(a)	(b)	(d)																	

HINTS AND SOLUTIONS

1. Percentage change for India

$$= \frac{0.72-1.71}{1.71} > 50\%$$

Percentage change for China

$$= \frac{5.46-4.8}{5.46} \approx 12\%$$

Percentage change for Korea

$$= \frac{3.12-2.8}{2.8} \approx 11\%$$

Percentage change for Malaysia

$$= \frac{0.33}{2.1} \approx 15\%$$

Hence, maximum % change is for India

2. For the year 1998

$$\text{For China, } \frac{FEI_{China}}{GDP_{China}} = 0.048 \qquad \ldots\ldots\ldots(1)$$

$$\text{For India, } \frac{FEI_{India}}{GDP_{India}} = 0.0072 \qquad \ldots\ldots\ldots(2)$$

$$\frac{FEI_{China}}{FEI_{India}} = 10$$

⇒ on dividing (1) by (2) :

$$\frac{GDP_{India}}{GDP_{China}} \times 10 = \frac{0.048}{0.0072} = \frac{20}{3}$$

$$\Rightarrow \frac{GDP_{China}}{GDP_{India}} = \frac{3}{2} = 1.5$$

So, percentage difference between China's GDP and India's GDP = 50%

3. Since the comparisons are for the same country in each case, assume the GDP as 100 in 1997 in all cases. So, GDP of China, India and Korea in 1998 is 105,107 and 98 respectively.

$$\frac{FEI_{1998}}{FEI_{1997}} = \frac{FEII_{1998} \times GDP_{1998}}{FEII_{1997} \times GDP_{1997}}$$

$$\text{For China : } \frac{FEI_{1998}}{FEI_{1997}} = \frac{4.8 \times 107}{5.46 \times 100} < 1$$

$$\text{For India : } \frac{FEI_{1998}}{FEI_{1997}} = \frac{0.72 \times 105}{1.71 \times 100} < 1$$

$$\text{For Korea : } \frac{FEI_{1998}}{FEI_{1997}} = \frac{3.12 \times 98}{2.8 \times 100} > 1$$

So, II and V are correct. Therefore option (b) is the correct answer

4. It can be seen that all the values (FEII) are expressed as a ratio of FEI to GDP, the absolute values of GDP cannot be determined.

5. Since only the ratios of GDP and FEI are known, the increase in one independent of the other cannot be determined.

6. You may note that apart from a few years for manufacturing and mining and quarrying, the annual growth rates in most years have been positive and near the 5%–6% mark for all the sectors. The other options (a), (c), and (d) are too far away to be considered. Alternatively, for ease of calculation take the 1989 values for Manufacturing as 400, mining and Quarrying and Electricity as 300 each and Chemicals as 200. then, the final value for manufacturing would be nearly 580; for Mining and Quarrying would be about 365; for Electricity would be around 550; and for chemicals would be around 335.

=> Initial value of production
= 400 + 300 + 300 + 200 = 1200

Final value of production

$= 580 + 365 + 550 + 335 = 1830$

⇒ Overall % increase ≈ 52.5%

⇒ Average annual growth = 52.5/9 = 5.85%

7. Electricity production has increased throughout (as growth rates are positive throughout) and the maximum production would be achieved in the last year, i.e. 1998.

8. The production for Manufacturing can be minimum either in 1989 or in 1992 (when the growth rate is negative).

   If Manufacturing started with a production of 100 in 1989, then production in 1992 will be $100 \times 1.04 \times 1.01 \times 0.97 > 1$

   So, the production in 1989 was the least

9. Electricity has shown the highest growth rate in all the years and therefore, would have the highest growth rate for the entire period.

10. Electricity in 1992:

    $100 \times 1.05 \times 1.07 \times 1.08 = 121.4$

    Chemicals in 1992:

    $100 \times 1.07 \times 1.03 \times 1.06 = 116.8$

    => In 1992, the index of Chemicals is less than that of Electricity by approximately 5.

11. Let Manufacturing in 1989 = 200 and

    Chemicals in 1989 = 100

    => Manufacturing in 1991:

    $200 \times 1.04 \times 1.01 = 210$

    and Chemicals in 1991:

    $100 \times 1.07 \times 1.03 = 110$

    $$\text{Required \%} = \frac{210-110}{110} \times 100 = 90.91\%$$

12. As can be seen directly from the graph, the highest increase (that of 8%) for Mining and Quarrying was in the year 1998, relative to 1997.

13. Hardware exports as a percentage of total revenues for corresponding years are:

    $$1994-1995: \frac{540}{2048} \times 100 < 33\%$$

    $$1995-1996: \frac{600}{2126} \times 100 < 33\%$$

    $$1996-1997: \frac{730}{2500} \times 100 < 33\%$$

    $$1997-1998: \frac{1770}{5048} \times 100 = 35.06\%$$

    $$1998-1999: \frac{1827}{4847} \times 100 = 37.7\%$$

    So, hardware exports lie between 35%–40% of total revenue in the last 2 years.

14. Checking the values from the tables, you can verify that option (b) is true.

15. The combined revenue from training and maintenance is never greater than 10% of total revenue for the corresponding year throughout the given period.

16.

Year	50% of software Revenue	Hardware Revenue
1994-95	1242 × 0.5=621	608
1995-96	1327 × 0.5=663	654
1996-97	1484 × 0.5=744	782
1997-98	2820 × 0.5=1410	1836
1998-99	2637 × 0.5=1318	1862

⇒ Hardware business provides revenues> 50% of the revenue from software business in 1996-1997, 1997-1998 and 1998-1999.

17. In 1999-2000:

    Total revenue = $0.95 \times 4847 = 4600$

    Software revenue = 2437

    Remaining revenue = 4600- 2437 = 2163

    In 1998-1999, revenue from heads other than software = 4847 – 2637 = 2210

    $$\text{\% decrease} = \frac{2210 - 2163}{2210} \times 100 = 2.12\%$$

18. Values added per worker for various sectors will be proportional to the following ratios:

Central	10/6
State	3/6
Central/state	12/3
Public	8/8

19. Fixed capital per factory will be proportional to the following ratios:

Central	5/8
State	4/4
Central/state	5/6
Public	6/12

20. If the variable is proportional to the number of employees and the production per employees, then we can say that the variable cost is directly proportional to the production. Since the variable cost is maximum for the private sector, it will also have the maximum production.

21. If 20% of the government fixed capital is equal to \$ 200 million => Total governtment sector fixed capital = \$ 1000 million which is equivalent to 14% of the total fixed capital. Now, 25% of government investment in joint sector i.e., 25% of the 8% of the total = 2% of total fixed capital will be given by:

    (1000 × 2) /14 = \$ 143 million

    Value in Rs is given by:

    143 × 45 = Rs 6450 million

22. From the above answer the money disinvested will be Rs 6450 million.

23. Neither of the statements (a) or (b) can be inferred, as we know only the percentages and not the actual values of the employees, factories or fixed capital.

6

C H A P T E R

Mastering Data Interpretation

PRACTICE EXERCISE 1

Direction for questions 1 to 4: Refer to the following chart and solve the questions based on it.

The following table gives the career record in the One-Day Internationals of few of the Indian batsmen:

Name	Matches Played	Innings Played	Runs Scored	Average
Yuvraj	50	48	1800	40
Sachin	50	50	2034	45.2
Kapil	46	46	1679	36.5
Saurav	44	44	1539	40.5
Rahul	40	38	1806	51.6
Sehwag	42	40	1445	42.5

The average is calculated by dividing the total runs scored by the total number of matches in which the batsman got out.

Q 1. Who among the following did not get out for the maximum amount of time in the total innings played?
(a) Yuvraj (b) Sachin
(c) Kapil (d) Sehwag

Q 2. To reflect the performance of the players better, it was decided that the average would be calculated on the basis of the total innings played. Whose average will see the maximum change due to this?

(a) Yuvraj (b) Sachin
(c) Kapil (d) Sehwag

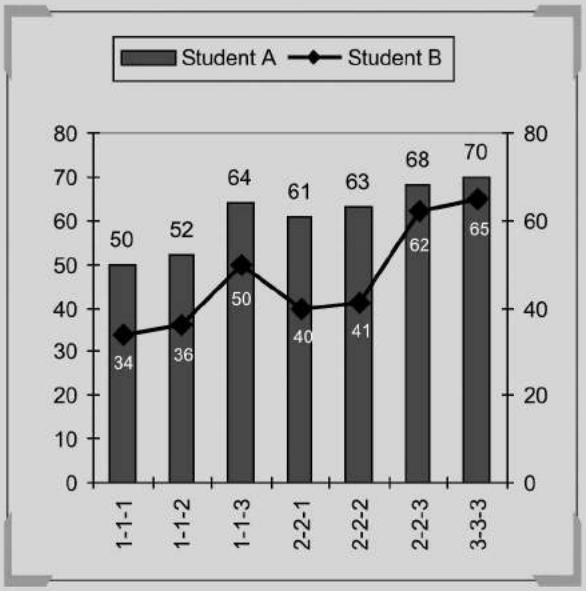
Q 3. How many of the batsmen got out in all the innings played?
(a) 0 (b) 1
(c) 2 (d) Cannot be determined

Q 4. To reflect the performance of the players better, it was decided that the average would be calculated on the basis of the total matches played. Whose average will see the minimum change due to this?
(a) Yuvraj (b) Sachin
(c) Kapil (d) Sehwag

Direction for questions 5 to 10: Refer to the following chart and solve the questions based on it.

The bar chart given below represents the performance of two students A and B while studying three different subjects with different groups of teachers. Ratings given to the teachers are 1, 2, 3 with 1 being the lowest and 3 being the highest.

The bar chart also represents the percentage marks obtained by A and B in three subjects. Each subject carries 100 marks. Students always get integral marks in each subject. For example, if A studies all the three subjects from a group of teachers so that all the three teachers have 1 rating (low rating); then he gets 50% marks of the total marks of the three subjects.



Q 5. What would be the change in the total marks obtained by student A when we compare the combination 1-1-1 and 1-1-3?
(a) 32 (b) 42
(c) 62 (d) Cannot be determined

Q 6. What is the percentage change in the total marks obtained by student B when we compare the combination 1-1-3 and 2-2-2?
(a) 15% (b) 16%
(c) 17% (d) 18%

Q 7. What can be the maximum marks obtained by student A in any of the three subjects with the combination of teachers being 1-1-3?
(a) 92
(b) 96
(c) 100
(d) None of these

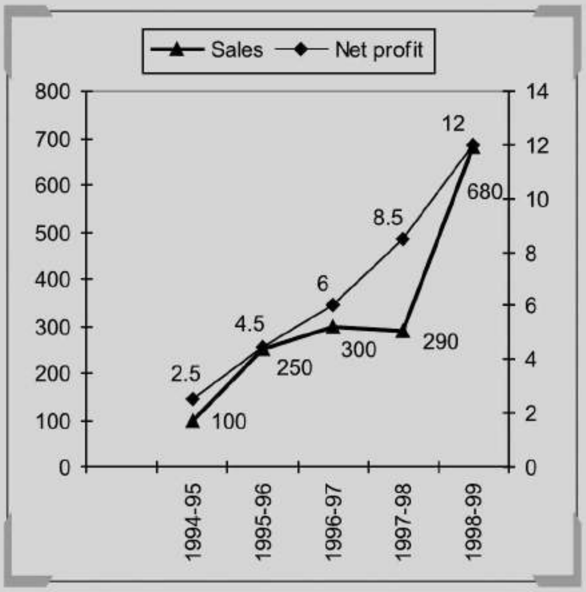
Q 8. For which of the following options is the difference in the marks scored by student B the maximum?
(a) 1-1-1 and 3-3-3 (b) 1-1-1 and 1-1-3
(c) 1-1-1 and 2-2-3 (d) 1-1-2 and 3-3-3

Q 9. For which of the following options are the marks scored by student A and student B the maximum?
(a) 1-1-1 (b) 1-1-3
(c) 2-2-3 (d) 2-2-2

Q 10. It is allowed to choose two different bars from the above given bars, and the percentage marks obtained become weighted average of the percentage marks obtained, in the ratio of the presence of 3-rating teacher in the bar. In the absence of 3-rating teacher, weighted average should be calculated in the ratio of the presence of 2-rating teachers. For example, if we mix the third bar and 7th bar, we will have to find out the weighted average of 64% and 70% in the ratio 1 : 3 for student A. What would be the percentage marks obtained by the student B if he opts for 4th bar and 5th bar?
(a) 40.2% (b) 40.6%
(c) 40.8% (d) None of these

Direction for questions 11 to 14: Refer to the following chart and solve the questions based on it.

The figure below represents sales and net profit (in Rs crore) of IVP Ltd, for five years from 1994-1995 to 1998-1999. During this period, the sales increased from Rs 100 crore to Rs 680 crore. Correspondingly, the net profit increased from Rs 2.5 crore to Rs 12 crore. (Net profit is defined as the excess of sales over total costs.)



Q 11. The highest percentage growth in sales, relative to the previous year occurred in
(a) 1995-1996 (b) 1996-1997
(c) 1997-1998 (d) 1998-1999

Q 12. The highest percentage growth in net profit, relative to the previous year, was achieved in
(a) 1998-1999 (b) 1997-1998
(c) 1996-1997 (d) 1995-1996

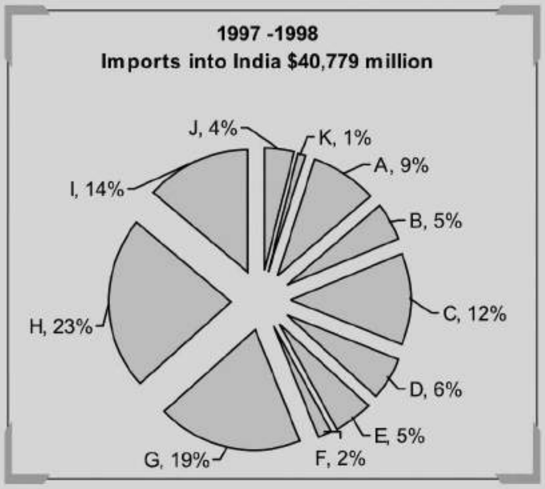
Q 13. Defining profitability as the ratio of net profit to sales, IVP Ltd recorded the highest profitability in
(a) 1998-1999 (b) 1997-1998
(c) 1994-1995 (d) 1996-1997

Q 14. With profitability as defined in question 3, it can be concluded that
- (a) Profitability is non-decreasing during the five years from 1994-1995 to 1998-1999.
- (b) Profitability is non-decreasing during the five years from 1994-1995 to 1998-1999.
- (c) Profitability remains constant during the five years from 1994-1995 to 1998-1999
- (d) None of above

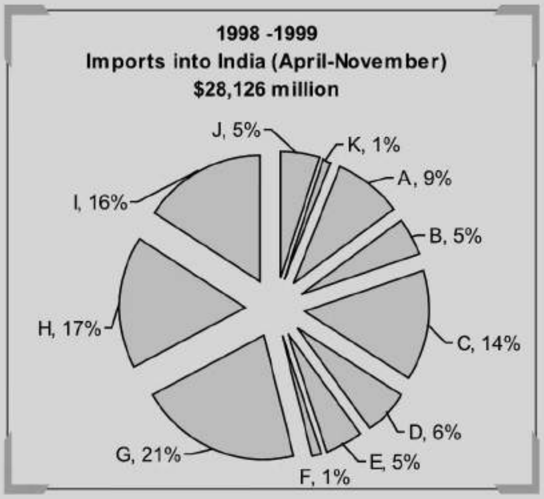
Direction for questions 15 to 18: Refer to the following chart and solve the questions based on it.

Consider the information provided in the figure below relating to India's foreign trade in 1997-1998 and the first eight months of 1998-1999. Total trade with a region is defined as the sum of exports and imports from that region. Trade deficit is defined as the excess of imports over exports; and it may be negative.

A. US B. Germany
C. Other EU D. UK
E. Japan F. Russia
G. Other East-European countries
H. OPEC I. Asia
J. Other LDCs
K. Other Source of Imports

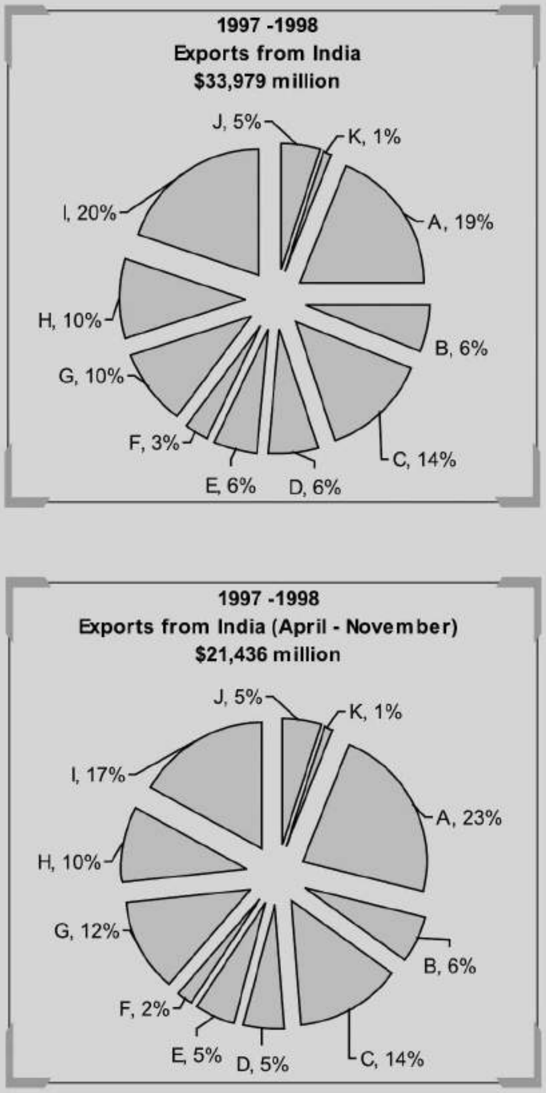


1997 -1998
Imports into India $40,779 million



1998 -1999
Imports into India (April-November)
$28,126 million

Destination of exports



Q 15. With which region does India have the highest total trade in 1997-1998?
(a) US (b) Other EU countries
(c) OPEC (d) Others

Q 16. In 1997-1998, the amount of Indian exports (in million US $) to the region with which India has the lowest total trade, is approximately:
(a) 750 (b) 340
(c) 220 (d) 440

Q 17. In 1997-1998, the trade deficit (in billion US $) for the region with which India has the highest trade deficit is approximately equal to
(a) 6.0 (b) 3.0
(c) 4.5 (d) 7.5

Q 18. Which region has the lowest trade deficit with India in 1997-1998?
(a) US (b) Asia
(c) Others (d) Other EU countries

Direction for questions 19 and 20: Answer the questions based on the following information:

Assume that the average monthly exports and imports to India during the remaining four months of 1998-1999 are the same as that for the first eight months of the year.

Q 19. To which region did India's exports register the highest percentage growth between 1997-1998 and 1998-1999?
(a) Other East-European countries
(b) USA
(c) Asia
(d) Exports have declined, therefore no growth

Q 20. What is the percentage growth rate in India's total trade deficit between 1997-1998 and 1998-1999?
(a) 43% (b) 47%
(c) 50% (d) 40%

Direction for questions 21 to 25: ***Refer to the following chart and solve the questions based on it.***

The table below presents the data on the percentage of population covered by drinking water and sanitation facilities in select Asian countries.

Percentage Population Covered by Drinking Water and Sanitation Facilities

	Drinking Water			Sanitation Facilities		
	Urban	**Rural**	**Total**	**Urban**	**Rural**	**Total**
India	85	79	81	70	14	29
Bangladesh	99	96	97	79	44	48
China	97	56	67	74	7	24
Pakistan	82	69	74	77	22	47
Philippines	92	80	86	88	66	77
Indonesia	79	54	62	73	40	51
Sri Lanka	88	52	57	68	62	63
Nepal	88	60	63	58	12	1

Country A is said to dominate B or A > B as A has a higher percentage in total coverage for both drinking water and sanitation facilities; and B is said to be dominated by A, or B < A.

A country is said to be on the coverage frontier if no other country dominates it. Similarly, a country is not on the coverage frontier if it is dominated by at least one other country.

Q 21. Which countries are on the coverage frontier?
(a) India and China
(b) Sri Lanka and Indonesia
(c) Philippines and Bangladesh
(d) Nepal and Pakistan

Q 22. Which of the following statements are true?

I. India > Pakistan and India > Indonesia
II. India > China and India > Nepal
III. Sri Lanka > China
IV. China > Nepal

(a) I and III (b) II and IV
(c) I, II and III (d) II, III and IV

Q 23. Using only the data presented under the 'Sanitation facilities' columns, it can be concluded that the rural population in India, as a percentage of its total population is approximately:

(a) 76 (b) 70
(c) 73 (d) Cannot be determined

Q 24. Again, using only the data presented under the 'Sanitation facilities' columns, China, Indonesia and Philippines are sequenced in an ascending order of the rural population as a percentage of their respective total population. The correct order is:

(a) Philippines, Indonesia, China
(b) Indonesia, China, Philippines
(c) Indonesia, Philippines, China
(d) China, Indonesia, Philippines

Q 25. India is not on the coverage frontier because

I. It is lower than Bangladesh in terms of the coverage of drinking water facilities.
II. It is lower than Sri Lanka in terms of the coverage of sanitation facilities.
III. It is lower than Pakistan in terms of the coverage of sanitation facilities.
IV. It is dominated by Indonesia.

(a) I and II (b) I and III
(c) IV (d) None of these

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12
Answer	(d)	(d)	(b)	(c)	(b)	(d)	(b)	(c)	(d)	(b)	(a)	(d)
Q. No.	13	14	15	16	17	18	19	20	21	22	23	24
Answer	(b)	(d)	(c)	(b)	(a)	(a)	(b)	(b)	(c)	(b)	(c)	(a)
Q. No.	25											
Answer	(d)											

HINTS AND SOLUTIONS

1. Yuvraj – 1800/40 = 45.
   Hence, not-out in 3 innings.
   Sachin – 2034/45.2 = 45.
   Hence, not-out in 5 innings.
   Kapil – 1679/36.5 = 46.
   Hence, not-out in 0 inning.
   Sehwag – 1445/42.5 = 34.
   Hence, not-out in 6 innings.
   Therefore, option (d) is the correct answer.
2. The difference is maximum in case of Sehwag.
3. Only Kapil got out in all the innings played. Take the help of the solution to Q 1.
4. Again, the difference is maximum in case of Sehwag.
5. The difference in the total score = 64 × 3 – 50 × 3 = 42 (Difference is caused by the teacher of the high rating only).
8. Go through the options.
9 Look at the bars for the maximum difference. It is the maximum in case of 2-2-2.
10. We would be required to find the weighted average of 40 and 41 in the ratio 2 : 3. Hence (40 × 2 + 41 × 3)/5 = 40.6%
11. Using the graph, the percentage growth in sales are:

	Previous Sales	Current Sales	Difference	Percentage
1995-96	100	250	150	150%
1996-97	250	300	50	20%
1997-98	300	290	–10	–3.33%
1998-99	290	680	390	134.5%

It is but obvious from the above table that the maximum percentage increase relative to previous year occurred in 1995-1996.

12. We can again calculate the growth in profits.

	Previous Profit	**Current Profit**	**Difference**	**Percentage**
1995-96	2.5	4.5	2	80%
1996-97	4.5	6	1.5	33.33%
1997-98	6.0	8.5	2.5	41.66%
1998-99	8.5	12	3.5	41.2%

The highest percentage growth in net profit relative to the previous year was achieved in 1995-1996.

13.

	Net Profit	**Net Sales**	**Ratio**
1994-95	2.5	100	0.025
1995-96	4.5	250	0.02
1996-97	6	300	0.02
1997-98	8.5	290	0.03
1998-99	12	680	0.018

Therefore, profitability is maximum for 1997-1998.

14. It can be seen that profitability does not follow a fixed pattern and the first three statements try to generalize the profitability.
Therefore, they are not applicable and option (d) is the correct answer.

15. Total trade with a region = Sum of exports and imports from that region. From the pie charts for 1997-1998, we have the following sectors occupying the maximum area.

	Country	**Exports**	**Imports**	**Trade**
H	OPEC	3397.9	9379.2	12795.1
I	Asia	6796	5709	12505
G	East Europe	3397	7748	11145.9
C	EU	4757	4893.5	9650.5
A	USA	6456	3670	10126

Therefore, H-OPEC has the maximum trade with India in 1997-1998

16. Using the pie chart, the region having the lowest trade is K =
1% of imports + 1% of exports
So, Indian exports are 1% of 3397.9 which is roughly 340 millon US $.

17.

	Imports	Exports	Trade Deficit
A	3670.11	6456.01	–2785.9
B	2038.95	2038.74	0.21
C	4893.48	4757.06	136.42
D	2446.74	2038.74	408
E	2038.95	2038.74	0.21
F	815.58	1019.37	–203.79
G	7748.01	3397.9	4350.11
H	9379.17	3397.9	5981.27
I	5709.06	6795.8	–1086.74
J	1631.16	1698.95	–67.79

So, it can be seen that region H has the highest trade deficit of approximately $ 6,000 million or $ 6 billion.

18. Using the pie chart for 1997-1998, we can see that US which is region A, has the lowest trade deficit.
(9% of imports – 19% of total exports)
(9/100 × 40779 – 19/100 × 33979)

19. Using the pie chart, we know that the exports have increased from three regions A, G and H as follows.

	Country	1998-99	1997-98
A	USA	7395.4	6456
G	East European Countries	3858.5	3397.9
H	OPEC	3215.4	3397.9

Now, the exports for 8 months have been given. According to new directions, we need to calculate exports for 12 months.
$\frac{21436 \times 12}{8}$ = $ 32,154 million
So, maximum percentage increase is from region A (USA).

20. Following is India's total trade deficit:

	Imports	Exports	Deficit
1997-98	40779	33979	6800
1998-99	42189	32154	10035

Total imports for 1998-1999
   $= 28126 \times 12/8$
   = $ 42,189 million
Similarly, exports for 1998-1999
   $= 28126 \times 12/8$
   = $ 32,154 million
Percentage growth rate
   $= 10035 - 6800/6800 \times 100$
   $= 47.6\%$
Therefore, option (b) is the correct answer.

21. Also, Bangladesh > Philippines (97 > 86) for drinking water.
And Philippines > any other country for sanitation facilities. Thus, countries are on the coverage frontier for two facilities.

22. Statement I > Statement II only if statement I has higher percentage in total coverage for both drinking water and sanitation facilities taken independently and not as a total of the two facilities.
Thus, only statement II and statement IV are true
India > China
(81 > 67 and 29 > 24)
India > Nepal
(81 > 63 and 29 > 18)
Also China > Nepal (67 > 63 and 24 >18)

23. Let the urban population be x and rural population be y.
Using the sanitation column, we have
$0.7x + 0.14y = 0.29(x + y)$
$0.41x = 0.15y$
So $x = 15/41y$
So, percentage of rural population
   $= y/x + y \times 100$
   $= y/15/41y + y \times 100$
   $= 41/56 \times 100$
   $= 73.2\%$

24. Use the method given in the last question to find the percentage of rural population for Philippines, Indonesia and China.

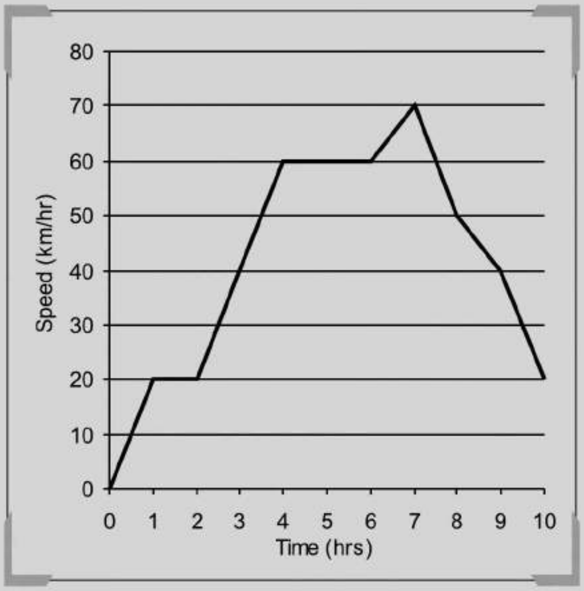
P	50%
I	66.66%
C	79.8%

Thus, P < I < C

25. India is not on the coverage frontier because
   (i) It is below Bangladesh and Philippines for drinking water.
   (ii) For sanitation facilities it is below Philippines, Sri Lanka, Indonesia and Pakistan.

PRACTICE EXERCISE 2

Direction for questions 1 to 5: Go through the data set given below and solve the questions based on it.



Q 1. What is the average speed represented in the above graph?
- (a) 32 km/h
- (b) 37 km/h
- (c) 43 km/h
- (d) None of these

Q 2. Considering two-hour slots from the beginning, the second highest distance covered in a two-hour slot was between
- (a) 0 – 2 h (b) 2 – 4 h
- (c) 4 – 6 h (d) 6 – 8 h

Q 3. What is the highest rate of acceleration achieved during the whole period?
- (a) 25 km/h^2
- (b) 20 km/h^2
- (c) 15 km/h^2
- (d) 10 km/h^2

Q 4. If the rate of deceleration of the 8th hour had continued further, the speed would have become zero in
- (a) 9 hrs 10 mins
- (b) 10 hrs 30 mins
- (c) 11 hrs 10 mins
- (d) None of these

Q 5. For which of the given hours in the graph was the rate of change of speed minimum?
- (a) 1st–2nd
- (b) 4th–5th
- (c) 5th–6th
- (d) All of these

Direction for questions 6 to 9: Go through the data set given below and solve the questions based on it.

In a management course, students of 2nd year are asked to select four optional subjects from the given list of seven optional subjects. There are ten students whose roll numbers are from 1 to 10. They have selected four optional subjects each. Their subjects selected are indicated by "X" marks.

Courses	**Roll Number**									
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
TA	X		X	X		X		X	X	X
MBC		X	X		X	X		X		
YCA			X			X	X		X	X
DMBC	X	X		X	X		X	X		
XA		X					X		X	X
PG	X		X	X	X		X	X		
PM	X	X		X	X	X			X	X

Q 6. Which of the following triple courses have maximum number of students in common?
(a) TA, MBC, YCA (b) DMBC, TA, PG
(c) YCA, PM, XA (d) DMBC, PG, XA

Q 7. Which roll numbers have the maximum common number of optional subjects?
(a) 1 and 8 (b) 3 and 6
(c) 9 and 10 (d) None of these

Q 8. Which optional subject was chosen by maximum number of students?
(a) XA (b) TA
(c) MBC (d) DMBC

Q 9. Which roll numbers have the same optional subjects?
(a) 3 and 6 (b) 1 and 3
(c) 6 and 8 (d) 9 and 10

Direction for questions 10 to 14: Go through the data set given below and solve the questions based on it.

A rating company rates the performance of three companies producing shoes. The points are allotted according to their sales. The point index (PI) of each of the companies = The number of lakh units sold during the month × points allotted.

Table 1

The number of lakh units sold = y	**Points allotted**
$y < 5$	3
$5 < y < 8$	4
$8 < y < 11$	5
$y > 11$	6

The following table shows the number of lakh units sold of each of the brands—Moon, Warle and Enivi—from Jan 07 to June 07.

Table 2

Name of month	Moon	Warle	Enivi	Rank of months according to PI
Jan 07	7	4	-	4
Feb 07	-	13	-	1
Mar 07	-	-	-	5
Apr 07	-	-	-	6
May 07	-	-	-	2
Jun 07	-	-	10	3

Note:

1. The number of lakh units sold by all the three companies in each of the months is identical.
2. The sum of the number of lakh units sold by each of the companies in all the six months together is identical.
3. The number of lakh units sold by any of the companies in any one of the months is at least 1.
4. The number of lakh units sold by exactly two companies in each of the months Feb 07, Mar 07 and Apr 07 is identical.
5. The Point Index of Jan 07 and May 07 is 26 and 17 less than Feb 07 respectively. Also, the PI in Jan 07 is one more than that of March 07.
6. The number of lakh units sold by Moon in Mar 07 and Apr 07 together is equal to that in May 07.
7. The sum of PI in any month is not greater than 100.
8. The least possible Point Index (PI) is in Apr 07 for the sum of lakh units sold by all three companies.

Q 10. What is the number of lakh units sold by Enivi in Mar 07?
(a) 5 (b) 6
(c) 4 (d) 7

Q 11. Find the PI of all the three companies in Apr 07.
(a) 59 (b) 60
(c) 61 (d) 58

Q 12. What is the number of lakh units sold by Moon in all the six months together?
(a) 34 (b) 32
(c) 36 (d) 38

Q 13. What is the number of lakh units sold by Enivi in May 07?
(a) 7 (b) 4
(c) 5 (d) 8

Q 14. What is the PI of Warle in all the six months together?
(a) 139 (b) 153
(c) 138 (d) 148

Direction for questions 15 to 20: Go through the passage given below and solve the questions based on it.

Five cricket coaches B, D, J, L and W were being judged for the award of best coach of the year. The selection committee ranked them on five different parameters viz., Empathy, Vision, Focus, Creativity, and Intuitions. The five coaches were given points on these five parameters. The best coach gets 5 points and the worst coach gets 1 point. There are no ties. The coach with highest total gets the award of best coach. B gets the award by scoring 24 points. W gets 5 points in Creativity and 3 points in Intuition. J got same score in four of the given five parameters. Their final standings were in alphabetical order of their names.

Q 15. What is the overall score of J?
(a) 12 (b) 13
(b) 14 (c) Indeterminable

Q 16. What is the rank of D in Creativity?
(a) 1st (b) 2nd
(c) 5th (d) Indeterminable

Q 17. What are the points scored by J in Intuition?
(a) 1 (b) 2
(c) 4 (d) Indeterminable

Q 18. If L scored his highest points in Empathy then what is the score of D in Vision?
(a) 1 (b) 2
(c) 3 (d) 4

Q 19. For how many of the coaches given above, is it possible to determine the exact points obtained in different parameters?
(a) 2 (b) 3
(c) 4 (d) 5

Q 20. If L gets a score of 2 in each—Empathy, Vision and Focus, then what is the rank of D in intuition?
(a) 2nd (b) 4th
(c) 3rd (d) 5th

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12
Answer	(c)	(d)	(b)	(b)	(d)	(b)	(c)	(b)	(d)	(b)	(d)	(a)
Q. No.	13	14	15	16	17	18	19	20	21	22	23	24
Answer	(c)	(d)	(b)	(c)	(a)	(b)	(c)	(b)				

HINTS AND SOLUTIONS

[illegible]

1. The average speed in first hour =10 km/hr
Average speed represented by the graph

$$= \frac{10 + 20 + 30 + 50 + 60 + 60 + 65 + 60 + 45 + 30}{10}$$

$$= \frac{430}{10} = 43\,\text{km/h}.$$

2. Distance covered in 0–2 hrs slot = 10 + 20 = 30 km
Distance covered in 2–4 hrs slot = 80 km
Distance covered in 4–6 hrs slot = 120 km
Distance covered in 6–8 hrs slot = 125 km

3. Acceleration = Change in velocity/h
= 20 km/h^2 in 1st, 3rd and 4th h

4. Deceleration in 8th hour = 20 km/h^2
Speed at the end of 8th hour = 50 km/h
Speed will be zero after ‘t’ h where 0 = 50 – 20 × t

$$= t = \frac{50}{20} = \frac{5}{2}\,\text{hours} = 2\frac{1}{2}\ \text{hrs.}$$

= Speed will be zero at 10 h 30 mts.

[illegible]

6. (1) Roll nos 3 and 6
(2) Roll nos 1, 4 and 8
(3) Roll nos 9, 10
(4) Roll no 7

7. (1) Roll nos 1 and 8 have TA, DMBC and PG as common.
(2) Roll nos 3 and 6 have TA, MBC, YCA as common.
(3) Roll nos 9 and 10 have all the four common options.
(4) Roll nos 2 and 5 have MBC, DMBC and PM as common.

8. TA → 7 students
XA → 4 students
MBCA → 5 students
DMBC → 6 students

Questions 1–3

Following table can be formed with the data given in the question:

	B	D	J	L	W
Empathy	5				
Vision	5				
Focus	5				
Creativity	4				5
Intuition	5				3
Total	24		Sum = 75 – 24 = 51		

We can see now that the minimum possible sum for W = 11.
Sum of points of the coaches are in the following order – B > D > J > L > W
And sum of the points of (D + J + L + W) = 5(1 + 2 + 3 + 4 + 5) – 24 (points of B) = 51
Hence, the point obtained by the remaining four coaches D, J, L and W will be 11, 12, 13, 15 respectively.
(Points obtained by any of J, L and W can not be other than 11, 12 and 13 because in that case, the sum of the points of the four coaches will be more than 51.)
Hence, the final points tally will be:

	B	D	J	L	W
Empathy	5	2/4	3	2/4	1
Vision	5	2/4	3	2/4	1
Focus	5	2/4	3	2/4	1
Creativity	4	1/2	3	1/2	5
Intuition	5	2/4	1	2/4	3
Total	**24**	**15**	**13**	**12**	**11**

Now all the questions can be answered.

PRACTICE EXERCISE 3

Direction for questions 1 to 7: Refer to the following table and solve the questions based on it.

The following tables and charts show the details of the advertising expenses and corresponding results for XYZ Ltd.

During 2000–2004, the company issues ads in three different newspapers— TOI, HT and others.

Table 1 shows the year wise cost incurred by XYZ Ltd during the given period.

Table 2 shows the number of advertisements released in each publication during the given period.

Chart 1 shows the total number of enquiries generated at XYZ Ltd in response to the advertisements in each publication during the given period.

Chart 2 gives the number of clients (out of the total who enquire) who finally provide business to XYZ Ltd, as a result of its advertising in each publication during the given period.

Table 3 shows the average amount of business per new client for XYZ Ltd as a result of its advertising in each publication in during the given period. (Assume that all new clients in a year are a result of the advertising in that specific year only and the clients responding to advertising in different publications are mutually exclusive)

Table 1

Total Money Spent (Rs Lacs)

Year	TOI	HT	Others
2000	3.5	3.2	2
2001	5	3.4	2.8
2002	6.2	4	5.2
2003	8	4.8	4.4
2004	14	3.5	3.1

Table 2

Number of Ads

Year	TOI	HT	Others
2000	10	12	15
2001	12	15	17
2002	15	20	36
2003	19	24	22
2004	38	18	30

Table 3

Average Business/Client (Rs lacs)

Year	TOI	HT	Others
2000	0.25	0.19	0.12
2001	0.19	0.28	0.09
2002	0.28	0.15	0.18
2003	0.31	0.55	0.22
2004	0.44	0.38	0.15

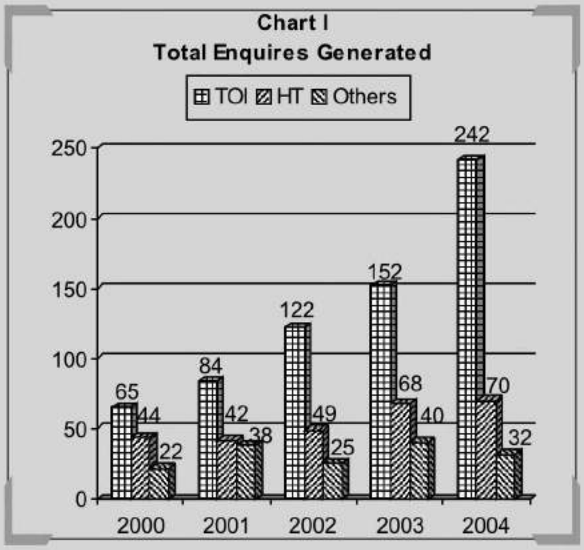


Chart I
Total Enquires Generated

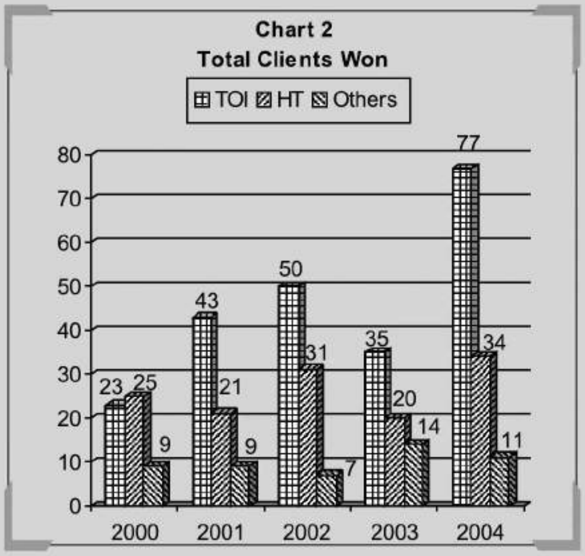


Chart 2
Total Clients Won

For advertising in any publication in any year (or period):

(i) Response ratio is the average number of enquiries generated per advertisement.

(ii) Conversion ratio is the percentage of clients who provide business out of those who enquire.

(iii) Success ratio is the average number of clients won per advertisement.

(iv) Return ratio is the ratio of business generated to the money spent on advertising.

Q 1. During the given period, the average cost per advertisement for XYZ Ltd was highest in
(a) TOI (b) HT
(c) Others (d) Cannot be determined

Q 2. The response ratio in a single year during the given period was highest for advertising in
(a) TOI in 2003 (b) HT in 2004
(c) Others in 2001 (d) TOI in 2002

Q 3. The response ratio during the given period was the lowest in which of the publications?
(a) TOI (b) HT
(c) Others (d) Cannot be determined

Q 4. The lowest conversion ratio for advertising by XYZ Ltd in HT was observed in
(a) 2001 (b) 2003
(c) 2002 (d) 2000

Q 5. Which of the following shows the correct order of the conversion ratios for advertising by XYZ Ltd in the three publications during the given period?
(a) TOI > HT > Others (b) HT > Others > TOI
(c) HT < TOI < Others (d) Others < TOI < HT

Q 6. For XYZ Ltd, the success ratio for advertising in the TOI during the given period was approximately what percentage of the success ratio for advertising in others in 2003?
(a) 525% (b) 425%
(c) 350% (d) 285%

Q 7. In how many years during the given period was the success ratio for advertising in the TOI greater than the success ratio for advertising in the HT?
(a) 5 (b) 4
(c) 3 (d) 2

Direction for questions 8 to 13: Refer to the following table and solve the questions based on it.

The tables given below show the currency exchange rates in April 2005 and April 2004 respectively. For any currency row, the value corresponding to each column shows that currency's exchange rate with the currency column. For example, in April 2005, one dollar was equivalent to Rs 43.750

Chart 3 and Chart 4 show the percentage break-up of rupee equivalent of currency reserves of a company in April 2005 and April 2004 respectively. The rupee equivalent reserves of a currency are the currency reserves calculated in rupee terms based on the prevalent exchange rate between the rupee and that currency. For example, reserves of 1000 euros in April 2004 would constitute rupee equivalent reserves of Rs 53,000. The rupee equivalent of currency reserves for the company was Rs 5 crores in April 2004, and grew by 40% in the next one year.

Table 1

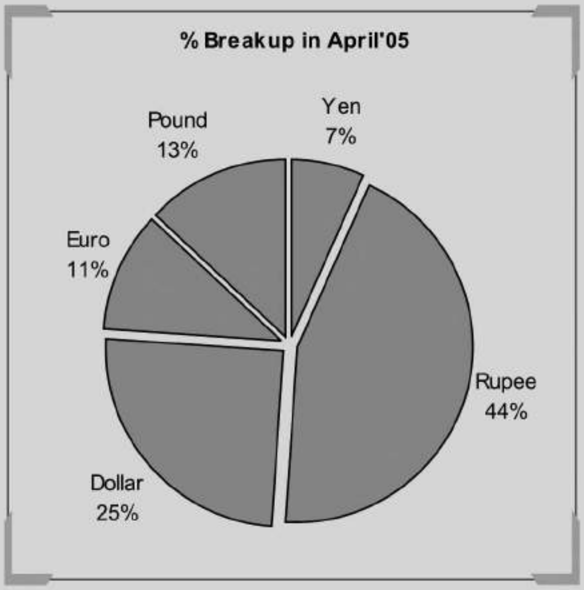
Currency Rates (April 2005)

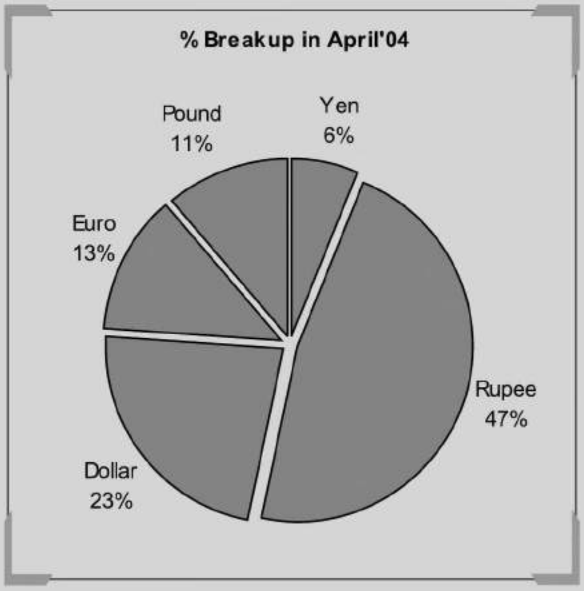
Currency	Rupee	Dollar	Euro	Pound	Yen
Rupee	----	0.023	0.018	0.012	0.024
Dollar	43.750	----	0.770	0.535	1.065
Euro	56.790	1.298	----	0.695	1.382
Pound	81.750	1.869	1.440	----	1.990
Yen	41.090	0.939	0.724	0.503	----

Table 2

Currency Rates (April 2004)

Currency	Rupee	Dollar	Euro	Pound	Yen
Rupee	----	0.020	0.019	0.013	0.029
Dollar	50.000	----	0.880	0.612	1.217
Euro	53.000	1.211	----	0.648	1.290
Pound	75.000	1.714	1.321	----	1.825
Yen	35.000	0.800	0.616	0.428	----





Q 8. The dollar currency reserves for the company (in $ million) in April 2004 was closest to
(a) 11.5 (b) 4.0
(c) 2.3 (d) 0.25

Q 9. The rupee equivalent (in rupees million) of the Yen reserves for the company in April 2005 was closest to

(a) 49 (b) 1.3

(c) 5 (d) 0.12

Q 10. The ratio of money in dollar reserves to the money in euro reserves for the company in April 2004 was

(a) 7 : 4 (b) 3 : 5

(c) 9 : 4 (d) Cannot be determined

Q 11. The money in pound reserves in April 2005 was how many times the money in Yen reserves in April 2004?

(a) 1.65 (b) 1.82

(c) 3 (d) Cannot be determined

Q 12. Between April 2004 and April 2005, by what percentage did the rupee reserves grow?

(a) 22% (b) 31%

(c) 49% (d) Cannot be determined

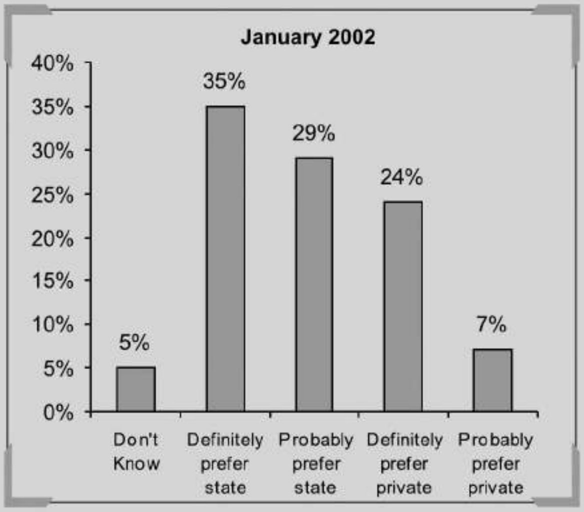
Q 13. Between April 2004 and April 2005, the company's reserves of which currency saw the maximum absolute increase (in rupee equivalent terms)?

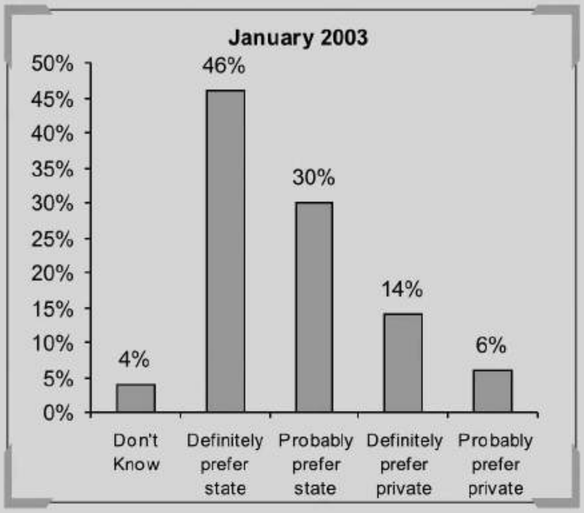
(a) Dollar (b) Euro

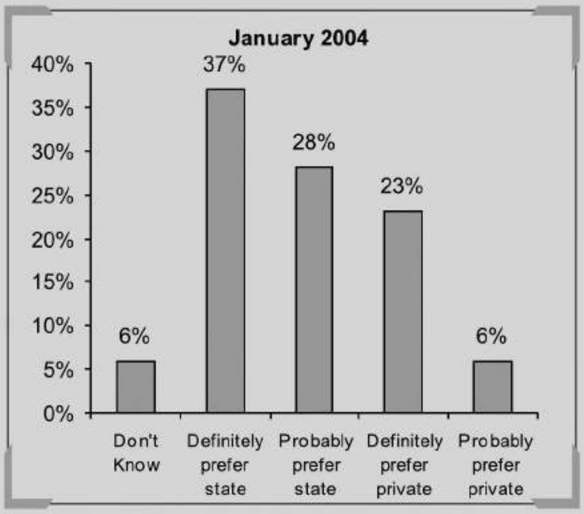
(c) Pound (d) Rupee

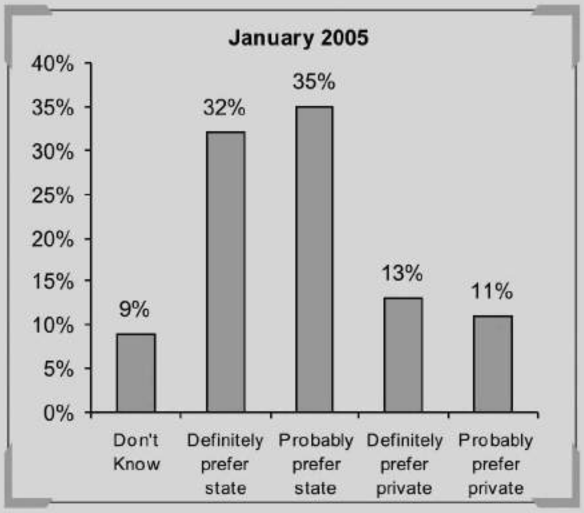
Direction for questions 14 to 20: Refer to the following table and solve the questions based on it.

The following charts show the responses of the population of the state of ABC over a four-year period, to the question—"Is State ownership the best way to run an enterprise OR an enterprise is best run by private entrepreneurs?" The eligible voting population of ABC was 2.4 million in January 2003 and can be assumed to have grown by 20% each year during the period January 2002–January 2005. The entire eligible population voted in each of the four years and every person casted a vote in favour of only one of the five options shown each year.









Q 14. For how many options during the given period did the number of people voting in favour of the options, increase in each year, relative to the the previous year?

(a) 2 (b) 3

(c) 1 (d) None

Q 15. For how many options during the given period did the number of people voting in favour of the options, decrease in each year, relative to the previous year.

(a) 2 (b) 3

(c) 1 (d) None

Q 16. The greatest absolute increase, between January 2002 and January 2005, in the number of people voting for an option was observed for:

(a) Definitely prefer private

(b) Probably prefer state

(c) Probably prefer private

(d) Don't know

Q 17. For which option did the number of people casting their vote decrease between January 2002 and January 2005?

(a) Definitely prefer state

(b) Probably prefer state

(c) Definitely prefer private

(d) None

Q 18. The greatest percentage increase, between January 2002 and January 2005, in the number of people voting for an option took place for

(a) Definitely prefer private

(b) Probably prefer state

(c) Probably prefer private

(d) Don't know

Q 19. The greatest number of votes cast in favour of an option in a single year were for

(a) Definitely prefer state

(b) Probably prefer private

(c) Probably prefer state

(d) Don't know

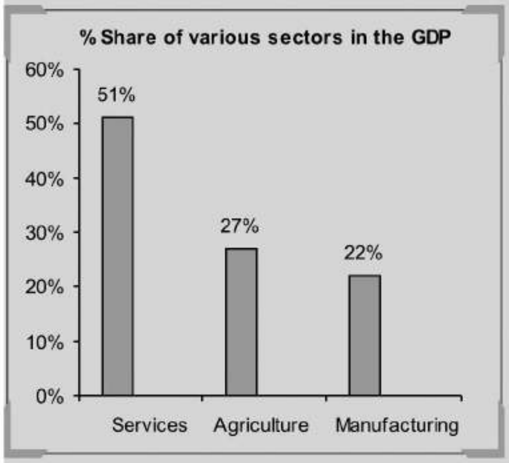
Q 20. What is (most nearly) the cumulative difference, over the entire period, between the number of votes in favour of 'state ownership'(definitely/probably) and the number of votes in favour of 'private ownership'(definitely/probably)?

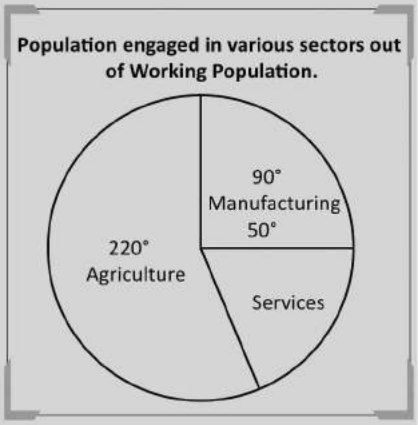
(a) 2.2 million (b) 4.5 million

(c) 3.3 million (d) 6.3 million

Direction for questions 21 to 25: Refer to the following table and solve the questions based on it.

The following charts show the sectorwise percentage break-up of $ 650 billion, which is the GDP of the country with a population of 12 billion people. 70% of the population of the country constitutes its working population, engaged in the various sectors of the economy and contributing to the GDP.





Productivity is defined as the average amount contributed to the GDP per person in that group or sector.

Q 21. Which of the following groups/sectors has the lowest productivity?

(a) Agriculture sector workers
(b) Manufacturing sector workers
(c) Services sector workers
(d) Non-working population

Q 22. What is the approximate difference in the number of people involved in the agriculture sector and the services sector?

(a) 1.8 billion (b) 2.5 billion
(c) 4 billion (d) 5.7 billion

Q 23. What percentage of the country's total population is engaged in the manufacturing sector?

(a) 17.5% (b) 35.7%
(c) 25% (d) Indeterminate

Q 24. The productivity for how many sectors is more than the productivity of the entire population of the country?

(a) 0 (b) 1
(c) 2 (d) 3

Q 25. In the next year, the work force engaged in each of the sectors maintains its productivity at the same level. If there is an increase of 5% in the work-force for each of the sectors, then the productivity of the country has:

(a) Increased (b) Decreased
(c) Remained same (d) Cannot be determined

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12
Answer	(a)	(d)	(c)	(b)	(d)	(d)	(d)	(d)	(c)	(a)	(c)	(b)
Q. No.	13	14	15	16	17	18	19	20	21	22	23	24
Answer	(d)	(a)	(d)	(b)	(c)	(d)	(c)	(b)	(d)	(c)	(a)	(c)
Q. No.	25											
Answer	(d)											

HINTS AND SOLUTIONS

Table 1

Total Money Spent (Rs Lacs)

Year	TOI	HT	Others
2000	3.5	3.2	2
2001	5	3.4	2.8
2002	6.2	4	5.2
2003	8	4.8	4.4
2004	14	3.5	3.1
Total	36.7	18.9	17.5

Table 2

Average of Ads

Year	TOI	HT	Others
2000	10	12	15
2001	12	15	17
2002	15	20	36
2003	19	24	22
2004	38	18	30
Total	94	89	120

Table 3

Average Business/Client (Rs Lacs)

Year	TOI	HT	Others
2000	0.25	0.19	0.12
2001	0.19	0.28	0.09
2002	0.28	0.15	0.18
2003	0.31	0.55	0.22
2004	0.44	0.38	0.15
Total	1.47	1.55	0.76

Table 4

Total Client Won

Year	TOI	HT	Others
2000	23	25	9
2001	43	21	9
2002	50	31	7
2003	35	20	14
2004	77	34	11
Total	228	131	50

Table 5

Total Enquiries Generated

Year	TOI	HT	Others
2000	65	44	22
2001	84	42	38
2002	122	49	25
2003	152	68	40
2004	242	70	32
Total	665	273	157

[illegible]

1. While the amount of money spent on ads in TOI is the maximum (nearly twice as that on ads in HT or others), the number of ads is nearly the same as that in HT and less than that in others.

2. Response ratio: Average number of enquiries generated (Table 4) per advertising (Table 2)
   (a) TOI in 2003:152/19 = 8
   (b) HT in 2004:70/18 < 4
   (c) Others in 2001:38/17 < 2.5
   (c) TOI in 2002:122/15 > 8
   So, amongst the given options, TOI in 2002 had the highest response ratio.

3. Amongst the three publications, the total number of clients (Table 5) to the number of enquiries generated was the least while the number of ads was the higher for others. So , the response ratio for others was the lowest during the given period.

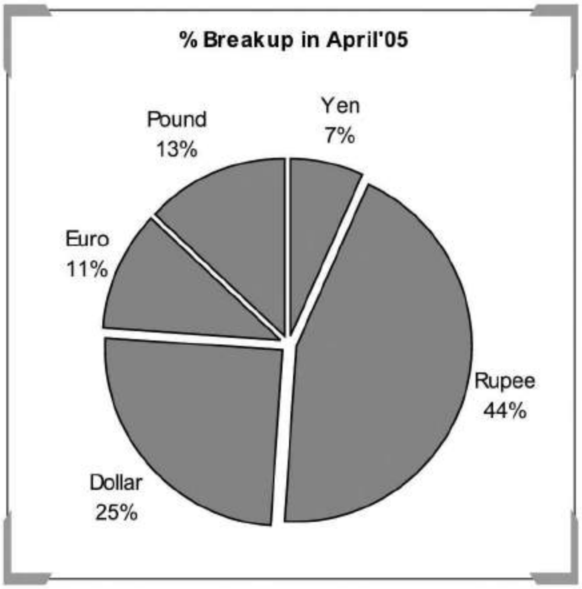
4.
   Conversion ratio: Ratio of number of clients (Table 5) to the number of enquires generated (Table 4)
   For HT:
   2001:21/42 = 50%   2003: 20/68 < 33% 2002:31/49 > 60%   2000:25/44 > 55%
   So, amongst the given options, HT had the lowest conversion ratio in 2003.

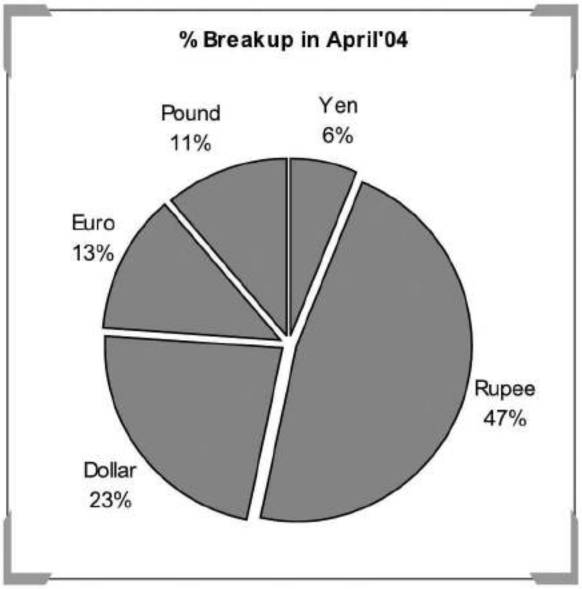
5. Conversion ratio: Ratio of number of clients (Table 5) to the number of enquiries generated (Table 4) over the entire period
   TOI: 228/665 > 33%   HT:131/273 = 48%   Others: 50/157 < 33%
   So, the conversion ratios are in the order: Others < TOI < HT

6. Success ratio of number of clients (Table 5) to number of ads (Table 2)
   Success ratio for TOI over the entire period = 228/94 = 2.43; Success Ratios for others in 2003 = 14/22 = 0.63
   Required percentage = 2.43 – 0.63/0.63 x 100 = 285% (You can see that it is < 300%)

7. You need not calculate the success Ratios in 2001,2002 and 2003 to compare since the number of clients won by TOI is higher than HT and the number of ads in TOI is lower than HT in each of these years. So, the success ratio for TOI is higher than for HT in these 3 years. Comparing the success ratios for 2000 (when number of clients is less for TOI) and 2004 (when number of ads in TOI is more):
   2000: Success ratio for TOI = 23/10 = 2.3;
   Success ratio for HT = 25/12 = 2.083 < 2.3
   2004: Success ratio for TOI = 77/38 > 2;
   Success ratio for HT = 34/18 < 2
   So, the success ratio for TOI > the success ratio for HT in each of the 5 years.

[illegible]





8. Dollar reserves in April 2004 = 0.23 × 5 =Rs 1.15 crores = Rs 11.5 million = \$ 11.5/50 million = \$ 0.23 million

9. Yen reserves in April 2005 = 7% of Rs 7 crores = Rs 0.49 crores = Rs 4.9 million

10. The ratio of the amount of reserves in dollar to that in euro is the same as the ratio of their respective percentages in rupee equivalent terms i.e., 23 : 13 which is very close to 7 : 4.

11. Rupee equivalent in pound reserves in April 2005 = 13% of 7 crores = 0.91 crores
Rupee equivalent Yen reserves in April 2005 = 6% of 5 crores = 0.3 crores
⇨ the money in pound reserves in April 2005 is nearly three times the money in Yen reserves in April 2004.

12. Rupee reserves in April 2004 = 47% of 5 crores = 2.35 crores
Rupee reserves in April 2005 = 44% of 7 crores = 3.08 crores
Percentage increase in Rupee reserves = $308 - 235/235 \times 100$
$= 73/235 \times 100 = 31\%$
The following table summarizes the status of reserves of various currencies (in Lacs)

	2004	**2005**	**Absolute Increase**	**% Increase**
Rupee	235.00	308.00	73.00	31.06
Dollar	2.30	4.00	1.70	73.91
Euro	1.23	1.36	0.13	10.56
Pound	0.73	1.11	0.38	51.79
Yen	0.86	1.19	0.34	39.13

13. Absolute change for Dollar reserves
$= 0.25 \times 700 - 0.23 \times 500 =$ Rs 60 lacs
Absolute change for Euro reserves
$= 0.11 \times 700 - 0.13 \times 500 =$ Rs 12 lacs
Absolute change for Pound reserves
$= 0.13 \times 700 - 0.11 \times 500 =$ Rs 36 lacs
Absolute change for Rupee reserves
$= 0.44 \times 700 - 0.47 \times 500 =$ Rs 73 lacs
Absolute change for Yen reserves
$= 0.07 \times 700 - 0.06 \times 500 =$ Rs 19 lacs
⇨ The maximum absolute increase is for the Rupee (change of 73 lacs)

[illegible]

The population of ABC in January 2002 was 2.4/1.2 = 2 million; in January 2003 was 2.4 million;

In January 2004 was $2.4 \times 1.2 = 2.88$ million; in January 2005 was $2.4 \times (1.2)^2 = 3.46$ million

14. Since the voting population increases by 20% each year, an increase in the number of people choosing an option would take place between any two years if:
    (i) the percentage of people choosing an option would remain the same or increase between the two years
    (ii) 1.2 × percentage in the second year ≥ percentage in the first year

    Increase in each year, from 2003 to 2005, happens only for 'probably prefer state' and 'probably prefer private'

15. Using the reverse of the above conditions, you can see that the actual percentage voting for each one of the options increased in at least one of the years. Hence, none of the options saw a continuous decrease over the period.

16. Assume that P_1 and P_2 are the percentage votings for an option in January 2002 and January 2005 respectively. Then, the greatest absolute increase in the number of people would be for the option for which $(3.46P_2 - 2P_1)$ is the highest. This is the highest for the option 'probably prefer state'.

17. Assume that P_1 and P_2 are the percentage voting for an option in January 2002 and January 2005 respectively. A decrease in the number of people would take place for the option for which $(3.46\ P_2 - 2\ P_1) < 0$. This is so for the option 'definitely prefer private'.

18. Assume that P_1 and P_2 are the percentage votings for an option in January 2002 and January 2005 respectively. Then, the greatest percentage increase in the number of people would be for the option for which $3.46P_2 - 2P_1/2P_1$ is the highest or $1.73 P_2/P_1$ or P_2/P_1 is the highest. This is clearly the case for the option 'don't know'.

[illegible]

Population = 12 billion = > working population = 0.7 X 12 = 8.4 billion.

	Contribution to GDP ($ billion)	**Number of People (billion)**	**Productivity ($/person)**
Agriculture	= 0.27 of 650 = 175.5	= (22/36) of 8.4 = 5.13	34.2
Manufacturing	= 0.22 of 650 = 143	= (9/36) of 8.4 = 2.1	68.1
Services	= 0.51 of 650 = 331.5	= (5/36) of 8.4 = 1.17	284.1
Working population	650	= 0.7 of 12 = 8.4	77.4
Total population	650	12	54.2

21. Non-working population contributes nothing to the GDP; hence, its productivity is zero.

22. Required percentage = $2.1/12 \times 100 = 17.5\%$

25. Since we do not know anything about the non-working population, we cannot determine the answer.

Module 2

Test Your Mettle

BENCHMARKING TEST 1

- There are 20 questions in this test.
- Each question carries 3 marks.
- There is 1/3rd negative marking for every wrong answer.
- Time allotted = 45 minutes

Time Given: 45 Minutes **Total Marks: 60** **Qualifying Marks: 38**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
48+	99+
45	98.5+
43	98
38	92
30	85

Direction for questions 1 to 3: Read the passage below and solve the questions based on it.

Following table gives the result of Hero Cup football competition. Matches were played round robin (each team playing against another once) and the teams participated were Bihar, Punjab and Haryana. The table has some data missing:

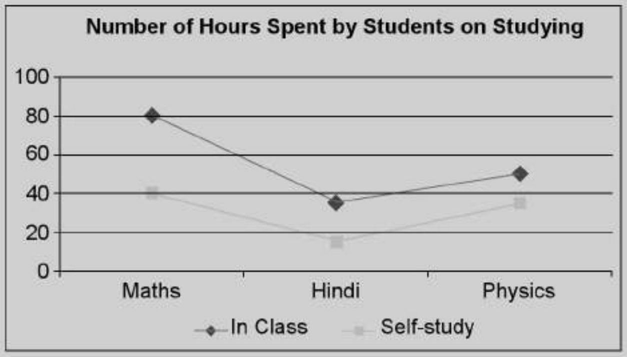
	Match played	*Won*	*Drawn*	*Lost*	*Goals for*	*Goals against*
Bihar	2		1		2	4
Punjab	2				3	7
Haryana	2					1

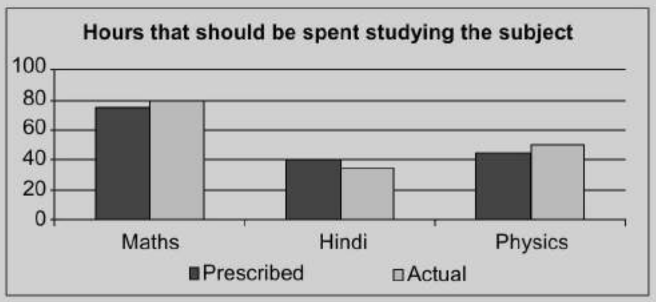
Q.1. What did the score card read in favour of Punjab in Punjab-Haryana match?
(a) 2 – 5 (b) 1 – 5
(c) 1 – 4 (d) 1 – 6

Q.2. How many matches did Punjab lose?
(a) 0 (b) 1
(c) 2 (d) 0 or 1

Q.3. How many goals did Haryana score?
(a) 5 (b) 1
(c) 3 (d) 7

Direction for questions 4 to 8: Go through the charts given below and solve the questions based on it.





Self-study is done always outside the class. All the values are the multiples of 5.

Q.4. Which of the following gives as many hours as those spent on Physics?
(a) Self-study - Math
(b) In Class - Physics and Hindi
(c) Self-study - Math and Hindi
(d) In Class - Math and Hindi

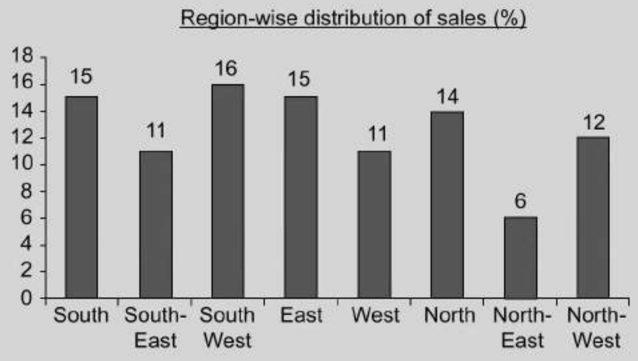
Q.5. What percentage of the hours is spent on learning in Class?
(a) 32% (b) 45%
(c) 65% (d) 80%

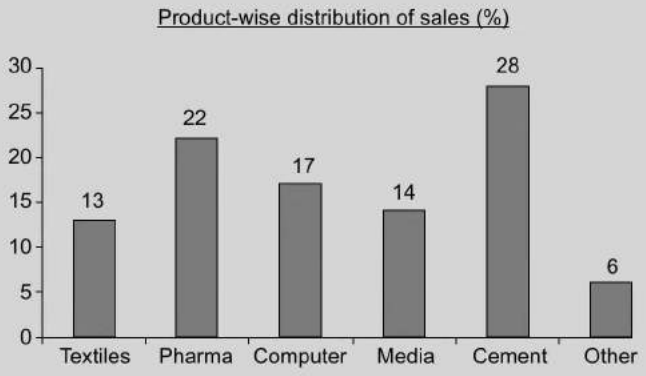
Q.6. The total number of hours spent on learning the subjects self-study is nearest to which of the following:
(a) The prescribed hours for Math
(b) The In Class hours for Math
(c) The sum of the Prescribed and Actual hours spent on Physics
(d) The total hours spent on Hindi

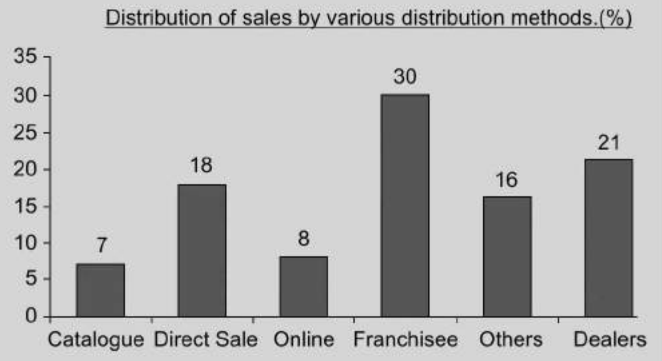
Q.7. If the total hours of study were to be 60 hours, then which of the following would account for approximately 30 hours?
(a) Physics (b) Math
(c) Hindi (d) In Class - Math and Hindi

Direction for questions 8 to 10: Read the passage below and solve the questions based on it.

The sales of a company in 2007–08 were Rs 1200 crore. Following charts represent the distribution of sales for various regions, product and distribution methods wise:


Region-wise distribution of sales (%)


Product-wise distribution of sales (%)


Distribution of sales by various distribution methods.(%)

Q.8. If the computers are sold only through the dealers, how much of the dealers sales are made by products other than computers?
(a) Rs 48 crore (b) Rs 43crore
(c) Rs 252 crore (d) Rs 204 crore

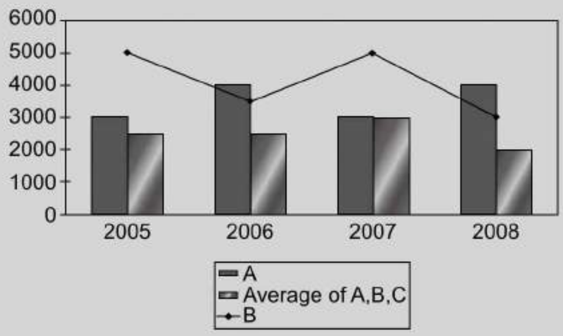
Q.9. In 2008–09 computers sales grow by 50 % while the sales of dealers grows by 30 %. If computers are sold only through dealers, what has been the percentage change in the sales of dealers of products other than computers?
(a) –20 % (b) –45 %
(c) –55% (d) +15%

Q.10. Cement sector has total sales of Rs 3215crore in the year. What is the market share of the company?
(a) 9.55% (b) 10.45%
(c) 8.71% (d) 11.55%

Direction for questions 11 to 15: Go through the bar chart below and solve the questions based on it.

There are three constituents of product D – A, B and C. When A, B and C are mixed in the ratio 1:2:3, D forms.

Following bar chart gives prices (in Rs) of 500 Kg of constituent A, 1000 Kg of constituent B and the average cost of 500 kgs each of constituent A, constituent B and constituent C. (All the values in the chart are in the multiples of 500).



Q.11. In which year was the price of C the highest?
(a) 2005 (b) 2006
(c) 2007 (d) 2008

Q.12. What was the ratio of price of C in 2005 to that in 2006?
(a) 13:14 (b) 8:7
(c) 27:29 (d) 4:3

Q.13. What was the percentage increase in price of C from 2006 to that in 2007?
(a) 100%
(b) 66.66%
(c) 124%
(d) 142%

Q.14. Vijay Shankar Pd. decides to buy 10 kg each of A, B and C. In which year will he pay the least amount of money?
(a) 2005 (b) 2006
(c) 2007 (d) 2008

Q.15. What was the cost of D in 2006 if 6000 kgs of D is to be made?
(a) Rs 17250 (b) Rs 36450
(c) Rs 25500 (d) Rs 20750

Direction for questions 16 to 20: Go through the table below and solve the questions based on it.

Following table gives the details pertaining to the company's employee appointment in three different wings at different levels and the salaries offered:

	Finance Wing			*Marketing Wing*			*H R D Wing*		
Details/Level of entry	*Entry Level*	*Middle Level*	*Top Level*	*Entry Level*	*Middle Level*	*Top Level*	*Entry Level*	*Middle Level*	*Top Level*
Average gross salary drawn per month (in Rs)	5000	8000	18000	4000	7000	17000	5500	7500	15000
Minimum Work Ex required for a graduate (in years)	0	2	NA	0	3	5	0	2	NA
Minimum Work Ex required for a PG (in years)	0	1	2	0	0	2	0	0	2
Number of Grads. Employed	1000	100	NA	1000	70	1	40	1	NA
Number of PG employed	200	100	5	Nil	80	2	80	4	1

Grads. Stands for graduate. A graduate has completed exactly 15 years of education. PG stands for post graduation and a post graduate has completed exactly 17 years of education.

A person is not allowed to shift from one wing to another wing.

Net salary = Gross Salary – taxes and other benefits

NA is given when it is not eligible.

Q.16. Ravi is a graduate and he joins in a wing at a particular level. What is the minimum time that Ravi will take to save Rs 200000, if 15% of his salary is deducted for tax and other benefits and he spends 50% of his net salary?
(a) 34 months (b) 44 months
(c) 56 months (d) None of these

Q.17. Ravi's friend Aftab, a post graduate joins at a middle management cadre. He joins that particular wing which will help him in saving maximum money in two years, given that 15% of his salary is deducted for taxation and other benefits and he spends 40% of his net income. Which wing he must have joined?
(a) Finance (b) Marketing
(c) HRD (d) None of these

Q.18. Ravi's another friend Indrajeet is a fresh graduate. He is confused about his next career move that should he go for post graduation or to start working. He finally decides to calculate her total earnings in 5 years' time in both cases for all wings and make his decision on that basis. Which of the following would be the best course of action assuming he keeps getting promoted immediately after meeting the minimum criteria? (cost of Post graduate program is Rs 250000 keeping all the other things constant.)
(a) Do post graduation and join marketing wing
(b) Do post graduation and join HRD wing
(c) Do not do post graduation and join HRD wing
(d) Do not do post graduation and join finance wing

Q.19. To facilitate the promotion of employees in a particular wing from lower level (L_1) to next higher level (L_2), a mechanism has been devised. The whole of this mechanism has been developed by Mr Thomas and hence, it is named as Thomas profile co-efficient. Thomas profile co-efficient (TPC) for any wing at any level is calculated as follows:

TPC = No. of the Employees in L_2 / No. of employees in L_1

Lower the TPC, higher is the chances of getting promoted.

What is the ratio of highest TPC of post graduate to the highest TPC of a graduate?

(a) 1:5
(b) 5:1
(c) 7:3
(d) 25:7

Q.20. Under a new policy to improve the profile of the company, it has been decided that no graduates will be recruited now and all graduates at entry level and middle level will be replaced with post graduates. Which wing has the best TPC at any level now (Use the definition of TPC from Q 19)?

(a) Finance
(b) HRD
(c) Marketing
(d) Finance or Marketing

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

* * *

■■■ Benchmarking Test 1

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(d)	(b)	(c)	(c)	(b)	(a)	(c)	(b)	(c)	(b)	(a)	(d)	(c)	(d)	(a)	(d)	(b)	(b)

HINTS AND SOLUTIONS

Questions 1 to 3

	Match Played	*Won*	*Drawn*	*Lost*	*Goals For*	*Goal Against*
Bihar	2	0	1	1	2	4
Punjab	2	0	1	1	3	7
Haryana	2	2	0	0	7	1

Score Card:

Bihar – (2 – 2), (0 – 2)

Punjab – (2 – 2), (1 – 5)

Haryana – (5 – 1), (2 – 0)

Questions 1 to 3

4. It can be observed that the value is same as [2] as in Class Hindi has the same value as Self-study Physics. Hence [2]

5. Hours In Class = 165. Hours self-study = 90. = 65% Hence [3]

6. Total Self-study = 90 hours. Option [3], 95 hours is the closest. Hence [3]

7. Total hours currently = 255. If we have 60 hours and are looking at 30 hours, we are looking at 50% of the time. This is around 127 hours.
We find that Math with 120 hours comes closest. Hence [2]

Questions 1 to 3

8. (1) Computer sales = 0.17 × 1200 = Rs 204 crore. Dealer sales = 0.21 × 1200 = Rs 252 crore
Hence, dealers sales by other products = Rs 48 crore

9. (3) In 2008–09, computer's sales = 1.5 × 204 = Rs 306 crore, Dealer sales = 1.3 × 252 = Rs 327.6 cross

2008–09 sales by dealers of other products = 327.6 – 306 = Rs 21.6 crore

From 48 to 21.6, change is negative and exceeds 50%.

10. (2) $\frac{0.28 \times 1200}{3215} = \frac{336}{3215} \approx 10.45\%$

[illegible]

For 2005 : 2500x3 – (3000+2500) = Rs 2000; For 2006: 7500 – 5750 = Rs 1750

For 2007: 9000 – 5500 = Rs 3500 ; For 2008: 6000 – 5500 = Rs 500

12. 2000:1750 = 8:7. So [2]

13. (3500–1750)/1750 = 100%. So [1]

15. Cost of 1000 kg of A in 2006 = 8000
Cost of 2000 kg of B in 2006 = 7000 and
Cost of 3000 kg of C in 2006 = 10500.
So, Cost of 6000 kgs of D in 2006 is Rs 25500.
So [3]

[illegible]

16. Minimum time will be if he earns maximum salary. This is possible if he works at the top level of Marketing Wing. He saves Rs 7225. Hence, time taken to save Rs 200000 is 28 months.
Hence, option [5] is the answer.

17. At the middle level, finance wing gives highest salaries. Hence he is bound to save most in this wing. Hence option [1] is the answer.

19. Highest TPC for a post-graduate is from entry level to middle level in finance wing (50%). Highest TPC for graduate is also from entry level to middle level in finance wing (10%). Hence, ratio = 5:1. Hence, option [2]is the answer.

20. Option 2, from middle level to top level in HRD.

BENCHMARKING TEST 2

- There are 20 questions in this test.
- Each question carries 3 marks.
- There is 1/3rd negative marking for every wrong answer.
- Time allotted = 45 minutes

Time Given: 45 Minutes **Total Marks: 60** **Qualifying Marks: 38**

Break-up of the Expected Percentile

(assuming this to be a CAT paper)

Marks	*Expected Percentile*
48+	99+
45	98.5+
43	98
38	92
30	85

Direction for questions 1 to 5: Go through the data set given below and solve the questions based on it.

Name of Stock	*Date of Auction*	*Date of Issue*	*Notified amount*	*Devolvement on PDs*	*Cut off price (in Rs)*	*Cut off yield (%)*	*Date of Maturity*
7.56% GS 2014	02-Apr-09	06-Apr-09	8000	0	103.47	6.80	03-Nov-14
7.94% GS 2021	02-Apr-09	06-Apr-09	4000	567	102.5	7.62	24-May-21
6.05% GS 2019	09-Apr-09	13-Apr-09	8000	0	95.05	6.75	02-Feb-19
7.50% GS 2034	09-Apr-09	13-Apr-09	4000	0	97.35	7.74	10-Aug-34
7.56% GS 2014	17-Apr-09	20-Apr-09	8000	0	106.75	6.10	03-Nov-14
8.24% GS 2027	17-Apr-09	20-Apr-09	4000	0	107.82	7.44	15-Feb-27
6.05% GS 2019	24-Apr-09	27-Apr-09	8000	0	99.38	6.13	02-Feb-19

Q.1. For how many stocks, date of maturity is minimum 5 years ahead of date of auction?

(a) All but 1 stock (b) All but 2 stocks
(c) All but 3 stocks (d) None of these

Q.2. Nomenclature of the stocks is done in the following way:

7.56% GS 2014 - It means that desired yield from the Government stock (GS) in 2014 is 7.56%.

For how many of the stocks Desired yield is more than cut-off yield?

(a) 3 (b) 4
(c) 5 (d) 2

Q.3. How many stocks with a cut-off price of more than Rs 100 have their cut-off yield more than its desired yield (use definition from above question if required)?

(a) 0 (b) 1
(c) 2 (d) 3

Q.4. Mr Robert is quite an ambitious person. He wants to obtain a minimum desired yield of 8%. For that to happen, he buys exactly two stocks in different ratio. How many combinations can he have, out of 7 stocks given, such that his minimum desired yield is atleast 8% (use information from previous qs)?

(a) Not possible (b) 6
(c) 7 (d) None of these

Q.5. If we arrange the stocks in the ascending order of their cut-off price, which stock will be ranked third?
(a) 7.50% GS 2034 (b) 6.05% GS 2019
(c) 7.94% GS 2021 (d) 7.56% GS 2014

Direction for questions 6 to 8: Go through the data set given below and solve the questions based on it.

Five women decided to go shopping to MG Road, Bangalore. They arrived at the designated meeting place in the following order: 1. Archana, 2. Chellamma, 3. Dhenuka, 4. Helen, and 5. Shahnaz. Each women spent at least Rs 1000. below are some additional facts about how much they spent during their shopping spree.

i. The woman who spent Rs 2234 arrived before the lady who spent Rs 1193.
ii. One woman spent Rs 1340 and she was not Dhenuka.
iii. One woman spent Rs 1378 more than Chellamma.
iv. One woman spent Rs 2517 and she was not Archana.
v. Helen spent more than Dhenuka.
vi. Shahnaz spent the largest amount and Chellamma the smallest.

Q.6. What was the amount spent by Helen?
(a) Rs 1193 (b) Rs 1340
(c) Rs 2234 (d) Rs 2517

Q.7. Which of the following amount was spent by one of them?
(a) Rs 1139 (b) Rs 1378
(c) Rs 2571 (d) Rs 2718

Q.8. The women who spent Rs 1193 is
(a) Archana (b) Chellamma
(c) Dhenuka (d) Helen

Direction for questions 9 to 11: Go through the data set given below and solve the questions based on it.

Five friends meet every morning at Sree sagar restaurant for an idli-vada breakfast. Each consumes a different number of idli and vada. The number of idli consumed are 1,4, 5,6 and 8, while the number of vadas consumed are 0,1,2,4 and 6. Below are some more facts about who eats what and how much:

i. The number of vadas eaten by Ignesh is three times the number of vadas consumed by the person who eats four idlis.
ii. Three persons, including the one who eats four vadas eat without chutney.
iii. Sandeep does not take any chutney.
iv. The one who eats one idki a day does not eat any vadas or chutney. Further, he is not Mukesh.
v. Daljits eats idli with chutney and also eats vada.
vi. Mukesh, who does not take chutney, eats half as many vadas as the person who eats twice as many idlis as he does.
vii. Bimal eats two more idlis than Ignesh, but Ignesh eats two more vadas than Bimal.

Q.9. Which of the following statement is true?
(a) Daljit eats 5 idlis (b) Ignesh eats 8 idlis
(c) Bimal eats 1 idli (d) Bimal eats 6 idlis.

Q.10. Which of the following statement is true?
(a) Sandeep eats 2 vadas.
(b) Mukesh eats 4 vadas.
(c) Ignesh eats 6 vadas.
(d) Bimal eats 2 vadas.

Q.11. Which of the following statements is true?
(a) Mukesh eats 8 idlis and 4 vada but not chutney
(b) The person who eats 5 idlis and 1 vada does not take chutney.
(c) The person who eats equal number of vadas and idlisalso takes chutney.
(d) The person who eats 4 idlis and 2 vada also takes chutney.

Direction for questions 12 to 16: Go through the data set given below and solve the questions based on it.

The table below presents data on percentage data on percentage covered by drinking water and sanitation facilities in selected Asian countries.

Population covered by drinking water and sanitation facilities (Percentage Coverage)

	Drinking water			Sanitation Facilities		
	Urban	**Rural**	**Total**	**Urban**	**Rural**	**Total**
India	85	79	81	70	14	29
Bangladesh	99	96	97	79	44	48
China	97	56	67	74	7	24
Pakistan	82	69	74	77	22	47
Philippines	92	80	86	88	66	77
Indonesia	79	54	62	73	40	51
Sri-Lanka	88	52	57	68	62	63
Nepal	88	60	63	58	12	1

Country A is said to dominate B or A>B if A has higher percentage in total coverage for both drinking water and sanitation facilities, and, B is said to be dominated by A, or B<A.

A country is said to be on coverage frontier if no other country dominates it. Similarly, a country is not on the coverage frontier if it is dominated by at least one other country.

Q12. Which countries are the countries on the coverage frontier?
(a) India and China
(b) Sri Lanka and Indonesia

(c) Philippines and Bangladesh
(d) Nepal and Pakistan

Q.13. Which of the following statements are true?
A India>Pakistan and India>Indonesia
B India>China and India> Nepal
C Sri Lanka>China
D China>Nepal
(a) A and C (b) B and D
(c) A, B and C (d) B, C and D

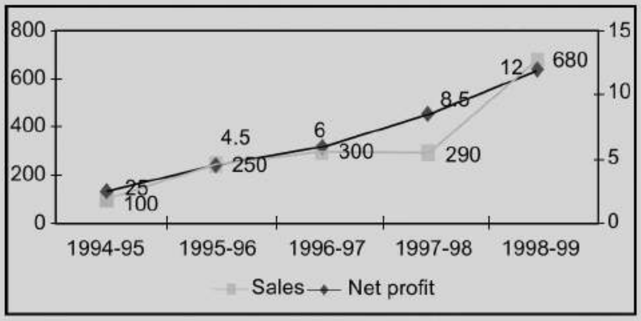
Q.14. Using only the data presented under 'sanitation facilities' columns, it can be concluded that rural population in India, as a percentage of its total population is approximately
(a) 76 (b) 70
(c) 73 (d) Cannot be determined

Q.15. Again, using only the data presented under 'Sanitation facilities' columns, sequence China, Indonesia and Philippines in ascending order of rural population as a percentage of their respective total population. The correct order is
1. Philippines, Indonesia, China
2. Indonesia, China, Philippines
3. Indonesia, Philippines, China
4. China, Indonesia, Philippines

Q.16. India is not on the coverage frontier because:
A. It is lower than Bangladesh in terms of coverage of drinking water facilities.
B. It is lower than Sri-Lanka in terms of coverage of sanitation facilities.
C. It is lower than Pakistan in terms of coverage of sanitation facilities.
D. It is dominated by Indonesia.
(a) A and B (b) A and C
(c) D (d) None of these.

Direction for questions 17 to 20: Go through the data set given below and solve the questions based on it.

The figure below represents states sales and net profit in Rs crore of IVP ltd For five years from 1994–95 to 1998–99. During this period the sales increased from Rs 100 crore to Rs 680 crore. Correspondingly, the net profit increased from Rs 2.5 crore to Rs 12 crore. Net profit is defined as the excess of sales over total costs.



Q17. The highest percentage of growth in sales, relative to the previous year occurred in
(a) 1995–96 (b) 1996–97
(c) 1997–98 (d) 1998–99

Q18. The highest percentage growth in net profit, relative to the previous year, was achieved in
(a) 1998–99 (b) 1997–98
(c) 1996–97 (d) 1995–96

Q.19. Defining the profitability as the ratio of net profit to sales, IVP Ltd, recorded the highest profitability in
(a) 1998–99 (b) 1997–98
(c) 1994–95 (d) 1996–97

Q.20. With profitability as defined in question 19, it can be concluded that:
(a) Profitability is non-decreasing during the five years from 1994–95 to 1998–99.
(b) Profitability is non-decreasing during the five years from 1994–95 to 1998–99.
(c) Profitability remained constant during the five years from 1994–95 to 1998–99
(d) None of above.

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■

■■■ Benchmarking Test 2

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(a)	(b)	(b)	(b)	(a)	(c)	(a)	(c)	(c)	(c)	(b)	(c)	(a)	(d)	(a)	(d)	(b)	(d)

HINTS AND SOLUTIONS

[illegible]

1. The least occurs in case of 7.56% GS 2014 which is more than 5 years. Hence, answer is – all the years. Hence answer is option (d) None of these.

2. For 4 stocks, the desired yield is more than cut off yield. Hence, option (b) is the answer.

3. Following stocks are having their cut-off price more than Rs 100:

Name of Stock	*Cut off price*
7.56% GS 2014	103.47
7.94% GS 2021	102.5
7.56% GS 2014	106.75
8.24% GS 2027	107.82

None of these stocks have their cut-off yield more than its desired yield. Hence, option (a) is the answer.

4. To obtain a minimum desired yield of 8%, Mr Robert has to take 8.24% GS, as for remaining all the stocks, minimum desired yield is 8%. (One cannot have above 8% yield by mixing two stocks with less than 7% stock. It means that one stock has to be higher than 8 % and other has to be less than 8%).
He can mix 8.24% GS with any one of the remaining 6 stocks to obtain minimum 8% desired yield.
Hence, option (b) is the answer.

5.

Rank 1	**6.05% GS 2019**	**95.05**
Rank 2	7.50% GS 2034	97.35
Rank 3	6.05% GS 2019	99.38
Rank 4	7.94% GS 2021	102.5
Rank 5	7.56% GS 2014	103.47
Rank 6	7.56% GS 2014	106.75
Rank 7	8.24% GS 2027	107.82

Hence option (b) is the answer.

[illegible]

One of the women spent 2517 – 1378 = 1139, she is Chellamma. This is the only possibility as we add Rs 1378 even to the least amount Rs 1193, we will not be able to satisfy all the conditions given simultaneously.
So, the table obtained is:

A	*C*	*D*	*H*	*S*
2234	1139	1193	1340	2517

[illegible]

Let us make a table of the information given:

	Idli	*Vada*
Ignesh	6	6
Sandeep	1	0
Mukesh	4	2
Daljit	5	1
Bimal	8	4

[illegible]

12. Bangladesh > Philippines (97>86) for drinking water. And Philippines > any other countries for sanitation facilities. Thus, these two countries are on the coverage frontier for two facilities.

13. Statement A > Statement B only if statement A has higher percentage in total coverage for both drinking water and sanitation facilities taken independently and not as a total of the two facilities.
Thus, only statement B and statement D are true
India > China (81>67 and 29>24)
India >Nepal (81>63 and 29>18)
Also china > Nepal (67>63 and 24>18)

14. Let the urban population be x and rural population be y.
From the sanitation column, we have 0.7x + 0.14y = 0.29(x+y), OR, 0.41x = 0.15y
Therefore $x = \frac{15y}{41}$
Therefore percentage of rural population

$$\frac{y}{x+y} \times 100 = \frac{y}{\frac{15y}{41}+y} \times 100 = \frac{41}{56} \times 100 = 73.2\%$$

15. In the same way as the previous questions, we can find percentage of rural population for Philippines, Indonesia and China.

P	50%
I	66.66%
C	79.8%

Thus, Philippines < Indonesia < China.

16. India is not on coverage frontier because
    (a) It is below Bangladesh and Philippines for drinking water.
    (b) For sanitation facilities it is below Philippines *inter alia.*

BENCHMARKING TEST 3

- There are 20 questions in this test.
- Each question carries 3 marks.
- There is 1/3rd negative marking for every wrong answer.
- Time allotted = 45 minutes

Time Given: 45 Minutes **Total Marks: 60** **Qualifying Marks: 38**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
48+	99+
45	98.5+
43	98
38	92
30	85

Direction for questions 1 to 5: Refer to the tables given below and answer the questions that follow.

Table 1 give the transportation cost of a product from six factories F1, F2, F3, F4, F5 and F6 to five warehouses W1, W2, W3, W4 and W5.

	W1	*W2*	*W3*	*W4*	*W5*
F1	60	25	45	40	80
F2	85	40	45	60	75
F3	80	20	45	50	90
F4	90	40	55	25	70
F5	95	20	40	45	80
F6	85	35	50	50	75

Table 2 give the transportation cost of the same product from five warehouses W1, W2, W3, W4 and W5 to six retailers R1, R2, R3, R4, R5 and R6.

	R1	*R2*	*R3*	*R4*	*R5*	*R6*
W1	40	20	40	30	85	25
W2	50	50	45	60	70	30
W3	20	10	50	50	80	35
W4	40	30	45	40	90	40
W5	50	30	50	40	70	30

In addition to the above tables given, following table gives the transportation cost incurred from different retailers to the five different colonies C1, C2, C3, C4 and C5.

	C1	*C2*	*C3*	*C4*	*C5*
R1	10	25	10	10	10
R2	15	10	15	10	15
R3	10	20	10	15	10
R4	20	10	15	15	10
R5	15	10	20	20	20
R6	10	15	15	15	15

Q.1. What is the minimum transportation cost incurred in transferring the product from any factory to any colony?
(a) Rs 45 (b) Rs 65
(c) Rs 55 (d) None of these

Q.2. What is the minimum transportation cost incurred in transferring the product from any factory to the colony C2?
(a) Rs 45 (b) Rs 60
(c) Rs 55 (d) None of these

Q.3. If one unit produced in each of the factories is to be sent to each of the colonies, then what will be the total transportation cost incurred?
(a) Rs 1705 (b) Rs 1980
(c) Rs 2185 (d) None of these

Q.4. $F_iW_jR_kC_l$ represents that the product has moved in this cycle – Factory i to warehouse j to retailer k to colony l. If i = 2 and k = 2, then what is the sum of (j + l) for the lowest transportation cost incurred in one product transferred from i = 2 to any of the colonies?
(a) 4 (b) 5
(c) 7 (d) cannot be determined

Q.5. In the above question, if j = 2, then how many different value/s of (i + k + l) is/are possible for the lowest transportation cost incurred in one product transferred from any of the factories to any of the colonies?
(a) 1 (b) 2
(c) 3 (d) cannot be determined

Direction for questions 6 to 10: Refer to the table given below and answer the questions that follow.

Given below are the forecasts of world and Asian energy demand for the years 1991, 2001 and 2011. The demand is given in million barrels per day, crude oil equivalent.

	1991		*2001*		*2011*	
	World	*Asia*	*World*	*Asia*	*World*	*Asia*
Petrol	50.0	4.0	70.0	10.0	80.0	15.0
Naphtha	30.0	0.5	40.0	2.5	50.0	5.0
Solid Fuels	50.0	4.0	60.0	5.0	75.0	10.0
Nuclear Fuel	10.0	0.5	20.0	1.0	25.0	1.3
Hydro Energy	10.0	1.0	10.0	1.5	20.0	2.0
Total	150.0	10.0	200.0	20.0	250.0	33.3

Q.6. What can be inferred from the given table?
(a) Percentage increase in the number of Petrol using vehicles/agents have increased in Asia than the world in the decade 1991–2001.
(b) Exploration of Naphtha is low in Asia is less than the exploration of Naphtha in the world throughout the period.
(c) Both (a) and (b)
(d) None of these

Q.7. Over 1991-2011, which two fuels meet more than 60% of the total energy demand of both the world and Asia?
(a) Petrol and Naphtha
(b) Petrol and Solid fuels
(c) Naphtha and Solid fuels
(d) None of these

Q.8. Which fuel's proportion in the total energy demand increases over the decade 1991–2001 and decreases over the decade 2001–2011 for both the world and Asia?
(a) Petrol (b) Naphtha
(c) Solid fuels (d) Nuclear Fuel

Q.9. Which is the fuel whose proportion in the total energy demand will decrease continuously over the period 1991–2011, in Asia?
(a) Naphtha (b) Solids fuels
(c) Nuclear Fuel (d) Hydro Energy

Q.10. Which is the fuel whose proportions to the total energy demand of the world will remain constant over the period 1991–2011 but whose proportion will increase in the total energy demand in Asia?
(a) Naphtha (b) Solids fuels
(c) Nuclear Fuel (d) Hydro Energy

Direction for questions 11 to 15: Refer to the table given below and answer the questions that follow.

Following table gives some incomplete information on the marks obtained by four persons Amit, Binit, Charu & Dilip in four different subjects.

	Physics	*Chemistry*	*Maths*	*English*	*Total*
Amit	80	70	–	80	–
Binit	60	–	60	–	–
Charu	–	–	60	–	220
Dilip	90	–	70	60	–

Additional Information:

(i) Amit's average marks was 72.5, in all the four subjects.
(ii) The average marks in Physics for all four persons was 67.5.
(iii) Binit's Average marks for all four subjects was 1.5 times Charu's marks in physics.
(iv) Binit & Dilip scored the same marks in English. The average English marks were the same as the average Physics marks.
(v) The total marks in Chemistry was 10 less than the total marks in physics for all four persons.

Q.11. Who scored the lowest total?
(a) Amit (b) Binit
(c) Charu (d) Dilip

Q.12. The average marks for all four persons (for all the four subjects) is (approx.)
(a) 65.625 (b) 81.225
(c) 99.5 (d) 105.6

Q.13. Who is the highest scorer in chemistry?
(a) Amit (b) Binit
(c) Charu (d) Dilip

Q.14. Who is the lowest scorer in physics?
(a) Amit (b) Binit
(c) Charu (d) Dilip

Directions for questions 15 to 17: In each question, there are two statements: A and B, either of which can be true or false on the basis of the information given below.

A research agency collected the following data regarding the admission process of a reputed management school in India.

Year	Gender	Number bought appli-cation forms	Number appeared for writ-ten test	Number called for in-terviews	Number selected for the course
2002	Male	61205	59981	684	171
	Female	19236	15389	138	48
2003	Male	63298	60133	637	115
	Female	45292	40763	399	84

Choose (1) if only A is true
Choose (2) if only b is true
Choose (3) if both A and B are true
Choose (4) if neither A nor b is true

Q.15. **Statement A:** The success rate of moving from written test to interview stage for males was worse than for females in 2003.

Statement B: The success rate of moving from written test to interview test to interview stage for females was better in 2002 than in 2003.

Q.16. **Statement A:** In 2002, the number of females selected for the course as a proportion of the number of females who bought application forms, was higher than the corresponding proportion for males.

Statement B: In 2002, among those called for interview, males had a greater success rate than females.

Q.17. **Statement A:** The percentage of absentees in the written test among females decreased from 2002 to 2003.

Statement B: The percentage of absentees in the written test among males was larger than among females in 2003.

Directions for questions 18 to 20: Answer the questions on the basis of the information given below.

The table below provides certain demographic details of 30 respondents who were part of a survey investigating the irregularities in NREGS. The demographic characteristics are: gender, number of children, and age of respondents. The first number in each cell is the number of respondents in that group. The minimum and maximum age of respondents in each group is given in brackets. For example, there are five female respondents with no children and among these five; the youngest is 34 years old, while the oldest is 49.

No. of children	*Male*	*Female*	*Total*
0	1(38,38)	5(34,49)	6
1	1(32,32)	8(35,57)	9
2	8(21,65)	3(37,63)	11
3	2(32,33)	2(27,40)	4
Total	12	18	30

Q.18. The percentage of respondents aged less than 40 years is at least
(a) 10% (b) 16.67%
(c) 20.0% (d) 30%

Q.19. Given the information above, the percentage of respondents older than 35 can be at most
(a) 30% (b) 73.33%
(c) 76.67% (d) 90%

Q.20. The percentage of respondents that fall into the 35 to 40 years age group (both inclusive) is at least
(a) 6.67% (b) 10%
(c) 13.33% (d) 26.67%

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■

■■■ Benchmarking Test 3

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(d)	(d)	(d)	(d)	(b)	(a)	(d)	(d)	(c)	(a)	(d)	(c)	(d)	(b)	(a)	(b)	(c)	(c)

HINTS AND SOLUTIONS

Questions 1 to 5

1. F3 to W2 to R2 to C1 is the lowest.
2. F3 to W2 to R6 to C2 is the minimum.
4. F2 – W3 – R2 – either C2/C4. Hence, two different values are possible.
5. There can be two values of i = 3/5, one value of j = 2, one value of k = 6 and one value of l = 1. Hence, two different sums are possible.

[illegible]

6. Obviously cannot be inferred. (1) seems probable, but number of vehicles/agents need not be increasing. There can be other reasons also like reduction in the efficiency of the vehicle etc.

7.

		1991		***2001***		***2011***	
		World	***Asia***	***World***	***Asia***	***World***	***Asia***
Total energy		150	10	200	20	250	33.3
Naphtha	Value	30	0.5	40	2.5	50	5
	Proportion	20%	5%	20%	12.50%	20%	15.01
Solid fuels	Value	50	4	60	5	75	10
	Proportion	33.30%	40%	30%	25%	30%	30.03
Petrol	Value	50	4	70	10	80	15
	Proportion	33.30%	40%	35%	50%	32%	45.04

Hence, solid fuels and Petrol combined constitute more than 60 percent of total energy in both the world and Asia for the given period.

8. It can be seen from the table that Petrol is the fuel whose proportion in the total energy demand increases during 1991-2001 and decreases during 2001-2011 for both the world and Asia.
9. For the answer choices given and for Asia we can take make the following table.

		1991	***2001***	***2011***
	Total energy	***10***	***20***	***33***
Naphtha	Value	0.5	2.5	5
	Proportion	5%	12.50%	15.15%
Solid fuels	Value	4	5	10
	Proportion	40%	25%	30.30%
Nuclear Fuel	Value	0.5	1	1.3
	Proportion	5%	5%	3.90%
Hydro Energy	Value	1	1.5	2
	Proportion	10%	7.50%	6.06%

Hence, we can see that the proportion of Hydro Energy goes on decreasing over the period.

10. For answer choices given and for the worlds, we can make the following table.

		1991	***2001***	***2011***
	Total energy	***150***	***200***	***250***
Naphtha	Value	30	40	50
	Proportion	20%	20%	20%
Solid Fuels	Value	50	60	75
	Proportion	33%	30%	30%
Nuclear Fuel	Value	10	20	25
	Proportion	6.66%	10%	10%
Hydro Energy	Value	10	10	20
	Proportion	6.66%	5%	8%

Hence, we can see that the proportion of Naphtha remains constant over the given period.

[illegible]

From the given information the following table can be prepared:

	Physics	***Chemistry***	***Maths***	***English***	***Total***
Amit	80	70	60	80	290
Binit	60	60	60	60	240
Charu	40	50	60	70	220
Dilip	90	80	70	60	300

[illegible]

19. (c) (23/30) × 100 = 76.67 %
20. (c) (4/30) × 100 = 13.33 %

BENCHMARKING TEST 4

- There are 20 questions in this test.
- Each question carries 3 marks.
- There is 1/3rd negative marking for every wrong answer.
- Time allotted = 45 minutes

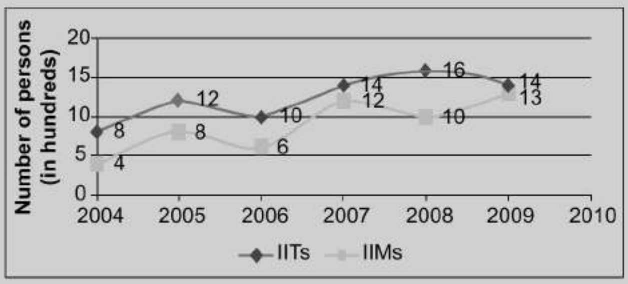
Time Given: 45 Minutes **Total Marks: 60** **Qualifying Marks: 38**

Break-up of the Expected Percentile
(assuming this to be a CAT paper)

Marks	*Expected Percentile*
48+	99+
45	98.5+
43	98
38	92
30	85

Direction for questions 1 to 4: Go through the chart given below and solve the questions based on it.

Chart given below represents total number of persons (in hundreds) who have passed out from IIMs and IITs in each year from 2004 to 2009. The IITs and IIMs are among the best known professional institutes in India (under top 10 rank) and offer courses in the areas of engineering and management respectively. Both the IIMs and IITs have six campuses located in various cities of India, none of the cities being common.



Q.1. How many times during the given period has the growth in pass outs from IITs exceeded 25% over previous year?

(a) Once	(b) Twice
(c) Thrice	(d) Never

Q.2. During the 2004-2008, the total pass outs from IIMs as a ratio of total pass outs from IIT is

(a) 41/60	(b) 10/13
(c) 6/9	(d) 4/5

Q.3. What is the ratio of [ratio of IIM graduates/IIT Graduates] in 2006 to that in 2008?

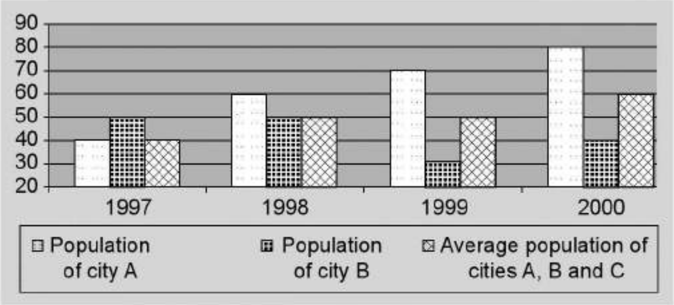
(a) 0.37	(b) 2.66
(c) 1.04	(d) 0.96

Q.4. On an average, every year 20% of the students who pass IITs join IIMs the same year (An IITian can join IIM but not the vice versa). What is the total number (in '000) of IITs pass outs who went to IIM in 2006?

(a) 2	(b) 2.5
(c) 1	(d) cannot be determined

Direction for questions 5 and 6: Go through the chart given below and solve the questions based on it.

There are only three cities—A, B and C—in a state. Following bar chart gives us the population figures of these cities (Figure in '000):

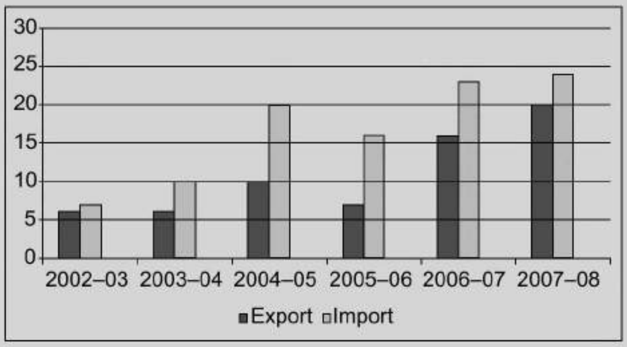


Q.5. If increase in the population of city can only be due to new births and decrease in population of city can only due to deaths, then number of new births in city A in year 2000 is (in '000):
(a) 20 (b) 40
(c) 38 (d) Cannot be determined

Q.6. For which year is the ratio of population of city A to the population of city B the highest?
(a) 1997 (b) 1998
(c) 1999 (d) 2000

Direction for questions 7 – 9: Go through the chart given below and solve the questions based on it.

Following Bar graph for the data related to the Export & Import in million Rupees of dueNorth Limited.



Q.7. Import in financial year 2007 – 08 has approximately
(a) Increased by 24% since base year of 2002 – 03
(b) Increased by 240% since base year of 2002 – 03
(c) Increased by 140% since base year of 2002 – 03
(d) Data is insufficient to find the answer.

Q.8. In which financial year difference between import and export is the highest?
(a) 2007 – 08 (b) 2006 – 07
(c) 2005 – 06 (d) 2004 – 05

Q.9. From year 2003 – 04 to 2007 – 08, the ratio of total Import during these five years to the total export in the same period is in the range of
(a) 0.75 – 1 (b) 1 – 1.25
(c) 1.25 – 1.5 (d) 1.5 – 1.75

Direction for questions 10 to 14: Go through the chart given below and solve the questions based on it.

A test was performed on various food items to check for protein content, fat content and Mineral content. Five food items viz., Butter, Apple, Dal, Chicken and Bournvita. The chart shows the quantity of foods checked:

Food Item	*Quantity Checked*	*Protein content*	*Fat Content*	*Mineral content*
Butter	2 gm	10 mg	1.0 gm	75 mg
Apple	1 gm	15 mg	35 mg	60 mg
Dal	2 gm	100 mg	15 mg	90 mg
Chicken	3 gm	75 mg	1 gm	150 mg
Bournvita	1 gm	25 mg	20 mg	80 mg
Veg Burger	2gm	5 mg	1.1 gm	15 mg

NOTE: 1gm = 1000 mg

Based on these results, the foods are classified into the following categories:

Healthy food	If percentage of proteins the highest and the percentage of fats is the least as compared to the other foods.
Junk food	If the percentage of fat is the highest and the percentage of protein is the least.
Not recommended	If the percentage of fats is more than the percentage of proteins and the mineral content is least as compared to the other foods

Answer the questions on the basis of the information given above.

Q.10. What is the highest percentage of protein any of the given foods?
(a) 0.5% (b) 5%
(c) 1% (d) 10%

Q.11. Which food can be referred to as Healthy?
(a) Butter (b) Apple
(c) Dal (d) Chicken

Q.12. Which food can be classified as junk from amongst the above mentioned foods?
(a) Veg burger (b) Apple
(c) Dal (d) Chicken

Q.13. Which food is not recommended as per the criteria defined above?
(a) Butter (b) Apple
(c) Veg Burger (d) Chicken

Q.14. In how many foods is the percentage of fat more than the percentage of protein?
(a) 1 (b) 2
(c) 3 (d) 4

Direction for questions 15 to 18: Go through the chart given below and solve the questions based on it.

Following grid presents the percentage composition of five verticals under different sectors.

Sector	*Factories*	*Employment*	*Fixed Capital*	*Variable Cost*	*Value Added*
Government	18	15	14	22	25
Central	8	6	5	7	10
State	4	6	4	8	3
Central/State	6	3	5	7	12
Public	12	8	6	19	8
Private	55	65	72	54	62
Joint	15	12	8	5	5
Total	100	100	100	100	100

Q15. If the total work force was 76 million whereas the total value added was 225 million, then which of the following had the maximum value addition per worker?
(a) Central (b) State
(c) Central/State (d) Public

Q16. Which of the following sectors has the maximum fixed capital invested per factory?
(a) Central (b) State
(c) Central/State (d) Public

Q17. If the variable is proportional to the number of employees and the production per employee, then for which of the following is the production highest?
(a) Government (b) Private
(c) Joint (d) Public

Q18. If the government has a fixed capital of $200 million in the Iron& Steel industry, which corresponds to 20.012% of its total investment as fixed capital, then how much did the government invest (in Rs million) in Maruti Udyog Ltd which forms 25% of the investment in the joint sector? (1US $ = Rs 45)
(a) 6500 (b) 2500
(c) 143 (d) 145

Direction for questions 19 to 20: Go through the chart given below and solve the questions based on it.

The following table shows the report card of a student of a management entrance exam. The exam had three sections Data Interpretation, Quantitative aptitude and Verbal Ability.

Test Area	*Number of questions*	*Questions Attempted*	*Incorrect attempts*
Data Interpretation	50	28	6
Quantitative Aptitude	50	30	11
Verbal Ability	50	38	13

For each correct answer the student gets 4 marks and for each wrong answer 1 mark is reduced from the total. Also, Accuracy is defined as (no. of questions attempted correctly)/(total no. of questions attempted).

Answer the following questions on the basis of the information given above.

Q.19. Arrange the three sections in the increasing order of the score obtained by the student?
(a) DI, VA, QA
(b) VA, DI, QA
(c) QA, DI, VA
(d) VA, QA, DI

Q.20. How much percentage of the total maximum score did the student score?
(a) 39% (b) 41.6%
(c) 47.2% (d) 50%

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■ Benchmarking Test 4

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(c)	(a)	(b)	(b)	(b)	(a)	(c)	(a)	(c)	(c)	(c)	(b)	(c)	(a)	(d)	(a)	(d)	(b)	(d)

HINTS AND SOLUTIONS

[illegible]

2. (c) IITs = 60; IIMs = 40 $\Rightarrow$ Ratio of IIM to IIT pass out = 4/6 = 6/9
3. (d) (6/10) / (10/16) = 0.96
4. Answer is ‘cannot be determined’ due to various reasons: Reason 1 – Data is given as ‘on an average’ and not the exact data. Reason 2 – There can be IITs pass outs from yesteryears making it to IIMs in a particular year. Hence, cannot be determined.
   Solution to Questions 5 – 6:
5. Population (2000) = Population (1999) + Number of new births (2000) – Number of deaths (2000). The no. of births figure cannot be determined as the no. of deaths is not known. Hence [d].
6. Look at the following table:

	1997	*1998*	*1999*	*2000*
City A	40	60	70	80
City B	50	50	30	40
Ratio	0.8	1.2	2.333333	2

Hence option (c) is the answer.

[illegible]

7. Import increased from 7 to 24, around 240% increase. Hence, option (b) is correct.
8. This question can be solved by looking the graph given. Hence, option (d) is correct.
9. $\frac{10+20+16+23+24}{6+10+7+16+20}$ x 100% = 157.6%. Hence, answer option is (4).

[illegible]

10. % of protein is highest in protein = (100/1000) × (1/2) × 100 = 5%. **Ans (b)**
11. % of protein is maximum in dal and % of fat is the least. Hence, is classified as ‘Healthy’. **Ans (c)**
12. Clearly, Veg Burger has the highest percentage of fat and the least percentage of protein. Hence, butter is the junk food out of the above mentioned foods. **Ans (a)**
14. Butter, Apple, Veg Burger and Chicken. Hence, **Ans (d)**

[illegible]

15. [c] Values added per worker for various sectors will be proportional to following ratios:

Central	10/6
State	3/6
Central/state	12/3
Public	8/8

16. [b] Fixed capital per factory will be proportional to the following ratios:

Central	5/8
State	4/4
Central/state	5/6
Public	6/12

17. [b] If the variables are proportional to the number of employees and the production per employee, then we can say that the variable cost is directly proportional to the production. Since the variable cost is maximum for private sector, it will also have the maximum production.
18. [a] If 20% of govt. fixed capital is equal to $200 million => Total govt. sector fixed capital = $ 1000 m which is equivalent to 14% of the total fixed capital. Now, 25% of Govt. investment in joint sector i.e., 25% of the 8% of the total = 2% 0f total fixed capital will be given by: (1000 × 2) /14 = $ 143 million
    Value in Rs is given by: 143 × 45 = Rs 6450 million

[illegible]

19. Marks obtained in the three sections are 65, 82 and 87 in QA, DI and VA respectively.
    Hence, option (3) is correct.
20. No. of correct attempts => 22 + 19 + 25 = 66. No. of wrong attempts = 30.
    Required percentage = $= \frac{66 \times 4 - 30}{150 \times 4} = \frac{234}{150 \times 4} = 30\%$
    Hence, option (1) is correct.

Module 3

Data Sufficiency

7

CHAPTER

Data Sufficiency

LEARNING OBJECTIVES

- **How does a typical Data Sufficiency question look like**
- **Similarity/Dissimilarity between Data Sufficiency questions and Quantitative Aptitude questions**
- **Various do's and don'ts of solving Data Sufficiency questions**
- **Process of solving the questions**

DATA SUFFICIENCY QUESTIONS

Data Sufficiency questions are designed to test the student's ability to reason with a limited set of data and to reach a valid conclusion. To solve data sufficiency questions, we should have a clear understanding of the different situations that might come with the questions. Besides, we should be clear with the format of the possible questions and various do's and don'ts related to the above situation and solution.

In most of the cases, a data sufficiency question will have the following structure:

Q. A question will be asked.
Statement 1 – One set of information will be given.
Statement 2 – Another set of information will be given.

On the basis of the statements, four or five different options will be provided. One of those options will be the most probable choice and hence the answer.

Significance of Data Sufficiency

In post MBA life, a manager is virtually burdened with the weight of the data—sometimes organized and sometimes unorganized. DS questions test the ability of the future managers to prune the data to make decisions. Apart from testing the decision making ability of the individual, DS questions also try to evaluate the ability of the student to see into the future that is visualize and predict what can happen in the next 2-3 minutes. Try to understand this—We might find the answer to the question asked in 2-3 minutes time with the help of the given statements, but can we find out the answer right away without actually investing those 2-3 minutes. So, the question now is—Can we predict correctly what is going to happen in the next 2-3 minutes without actually living those 2-3 minutes?

A data sufficiency question is designed—by virtue of its design and content—in such a way that it essentially checks students' ability to reason out. In most of the cases of

DS questions, basic concepts of Quantitative Aptitude will be required.

Various do's and don'ts of Data Sufficiency

The process to solve the DS questions is a bit different from the process involved in solving the QA or LR/DI questions. There are certain key things that we should keep in mind before solving the DS questions. Let us look at them one by one:

1. Understanding the options

This is the first step in solving the DS questions. One must go through the options and understand them clearly. Sometimes, the options given in data set themselves can become a perennial source of problem.

Let us understand the statements given in the options:

If the option statement reads like — "If the question can be solved by using only one of the statements alone, but not by the other statement alone".

It means—Out of the given two (or, more) statements, only one statement can answer the question given, and the other statement cannot give the answer to the question given. In this case either the first statement will give the answer or the second one, but not both simultaneously i.e., if the first statement can give the answer, then second statement cannot give the answer and vice-versa.

If the option statement reads like— "If the question can be solved by using either of the statements alone".

It means—Out of the given two (or more) statements, the question can be solved by using any one of the statements alone. In this case either first statement will give the answer or the second one used individually and independently.

If the option statement reads like — "If the question can be solved by using both (or all) the statements together, but not by either of the statements alone".

It means—Out of the given two (or more) statements, the question can be solved only if the information given in both (or all) the statements is used together. And, using only one of the statements alone cannot solve the given question.

If the statement reads like— "If the question cannot be solved by even using both the statements together".

It means—Out of the given two (or more) statements, the question cannot be solved even by using the information given in both (or all) the statements together.

Once you have understood the statements given in the options, you must not forget to see which statement corresponds to which option. CAT is known to introduce the surprise element quite often, and this also can be one of those surprises which might catch you unaware if you are not careful about this basic fact.

2. General awareness/assumptions are not allowed

As an unproclaimed rule, the element of general awareness cannot be used to solve the DS questions. However, this is also true for QA and LR /OI, but its importance in DS increases manifold.

Let us understand this with an example:

Example 1 Is New Delhi the most polluted city in India?

Statement 1 – The Capital of India is the most polluted city in India.

Statement 2 – New Delhi is the capital.

Solution Well, we can understand that statement 1 alone cannot answer the question, as it is not given that New Delhi is the capital of India.

If your answer is "Both the statements together can answer the question", think again. It is not given in statement 2 that New Delhi is the capital of India.

So, answer to this question is "Cannot be determined."

However, we should have a clear idea regarding the differences between a universal fact like "Pythagoras theorem" and general awareness like "New Delhi is the capital of India".

Universal facts like mathematical principles are not the general awareness stuff, and hence can be used.

Example 2 In how many days A and B working together can finish a work?

Statement I— A can finish the same work alone in 10 days.

Statement 2— B can finish the same work alone in 15 days.

Solution Understandably, we cannot have a unique answer to this question by using the statements alone. However, in this case we cannot solve this question even by using both the statement together.

FLAW DETECTOR—Had this question been a question in Quantitative Aptitude, we must have got the answer by using the methods of time and work. However, in this case of data sufficiency, since it is not given that A and B start working together and kept on working together till the work is finished, we cannot solve this question even by using both the statements together.

And we cannot make assumptions that they started working together and kept on working till the work is finished.

Answer should be unique

This is one aspect that makes the process of solving the DS questions different from the process of solving QA or DI/LR questions. While in case of QA, we can have multiple values of a variable; in DS there should be only one value of any variable or constant given.

Example 3 What is the value of x?

Statement 1— $x^2 - 5x + 6 = 0$

Statement 2— $x^2 + 2x - 15 = 0$

Solution Using statement 1, value of $x = 2, 3$

Since we are not getting unique value of x using statement 1 alone, we will say that statement 1 alone is not sufficient to give the answer.

Using statement 2, value of $x = -5, 3$

Since we are not getting unique value of x using statement 2 alone, we will say that statement 2 alone is not sufficient to give the answer.

Till now we have seen that using the statements alone is not sufficient to find the answer. Hence, we will move on to using both the statements together.

Using statement 1 and 2 simultaneously, we get a common unique value of $x = 3$.

Hence, both the statements together are needed to answer the question.

Example 4 What is the value of x, where x is the length of a rectangle?

Statement 1— $x^2 - 5x + 6 = 0$

Statement 2— $x^2 + 2x - 15 = 0$

Solution Using statement 1, value of $x = 2, 3$

Since we are not getting unique value of x using statement 1 alone, we will say that statement 1 alone is not sufficient to give the answer.

Using statement 2, value of $x = -5, 3$

x being the length of the rectangle, it cannot be negative. Hence, $x = 3$ is the unique answer that we are getting using statement 2 alone.

Concern for the answer, and not the technicalities

Quite a few times it has been observed that students, rather than solving the questions, tend to get into the technicalities of the data. This furthers students' 'emotional attachment' with the question ending up in the wastage of time and efforts.

Example 5 Can we fill up a big drum of 2000 litres capacity with exactly 88 litres of water (there is unlimited supply of water)?

Statement 1—There is a bucket having a of capacity 20 litres.

Statement 2—There is a bucket having a of capacity 50 litres and a mug of 3 litres capacity.

Solution Using statement 1, we can fill anything in the multiples of 20 litres only. And it can never be ascertained that the drum has exactly 88 litres of water. So, answer to the question is—We may or may not be able to fill up a big drum with exactly 88 litres of water. And whatever the quantity of the water in the drum is, we can not be sure if this is exactly 88 litres of water.

Using statement 2, we can fill the drum exactly with 88 litres of water ($50 + 50 - 3 - 3 - 3 - 3$). Hence, answer to the question is—Yes, we can fill up a big drum with exactly 88 litres of water.

So, we can find the answer using statement 2 alone.

Example 6 Is $y > x$?

Statement 1— $x^2 > y^2$.

Statement 2— $x^3 > y^3$.

Solution Using statement 1, assuming some of the values of x and y can tell us that in some cases $x > y$ and in some of the cases $y > x$.

x	y	Remarks
5	4	$x > y$
–5	–4	$y > x$

While assuming the values, we can take only those values of x and y that satisfy the given statement.

Using statement 2,

$$x^3 > y^3$$
$$\Rightarrow \quad x > y$$

Now look at the question. Obviously, we are getting an answer that y is not greater than x.

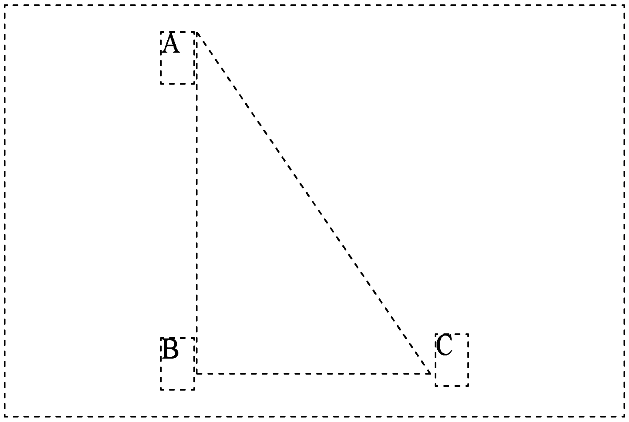
Hence, statement 2 alone is sufficient to answer the question.

Look for the obvious, but don't miss the hidden

Looking at any of the statements gives us an idea of the facts given, and sometimes we get so enamored with these obvious facts that we fail to see something hidden. Evidently, there is no point saying that "You should not make silly mistakes" (because nobody does it intentionally anyway. However, we can reduce the incidences of these silly mistakes or missing some hidden information.

Some of the precautions that must be taken while tackling any DS question:

1. While assuming the values, take different set of values (like positive, negative, fractions between 0 and 1) to make an informed decision.
2. Any geometrical figure should not be taken as accurate unless it is given. For example, the below given triangle ABC given below cannot be taken as a right-angled triangle even though it appears to be so.



Hence, no conclusion should be drawn by finding proportionate value of length or the angle in any geometrical figure unless such are clearly stated.

Example 7 Is Arvind the father of Puja?

Statement 1 — Puja is the daughter of Arvind.

Statement 2 — Arvind is the aunt of Abhishek.

Solution Using statement 1, despite the fact that Puja is the daughter of Arvind, we cannot conclude that Arvind is the father of Puja. Arvind can be the mother too.

Using statement 2, in first impression statement 2 seems to be irrelevant, but looking closely at this statement gives us an idea that Arvind is a woman (only women can be aunts). And hence, we can conclude that Arvind is not the father of Puja.

Therefore, statement 2 alone is sufficient to answer the question.

PRACTICE EXERCISE 1

Direction for questions 1 to 25: Each item is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a): If the question can be answered by one of the statements alone and not by the other.

Choose (b): If the question can be answered by using either statement alone.

Choose (c): If the question can be answered by using both the statements together, but cannot be using either statement alone.

Choose (d): If the question cannot be answered even by using both the statements together.

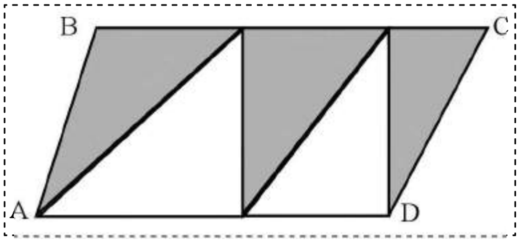
Q 1. Is the point P on the circle with center O?
- A. Q is a point on the circle and the distance from P to Q is equal to the distance from O to Q.
- B. Q is on the circle and PQO is an equilateral triangle.

Q 2. Is Arvind the father of Puja?
- A. Puja is the daughter of Arvind.
- B. Arvind is the brother of Abhishek.

Q 3. If the ratio of boys to girls attending school in 2000 was 1/3, what was the ratio of boys to girls attending school in 2001?
- A. 100 more boys were attending school in 2001 than in 2000.
- B. 150 more girls were attending school in 2001 than in 2000.

Q 4. If a and b are positive numbers, is b > a?
- A. $a^2 > b$
- B. $a^2 > b^2$

Q 5. What is the area of the unshaded region in the given figure (Points E, F are on the line BC and point G is on the line AD)?



- A. ABCD is a parallelogram.
- B. Area of ABCD is 50 square units.

Q 6. If the integer P divisible by 15?
- A. Sum of the digits of P equals 15.
- B. Unit digit of P is 6.

Q 7. Is Q a positive number?
- A. $4Q + 24 > 0$
- B. $4Q - 24 < 0$

Q 8. How many chocolate bars 2 inches wide and 4 inches long can be packed into carton C?
- A. The inside dimension of carton C are 8 cms by 12 cms.
- B. The width of carton C is equal to the height and 3/4 of the length.

Q 9. What are the values of x and y?
- A. $3x + 2y = 45$
- B. $y = 22.5 - 1.5x$

Q 10. Is x an even number?
- A. $4x + 3y$ is an even number.
- B. $3x + 4y$ is an even number.

Q 11. What is the ratio of the number of boys and girls in a school (Number of boys and girls are natural numbers)?
- A. Number of boys is 40 more than girls.
- B. Number of girls is 80% of the number of boys.

Q 12. What is the difference between two numbers?
- A. First number is 60% of the other number.
- B. 50% of the sum of first and second number is 24.

Q 13. How many marks did Arpit Sinha obtain in mathematics?
- A. Arpit Sinha secured on an average 55% marks in mathematics, physics, and chemistry together.
- B. Arpit Sinha secured 10% marks more in mathematics than the average of mathematics, physics and chemistry.

Q 14. Is x > y?
- A. $\dfrac{(x^4 - y^4)}{x^3 + y^3} > 0$
- B. $\dfrac{x^3 - y^3}{x^4 + y^4} > 0$

Q 15. What is a two digits number?
- A. The number obtained by interchanging the digits is smaller than the original number by 63.
- B. Sum of the digits is 11.

Q 16. x, y, and z are integers, is x an odd number?
- A. an odd number is obtained when x is divided by 5.
- B. (x+y) is an odd number.

Q 17. What is the number x?
- A. The LCM of x and 18 is 36.
- B. The HCF of x and 18 is 2.

Q 18. Is y greater than x?
A. x+y = 2
B. x/y = 2

Q 19. Which of the four numbers w, x, y, and z is the largest?
A. The average of w, x, y and z is 25.
B. The numbers w, x, and y are each less than 24.

Q 20. P, Q, R, and S are four consecutive even integers. What is the value of the largest integer among these?
A. The average of the four numbers is the first prime number greater than 10.
B. The ratio between the largest and the smallest of the number is less than 10.

Q 21. Is A greater than B?
A. (A + 3) is greater than (B + 2).
B. Fourth power of A is greater than third power of B.

Q 22. Which of the following is greater: $\frac{a+x}{b+x}$ or $\frac{a}{b}$
A. $\frac{a}{b} > 1$
B. x is positive.

Q 23. How many boys and girls are there in the family of 7 children?
A. Each boy in the family has as many sisters as brothers.
B. Each girl in the family has as many brothers as sisters.

Q 24. In how many days does Binod finish a certain work?
A. Binod is twice as efficient as Amar.
B. Akshay is twice as efficient as Binod.

Q 25. What is the equation of the straight line AB?
A. Straight line AB is perpendicular to another straight line 3x + 5y = 10
B. Straight line AB passes through the point (1, 2).

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(c)	(a)	(b)	(a)	(d)	(c)	(d)	(d)	(a)	(c)	(d)	(a)	(c)	(a)	(d)	(c)	(c)	(a)
Q. No.	21	22	23	24	25															
Answer	(d)	(c)	(b)	(d)	(c)															

HINTS AND SOLUTIONS

1. Using statement A alone, since it is given that the centre of the circle is O, hence OQ is the radius of the circle. But point Q can be inside the circle or on the circle or outside the circle and in all these cases OQ = PQ. Hence, statement A alone does not give the answer.

   Using statement B alone, PQ = QO = PO = Radius of the circle.

   Hence, point P is on the circle.

2. Using statement A alone, Arvind can be father or mother of Puja.

   Using statement B alone, obviously it cannot be solved.

   Using both the statements together, since Arvind is the brother of Abhishek, hence Arvind is a male. Hence Arvind is the father of Puja.

3. Using statement A alone, nothing can be concluded.

   Using statement B alone, nothing can be concluded.

   Using both the statements together, if the ratio of boys to girls in 2000 = x/3x, then the new ratio of boys to girls in 2001 = $\frac{x+100}{3x+150}$

   Now everything depends upon the value of x. Hence cannot be determined uniquely.

4. Using statement A alone, depending upon the values of A and B, either can be greater (A = 10, B = 1 or A= 5, B = 10) according to the condition given in the statement.

   Using statement B alone, since both the numbers a and b are positive, and a2 > b2, hence a > b. So, the answer to the question is – No, b is not greater than a.

5. Using both the statements together, since the base and height of ABCD and unshaded region are same, hence the area of unshaded region is half of area (ABCD).

6. Using statement A alone, nothing can be concluded. Numbers can be 195 or 159 etc. Otherwise also, we know that if any number is divisible by 5 and 3 both, then the number will be divisible by 15. If the sum of the digits is 15, then the number is divisible by 3 but nothing can be said about the divisibility of this number by 5.

   Using statement B alone, since the unit digit of P is 6, hence this number is not divisible by 5, and hence not divisible by 15.
   Answer (a)

7. A. $4Q + 24 > 0$ Or, $4Q > -24$, or, $Q > -6$

   Hence, Q can be negative or positive or zero as well.

   B. $4Q - 24 < 0$

   Or, $4Q < 24$, or, $Q < 6$

   Hence, Q can be negative or positive or zero as well.

9. Using statement A alone, since the equation is in two variables and there is one equation nothing can be concluded.

   Using statement B alone, since the equation is in two variables and there is one equation nothing can be concluded.

   Using both the equations together, both the equations are same. Hence, cannot be determined.

10. Is x an even number?

    A. $4x + 3y$ is an even number.

    B. $3x + 4y$ is an even number.

    Using statement A alone, if $4x + 3y$ is an even number then both 4x and 3y should be either odd or both should be even. Both 4x and 3y cannot be odd (as 4x is even) and hence both of 4x and 3y are even.

    Now even if 4x is even, then we cannot conclude that x is even or odd.

    Using statement B alone, if $3x + 4y$ is an even number then both 3x and 4y should be either odd or both should be even.

    If each of 3x and 4y is even, then x can take values like $x = 4/3$, which is neither odd nor even.

    If each of 3x and 4y is odd, then 3x is odd. But using this we cannot conclude that x is also odd. X may be a fraction also like $x = 1/3$ or even it can be an odd number.

FLAW DETECTOR—Whenever we are required to assume the values, we should assume the different values viz. Positive numbers, negative numbers, fractional values and definitely the fractional values between O and 1 before validating our result.

11. Using statement A alone, we cannot determine the ratio of boys and girls.

    Using statement B alone, Number of girls = 4/5 Number of boys, now we can find out the ratio of boys and girls in the school.

12. Using statement A, we get the ratio between the two numbers but not the difference between the numbers. Statement B gives the sum of the numbers but not the difference. Using both the statements together gives the difference between the numbers.

13. In none of the statements given marks have been given in terms of numbers, hence we cannot find the marks obtained in mathematics.

14. Using statement A, we cannot determine if $(x^4 - y^4)$ is greater than zero or less than zero since we do not have the sign of $(x^3 + y^3)$. Hence we cannot determine anything.

    Using statement B, we cannot determine if

    $(x^3 - y^3) > 0$ since $(x^4 + y^4) > 0$.

    $(x^3 - y^3) > 0$, so, $x > y$.

15. Using statement A alone gives that the difference between the digits of the number is 7. This gives the possibility of many numbers like 81, 92 etc.
    Using statement B alone also gives the possibility of many numbers.
    Using both the statements together, we get a unique number.

16. Using statement A alone, we get that the quotient obtained when x is divided by an odd number is odd. Hence x has to be odd.
    Using statement B alone, one of x and y will be odd and other one will be even. So we cannot determine that which of x or y is even?

17. Using statement A, value of x can be 22 × 30–1. Hence, two values of x are possible.

    Using statement B, value of x can be

    21 × 30 × 5N × 7N × …,

    where value of N can be any real number.

    Using both the statements together, we can see that no unique value of X is possible.

18. Using statement A alone cannot give us any result.
    Using statement B alone cannot give us any result since the values of x and y can be positive as well as negative too.

Using both the statements together will give us the result.

19. Using statement A alone cannot give us any result.

Using statement B alone gives that value of three of the given numbers is less than the average. Hence, fourth number will be greater than the average. So, this fourth number is largest.

20. Using statement A alone, average of the given four numbers = 11, hence, their sum = 44, hence, the numbers will be 8, 10, 12, 14. Hence, the value of largest integer, whichever is that, is 14.

Using statement B alone is not going to give us the result.

PRACTICE EXERCISE 2

Direction for questions 1 to 20: Each item is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a): If the question can be answered by one of the statements alone and not by the other.

Choose (b): If the question can be answered by using either statement alone.

Choose (c): If the question can be answered by using both the statements together, but cannot be using either statement alone.

Choose (d): If the question cannot be answered even by using both the statements together.

Q 1. A, B, C, D are playing cricket and scored 96 runs together. Did A score the maximum runs?
A. A scored 23 runs.
B. B and C together scored 42 runs.

Q 2. Is ab an even number?
A. a is divisible by 3.
B. (b + 1) is divisible by 4.

Q 3. Three teams of woodcutters took part in a competition (to cut the maximum quantity of wood). Which team could win the competition?
A. The first and the third teams cut twice as much as the second team cut.
B. The second and the third teams cut three times as much as the first team cut.

Q 4. If m and n are integers, is m divisible by 11?
A. mn is divisible by 110.
B. n is divisible by 2.

Q 5. On a fishing trip, Sanju and Ajay each caught some fishes. Who caught more fish?
A. Sanju and Ajay caught fishes in the ratio 2 : 3.
B. After Sanju stopped fishing, Ajay continued fishing until he caught 12 fishes.

Q 6. What are the ages of three brothers (ages are natural numbers)?
A. The product of their ages is 21.
B. The sum of their ages is not divisible by 3.

Q 7. Is 5x + 25 is divisible by 50?
A. x is divisible by 5.
B. x is divisible by 10.

Q 8. How many of a, b, c, d are odd, given that all of them are positive integers?
A. ad + bc is odd.
B. ac + bd is odd.

Q 9. A is not shorter than B, who in turn, is not taller than C who is shorter than D. who among A, B, C and D is the tallest?
A. C and A are of equal height.
B. B and A are of equal height.

Q 10. If Bhushan or Charan passed the examination, then neither Sujan nor Tarun passed the examination. Did Bhushan pass the examination?
A. Tarun did not pass the examination.
B. Sujan passed the examination.

Q 11. Six people—A through F—sit around a circular table, not necessarily in the same order. B and E sit opposite each other. Does C sit opposite D?
A. If C and E interchange their positions, then E will be to the immediate left of B.
B. If A and B interchange their positions, then B will be to the immediate left of E.

Q 12. Average age of a, b, c, and d is 46 years. Who is the oldest among them?
A. c is 91 years old.
B. Ages (in years) of all of them are distinct natural numbers.

Q 13. Product of a, b, c, and d is 1003 (a, b, c and d are all natural numbers). Which one is the largest among them?
A. $a > b > c$
B. $a > b > d$

Q 14. What is the least common multiple of a x b, b x c and c x a (a, b, c are all natural numbers)?
A. least common multiple of a, b and c is 30.
B. The highest common factor of a, b and c is 2.

Q 15. What are the values of m and n?
A. n is an even integer, m is an odd integer, and m is greater than n.
B. Product of m and n is 30.

Q 16. On a given day a boat ferried 1500 passengers across the river in 12 hours. How many round trips did it make?
A. The boat can carry 200 passengers at any time.
B. It takes 40 min each way and 20 min of waiting time at each terminal.

Q 17. What is the value of x?
A. x and y are unequal even integers, less than 10, and x/y is an odd integers.
B. x and y are even integers, each less than 10, and product of x and y is 12.

Q 18. What are the ages of two individuals, x and y?
A. The age difference between them is 6 years.
B. The product of their ages is divisible by 6.

Q 19. If m and n are consecutive positive integers, is m > n?
A. m – 1 and n + 1 are consecutive positive integers.
B. m is an even integer.

Q 20. Which of a, b, c and d is/are odd?
A. ad + bc is odd.
B. ac + bd is odd.

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(c)	(d)	(a)	(d)	(a)	(c)	(b)	(a)	(c)	(c)	(a)	(d)	(d)	(a)	(a)	(d)	(a)	(a)

HINTS AND SOLUTIONS

1. Using statement A alone, since the average runs scored by all of them = 24, and runs scored by A = 23, hence we can conclude that atleast one of remaining three persons must have scored more than the score of A. So, A didn't score maximum runs.

    Using statement B alone cannot give us any result.

2. Using statement A alone cannot give us any result.

    Using statement B alone gives that b is an odd number.

    Using both the statements together we get ab is an odd number.

4. Using statement A, atleast one of m or n will be divisible by 11, but we are not sure that which of m or n will be divisible by 11.

    Using statement B, n is divisible by 2 but it does not give that which of m or n is divisible by 11.

    Using both the statements together, we cannot find if m is divisible by 11 or not.

5. Using statement A alone, we can find out that Sanju caught more fishes than Ajay whereas using statement B alone, we cannot find that who caught more fishes.

6. Using statement A alone, ages of the brothers can be–21, 1, 1 or 1, 7, 3. Hence, using statement A alone will not give us the unique answer.

    Using statement B alone also will not give us the result.

    Even after using both the statements together, we cannot find the ages of the brothers as both the possibilities (21 + 1 + 1) and (1 + 7 + 3) are not divisible by 3.

7. Using statement A alone, assuming some values of x (= 5 or = 10) gives that 5x + 25 is sometimes divisible and sometimes not divisible.

    Using statement B alone, whatever values of x we take, we see that 5x + 25 = 5 (x + 5) is not divisible by 10.

8. Using both the statements together, we can find the answer.

9. This question can be answered by using either statement alone.

10. Question can be answered by using the statement B alone but not by statement A alone.

11. Question can be answered by using both the statements together but not by any of the statements alone.

12. Sum of their ages = 184 years.

    Using statement A alone, since the age of C = 91 years, hence sum of ages of rest of them = 93 years. Using this we cannot find that who is oldest of them.

    Using statement B alone we cannot get the answer.

    Using both the statements together, since the age has to be natural number, so minimum possible age of any one of them = 1 year. So, the youngest of them can be 1 year old, second youngest of them can be 2 years old, hence the maximum age of third person can be 90 years.

    So, C is the oldest of them.

13. 1003 = 1 × 1 × 17 × 59

    Using both the statements together, 'a' is the largest one.

14. Using statement A alone, values of a, b and c can be- 2, 3, 5 or 2, 3, 30. Hence, answer cannot be obtained using statement A only.

    Using statement B alone, values of a, b and c can be 2, 4, 6 or 4, 6, 8 etc. Hence, answer cannot be obtained using statement B only.

    Using both the statements together, values of a, b and c can be – 2, 6, 10 or 6, 10, 30. And we can see that we are not getting a unique value of a × b, b × c and a × c.

15. (Don't forget to take the negative values of m and n)

16. This question can be answered by using statement B alone but not by using statement A alone.

17. (Don't forget to take the negative values of x and y)

18. Using statement A alone will not give us the answer.
    Using statement B alone will not give us the answer.
    Using both the statements together also will not give us the answer.

19. This question can be solved by using statement A alone but not by using statement B alone.

PRACTICE EXERCISE 3

Direction for questions 1 to 17: Each item is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a): If the question can be answered by first statement (statement A) alone but not by the second statement (statement B) alone.

Choose (b): If the question can be answered by second statement (statement B) alone but not by the first statement (statement A) alone.

Choose (c): If the question can be answered by using either statement alone.

Choose (d): If the question can be answered by using both the statements together, but cannot be using either statement alone.

Choose (e): If the question cannot be answered even by using both the statements together.

Q 1. Is the integer y greater than 4?
A. 2y is a factor of 8
B. 24 is a factor of y
(a) 1 (b) 3
(c) 4 (d) 2
(e) 5

Q 2. A sum of Rs 215 is distributed among three persons A, B and C. Who gets the least?
A. A gets 2/3rd of what (B + C) are getting.
B. B gets 1/5th of what (A + C) are getting.
(a) 1 (b) 3
(c) 4 (d) 2
(e) 5

Q 3. What is the HCF of natural numbers A and B?
A. LCM of the natural numbers A and B is 101.
B. A and B are distinct natural numbers.
(a) 1 (b) 4
(c) 2 (d) 3
(e) 5

Q 4. Ram is the son of Shyam and father of Dharam. All of these people—Ram, Shyam and Dharam are politicians. Is Shyam an honest politician?
A. Sons of all the honest politicians are honest politicians. Ram is an honest politician.
B. Son of only a dishonest politician is a dishonest politician. Dharam is a dishonest politician.
(a) 1 (b) 4
(c) 2 (d) 3
(e) 5

Q 5. $y = f(x)$ is said to be an even function if $f(x) = f(-x)$. Is $y = f(x)$ an even function?
A. Graph of $y = f(x)$ lies in only two quadrants and both these quadrants are adjacent to each other.
B. Graph of $y = f(x)$ lies in only two quadrants and both these quadrants are opposite to each other.
(a) 1 (b) 2
(c) 3 (d) 4
(e) 5

Q 6. In an examination of five papers, marks obtained by Pranav are in the ratio 4 : 5 : 6 : 7 : 8 across the five papers. Each paper carries the same maximum marks. In how many papers did Pranav obtain more than 50% of the maximum marks?
A. Total marks obtained by Pranav in all the papers together is 300 marks.
B. Lowest percentage of marks obtained by Pranav in any of the papers of this examination is 30%.
(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 7. N is a natural number. Is N a perfect square?
A. When N is divided by 100, remainder obtained is 2.
B. Total number of factors of N is odd.
(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 8. What is the respective speed of two trains of lengths 70 m and 75 m?
A. They take 35 seconds to cross each other when they are running in the same direction.
B. They take 10 seconds to cross each other when they are running in the opposite direction.
(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 9. Vijay and Rupesh are two friends working in the same company. Sum of their monthly salaries is more than Rs 1 lac per annum. Who saves more per month?
A. Ratio of monthly income of Vijay and Rupesh is 3 : 5.
B. Ratio of monthly expenses of Vijay and Rupesh is 1 : 2.
(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 10. Is triangle ABC a right-angled triangle?
A. Ratio of length of side AB and side BC = 3 : 2
B. Ratio of length of side BC and side CA = 4 : 5

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 11. A bus is running with a uniform speed from Patna to Gaya. It met with an accident somewhere enroute. What is the speed of this bus?

A. Due to this accident, speed of bus got reduced by 22%. And Due to this one bus reached Gaya 40 minutes late.

B. Had the accident occurred 5 kms ahead of the point where the accident occurred, it would have reached Gaya only 20 minutes late.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 12. There are five members—P, Q, R, S and T in a family.How many members of this family are male?

A. T has two sisters—Q and S.

B. P is the father of T and R is the only son of T.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 13. Which month is the current month?

A. Sum of dates of last Monday of previous month and 1st Thursday of next month is 38 and both the dates are of the same year.

B. Current year is a leap year and the month in the question is from current year.

(a) 5 (b) 4
(c) 3 (d) 2
(e) 1

Q 14. 1st January 19PQ is Monday, where P and Q are the last two digits of the year in such a way that maximum value of P and Q each is 6. Which is the next year when 1st January will be a Monday?

A. 19PQ is a leap year.

B. 19PQ is a non-leap year.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 15. A triangle is circumscribed by a circle. Is this triangle a right-angled triangle?

A. Triangle is isosceles triangle.

B. One side of the triangle is the largest possible chord of the circle.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 16. Triangle ABC is a right angled triangle. What is the distance between circum centre and the centroid of this triangle ABC?

A. Inradius of Δ ABC = 5 units

B. Two smaller sides of the Δ ABC are 3 units and 4 units.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Q 17. What is the HCF of $(5a - 1)$ and $(5b - 1)$?

A. HCF of a and b = 5

B. a and b are natural numbers.

(a) 1 (b) 3
(c) 2 (d) 4
(e) 5

Direction for questions 18 to 20: Each item is followed by three statements, A, B and C. Answer each question using the following instructions.

Mark (a): If the question can be answered by using any two statements together but not by using any statement alone

Mark (b): If the question can be answered by using either of the statements alone.

Mark (c): If the question can be answered only by using all the three statements together and neither by any pair of statements nor by any statement independently.

Mark (d): If the question cannot be answered using the statements in any case.

Mark (e): If none of the above four options are correct.

Q 18. A football is kept inside a circular field. What is the probability of this ball being closer to the center of the circle than to the periphery of the ground?

(a) Radius of the circular field is 15 m.

(b) Radius of the football is 5 cm.

(c) Ratio of circumference and diameter of the circle is constant.

Q 19. A and B are running continuously around a circular track with respective uniform speed. They start running from the same point at the same time. After how much time will they meet at the diametrically opposite point of their starting point?

(a) Speed of A is 20 m/s and speed of B is 35 m/s.

(b) Length of the track is 1000 m.

(c) Both A and B are running in the same direction.

Q 20. At how many different points, graph of $f(x) = ax^2 + bx + c = 0$ will cut X- axis?

(a) $a > 0$, $b > 0$ and $c > 0$.

(b) Discriminant of $f(x) < 0$

(c) $a < 0$.

ANSWERS

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(d)	(b)	(d)	(c)	(b)	(c)	(b)	(e)	(d)	(d)	(e)	(e)	(e)	(a)	(c)	(c)	(a)	(e)	(e)	(e)

HINTS AND SOLUTIONS

1. Using statement A, y can have any integer value from 0-7, hence cannot be determined using statement A alone.

   Using statement B, minimum value of y can be 6. And hence y is greater than 4.

2. Obviously, using the statements alone will not give the result. However, if we use both the statements together, then we will get the result that who is earning the least.

3. Using statement A only gives us two different sets of values of A and B – (101, 101) and (101, 1). So, unique value of HCF cannot be obtained.

   Using the statement B only also does not give us a unique number.

   However, using both the statements together gives us the values of A and B as – (101, 1). And HCF of these two numbers = 1.

4. Using statement A, we cannot find if the fathers of all the honest politicians are honest or not? It is only said that if the father is honest, then the son will be honest; however, it does not mean that son of dishonest politicians cannot be honest. [For more, see the chapter—Logical Link in the 1st part of this book]. So, nothing can be determined on the basis of this statement alone.

   Using statement B, since son of only a dishonest politician can be a dishonest politician, hence we can conclude that if a politician is dishonest, then his father will also be dishonest. On the basis of this, if Dharam is dishonest politician then his father Ram is dishonest politician and if Ram is dishonest politician then his father Shyam is dishonest politician. And this answers our question.

5. Using statement A, nothing can be inferred about the even or odd nature of the function y = f(x).

   Using statement B, since the graph lies in only two quadrants opposite to each other, it cannot be the graph of any even function. Hence, y = f(x) is not an even function.

6. Using statement A, since it is not given that what are the maximum marks for each of the papers, hence just by knowing that 300 marks are obtained by Pranav does not provide us with any other information.

   Using statement B, Let us assume that the marks obtained by Pranav are 4x, 5x, 6x, 7x and 8x. Now, 4x = 30%. Hence, now we can find out that in how many papers Pranav has scored more than 50% marks.

7. Using statement A, if the remainder obtained when N is divided by 100 is 2, then the unit digit of N is 2. Hence, N cannot be a perfect square.

   Using statement B, since the total number of factors of N is odd, hence N is a perfect square. [Total number of factors of any perfect square is odd and vice-versa is also true.]

8. Using statement A and B independently, we cannot find the speed of the trains. Even using both the two statements together, we get two speeds but we cannot find that which speed corresponds to which train.

   Hence, the speed of the trains cannot be determined.

9. Obviously, it can be seen that using the statements alone will not give us the solution.

   Using both the statements together, since the ratio of expenses is more than the ratio of income and both the fractions are proper fractions, hence it can be concluded on the basis of the data given that Rupesh is saving more than Vijay in all the cases.

10. Obviously, it can be seen that using the statements alone will not give us the solution.

    Using both the statements together will give us the ratio of sides of the triangle. Using that we can find out if the triangle ABC is right-angled or not?

13. Using statement A alone,

    Sum of dates of last Monday of previous month and 1st Thursday of next month is 38 is possible only if last Monday is 31st and 1st Thursday is 7th. (Since if we take 30 + 8 = 38, then 30 can be last Monday of any month but 8th can not be the 1st Thursday of any month)

So, 31st of last month is a Monday. Hence 7th of current month—14th of current month—21st of current month—28th of current month will be a Monday.

Now, if current month is a month with 30 days, then 5th of next month will be a Monday, so 7th of next month cannot be a Thursday.

If current month is a month with 31 days, then 4th of next month will be a Monday, so 7th of next month will be a Thursday.

Finally, we can conclude that previous month and current month, both are having 31 days. Since both the dates are of the same year, so current month is August.

We cannot find the answer using statement B alone.

14. We know that any leap year and leap year +5 will have same 1st January. Hence, 19(PQ + 5) will have its 1st January as Monday. Hence, question can be solved using statement A alone. However, statement B alone will not give us a unique result.

Nature of year	No. of years after which 1st January will be same
Leap year	5
Leap year + 1	6
Leap year + 2	6
Leap year + 3	11

15. Using statement A alone will not answer this question. Using statement B alone tells that one side of the triangle is the diameter of the circle and hence will be the hypotenuse of the circle. Hence, the triangle is right-angled triangle.

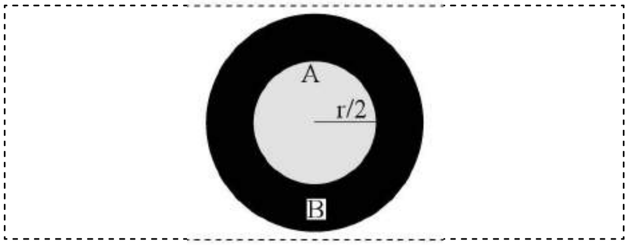
16. Using statement A alone, we cannot find the circum radius of the triangle ABC.

Using statement B alone, since the circum radius of the right-angled triangle is half the hypotenuse of the triangle, so we can find out the circum radius of the triangle. And 1/3rd of this circum radius will be the distance between circum centre and the centroid. In short, 1/6th of the hypotenuse will be the distance between circum centre and the centroid.

18. This question can be solved without using any of the statements given.

Football will be closer to the center than to the circumference if point is lying in the segment A.

Area of segment A = $1/4\pi r2$



So, probability of point being closer to the circumference = $1/4\ \pi r2/\pi r2 = 1/4$

[The probability of ball being closer to the center = Area of the smaller circle inside/Area of the larger circle (A/ A+ B) = 1/4]

19. This question can be solved by using statement A individually. Since ratio of speed of A and speed of B = 4 : 7, hence total number of meeting points = 3 (if they are running in the same direction) and total number of meeting points = 11 (if they are running in the opposite direction). In none of these cases, A and B will meet at diametrically opposite point of their starting point.

Using statement B or statement C alone will not give us the result.

20. This question can be solved by using statement B alone. Since discriminant < 0, hence this equation f(x) = 0 will not have any real root, and hence graph of f(x) = 0 will not cut X-axis.

So, graph of f(x) = 0 cuts X-axis at 0 point.

Using statement B or statement C alone will not give us the unique result.

PART

3

CAT Papers

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CAT 2002

SECTION III

There are 50 questions in this paper. All the questions carry equal mark.

There is negative marking.

Q.51. If there are 10 positive real numbers $n\,1 < n2 < n3 \ldots < n10$, How many triplets of these numbers ($n1$, $n2$, $n3$), ($n2$, $n3$, $n4$),… can be generated such that in each triplet the first number is always less than the second number, and the second number is always less than the third number?

(a) 45 (b) 90 (c) 120 (d) 180

Q.52. In ∆ABC, the internal bisector of ∆A meets BC at D. If AB = 4, AC = 3 and DA = 60°, then the length of AD is

(a) $2\sqrt{3}$ (b) $\frac{12\sqrt{3}}{7}$

(c) $\frac{15\sqrt{3}}{8}$ (d) $\frac{6\sqrt{3}}{7}$

Q.53. The length of the common chord of two circles of radii 15 cm and 20 cm, whose centres are 25 cm apart, is

(a) 24 cm (b) 25 cm

(c) 15 cm (d) 20 cm

Q.54. If $f(x) = \log\left\{\frac{(1+x)}{(1-x)}\right\}$ then $f(x) + f(y)$ is

(a) $f(x+y)$ (b) $f\left\{\frac{(x+y)}{(1+xy)}\right\}$

(c) $(x+y)f\left\{\frac{1}{(1+xy)}\right\}$ (d) $\frac{f(x)+f(y)}{(1+xy)}$

Q.55. Four horses are tethered at four corners of a square plot of side 14 m so that the adjacent horses can just reach one another. There is a small circular pond of area 20 m^2 at the centre. Find the ungrazed area.

(a) 22 m^2 (b) 42 m^2

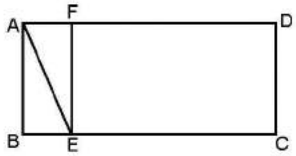
(c) 84 m^2 (d) 168 m^2

Q.56. On a straight road XY, 100 m long, five heavy stones are placed 2 m apart beginning at the end X. A worker, starting at X, has to transport all the stones to Y, by carrying only one stone at a time. The minimum distance he has to travel is

(a) 472 m (b) 422 m

(c) 744 m (d) 860 m

Q.57. In the figure given below, ABCD is a rectangle. The area of the isosceles right triangle ABE = 7 cm^2, EC = 3(BE). The area of ABCD (in cm^2) is



(a) 21 cm^2 (b) 28 cm^2

(c) 42 cm^2 (d) 56 cm^2

Q.58. The area of the triangle whose vertices are (a, a), (a + 1, a + 1) and (a + 2, a) is

(a) a^3 (b) 1 (c) $2a$ (d) $2^{1/2}$

Q.59. Instead of walking along two adjacent sides of a rectangular field, a boy took a short cut along the diagonal and saved a distance equal to half the longer side. Then the ratio of the shorter side to the longer side is

(a) $\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $\frac{1}{4}$ (d) $\frac{3}{4}$

Q.60. Only a single rail track exists between stations A and B on a railway line. One hour after the northbound super fast train N leaves station A for station B, a south-bound passenger train S reaches station A from station B. The speed of the super fast train is twice that of a normal express train E, while the speed of a passenger train S is half that of E. On a particular day, N leaves for B from A, 20 min behind the normal schedule. In order to maintain the schedule, both N and S increased their speeds. If the super fast train doubles its speed, what should be the ratio (approximately) of the speeds of passenger train to that of the super fast train so that the passenger train S reaches exactly at the scheduled time at A on that day?

(a) 1 : 3 (b) 1 : 4 (c) 1 : 5 (d 1 : 6

Q.61. On a 20 km tunnel, connecting two cities A and B, there are three gutters (1, 2 and 3). The distance between gutters 1 and 2 is half the distance between gutters 2 and 3. The distance from city A to its nearest gutter, gutter 1, is equal to the distance of city B from gutter 3. On a particular day, the hospital in city A receives information that an accident has happened at gutter 3. The victim can be saved only if an operation is started within 40 min. An ambulance started from city A at 30 km/h and crossed gutter 1 after 5 min. If the driver had doubled the speed after that, what is the maximum amount of time would the doctor get to attend the patient at the hospital. Assume 1 min is elapsed for taking the patient into and out of the ambulance?

(a) 4 min
(b) 2.5 min
(c) 1.5 min
(d) The patient died before reaching the hospital

Q.62. Number S is obtained by squaring the sum of the digits of a two-digit number D. If difference between S and D is 27, then the two-digit number D is

(a) 24 (b) 54 (c) 34 (d) 45

Q.63. The nth element of a series is represented as $X_n = (-1)^n X_{n-1}$

If $X_0 = x$ and $x > 0$, then which of the following is always true?

(a) X_n, is positive if n is even
(b) X_n, is positive if n is odd
(c) X_n is negative if n is even
(d) None of these

Q.64. If x, y and z are real numbers such that $x + y + z = 5$ and $xy + yz + zx = 3$, what is the largest value that x can have?

(a) $\frac{5}{3}$ (b) $\sqrt{19}$

(c) $\frac{13}{3}$ (d) None of these

Q.65. Neeraj has agreed to mow a lawn, which is a 20 m × 40 m rectangle. He mows it with 1 m wide strip. If Neeraj starts at one corner and mows around the lawn toward the centre, about how many times would he go round before he has mowed half the lawn?

(a) 2.5 (b) 3.5
(c) 3.5 (d) 4

Q.66. The owner of a local jewellery store hired three watchmen to guard his diamonds, but a thief still got in and stole some diamonds. On the way out, the thief met each watchman, one at a time. To each he gave $\frac{1}{2}$ of the diamonds he had then, besides 2 more. He escaped with one diamond. How many did he steal originally?

(a 40 (b) 36
(c) 25 (d) None of these

Q.67. Mayank, Mirza, Little and Jaspal bought a motorbike for $ 60. Mayank paid one-half of the sum of the amounts paid by the other boys. Mirza paid one-third of the sum of the amounts paid by the other boys. Little paid one-fourth of the sum of the amounts paid by the other boys. How much did Jaspal have to pay?

(a) $ 15 (b) $ 13
(c) $ 17 (d) None of these

Q.68. A rich merchant had collected many gold coins. He did not want anybody to know about his collection. One day, his wife asked, “How many gold coins do we have?” After a brief pause, he replied, “Well! if I divide the coins into two unequal numbers, then 48 times the difference between the two numbers equals the difference between the squares of the two numbers.” The wife looked puzzled. Can you help the merchant’s wife by finding out how many gold coins the merchant has?

(a) 96 (b) 53
(c) 43 (d) None of these

Q.69. Shyam visited Ram during his brief vacation. In the mornings, they both would go for yoga. In the evenings they would play tennis. To have more fun, they indulge in only one activity per day, i.e., either they went for yoga or played tennis each day. There were days when they were lazy and stayed home all day long. There were 24 mornings when they did nothing, 14 evenings when they stayed at home, and a total of 22 days when they did yoga or played tennis. For how many days Shyam stayed with Ram?

(a) 32 (b) 24
(c) 30 (d None of these

Q.70. Let S denotes the infinite sum $2 + 5x + 9x^2 + 14x^3 + 20x^4 + \ldots$, where $|x| < 1$ and the coefficient of x^{n-1} is $\frac{1}{2} n(n + 3)$, $(n = 1, 2, \ldots)$. Then S equals:

(a) $\frac{2-x}{(1-x)^3}$ (b) $\frac{2-x}{(1+x)^3}$

(c) $\frac{2+x}{(1-x)^3}$ (d) $\frac{2+x}{(1+x)^3}$

Q.71. If $x^2 + 5y^2 + z^2 = 2y(2x + z)$, then which of the following statements is/are necessarily true?

(a) $x = 2y$ (b) $x = 2z$ (c) $2x = z$

(a) Only A (b) B and C

(c) A and B (d) None of these

Q.72. Amol was asked to calculate the arithmetic mean of 10 positive integers, each of which had 2 digits. By mistake, he interchanged the 2 digits, say a and b, in one of these 10 integers. As a result, his answer for the arithmetic mean was 1.8 more than what it should have been. Then b–a equals:

(a) 1 (b) 2

(c) 3 (d) None of these

Q.73. A car rental agency has the following terms. If a car is rented for 5 h or less, then, the charge is Rs 60 per h or Rs 12 per km whichever is more. On the other hand, if the car is rented for more than 5 h, the charge is Rs 50 per h or Rs 7.50 per km whichever is more. Saurabh Gandhi rented a car from this agency, drove it for 30 km and ended up paying Rs 300. For how many hours did he rent the car?

(a) 4 h (b) 5 h

(c) 6 h (d) None of these

Q.74. A child was asked to add first few natural numbers (i.e., 1 + 2 + 3 + …) so long as his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result as wrong, the child discovered he had missed one number in the sequence during addition. The number he missed was

(a) Less than 10 (b) 10

(c) 15 (d) More than 15

Q.75. Suppose for any real number x, $[x]$ denotes the greatest integer less than or equal to x. Let $L(x, y) = [x] + [y] + [x + y]$ and $R(x, y) = [2x] + [2y]$. Then it is impossible to find any two positive real numbers x and y for which

(a) $L(x, y) = R(x, y)$

(b) $L(x, y) \neq R(x, y)$

(c) $L(x, y) < R(x, y)$

(d) $L(x, y) > R(x, y)$

Q.76. Ten straight lines, no two of which are parallel and no three of which pass through any common point, are drawn on a plane. The total number of regions (including finite and infinite regions) into which the plane would be divided by the lines is

(a) 56 (b) 255

(c) 1024 (d) Not unique

Q.77. When 2^{256} is divided by 17, then the remainder would be

(a) 1 (b) 16

(c) 14 (d) None of these

Q.78. The number of real roots of the equation $\frac{A^2}{x} + \frac{B^2}{x-1} = 1$, where A and B are real numbers and simultaneously not equal to zero, is

(a) None (b) 1

(c) 2 (d) 1 or 2

Q.79. At a bookstore, 'Modern Book Store' is a flashed using neon lights. The words as individually flashed at the intervals of $2\frac{1}{2}$s, $4\frac{1}{4}$s and $5\frac{1}{8}$s respectively, and each word is put off after a second. The least time after which the full name of the bookstore can be read again is

(a) 49.5 s (b) 73.5 s

(c) 1744.5 s (d) 855 s

Q.80. Three pieces of cakes of weight $4\frac{1}{2}$lb, $6\frac{3}{4}$lb and $7\frac{1}{5}$lb respectively are to be divided into parts of equal weight. Each part must be as heavy as possible. If one such part is served to each guest, then what is the maximum number of guests that could be entertained?

(a) 54 (b) 72

(c) 20 (d) None of these

Q.81. After the division of a number successively by 3, 4 and 7, the remainders obtained are 2, 1 and 4 respectively. What will be the remainder if 84 divides the same number?

(a) 80 (b) 75 (c) 41 (d) 53

Q.82. Six persons are playing a card game. Suresh is facing Raghubir who is to the left of Ajay and to the right of Pramod. Ajay is to the left of Dhiraj. Yogendra is to the left of Pramod. If Dhiraj exchanges his seat with Yogendra and Pramod exchanges with Raghubir, who will be sitting to the left of Dhiraj?

(a) Yogendra (b) Raghubir

(c) Suresh (d) Ajay

Direction for questions 83 and 84: Answer the questions based on the following information.

A boy is asked to put one mango in a basket when ordered 'one', one orange when ordered 'two', one apple when ordered three, and is asked to take out from the basket one mango and an orange when ordered four. A sequence of orders is given as: 1 2 3 3 2 1 4 2 3 1 4 2 2 3 3 1 4 1 1 3 2 3 4

Q.83. How many total oranges were in the basket at the end of the above sequence?

(a) 1 (b) 4
(c) 3 (d) 2

Q.84. How many total fruits will be in the basket at the end of the above order sequence?

(a) 9 (b) 8 (c) 11 (d) 10

Direction for questions 85 and 86: Answer the questions based on the following information.

Each of the 11 letters A, H, I, M, 0, T, U, V, W, X and Z appears same when looked at in a mirror. They are called symmetric letters. Other letters in the alphabet are asymmetric letters.

Q.85. How many four-letter computer passwords can be formed using only the symmetric letters (no repetition allowed)?

(a) 7,920 (b) 330
(c) 14,640 (d) 4,19,430

Q.86. How many three-letter computer passwords can be formed (no repetition allowed) with at least one symmetric letter?

(a) 990 (b) 2,730
(c) 12,870 (d) 15.600

Q.87. A train approaches a tunnel AB. Inside the tunnel is a cat located at a point that is $\frac{3}{8}$ of the distance AB measured from the entrance A. When the train whistles the cat runs. If the cat moves to the entrance of the tunnel A, the train catches the cat exactly at the entrance. If the cat moves to the exit B, the train catches the cat at exactly the exit. The speed of the train is greater than the speed of the cat by what order?

(a) 3:1 (b) 4:1
(c) 5:1 (d) None of these

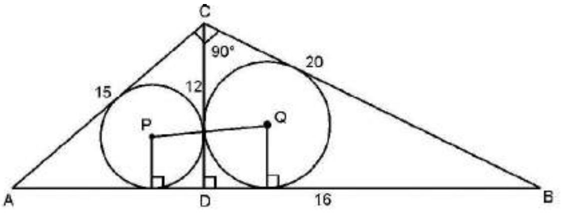
Q.88. A piece of string is 40 cm long. It is cut into three pieces. The longest piece is three times as long as the middle-sized and the shortest piece is 23 cm shorter than the longest piece. Find the length of the shortest piece.

(a) 27 cm (b) 5 cm (c) 4 cm (d) 9 cm

Q.89. Three travellers are sitting around a fire, and are about to eat a meal. One of them has 5 small loaves of bread, the second has 3 small loaves of bread. The third has no food, but has 8 coins. He offers to pay for some bread. They agree to share the 8 loaves equally among the three travellers, and the third traveller will pay 8 coins for his share of the 8 loaves. All loaves were of the same size. The second traveller (who had 3 loaves) suggests that he will be paid 3 coins, and that the first traveller be paid 5 coins. The first traveller says that he should get more than 5 coins. How much should the first traveller get?

(a) 5 (b) 7
(c) 1 (d) None of these

Q.90.



In the above figure, ACB is a right-angled triangle. CD is the altitude. Circles are inscribed within the ΔACD and ΔBCD. P and Q are the centres of the circles. The distance PQ is

(a) 5 (b) 50
(c) 7 (d) 8

Q.91. If u, v is the weighted average and m is the natural number such that $u^m + v^m = w^m$, then which one of the following is true?

(a) $m \geq \min(u, v, w)$
(b) $m \geq \max(u, v, w)$
(c) $m < \min(u, v, w)$
(d) None of these

Q.92. In how many ways is it possible to choose a white square and a black square on a chessboard so that the squares must not lie in the same row or column?

(a) 56 (b) 896
(c) 60 (d) 768

Q.93. $7^{6n} - 6^{6n}$, where n is an integer > 0, is divisible by

(a) 13 (b) 127
(c) 559 (d) All of these

Q.94. If $pqr = 1$, the value of the expression

$$\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$$ is equal to

(a) $p + q + r$ (b) $\frac{1}{p+q+r}$
(c) 1 (d) $p^{-1} + q^{-1} + r^{-1}$

Q.95. It takes six technicians a total of 10 h to build a new server from a direct computer, with each working at the same rate. If six technicians start to build the server at 11 am, and one technician per h is added beginning at 5 pm, at what time will the server be completed?

(a) 6.40 pm (b) 7 pm
(c) 7.20 pm (d) 8 pm

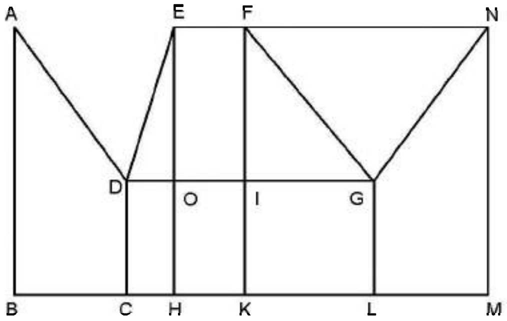
Q.96. Davji Shop sells samosas in boxes of different sizes. The samosas are priced at Rs 2 per samosa up to 200 samosas. For every additional 20 samosas, the price of the whole lot goes down by 10 paise per samosa. What should be the maximum size of the box that would maximize the revenue?

(a) 240
(b) 300
(c) 400
(d) None of these

Q.97. Three small pumps and a large pump are filling a tank. Each of the three small pump works at $\frac{2}{3}$rd the rate of the large pump. If all four pumps work at the same time, they should fill the tank in what fraction of the time that it would have taken the large pump alone?

(a) $\frac{4}{7}$ (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{3}{4}$

Directions for questions 98 to 99: Answer the questions based on the following diagram.



In the above diagram, ∠ABC = 90° = ∠DCH = ∠DOE = ∠EHK = ∠FKL = ∠GLM = ∠LMN

AB = BC = 2CH = 2CD = EH = FK = 2HK = 4KL = 2LM = MN

Q.98. The magnitude of ∠FGO =

(a) 30° (b) 45°
(c) 60° (d) None of these

Q.99. What is the ratio of the areas of the two quadrilaterals ABCD to DEFG?

(a) 1:2 (b) 2:1
(c) 12:7 (d) None of these

Q.100. How many numbers greater than 0 and less than a million can be formed with the digits 0, 7 and 8?

(a) 486 (b) 1084
(c) 728 (d) None of these

ANSWERS

■■■ CAT 2002

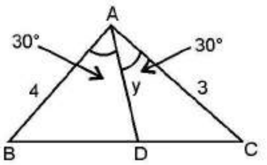
Q. No.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
Answer	(c)	(b)	(a)	(b)	(a)	(d)	(d)	(b)	(d)	(d)	(c)	(b)	(d)	(d)	(c)	(b)	(b)	(d)	(c)	(a)
Q. No.	71	72	73	74	75	76	77	78	79	80	81	82	83	84	85	86	87	88	89	90
Answer	(c)	(b)	(c)	(d)	(d)	(a)	(a)	(d)	(b)	(d)	(d)	(c)	(d)	(c)	(a)	(c)	(a)	(c)	(b)	(b)
Q. No.	91	92	93	94	95	96	97	98	99	100										
Answer	(d)	(d)	(d)	(c)	(d)	(b)	(b)	(d)	(c)	(c)										

HINTS AND SOLUTIONS

51. (c) Total possible arrangements = 10 × 9 × 8
Now 3 number can be arranged among themselves in 3! way = 6 ways

Given condition is satisfied by only 1 out of 6 ways hence, the required number of arrangements= $\frac{10\times9\times8}{6}=120$

52. (b)



Let BC = x and AD = y

As per bisector theorem, $\frac{BD}{DC}=\frac{AB}{AC}=\frac{4}{3}$

Hence, BD = $\frac{4x}{7}$; DC = $\frac{3x}{7}$

In ΔABD, cos 30° = $\frac{(4)^2+y^2-\frac{16x^2 49}{49}}{2\times4\times y}$

$$2\times4\times y\times\frac{\sqrt{3}}{2}=16+y^2-\frac{16x^2}{49}$$

$$4\sqrt{3}y=16+y^2-\frac{16x^2}{49} \qquad \text{...(i)}$$

Similarly, from DADC, Cos 30°

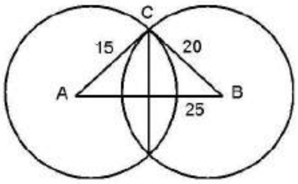
$$=\frac{9+y^2-\frac{9x^2}{49}}{2\times3\times y}$$

$$\Rightarrow 3\sqrt{3}y=9+y^2-\frac{9x^2}{49} \qquad \text{...(ii)}$$

Now (i) x 9 −16 x (ii), we get

$$36\sqrt{3}y-48\sqrt{3}y=9y^2-16y^2\Rightarrow y=\frac{12\sqrt{3}}{7}$$

53. (a)



Let the chord be = x cm

$$\therefore \frac{1}{2}(15\times20)=\frac{1}{2}\times25\times\frac{x}{2}\Rightarrow x=24\,\text{cm}$$

54. (b) $f(x)+f(y)=\log\left(\frac{1+x}{1-x}\right)+\log\left(\frac{1+y}{1-y}\right)$

$$=\log\left(\frac{(1+x)(1+y)}{(1-x)(1-y)}\right)$$

$$=\log\left(\frac{1+x+y+xy}{1+xy-(x+y)}\right)$$

$$=\log\left(\frac{1+\left(\frac{x+y}{1-xy}\right)}{1-\left(\frac{x+y}{1-xy}\right)}\right)$$

$$=f\left(\frac{x+y}{1+xy}\right)$$

55. (a) Total area=14 × 14 = 196 m^2

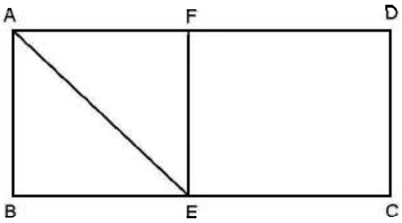
Grazed area = $\left(\frac{\pi\times r^2}{4}\right)\times4=\pi r^2=22\times7$
= 154 m^2
The ungrazed area is less than (196 – 154) = 42 m^2 for which there is only one option.

56. (d) Every trip will need more than 180 m and there are $4\frac{1}{2}$ trips. hence, the distance covered will be greater than 750 m for which there is only one option = 860.

Alternative Method

For the first stone, he will cover 100 m.
For second, 200 – 4 = 196
For third, 200 – 8 = 192
For fourth, 200 – 12 = 188
For fifth, 200 – 16 = 184
Hence, the total distance = 860 m

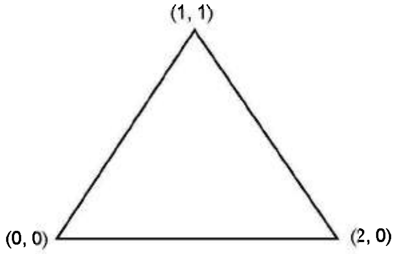
57. (d)



Area of ΔABE = 7cm^2

Area of ABEF = 14 cm^2
Area of ABCD = 14 × 4
= 56 cm^2

58. (b)



Hence, area = $\frac{1}{2}(2)(1) = 1$

Note: Answer should be independent of a and area of the triangle does not have a square root.

59. (d) Check choices, e.g., $\frac{1}{2}$ Diagonal = $\sqrt{5}$

Distance saved = 3 – $\sqrt{5}$ = 0.75 ≠ Half the larger side hence, incorrect.

$\frac{3}{4} \Rightarrow$ Diagonal = 5
Distance saved = (4 + 3) – 5 = 2 = Half the larger side.

60. (d) If speed of N = 4, speed of g = 1,

Average speed = $\frac{2\times4\times1}{4+1} = 1.6$

Because time available is $\frac{2}{3}$, speed = $\frac{3}{2}$

Now average speed = 2, 4

Now speed of N = 8

Now speed of S = y

$\frac{2\times8\times y}{8+y} = 2.4 \Rightarrow y = 1.3$

Required ratio = 1.3 : 8 = 1:6

61. (c) AG$_1$=5 min at 30 km/h = 2.5 km
G$_1$G$_3$ = 15 km
Time for AG$_1$ = 5 min
Time for
G$_1$G$_3$ + G$_3$A = 32.5 min = total of 37.5 min 1 min is taken for transferring the patient into and out of the ambulance.
Hence, (40 – 37.5 – 1) = 1.5 min is remaining.

62. (b) Check choices
Choice (b) 54 S = (5 + 4)2 = 81
⇒ D – S = 81 – 54 = 27. Hence, the number = 54

63. (d) $x_0 = x$
$x_1 = -x$
$x_3 = x$
$x_4 = x$
$x_5 = -x$
$x_6 = -x$
....
⇒ Choices (a), (b), (c) are incorrect.

64. (d) $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + zy + zx)$
$\Rightarrow x^2 + y^2 + z^2 = 19$
$\Rightarrow y + z$ cannot be simultaneously = 0
else $xy + zy + zx = 0 \Rightarrow x^2 < 19 \Rightarrow x < \sqrt{19} \cong 4.4$

65. (c) Area = 40 × 20 = 800
If 3 rounds are done, area = 34 × 14 = 476
Area > 3 rounds
If 4 rounds ⇒ Area left = 32 × 12 = 347
Hence, the area should be slightly less than 4 rounds.

66. (b) Since thief escaped with 1 diamond,
Before 3rd watchman he had (1 + 2) × 2 = 6
Before 2nd watchman he had (6 + 2) × 2 = 16
Before 1st watchman he had (16 + 2) × 2 = 36
Alternative Method
Check with choices.

67. (b) Mayank paid $\frac{1}{2}$ of what others paid.

⇒ Mayank paid $\frac{1}{3}$ rd of the total amount
= $ 20.

Similarly, Mirza paid $ 15 and Little paid $ 12.

Remaining amount of $60 – $20 – $15 – $12 = $13 is paid by Jaspal.

68. (d) Let the number of gold coins = $x + y$
$48(x - y) = x^2 - y$?
$48(x - y) = (x - y)(x + y) = -x + y = 48$
Hence, the correct choice would be none of these.

69. (c) Let us assume that
p days : they played tennis
y days:they went for yoga
T days:total duration for which Ram and Shyam stayed together.

⇒ p + y = 22

(T – y) = 24

And (T – p) = 14

Adding all of them,

2T = 22 + 24 + 14 – T = 30 days.

70. (a) Coefficient of $x = -(n + 1)\,(n + 4)$

$S = 2 + 5x + 9x^2 + 14x^3 +$

$xS = 2x + 5x^2 + \ldots$

$S(1 - x) = 23 + 3x + 4x^2 + 5x^3 + \ldots$

Let $S_1 = S(1 - x) \Rightarrow S_1 = 2 + 3x + 4x^2 + \ldots$

$x\,S_1 = 2x + 3x^2 + \ldots$

$S_1(1 - x) = 2 + x + x^2 + \ldots$

$$S_1\,(1 - x^2) = 2 + \frac{x}{1-x}$$

$$S(1 - x^2) = 2 + \frac{x}{1-x} \Rightarrow S = \frac{2-x}{(1-x)^3}$$

71. (c) $x^2 + 5y^2 + z^2 - 4yx + 2yz$

$(x^2 + 4y^2 - 4yx) + z^2 + y^2 - 2yz = 0$

$(x - 2y)^2 + (z - y)^2 = 0$

It can be true only if $x = 2y$ and $z = y$

72. (b) Arithmetic mean is more by 18 means sum is more by 18. So $ba\text{-}ab = 18$

$b > a$ because sum has gone up, e.g., $31 - 13 = 18$

Hence, $b - a = 2$

73. (c) By trial and error:

$30 \times 12 = 360 > 300$

$30 \times 7.5 = 225 < 300$

$50 \times 6 = 300$. Hence, he rented the car for 6 h.

74. (d) $575 = -x$

$1150 = n^2 + n - 2x$

$n(n + 1) \geq 1150$

$n + n \geq 1150$

The smallest value for it is $n = 34$.

For $n = 34$

$40 = 2x \Rightarrow x = 20$

75. (d) $x - 1 \leq [x] \leq x$

$2x + 2y - 3 \leq L(x, y) \leq 2x + 2y \Rightarrow a - 3 \leq L \leq a$

$2x + 2y - 2 \leq R(x, y) \leq 2x + 2y \Rightarrow a - 2 \leq R \leq a$

Therefore, $L \leq R$

Note Choice (b) is wrong, otherwise choice (a) and choice (c) are also not correct. Choose the numbers to check.

76. (a) Number of regions $\frac{n(n+1)}{2} + 1$ where n = Number of lines, i.e., for 0 line we have region = 1.

For 1 line we have region = 2.

If can be shown as:

Number of lines	0	1	2	3	4	5	…	10
Number of regions	1	2	4	7	11	16	…	56

Therefore, for $n = 10$, it is 2 + 1= 56

77. (a) $(2^4)^{64} = (17-1)^{64} = 17n + (-1)^{54} = 17n + 1$ Hence, remainder = 1

78. (d) $\frac{A^2}{x} + \frac{B^2-1}{x-1} \Rightarrow A^2(x-1)\,B^2x = x^2 - x$

This is a quadratic equation. Hence, the number of roots = 2 or 1 (1 in the case when both roots are equal).

79. (b) Because each word is lit for a second,

$$\text{LCM}\left(\frac{5}{2}+1, \frac{17}{4}+1, \frac{41}{8}+1\right) = \text{LCM}\left(\frac{7}{2}, \frac{21}{4}, \frac{49}{8}\right)$$

$$\frac{\text{LCM}(7, 21, 49)}{\text{HCM}(2, 4, 8)} = \frac{49 \times 3}{2} = 73.5\text{S}$$

80. (d) $HCF\left(\frac{9}{2}, \frac{27}{4}, \frac{36}{5}\right) = \frac{HCF(9, 27, 36)}{LCM(2, 4, 5)} = \frac{9}{20} lb$

= weight of each piece total = weight = 18.45 lb

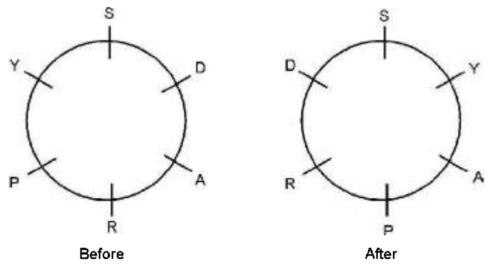
Maximum number of guests

$$= \frac{18.45 \times 20}{9} = 41$$

81. (d) $3(4(7x + 4) + 1) + 2 = 84x + 53$

Therefore, remainder is 53.

82. (c)



Before After

Suresh is sitting to the left of Dhiraj.

83. (d) Number of oranges at the end of the sequence = Number of 2s – Number of 4s = 6 – 4 = 2

84. (c) Number of (1s + 2s + 3s) – 2 (Number of 4s) = 19 – 8

85. (a) $11 \times 10 \times 9 \times 8 = 7920$

86. (c) Total number of passwords using all letters – Total number of passwords using no symmetric letters = $(26 \times 25 \times 24) - (15 \times 14 \times 13) = 12870$

87. (a) Let tunnel be = 8 km and speed of cat = 1 km/h

Time taken to reach the entrance of the tunnel by the cat = 3 h

Time taken to reach the exit of the tunnel by the cat = 5 h

Train will cover the sum (length of tunnel) = 2 h

Therefore, the ratio of the speeds of the train and the cat = 4 : 1

Speed of the train is greater by 3 : 1 than that of the cat.

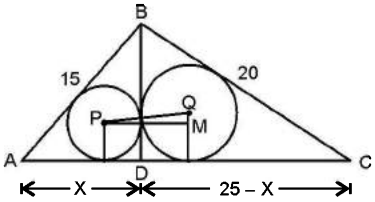
88. (c) Let the largest piece = $3x$ Middle = x
Shortest = $3x - 23$
or $3x + x + (3x - 23) = 40$
or $x = 9$
or the shortest piece = $3(9) - 23 = 4$
Check choices:
The shortest piece has to be < 20 cm.
27 is wrong choice.
The largest piece is a multiple of 3.
Or (23 + shortest) should be a multiple of 3.
Answer = 4 cm (Among other choices)

89. (b) Each traveller had $\frac{8}{3}$ loaves.

First traveller has given $5 - \frac{8}{3}$ loaves to the third.

Second traveller sacrificed only $3 - \frac{8}{3} = \frac{1}{3}$rd of a loaf. So, first should get 7 coins.

90. (b)



$(15)^2 - x^2 = (20)^2 - (25 - x)^2$
$\Rightarrow x = 9$
$\Rightarrow BD = 12$

Area of $\Delta ABD = \frac{1}{2} \times 12 \times 9 = 54$

$s = \frac{1}{2}(15 + 12 + 9) = 18$

$r_1 = \frac{Area}{s} \Rightarrow r_1 = 3$

Area of $\Delta BCD = \frac{1}{2} = 96$

$s = \frac{1}{2}(16 + 20 + 12) = 24$

$r_2 = \frac{Area}{s} \Rightarrow r_2 = 4$

in ΔPQM, $PM = r_1 + r_2 = 7$ cm
$QM = r_2 - r_1 = 1$ cm
Hence, $PQ = \sqrt{50}$ cm

91. (d) $u^m + v^m = w^m$
$u^2 + v^2 = w^2$
Taking Pythagorean triplet 3, 4 and 5, we see in $<$ mm (u, v, w)
Also $1^1 + 2' = 3'$ and hence in $\leq$ mm (u, v, w)

92. (d) A black square can be chosen in 32 ways. Once a black square is there, you cannot choose the 8 white squares in its row or column. So the number of white squares available = 24
Number of ways = $32 \times 24 = 768$

93. (d) $7^{6n} - 6^{6n}$
put $n = 1$
$7^6 - 6^6 = (7^3 - 6^3)(7^3 + 6^3)$
This is a multiple of $7^3 + 6^3 = 559$ and $7 + 6 = 13$

94. (c) If $p = q = r = 1$, then expression = 1
Check the choice only, one choice gives the value of expression = 1.

95. (d) Total amount of work = 60 man hours
From 11 am to 5 pm, 6 technicians = 36 man hours
From 5 pm to 6 pm, 7 technicians = 7 man hours
From 6 pm to 7 pm, 8 technicians = 8 man hours
From 7 am to 8 pm, 9 technicians = 9 man hours
Total = 60 man hours.

96. (b) Number of samosas = 200 + 20n,
n is a natural number.
Price per samosa = Rs (2 – 0.1n)
Revenue = $(200 + 20n)(2 - 0.ln) = 400 + 20n - 2n^2$
For maxima $20 - 4n = 0$; by differentiation $n = 5$
$\Rightarrow$ Maximum revenue will be at
$(200 + 20 \times 5) = 300$ samosas

97. (b) Three small pumps = Two large pumps
Three small + One large pumps
= Three large pump $\Rightarrow \frac{1}{3}$

98. (d) It KL = 1, then lG = 1 and FI = 2
Hence, $\tan q = \frac{2}{1} = 2$
Thus, q none of 30. 45 and 600.

99. (c) Area of quadrilateral ABCD = $\frac{1}{2}(2x + 4x) \times 4x = 12x^2$

Area of quadrilateral DEFG = $\frac{1}{2}(5x + 2x) \times 2x = 7x^2$

100. (c) Number of ways for single digit = 2
2 digits = $2 \times 3 = 6$
3 digits = $2 \times 3 \times 3 = 18$
4 digits = $2 \times 3 \times 3 \times 3 = 54$
5 digits = $2 \times 3 \times 3 \times 3 \times 3 = 162$
6 digits = $2 \times 3 \times 3 \times 3 \times 3 \times 3 = 486$
Total = 728

CAT 2003

LEAKED PAPER

Direction for questions 101 and 102: Answer the questions on the basis of the information given below.

A certain perfume is available at a duty-free shop at the Bangkok International Airport. It is priced in the Thai currency Baht but other currencies are also acceptable. In particular, the shop accepts Euro and US Dollar at the following rates of exchange:

US Dollar 1 = 41 Bahts and Euro 1 = 46 Bahts

The perfume is priced at 520 Bahts per bottle. After one bottle is purchased, subsequent bottles are available at a discount of 30%. Three friends S, R and M together purchase three bottles of the perfume, agreeing to share the cost equally. R pays 2 Euros M pays 4 Euros and 27 Thai Bahts and S pays the remaining amount in US Dollars.

Q.101. How much does R owe to S in Thai Baht?
(a) 428 (b) 416 (c) 334 (d) 324

Q.102. How much does M owe to S in US Dollars?
(a) 3 (b) 4 (c) 5 (d) 6

Direction for questions 103 and 104: Answer the questions on the basis of the information given below.

New Age Consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani, and 3 projects with Gyani but without Medha. The total number of projects for the New Age Consultants is one less than twice the number of projects in which more than one consultant is involved.

Q.103. What is the number of projects in which Gyani alone is involved?
(a) Uniquely equal to zero
(b) Uniquely equal to 1
(c) Uniquely equal to 4
(d) Cannot be determined uniquely

Q.104. What is the number of projects in which Medha alone is involved?
(a) Uniquely equal to zero
(b) Uniquely equal to 1
(c) Uniquely equal to 4
(d) Cannot be determined uniquely

[redacted]

Q.105. The number of non-negative real roots of $2^x - x - 1 = 0$ equals
(a) 0 (b) 1 (c) 2 (d) 3

Q.106. When the curves $y = \log_{10}x$ and $y = x^{-1}$ are drawn in the x-y plane, how many times do they intersect for values $x \geq 1$?
(a) Never (b) Once
(c) Twice (d) More than twice

Q.107. Let A and B be the two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be
(a) 85.5 (b) 92.5 (c) 90.5 (d) 87.5

Q.108. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p + q + r \neq 0$?

$x + 2y - 3z = p$

$2x + 6y - 11z = q$
$x - 2y + 7z = r$

(a) $5p - 2q - r = 0$ (b) $5p + 2q + r = 0$
(c) $5p + 2q - r = 0$ (d) $5p - 2q + r = 0$

Q.109. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs 20 on a standard bag and Rs 30 on a deluxe bag. Every bag must be processed on machine A and on Machine B. The processing times per bag on the two machines are as follows:

	Time required (h/bag)	
	Machine A	*Machine B*
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 h and on machine B is 1250 h. Among the following production plans, which one meets the machine availability constraints and maximizes the profit?

(a) Standard 75 bags, Deluxe 80 bags
(b) Standard 100 bags, Deluxe 60 bags
(c) Standard 50 bags, Deluxe 100 bags
(d) Standard 60 bags, Deluxe 90 bags

Q.110. The sum of the 3rd and the 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero?

(a) 1st (b) 9th
(c) 12th (d) None of the above

Directions for questions 111 to 113: Answer the questions on the basis of the information given below.

A city has two perfectly circular and concentric ring roads, the outer ring road (OR) being twice as long as the inner ring road (IR). There are also four (straight line) chord roads from E1, the east end point of OR to N2, the north end point of IR; from N1, the north end point of OR to W2, the west end point of IR; from W1, the west end point of OR, to S2, the south end point of IR; and from S1 the south end point of OR to E2, the east end point of IR. Traffic moves at a constant speed of 30 p km/h on the OR road, 20 p km/h on the IR road, and $15\sqrt{5}$ km/h on all the chord roads.

Q.111. The ratio of the sum of the lengths of all the chord roads to the length of the outer ring road is

(a) √5:2 (b) √5:2p
(c) √5:p (d) None of these

Q.112. Amit wants to reach N2 from S1. It would take him 90 min if he goes on minor arc S1 – E1 on or, and then on the chord road E1 – N2. What is the radius of the outer ring road in kms?

(a) 60 (b) 40 (c) 30 (d) 20

Q.113. Amit wants to reach E2 from N1 using first the chord N1 – W2 and then the inner ring road. What will be his travel time in min on the basis of information given in the above question?

(a) 60 (b) 45 (c) 90 (d) 105

Directions for questions 114 to [illegible]: Answer the questions independently of each other.

Q.114. A test has 50 questions. A student scores 1 mark for a correct answer, –1/3 for a wrong answer, and –1/6 for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by that student cannot be less than

(a) 6 (b) 12 (c) 3 (d) 9

Q.115. Twenty-seven persons attend a party. Which one of the following statements can never be true?

(a) There is a person in the party who is acquainted with all the twenty-six others.
(b) Each person in the party has a different number of acquaintances.
(c) There is a person in the party who has an odd number of acquaintances.
(d) In the party there is no set of three mutual acquaintances.

Q.116. Let $g(x) = \max(5 - x, x + 2)$. The smallest possible value of $g(x)$ is

(a) 4.0 (b) 4.5
(c) 1.5 (d) None of the above

Q.117. The function $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is a real number, attains a minimum at:

(a) $x = 2.3$ (b) $x = 2.5$
(c) $x = 2.7$ (d) None of these

Q.118. How many even integers n, where $100 \le n \le 200$, are divisible neither by seven nor by nine?

(a) 40 (b) 37 (c) 39 (d) 38

Q.119. A positive whole number M less than 100 is represented in base 2 notation, base 3 notation, and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals

(a) 31 (b) 63 (c) 75 (d) 91

Q.120. In a 4000 m race around a circular stadium having a circumference of 1000 m, the fastest runner and the slowest runner reach the same point at the end of the 5th min for the first time after the start of the race. All the runners have the same starting point and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of the slowest runner, what is the time taken by the fastest runner to finish the race?

(a) 20 min (b) 15 min
(c) 10 min (d) 5 min

Direction for questions 121 to 125: Each question is followed by two statements, A and B. Answer each question using the following instructions.

Choose (a): If the question can be answered by one of the statements alone but not by the other.
Choose (b): If the question can be answered by using either statement alone.
Choose (c): If the question can be answered by using both the statements together, but cannot be answered by using either statement alone.
Choose (d): If the question cannot be answered even by using both the statements together.

Q.121. Is $a^{44} < b^{11}$, given that $a = 2$ and b is an integer?

(A) b is even
(B) b is greater than 16

Q.122. What are the unique values of b and c in the equation $4x^2 + bx + c = 0$ if one of the roots of the equation is $(-1/2)$?

(A) The second root is 1/2.
(B) The ratio of c and b is 1.

Q.123. AB is a chord of a circle. AB = 5 cm. A tangent parallel to AB touches the minor arc AB at E.
What is the radius of the circle?

(A) AB is not a diameter of the circle
(B) The distance between AB and the tangent at E is 5 cm

Q.124. Is $\left(\frac{1}{a^2}+\frac{1}{a^4}+\frac{1}{a^6}+\ldots\right) > \left(\frac{1}{a}+\frac{1}{a^3}+\frac{1}{a^5}+\ldots\right)$?

(A) $-3 \le a \le 3$
(B) One of the roots of the equation $4x^2 - 4x + 1 = 0$ is a.

Q.125. D, E, F are the mid points of the sides AB, BC and CA of triangle ABC respectively. What is the area of DEF in square centimeters?

(A) AD = 1 cm, DF = 1 cm and perimeter of DEF = 3 cm
(B) Perimeter of ABC = 6 cm, AB = 2 cm, and AC = 2 cm.

Direction for questions 126 to [illegible]: Answer the questions independently of each other.

Q.126. At the end of year 1998, Shepard bought nine dozen goats. Henceforth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where p > 0 and q > 0. If Shepard had nine dozen goats at the end of year 2002 after making the sales for that year, which of the following is true?

(a) p = q (b) p<q (c) p>q (d) p = q/2

Q.127. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be

(a) 20 (b) 0 (c) 21 (d) 22

Q.128. The 288th term of the series a, b, b, c, c, c, d, d, d, d, e, e, e, e, e, f, f, f, f, f, f...is

(a) u (b) v (c) w (d) x

Q.129. Let p and q be the roots of the quadratic equation $x^2 - (a-2)x - a - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

(a) 0 (b) 3 (c) 4 (d) 5

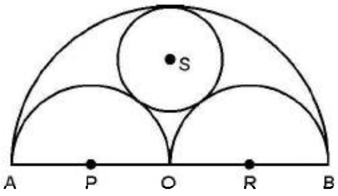
Q.130. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of the outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 cm² then the area (in sq cm) of the triangle ABC would be

(a) $\pi\sqrt{12}$ (b) $\frac{9}{\pi}$ (c) $\frac{9\sqrt{3}}{\pi}$ (d) $\frac{6\sqrt{3}}{\pi}$

Q.131. Let a, b, c and d be four integers such that $a + b + c + d = 4m + 1$ where m is a positive integer. Given m, which one of the following is necessarily true?

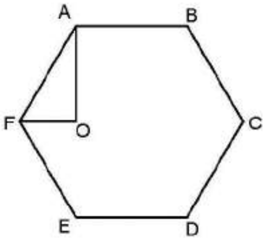
(a) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$
(b) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$
(c) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$
(d) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

Q.132. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semicircular field with center at O. Horses are tied up at P, R and S such that PO and RO are the radii of semicircles with centres at P and R respectively, and S is the center of the circle touching the two semicircles with diameters AO and OB. The horses tied at P and R can graze within the respective semicircles and the horse tied at S can graze within the circle centred at S. The percentage of the area of the semicircle with diameter AB that cannot be grazed by the horses is nearest to



(a) 20 (b) 28 (c) 36 (d) 40

Q.133. In the figure given below, ABCDEF is a regular hexagon and $\angle AOF = 90°$. FO is parallel to ED. What is the ratio of the area of the triangle AOF to that of the hexagon ABCDEF?



(a) $\frac{1}{12}$ (b) $\frac{1}{6}$

(c) $\frac{1}{24}$ (d) $\frac{1}{18}$

Q.134. How many three digit positive integers, with digits x, y and z in the hundreds, tens and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$?

(a) 245 (b) 285 (c) 240 (d) 320

Q.135. A vertical tower OP stands at the centre O of a square ABCD. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60°$, then the relationship between h and b can be expressed as

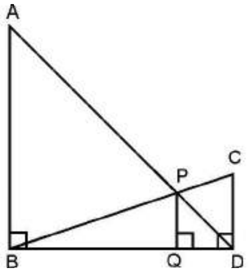
(a) $2b^2 = h^2$ (b) $2h^2 = b^2$

(c) $3b^2 = 2h^2$ (d) $3h^2 = 2b^2$

Q.136. In the triangle ABC, AB = 6, BC = 8 and AC = 10. A perpendicular dropped from B, meets the side AC at D. A circle of radius BD (with center B) is drawn. If the circle cuts AB and BC at P and Q respectively, the AP : QC is equal to

(a) 1:1 (b) 3:2 (c) 4:1 (d) 3:8

Q.137. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90°$. If AB:CD = 3:1, the ratio of CD: PQ is



(a) 1:0.69 (b) 1:0.75

(c) 1:0.72 (d) None of the above

Q.138. There are 8436 steel balls, each with a radius of 1 cm, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is

(a) 34 (b) 38 (c) 36 (d) 32

Q.139. If the product of n positive real numbers is unity, then their sum is necessarily

(a) A multiple of n (b) Equal to $n + \frac{1}{n}$

(c) Never less than n (d) A positive integer

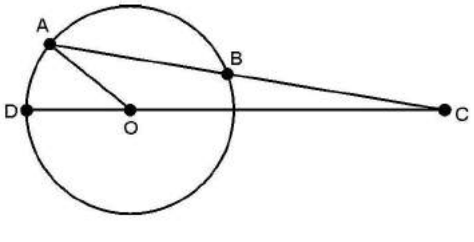
Q.140. If $\log_3 2$, $\log_3(2^x - 5)$, $\log_3(2^x - 7/2)$ are in an arithmetic progression, then the value of x is equal to

(a 5 (b) 4 (c) 2 (d) 3

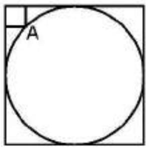
Q.141. In the figure given below, AB is the chord of a circle with center O. AB is extended to C such that BC = OB. The straight line CO is produced to meet the circle at D. If $\angle ACD = y$ degrees and $\angle AOD = x$ degrees such that $x = ky$, then the value of k is

(a) 3 (b) 2

(c) 1 (d) None of the above



Q.142. In the figure given below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm?



(a) 10 cm (b) 40 cm

(c) 50 cm (d) None of these

Q.143. Given that $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$ and $w = vz/u$, then which of the following is necessarily true?
(a) $-0.5 \le w \le 2$ (b) $-4 \le w \le 4$
(c) $-4 \le w \le 2$ (d) $-2 \le w \le -0.5$

Q.144. There are 6 boxes numbered 1, 2, … 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is
(a) 5 (b) 21 (c) 33 (d) 60

Q.145. Consider the following two curves in the *x*-*y* plane:
$y = x^3 + x^2 + 5$
$y = x^2 + x + 5$
Which of following statements is true for $-2 \le \times \le 2$?
(a) The two curves intersect once
(b) The two curves intersect twice
(c) The two curves do not intersect
(d) The two curves intersect thrice

Q.146. In a certain examination paper, there are n questions. For j = 1, 2 … n, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is
(a) 12 (b) 11 (c) 10 (d) 9

Q.147. If *x*, *y*, *z* are distinct positive real numbers the $\frac{x^2(y+z)+y^2(x+z)+z^2(x+y)}{xyz}$ would be:
(a) Greater than 4 (b) Greater than 5
(c) Greater than 6 (d) None of the above

Q.148. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any point through a sequence of edges. The number of edges, e, in the graph must satisfy the condition
(a) $11 \le e \le 66$ (b) $10 \le e \le 66$
(c) $11 \le e \le 65$ (d) $0 \le e \le 11$

Q.149. The number of positive integers n in the range $12 \le n \le 40$ such that the product $(n-1)(n-2) \ldots 3.2.1$ is not divisible by *n* is
(a) 5 (b) 7 (c) 13 (d) 14

Q.150. Let T be the set of integers {3, 11 19, 27, 451, 459, 467} and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is
(a) 32 (b) 28 (c) 29 (d) 30

ANSWERS

■■■ CAT 2003

Q. No.	101	102	103	104	105	106	107	108	109	110	111	112	113	114	115	116	117	118	119	120
Answer	(d)	(c)	(d)	(b)	(c)	(b)	(d)	(a)	(a)	(c)	(c)	(c)	(d)	(c)	(b)	(d)	(b)	(c)	(d)	(c)
Q. No.	121	122	123	124	125	126	127	128	129	130	131	132	133	134	135	136	137	138	139	140
Answer	(a)	(b)	(a)	(a)	(b)	(c)	(c)	(d)	(d)	(c)	(b)	(b)	(a)	(c)	(b)	(d)	(b)	(c)	(c)	(c)
Q. No.	141	142	143	144	145	146	147	148	149	150										
Answer	(a)	(c)	(b)	(b)	(d)	(a)	(c)	(a)	(b)	(d)										

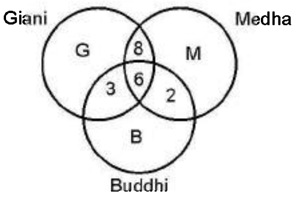
HINTS AND SOLUTIONS

101. (d)

102. (c)

103. (d) Putting the value of M in either equation, we get G + B = 17.
Hence, neither of two can be uniquely determined.

104. (b) As per the given data we get the following:



G + B = M + 16
Also, M + B + G + 19 = (2 × 19) – 1 i.e., (G + B) = 18 – M. Thus, M + 16 = 18 – M i.e., M = 1

105. (c) $2^x - x - 1 = 0$
$\Rightarrow 2^x - 1 = x$
If we put $x = 0$, then this is satisfied and if we put $x = 1$, then also this is satisfied.
Now we put $x = 2$, then this is not valid.

106. (b) For the curves to intersect, $\log_{10}x = x^{-1}$
Thus, $\log_{10}x = \frac{1}{x}$ or $x^x = 10$
This is possible for only one value of x ($2 < x < 3$).

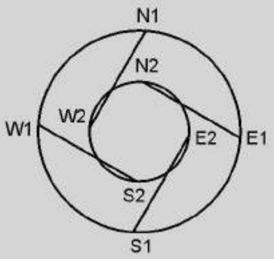
107. (d) The surface area of a sphere is proportional to the square of the radius.
Thus, $\frac{S_B}{S_A} = \frac{4}{1}$ (SA of B is 300% higher than A)
5A 1
$\therefore \frac{r_B}{r_A} = \frac{2}{1}$
The volume of a sphere is proportional to the cube of the radius.
Thus, $\frac{V_B}{V_A} = \frac{8}{1}$
Or, V_A is $\frac{7}{8}$th less than B i.e., $\left(\frac{7}{8}\times 100\right)$ 87.5%

108. (a) It is given that $p + q + r \neq 0$, if we consider the first option, and multiply the first equation by 5, second by -2 and third by -1, we see that the coefficients of x, y and z all add up-to zero.
Thus, $5p - 2q - r = 0$
No other option satisfies this.

109. (a) Let 'x' be the number of standard bags and y be the number of deluxe bags.
Thus, $4x + 5y \leq 700$ and $6x + by \leq 1250$
Among the choices, (c) and (d) do not satisfy the second equation.
Choice (b) is eliminated as, in order to maximize profits the number of deluxe bags should be higher than the number of standard bags because the profit margin is higher in a deluxe bag.

110. (c) Let the 1st term be 'a' and common difference be 'd' then we have 3rd term = $a + 2d$
15th term = $a + 14d$
6th term = $a + 5d$
11th term = $a + 10d$
13th term = $a + 12d$
Since sum of 3rd and 15th term – sum of 6th, 11th and 13th term, therefore we have $2a + 16d = 3a + 27d$
$\Rightarrow a + 11d = 0$
Which is the 12th term

Questions 111 to 113



If the radius of the inner ring road is r, then the radius of the outer ring road will be 2r (since the circumference is double).

The length of IR = 2πr, that of OR = 4πr and that of the chord roads are r$\sqrt{5}$ (Pythagoras theorem)

The corresponding speeds are
20π, 30π and 15$\sqrt{5}$ km/h.

Thus time taken to travel one circumference of IR = $\frac{r}{10}$h, one circumference or or = $\frac{r}{7.5}$h

111. (c) Sum of the length of the chord roads = $4r\sqrt{5}$ and the length of OR = $4\pi r$.

Thus the required ratio = $\sqrt{5} : \pi$

112. (c) The total time taken by the route given

$= \frac{r}{30} + \frac{r}{15} = \frac{3}{2}$ (i.e., 90 min.)

Thus, r = 15 km.

The radius of OR = 2r = 30 km

113. (d) The total time taken = $\frac{r}{20} + \frac{r}{15} = \frac{7r}{60}$

Since r = 15, the total time taken = $\frac{7}{4}$h

= 105 min.

114. (c) Let the number of correct answers be x’, number of wrong answers be y and number of questions not attempted be ‘z’.

Thus, $x + y + z = 50$ ... (i)

And $x - \frac{y}{3} - \frac{z}{6} = 32$

The second equation can be written as,

$6x - 2y - z = 192$ ... (ii)

Adding the two equations we get,

$7x - y = 242$ or $x = \frac{242}{7} + y$

Since, x and y are both integers, y cannot be 1 or 2. The minimum value that y can have is 3.

115. (b) The number 27 has no significance here.

Statement b, will never be true for any number of people.

Let us take the case of 2 people.

If A knows B and B only knows A, both of them have 1 acquaintance each. Thus, B should be knowing atleast one other person.

Let us say he knows ‘C’ as well. So now ‘B has two acquaintances (A and C), but C has only acquaintance (B), which is equal to that of A.

To close this loop, C will have to know A as well. In which case he will have two acquaintances, which is the same as that of C.

Thus the loop will never be completed unless atleast two of them have the same number of acquaintances. Besides, statements 1, 3 and 4 can be true.

Note If we consider the situation other wise, to satisfy condition 2, the first person must have 26 acquaintances, the second 25, third 24 and so on. If we continue, the last one should have 0 acquaintance, which is not possible.

116. (d) We can see that $x + 2$ is an increasing function and $5 - x$ is a decreasing function. This system of equation will have smallest value at the point of intersection of the two i.e., $5 - x = x + 2$ or $x = 1.5$. Thus, the smallest value of $g(x) = 3.5$

117. (b) **Case 1:** If $x < 2$, then $y = 2 - x + 2.5 - x + 3.6 - x = 8.1 - 3x$.

This will be the least if x is the highest i.e., just less than 2.

In this case y will be just more than 2.1

Case 2: If $2 \le x < 2.5$, then $y = x - 2 + 2.5 - x + 3.6 - x = 4.1 - x$

Again, this will be the least if x is the highest in this case y will be just more than 1.6.

Case 3: If $2.5 \le x < 3.6$, then $y = x - 2 + x - 2.5 + 3.6 - x = x - 0.9$

This will be the least if x is least i.e., the $X = 2.5$.

Case 4: If $x \ge 3.6$, then

$y = x - 2 + x - 2.5 + x - 3.6 = 3x - 8.1$

The minimum value of this will be at $x = 3.6$ and $y = 2.7$

Hence, the minimum value of y is attained at $x = 2.5$

118. (c) There are 101 integers in all, of which 51 are even.

From 100 to 200, there are 14 multiples of 7, of which 7 are even.

There are 11 multiples of 9, of which 6 are even. But, there is one integer (i.e., 126) that is a multiple of both 7 and 9 and also even.

Hence, the answer is (51 − 7 − 6 + 1) = 39

119. (d) Since the last digit in base 2, 3 and 5 is 1, the number should be such that on dividing by either 2, 3 or 5 we should get a remainder 1. The smallest such number is 31. The next set of numbers are 61, 91. Among these only 31 and 91 are a part of the answer choices.

Among these, $(31)_{10} = (11111)_2 = (1011)_3 = (111)_5$

Thus, all three forms have the leading digit 1.

Hence, the answer is 91.

120. (c) The ratio of the speeds of the fastest and the slowest runners is 21. Hence they should meet at only one point on the circumference i.e., the starting point (as the difference in the ratio in the reduced form is 1). For the two of them to meet for the first time, the faster should have completed one complete round over the slower one. Since the two of them meet for the first time after 5 min, the faster one should have completed 2 rounds

(i.e., 2000 m) and the slower one should have completed 1 round. (i.e., 1000 m) in this time. Thus, the faster one would complete the race (i.e., 4000 m) in 10 min.

121. (a) Solution cannot be found by using only Statement A since b can take any even number 2, 4, 6… But we can arrive at solution by using statement B alone.

If $b > 16$, say $b = 17$

Hence, $2^{44} < (16 + 1)^{11}$

$2^{44} < (2^4 + 1)^{11}$

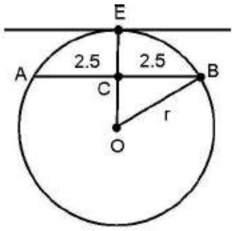
122. (b) Solution can be found using Statement A as we know both the roots for the equation $\left(\text{viz.}, \frac{1}{2} \text{ and} -\frac{1}{2}\right)$

Also statement B is sufficient.

Since ratio of c and $b = 1$, $c = b$.

Thus the equation $= 4x^2 + bx + b = 0$. Since $x = -\frac{1}{2}$ is one of the roots, substituting x we get $1 - \frac{b}{2} + b = 0$ or $b = -2$. Thus, $c = -2$.

123. (a)



We can get the answer by using the second statement only. Let the radius be r. AC = CB = 2.5 and using statement B, CE = 5, thus OC = (r – 5).

Using Pythagoras theorem, $(r - 5)^2 + (2.5)^2 = r^2$

We get r = 3.125

Note: You will realize that such a circle is not possible (if r = 3.125 how can CE be 5). However, we need to check for data sufficiency and not for data consistency. Since we are able to find the value of r uniquely using second statement the answer is (a).

124. (a) Both the series are infinitely diminishing series.

For the first series: First term $\frac{1}{a^2}$ and $r = \frac{1}{a^2}$

For the second series: First term $= \frac{1}{a}$ and $r = \frac{1}{a^2}$

The sum of the first series $= \dfrac{\frac{1}{a^2}}{1 - \frac{1}{a^2}} = \dfrac{1}{a^2 - 1}$

The sum of the second series $= \dfrac{\frac{1}{a^2}}{1 - \frac{1}{a^2}} = \dfrac{a}{a^2 - 1}$

Now, from the first statement, the relation can be anything (depending on whether 'a' is positive or negative).

But the second statement tells us, $4a^2 - 4a + 1 = 0$ or $a = \frac{1}{2}$. For this value of a, the sum of second series will always be greater than that of the first.

125. (b) The question tells us that the area of a triangle DEF will be the area of a triangle ABC. Thus by knowing either of the statements, we can get the area of the triangle DEF.

126. (c) The number of goats remain the same. If the percentage that is added every time is equal to the percentage that is sold, then there should be a net decrease. The same will be the case if the percentage added is less than the percentage sold.

The only way, the number of goats will remain the same is if $p > q$.

127. (c) In this kind of polygon, the number of convex angles will always be exactly 4 more than the number of concave angles.

Note The number of vertices have to be even. Hence, the number of concave and convex corners should add up to an even number. This is true only for the answer choice (c).

128. (d) The number of terms of the series forms the sum of first n natural numbers i.e., $n(n + 1)$

Thus the first 23 letters will account for the first 23×24

= 276 terms of the series.

The 288 m term will be the 24 m letter which is x.

129. (d) $p + q = a - 2$ and $pq = -a - 1$

$(p + q)^2 = p^2 + q^2 + 2pq$,

Thus, $(a - 2)^2 = p^2 + q^2 + 2(-\alpha - 1)$

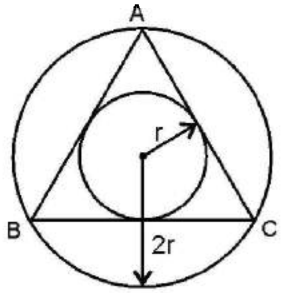
$p^2 + q^2 = \alpha^2 - 4\alpha + 4 + 2\alpha + 2$

$p^2 + q^2 = \alpha^2 - 2\alpha + 6$

$p^2 + q^2 = -2a + 1 + 5$

$p^2 + q^2 - (\alpha - 1)^2 + 5$ Thus, the minimum value of $p2 + q2$ is 5.

130. (c)



Since the area of the outer circle is 4 times the area of the inner circle, the radius of the outer circle should be 2 times that of the inner circle.

Since AB and AC are the tangents to the inner circle, they should be equal. Also, BC should be a tangent to inner circle. In other words, triangle ABC should be equilateral.

The area of the outer circle is 12. Hence, the area of the inner circle is 3 or the radius is $\sqrt{\frac{3}{\pi}}$. The area of the equilateral triangle $= 3\sqrt{3}\,r^2$, where r is the in radius. Hence the answer is c.

131. (b) $(a + b + c + d)^2 - (4\,m + 1)^2$

Thus, $a^2 + b^2 + c^2 + d^2 + 2\,(ab + ac + ad + bc + bd + cd)$

$- 16\,m^2 + 8\,m + 1$

$a^2 + b^2 + c^2 + d^2$ will have the minimum value if $(ab + ac + ad + bc + bd + cd)$ is the maximum.

This is possible if $a = b = c = d = (m + 0.25)$ … since $a + b + c + d = 4\,m + 1$

In that case $2\,(ab + ac + ad + bc + bd + cd)$

$= 12\,(m + 0.25)^2 - 12\,m^2 + 6\,m + 0.75$

Thus, the minimum value of $a^2 + b^2 + c^2 + d^2$

$= (16\,m^2 + 8\,m + 1) - 2\,(ab + ac + ad + bc + bd + cd)$

$= (16\,m\,2 + 8\,m + 1) - (12\,m^2 + 6\,m + 0.75)$

$= 4\,m^2 + 2\,m + 0.25$

Since it is an integer, the actual minimum value $= 4\,m^2 + 2\,m + 1$

132. (b) If the radius of the field is r, then the total area of the field $= \frac{\pi r^2}{2}$

The radius of the semi-circles with centre's P and R $= \frac{r}{2}$

Hence, their total area $= \frac{\pi r^2}{4}$

Let the radius of the circle with centre S be x.

Thus, OS = $(r - x)$, OR = and RS $= \left(\frac{r}{2} + x\right)$.

Applying Pythagoras theorem, we get

Solving this, we get $x = \frac{r}{3}$

Thus the area of the circle with centre S $= \frac{\pi r^2}{9}$

The total area that can be grazed

$= \pi r^2\left(\frac{1}{4} + \frac{1}{9}\right) = \frac{13\pi r^2}{36}$

Thus the fraction of the field that can be grazed

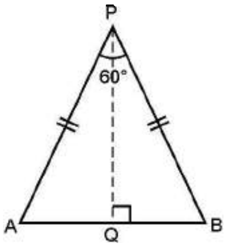
$= \frac{26}{36}\left(\frac{\text{Area that can be grazed}}{\text{Area of the field}}\right)$

The fraction that can not be grazed $= \frac{10}{36} = 28\%$

133. (a) It is very clear that a regular hexagon can be divided into six equilateral triangles. And triangle AOF is half of an equilateral triangle. Hence, the required ratio = 1 : 12

134. (c) If $y = 2$ (it can not be 0 or 1), then x can take 1 value and z can take 2 values. Thus with $y = 2$, a total of $1 \times 2 = 2$ numbers can be formed. With $y = 3$, $2 \times 3 = 6$ numbers can be formed. Similarly checking for all values of y from 2 to 9 and adding up we get the answer as 240.

135. (b)



Given ∠APB = 60° and AB = b.

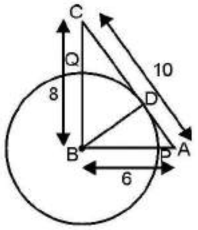
$\therefore\ PQ = \frac{b}{2} \times \sqrt{3}$

Next, $\frac{b}{2}$, h and PQ from a right angle triangle.

$\therefore \frac{b^2}{4} + h^2 = \frac{3b^2}{4}$

$\therefore\ 2h^2 = b^2$

136. (d)



Triangle ABC is a right angled triangle.

Thus $\frac{1}{2} \times CB \times AB = \frac{1}{2} \times BD \times AC$

Or $6 \times 8 = BD \times 10$. Thus BD = 4.83.

Therefore, BP = BQ

So, AP = AB – BP = 6 – 4.8 = 1.2 and CQ = BC – BQ = 8 – 4.8 = 3.2

Thus, AP : CQ = 1.2 : 3.2 = 3:8

137. (b) Using the Basic Proportionality Theorem, $\frac{AB}{PQ} = \frac{BQ}{QD}$ and $\frac{PQ}{CD} = \frac{BQ}{BD}$

Multipying the two we get, $\frac{AB}{CD} = \frac{BQ}{QD} = 3:1$.

Thus CD : PQ = BD:BQ = 4:3 = 1:0.75

138. (c) Assume the number of horizontal layers in the pile be n.

So $\sum \frac{n(n+1)}{2} = 8436$

$\Rightarrow \frac{1}{2}\left[\sum n^2 + \sum n\right] = 8436$

$\Rightarrow n(n+1)\left[\frac{2n+4}{12}\right] = 8436$

$\Rightarrow n(n+1)(n+2) = 36 \times 37 \times 38$

So, $n = 36$

139. (c) The best way to do this is to take some value and verify.

e.g., 2, $\frac{1}{2}$ and 1. Thus, $n = 3$ and the sum of the three numbers = 3.5.

Thus options 1, 2 and 4 get eliminated.

Alternative method

Let the n positive numbers be $a1, a2, a3 \dots a_n$

We know that AM ≥ GM

Hence, $\frac{1}{n}(a_1+a_2+a_3+\dots a_n) \ge (a_1 a_2 \dots an)^{1/n}$

or $(a_1+a_2+a_3+\dots a_n) \ge n$

140. (c) Using $\log a - \log b = \log\left(\frac{a}{b}\right), \frac{2}{y-5}$

$= \frac{y-5}{y-3.5}$, where, $y = 2^x$

Solving we get $y = 4$ or 8 i.e., $x = 2$ or 3. It cannot be 2 as log of negative number is not defined (see the second expression).

141. (a)

∠BOC = 10° (Opposite equal sides)

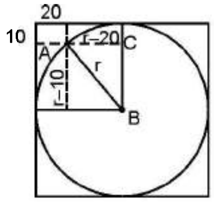
∠OBA = 20° (External angle of ABOC)

∠OAB = 20° (Opposite equal sides)

∠AOD = 30° (External angle of AAOC)

Thus, k = 3

142. (c)



Let the radius be r. Thus by Pythagoras' theorem for ΔABC we have

$(r-10)^2 + (r-20)^2$

i.e., $r^2 - 60r + 500 = 0$. Thus, r = 10 or 50.

It would be 10, if the corner of the rectangle had been lying on the inner circumference. But as per the given diagram, the radius of the circle should be 50 cm.

143. (b) u is always negative. Hence, for us to have a minimum vz value of $\frac{vz}{u}$, vz should be positive.

Also for the least u value, the numerator has to be the maximum positive value and the denominator has to be the smallest negative value. In other words, vz has to be 2 and u has to be −0.5.

Hence, the minimum value of $\frac{vz}{u} = \frac{2}{-0.5} = 4$.

For us to get the maximum value, vz has to be the smallest negative value and u has to be the highest negative value. Thus, vz has to be - 2 and u has to be −0.5.

Hence, the maximum value of $\frac{vz}{u} = \frac{-2}{-0.5} = 4$

144. (b) GRRRRR, RGRRRR, RRGRRR, RRRGRR, RRRRGR, RRRRRG

GGRRRR, RGGRRR, RRGGRR, RRRGGR, RRRRGG, GGGRRR, RGGGRR, RRGGGR, RRRGGG, GGGGRR, RGGGGR, RRGGGG GGGGGR, RGGGGG, GGGGGG

Hence, 21 ways.

145. (d) When we substitute two values of x in the above curves, at $x = -2$, we get

$y = -8 + 4 + 5 = 1$

$y = 4 - 2 + 5 = 7$

Hence, at $x = -2$ the curves do not intersect.

At $x = 2$, $y_1 = 17$ and $y_2 = 11$

At $x = -1$, $y_1 = 5$ and 2 and $y_2 = 5$

When $x = 0$, $y_1 = 5$ and $y_2 = 5$

And at $x = 1, y_1 = 7$ and $y_2 = 7$

Therefore, the two curves meet thrice when $x = -1, 0$ and 1.

146. (a) Let us say there are only 3 questions. Thus there are $2^{3-1} = 4$ students who have done 1 or more questions wrongly, $2^{3-2} = 2$ students who have done 2 or more questions wrongly and $2^{3-3} = 1$ student who must have done all 3 wrongly. Thus the total number of wrong answers $= 4 + 2 + 1 = 7 = 2^3 - 1 = 2^n - 1$.
In our question, the total number of wrong answers $= 4095 = 2^{12}-1$.
Thus $n = 12$.

147. (c) Here x, y, z are distinct positive real number

So, $\frac{x^2(y+z)+y^2\ (x+2)+z^2(x+y)}{xyz}$

$=\frac{x}{y}+\frac{x}{z}+\frac{y}{x}+\frac{y}{z}+\frac{z}{x}+\frac{z}{y}$ (we know that

$=\left(\frac{x}{y}+\frac{y}{x}\right)+\left(\frac{y}{z}+\frac{z}{y}\right)+\left(\frac{z}{x}+\frac{x}{z}\right)$ that $\frac{a}{b}+\frac{b}{a}>2$ if

a and b are distinct numbers

$> 2 + 2 + 2$

> 6

148. (a) The least number of edges will be when one point is connected to each of the other 11 points, giving a total of 11 lines. One can move from any point to any other point via the common point. The maximum edges will be when a line exists between any two points. Two points can be selected from 12 points in $^{12}C_2$ i.e., 66 lines.

149. (b) From 12 to 40, there are 7 prime number, i.e., 13, 17, 19, 23, 29, 31, 37, which is not divisible by $(n - 1)!$

150. (d) $T_n = a + (n - 1)d$

$467 = 3 + (n - 1)\ 8$

$n = 59$

Half of $n = 29$ terms

29th term is 227 and 30th term is 235 and when these two terms are added the sum is less than 470.

Hence, the maximum possible values the set S can have, is 30.

RETEST PAPER

Direction for questions 51 to 53: Answer the questions on the basis of the information given below.

The seven basic symbols in a certain numeral system and their respective values are as follows:

I = 1, V = 5, X = 10, L = 50, C = 100, D = 500 and M = 1000

In general, the symbols in the numeral system are (read from left to right), starting with the symbol representing the largest value; the same symbol cannot occur continuously more than three times: the value of the numeral is the sum of the values of the symbols. For example, XXVII = 10 + 10 + 5 + 1 + 1 = 27. An exception to the left-to-right reading occurs when a symbol is' followed immediately by a symbol of greater value; then the smaller value is subtracted from the larger.

For example, XLVI (50 — 10) + 5 + 1 = 46.

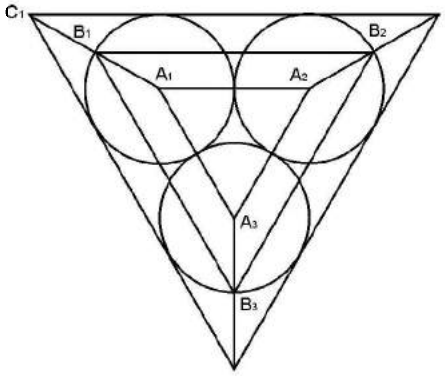
Q.51. The value of the numeral MDCCLXXXVII is
(a) 1687 (b) 1787
(c) 1887 (d) 1987

Q.52. The value of the numeral MCMXC IX is
(a) 1999 (b) 1899
(c) 1989 (d) 1889

Q.53. Which of the following represent the numeral for 1995?
I. MCMLXXV II. MCMXCV
III. MVD IV. MVM
(a) Only I and II (b) Only Ill and IV
(c) Only II and IV (d) Only IV

Direction for questions 54 to 56: Answer the questions on the basis of the information given below.

Consider three circular parks of equal size with centres at A_1, A_2 and A_3 respectively. The parks touch each other at the edge as shown in the figure (not drawn to scale). There are three paths formed by the triangles $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$ as shown. Three sprinters A, B, and C begin running from points A_1, B_1 and C_1 respectively. Each sprinter traverses her respective triangular path clockwise and returns to her starting point.



Q.54. Let the radius of each circular park be r, and the distances to be traversed by the sprinters A, B and C be a, b and c respectively. Which of the following is true?

(a) $b-a=c-b=3\sqrt{3}r$

(b) $b-a=c-b=\sqrt{3}r$

(c) $b=\frac{a+c}{2}=2\left(1+\sqrt{3}\right)r$

(d) $c=2b-a=\left(2+\sqrt{3}\right)r$

Q.55. Sprinter A traverses distance A_1A_2, A_2A_3 and A_3A_1 at average speeds of 20, 30 and 15 respectively. B traverses her entire path at a uniform speed of $\left(10+\sqrt{3}+20\right)$. C traverses distances C_1C_2, C_2C_3 and C_3C_1 at an average speed of $\frac{40}{3}\left(\sqrt{3}+1\right), \frac{40}{3}\left(\sqrt{3}+1\right)$ and 120 respectively. All speeds are in the same unit. Where would B and C be respectively when A finishes her sprint?

(a) B_1, C_1

(b) B_3, C_3

(c) B_1, C_3

(d) B_1, somewhere between C_3 and C_1

Q.56. Sprinters A, B and C traverse their respective paths at uniform speeds of u, v and w respectively. It is known that $u^2:v^2:w^2$ is equal to Area A:Area B: Area C, where Area A, Area B and Area C are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$, and $C_1C_2C_3$ respectively. Where would A and C be when B reaches point B_3?

(a) A_2, C_3

(b) A_3, C_3

(c) A_3, C_2

(d) Some where between A_2 and A_3, somewhere between C_3 and C_1

Direction for questions 57 to 59: Answer the questions on the basis of the information given below.

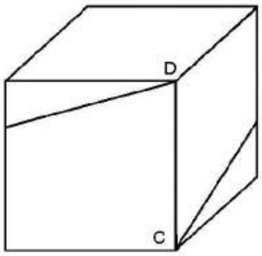
Consider a cylinder of height h cm and radius r = $\frac{2}{\pi}$ cm as shown in the figure (not drawn to scale). A string of a certain length, when wound on its cylindrical surface, starting at point A and ending at point B, gives a maximum of n turns (in other words, the string's length is the minimum length required to wind n turns).

Q.57. What is the vertical spacing between the two consecutive turns?

(a) $\frac{h}{n}$ cm (b) $\frac{h}{\sqrt{n}}$ cm

(c) $\frac{h}{n^2}$ cm (d) Cannot be determined

Q.58. The same string, when wound on the exterior four walls of a cube of side n cm, starting at point C and ending at point D, can give exactly one turn (see figure, not drawn to scale). The length of the string is



(a) $\sqrt{2}$ n cm (b) $\sqrt{17}$ n cm

(c) n cm (d) $\sqrt{13}$ n cm

Q.59. In the set-up of the previous two questions, how is h related to n?

(a) $h=\sqrt{2}\ n$ (b) $h=\sqrt{17}\ n$

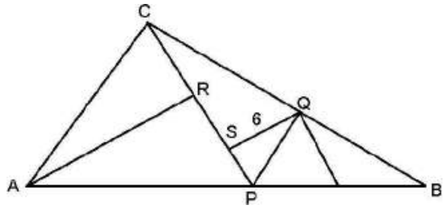
(c) $h=n$ (d) $h=\sqrt{13}\ n$

[illegible]

Q.60. There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with a telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required?

(a) 72 (b) 90 (c) 96 (d) 144

Q.61. In the figure (not drawn to scale) given below, P is a point on AB such that AP: PB = 4:3. PQ is parallel to AC and QD is parallel to CP. In ΔARC, ∠ARC = 90° and in ΔPQS, ∠PSQ = 90°. The length of QS is 6 cm. What is the ratio of AP:PD?



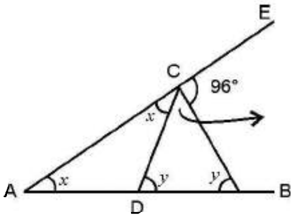
(a) 10:3 (b) 2:1 (c) 7:3 (d) 8:3

Q.62. A car is being driven, in a straight line and at a uniform speed, towards the base of a vertical tower. The top of the tower is observed from the car and, in the process, it takes 10 min for the angle of elevation to change from 45° to 60°. After how much more time will this car reach the base of the tower?

(a) $5\left(\sqrt{3}+1\right)$ (b) $6\left(\sqrt{3}+\sqrt{2}\right)$

(c) $7\left(\sqrt{3}-1\right)$ (d) $8\left(\sqrt{3}-2\right)$

Q.63. In the figure (not drawn to scale) given below. If AD = CD= BC and ∠BCE = 96°, how much is the value of ∠DBC?



(a) 32° (b) 84°

(c) 64° (d) Cannot be determined

Q.64. If both *a* and b belong to the set {1, 2, 3, 4}, then the number of equations of the of $ax^2 + bx + 1 = 0$ having real roots is

(a) 10 (b) 7

(c) 6 (d 12

Q.65. If $\log_{10}x - \log_{10}\sqrt{x} = 2\log_x 10$, then the possible value of *x* is given by

(a) 10 (b) $\frac{1}{100}$

(c) $\frac{1}{1000}$ (d) None of the

Q.66. What is the sum of all two-digit numbers that give a remainder of 3 when they are divided by 7?

(a) 666 (b) 676 (c) 683 (d) 777

Q.67. An intelligence agency forms a code of two distinct digits selected from 0, 1, 2,…, 9 such that the first digit of the code is non-zero. The code, handwritten on a slip, can however potentially create confusion when read upside down, for example, the code 91 may appear as 16. How many code are there for which no such confusion can arise?

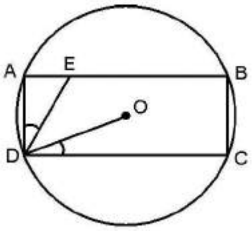
(a) 80 (b) 78 (c) 71 (d) 69

Q.68. Consider two different cloth-cutting processes. In the first one, n circular cloth pieces are cut from square cloth piece of side a in the following steps: the original square of side a is divided into smaller squares, not necessarily of the same size, then a circle of maximum possible area is from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side a and the process ends there. The cloth pieces remaining after cutting the circles are scrapped in both the processes: The ratio of the total area of scrap cloth generated in the former to that in the latter, is

(a) 1:1 (b) $\sqrt{2}:1$

(c) $\frac{n(4-\pi)}{4n-\pi}$ (d) $\frac{4n-\pi}{n(4-\pi)}$

Q.69. In the figure below (not drawn to scale), rectangle ABCD is inscribed in the circle with centre at O.The length of side AB is greater than side BC. The ratio of the area of the circle to the area of the rectangle ABCD is $\pi : \sqrt{3}$. The line segment DE intersects AB at E such that ∠ODC = ∠ADE. Find ratio AE : AD.



(a) $1:\sqrt{3}$ (b) $1:\sqrt{2}$

(c) $1:2\sqrt{3}$ (d) 1:2

Q.70. If $\frac{1}{3}\log, M + 3\log_3 N = 1 + \log_{0.008} 5$, then

(a) $M^9 = \frac{9}{N}$ (b) $N^9 = \frac{9}{M}$

(c) $M^3 = \frac{3}{N}$ (d) $N^9 = \frac{3}{M}$

Q.71. Using only 2, 5, 10, 25, and 50 paisa coins, what will be the minimum number of coins required to pay exactly 78 paise, 69 paise and Rs 1.01 to three different persons?

(a) 19 (b) 20 (c) 17 (d) 18

Q.72. The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are c, t and s, respectively. Then,

(a) s > t > c (b) c > t > s

(c) c > s > t (d) s > c > t

Q.73. What is the remainder when 4^{96} is divided by 6?

(a) 0 (b) 2 (c) 3 (d) 4

Q.74. If *x* and *y* are integers, then the equation $5x + 1\ 9y = 64$ has

(a) No solution for $x < 300$ and $y < 0$

(b) No solution for $x > 250$ and $y > -100$

(c) A solution for $250 < x < 300$

(d) A solution for $-59 < y < -56$

Q.75. What is the sum of 'n' terms in the series

$$\log m+\log\left(\frac{m^2}{n}\right)+\log\left(\frac{m^3}{n^2}\right)+\log\left(\frac{m^4}{n^3}\right)+\ldots?$$

(a) $\log\left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{\frac{n}{2}}$ (b) $\log\left[\frac{m^m}{n^n}\right]^{\frac{n}{2}}$

(c) $\log\left[\frac{m^{(1-n)}}{n^{(1-m)}}\right]^{\frac{n}{2}}$ (d) $\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{\frac{n}{2}}$

Q.76. Let S_1 be a square of side a. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3 and so on. If $A_1, A_2 A_3 \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of S_1, S_2, S_3 respectively, then the ratio $\frac{P_1+P_2+P_3+\ldots}{A_1+A_2+A_3+\ldots}$ equals

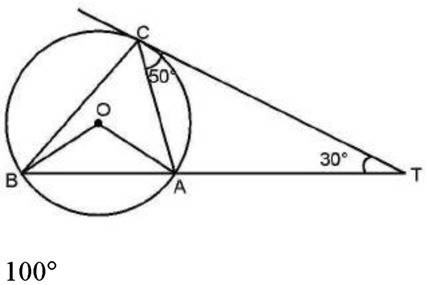
(a) $\frac{2\left(1+2\sqrt{2}\right)}{a}$ (b) $\frac{2\left(2-2\sqrt{2}\right)}{a}$

(c) $\frac{2\left(2+2\sqrt{2}\right)}{a}$ (d) $\frac{2\left(1+2\sqrt{2}\right)}{a}$

Q.77. If three positive real numbers x, y and z satisfy $y-x = z-y$ and $xyz = 4$, then what is the minimum possible value of y?

(a) $21^{1/3}$ (b) $2^{2/3}$ (c) $2^{1/4}$ (d) $2^{3/4}$

Q.78. In the figure given below (not drawn to scale), A, B and C are three points on a circle with centre O. The chord BA is extended to a point T such that CT becomes a tangent to the circle at point C. If ∠ATC = 30° and ∠ACT = 50°, then the angle ∠BOA is



(a) 100°
(b) 150°
(c) 80°
(d) Not possible to determine

Q.79. The infinite sum $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+\ldots\ldots$ equals

(a) $\frac{27}{14}$ (b) $\frac{21}{13}$ (c) $\frac{49}{27}$ (d) $\frac{256}{147}$

Q.80. Consider the sets $T_n = \{n, n+1, n+2, n+3, n+4\}$, where $n = 1, 2, 3 \ldots 96$. How many of these sets contain 6 or any integral multiple thereof (i.e. any one of the numbers 6, 12, 18, ...)?

(a) 80 (b) 81 (c) 82 (d) 83

Q.81. Let ABCDEF be a regular hexagon. What is the ratio of the area of the DACE to that of the hexagon ABCDEF?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{5}{6}$

Q.82. The number of roots common between the two equations $x^3+3x^2+4x+5=0$ and $x^3+2x^2+7x+3=0$ is

(a) 0 (b) 1 (c) 2 (d) 3

Q.83. A real number x satisfying $1-\frac{1}{n}<x\leq 3+\frac{1}{n}$, for every positive integer n, is best described by

(a) $1<x<4$ (b) $1<x\leq 3$
(c) $0<x\leq 4$ (d) $1\leq x\leq 3$

Q.84. If n is such that $36 \leq n \leq 72$, then $x = \frac{n^2+2\sqrt{n}(n+4)+16}{n+4\sqrt{n}+4}$ satisfies

(a) $20<x<54$ (b) $23<x<58$
(c) $25<x<64$ (d) $28<x<60$

Q.85. If $13x+1<2z$ and $z+3=5y^2$, then,

(a) x is necessarily less than y
(b) x is necessarily greater than y
(c) x is necessarily equal to y
(d) None of the above is necessarily true

Q.86. Let n (>1) be a composite integer such that $\sqrt{n}$ is not an integer. Consider the following statements:

(A) n has a perfect integer-valued divisor which is greater than 1 and less than $\sqrt{n}$
(B) n has a perfect integer-valued divisor which is greater than $\sqrt{n}$ but less than n

(a) Both A and B are false
(b) A is true but B is false
(c) A is false but B is true
(d) Both A and B are true

Q.87. If $|b|\geq 1$ and $x=-|a|b$, then which one of the following is necessarily true?

(a) $a-xb<0$ (b) $a-xb\geq 0$
(c) $a-xb>0$ (d) $a-xb\leq 0$

Q.88. A piece of paper is in the shape of a right-angled triangle and is cut along a line that is parallel to the hypotonuse, leaving a smaller triangle. There was 35% reduction in the length of the hypotonuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle?

(a) 16.665 (b) 16.565
(c) 15.465 (d) 14.365

Q.89. Two straight roads R_1 and R_2 diverge from a point A at an angle of 120°. Ram starts walking from point A along R_1 at a uniform speed of 3 km/h. Shyam starts walking at the same time from A along R_2 at a uniform speed of 2 km/h. They continue walking for 4 h along their respective roads and reach points B and C on R_1 and R_2 respectively. There is a straight-line path connecting B and C. Then Ram returns to point A after walking along the line segments BC and CA. Shyam also returns to A after walking along line segments BC and CA. Their speeds remain unchanged. The time interval (in h) between Ram's and Shyam's return to the point A is

(a) $\frac{10\sqrt{19}+26}{3}$ (b) $\frac{2\sqrt{19}+10}{3}$

(c) $\frac{\sqrt{19}+26}{3}$ (d) $\frac{\sqrt{19}+10}{3}$

Q.90. A square tin sheet of side 12 inches is converted into a box with open top in the following steps. The sheet is placed horizontally. Then, equal-sized squares, each of side x inches, are cut from the four corners of the sheet. Finally, the four resulting sides are bent vertically upwards in the shape of a box. If x is an integer, then what value of x maximizes the volume of the box?
(a) 3 (b) 4
(c) 1 (d) 2

Q.91. If a, $a+2$ and $a+4$ are prime numbers, then the number of possible solutions for a is
(a) One (b) Two
(c) Third (d) More than three

Q.92. Let a, b, c, d and e be integers such that $a = 6b = 12c$, and $2b = 9d = 12e$. Then which of the following pairs contains a number that is not an integer?

(a) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$

(c) $\left(\frac{a}{12}, \frac{bd}{18}\right)$ (d) $\left(\frac{a}{6}, \frac{c}{d}\right)$

Q.93. In a coastal village, every year floods destroy exactly half of the huts. After the flood water recedes, twice the number of huts destroyed are rebuilt. The floods occurred consecutively in the last three years—2001, 2002 and 2003. If the floods are expected again in 2004, the number of huts expected to be destroyed is

(a) Less than the number of huts existing at the beginning of 2001
(b) Less than the total number of huts destroyed by the floods in 2001 and 2003
(c) Less than the total number of huts destroyed by the floods in 2002 and 2003
(d) More than the total number of huts built in 2001 and 2002

Direction for questions 94 to 96: Answer the questions on the basis of the tables given below.

Two binary operations ⊕ and * are defined over the set {a, e, f, g, h} as per the following tables:

⊕	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

⊕	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	f	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first table f ⊕ g = a, while according to the second table g * h = f, and so on. Also, let $f^2 = f * f$, $g^3 = g*g*g$, and so on.

Q.94. What is the smallest positive integer n such that $g^n = e$?
(a) 4 (b) 5 (c) 2 (d) 3

Q.95. Upon simplification, f ⊕ [f * {f ⊕ (f * f)}] equals
(a) e (b) f (c) g (d) h

Q.96. Upon simplification, $\{a^{10} * (f^{10} \oplus g^{g})\} \oplus e^{8}$ equals
(a) e (b) f (c) g (d) h

Direction for questions 97 and 98: Answer the questions on the basis of the information given below.

A string of three English letters is formed as per the following rules:

I. The first letter is any vowel.
II. The second letter is m, n or p.
III. If the second letter is m, then the third letter is any vowel which is different from the first letter.
IV. If the second letter is n, then the third letter is e or u.
V. If the second letter is p, then the third letter is the same as the first letter.

Q.97. How many strings of letters can possibly be formed using the above rules?

(a) 40 (b) 45 (c) 30 (d) 35

Q.98. How many strings of letters can possibly be formed using the above rules such that the third letter of the string is e?

(a) 8 (b) 9 (c) 10 (d) 11

[redacted]

Q.99. Let x and y be positive integers such that x is a prime and y is a composite. Then,

(a) $y - x$ cannot be an even integer

(b) xy cannot be an even integer

(c) $\frac{(x+y)}{x}$ cannot be an even integer

(d) None of these

Q.100. A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options —— air conditioning, radio and power windows were already installed.

Following were the observation of the survey:

I. 15 had air conditioning
II. 2 had air conditioning and power windows but no radios
III. 12 had radio
IV. 6 had air conditioning and radio but no power windows
V. 11 had power windows
VI. 4 had radio and power windows
VII. 3 had all three options

What is the number of cars that had none of the options?

(a) 4 (b) 3 (c) 1 (d) 2

ANSWERS

■■■

■■■ CAT 2003

Q. No.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
Answer	(b)	(a)	(c)	(a)	(c)	(b)	(a)	(b)	(c)	(b)	(c)	(a)	(c)	(b)	(b)	(b)	(c)	(a)	(a)	(b)
Q. No.	71	72	73	74	75	76	77	78	80	81	82	83	84	85	86	87	88	90	91	92
Answer	(a)	(c)	(d)	(c)	(d)	(c)	(b)	(a)	(a)	(b)	(a)	(c)	(d)	(d)	(d)	(b)	(d)	(d)	(a)	(d)
Q. No.	93	94	95	96	97	98	99	100												
Answer	(c)	(a)	(d)	(a)	(d)	(c)	(d)	(d)												

HINTS AND SOLUTIONS

51. (b) MDCCLXXXVII = 1000 + 500 + 100 + 100 + 50 + 10 + 10 + 10 + 5 + 1 + 1 = 1787

52. (a) MCMXCIX = 1000 + (1000 – 100) + (100 – 10) + (10 – 1) = 1000 + 900 + 90 + 9 = 1999

53. (c) (I) MCMLXXV = 1000 + (1000 – 100) + 50 + 10 + 10 + 5 = 1975

(II) MCMXCV = 1000 + (1000 – 100) + (100 – 10) + 5 = 1995

(III) MVD = 1000 + (500 – 5) = 1495

(IV) MVM = 1000 + (1000 – 5) = 1995

Therefore, the answer is (II) and (IV), i.e. option (c).

Questions 54 to 56

$A_1A_2 = 2r$, $B_1B_2 = 2r + r\sqrt{3}$, $C_1C_2 = 2r + 2r\sqrt{3}$ r.

Hence, $a = 3 \times 2r$

$b = 3 \times (2r + r\sqrt{3})$

$c = 3 \times (2r + 2r\sqrt{3})$

54. (a) Difference between (1) and (2) is $3\sqrt{3}$ r and that between (2) and (3) is $3\sqrt{3}$ r. Hence, (1) is the correct choice.

55. (c) Time taken by A $= \frac{2r}{20} + \frac{2r}{30} + \frac{2r}{15} = \left(\frac{2r \times 9}{60}\right) = \frac{3}{10}r$

Therefore, B and C will also travel for time $\frac{3}{10}$r

Now the speed of B $= \left(10\sqrt{3} + 20\right)$

Therefore the distance covered $= \left(10\sqrt{3} + 20\right) \times \frac{3}{10}r = \left(\sqrt{3} + 2\right) \times 10 \times \frac{3}{10}$r

$= \left(2r + \sqrt{3}r\right) \times 3 = B_1B_2 + B_2B_3 + B_3B_1$

$\therefore$ B will be at B_1.

Now the time taken by C for each distance is

$$\frac{C_1C_2}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_2C_3}{\frac{40}{3}\left(\sqrt{3}+1\right)}, \frac{C_3C_1}{120}$$

$$\frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{3}{40} \times \frac{\left(2+2\sqrt{3}\right)r}{\left(\sqrt{3}+1\right)}, \frac{\left(2+2\sqrt{3}\right)}{120}$$

i.e., $\frac{3}{40} \times 2r, \frac{3}{40} \times 2r, \frac{\left(1+\sqrt{3}\right)}{60}r$

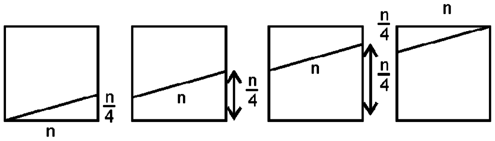
i.e., $\frac{3}{20}r, \frac{3}{20}r, \frac{\left(1+\sqrt{3}\right)}{60}r$

We can observe that time taken for C_1C_2 and C_2C_3 combined is $\frac{3}{20}r + \frac{3}{20}r = \frac{3}{10}r$, which is same as the time taken by A. Therefore, C will be at C_3.

56. (b) In similar triangles, the ratio of the area = ratio of the squares of the corresponding sides.
Hence, A and C reach A_3 and C_3 respectively.

57. (a) The whole height h will be divided into n equal parts. Therefore, the spacing between two consecutive turns is $\frac{h}{n}$.

58. (b) The four faces through which a string is passing can be shown as



Therefore, the length of the string in each face

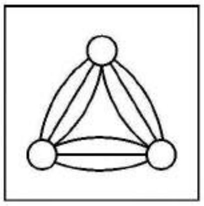
$$= \sqrt{n^2 + \left(\frac{n}{4}\right)^2}$$

$$= \sqrt{n^2 + \frac{n^2}{16}} = \frac{\sqrt{17}n}{4}$$

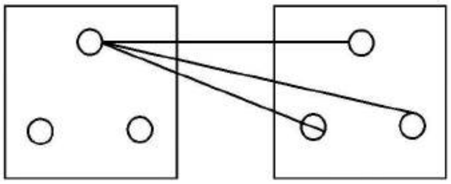
Therefore, the length of the string through four faces $= \frac{\sqrt{17}n}{4} \times 4 = \sqrt{17}n$

59. (c) As h/n = Number of turns = 1 (as given). Hence h = n.

60. (b) Consider the first zone. The number of telephone lines can be shown a follows.



Therefore, the total number of lines required for internal connections in each zone = 9 × 4 = 36 lines.
Now, consider the connection between any two zones.



Each town in the first zone can be connected to the three towns in the second zone.
Therefore, the lines required = 3 × 3 = 9
Therefore, the total number of lines required for connecting the towns of different zones $= {}^4C_2 \times 9 = 6 \times 9 = 54$.
Therefore, the total number of lines in all = 54 + 36 = 90

61. (c) PQ || AC

$$\therefore \frac{CQ}{QB} = \frac{AB}{PB} = \frac{4}{3}$$

QD||PC

$\therefore \frac{PD}{DB} = \frac{CQ}{QB} = \frac{4}{3}$

As $\frac{PD}{DB} = \frac{4}{3}$

$\therefore$ PD = $\frac{4}{7}PB$

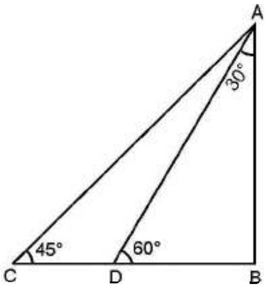
$\therefore \frac{AP}{PD} = \frac{AR}{\frac{4}{7}PB}$

$= \frac{7}{4} \times \frac{AP}{PB}$

$= \frac{7}{4} \times \frac{4}{3}$

= 7:3

62. (a)



Let AB = 1

Therefore, BC = 1

$\therefore$ tan 60 = $\frac{AB}{BD} \therefore \sqrt{3} = \frac{1}{BD}$

$\therefore \text{BD} = \frac{1}{\sqrt{3}}$

$\therefore$ CD = BC – BD

$= 1 - \frac{1}{\sqrt{3}}$

As time for travelling CD, i.e., $1 - \frac{1}{\sqrt{3}}$ is 10 min.

$\therefore$ Time required for travelling BD

$= \frac{\frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}} \times 10 \times 10$

$= \frac{1}{\sqrt{3}-1} \times 10$

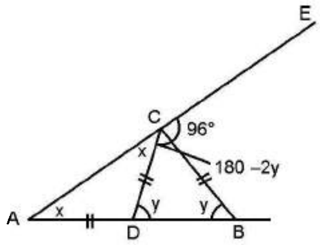
$= \frac{10}{\sqrt{3}-1}$

$= \frac{10}{\sqrt{3}-1} \times \frac{\sqrt{3}+1}{\sqrt{3}+1}$

$= \frac{10\left(\sqrt{3}+1\right)}{2}$

$= 5\left(\sqrt{3}+1\right)$

63. (c)



Using exterior angle theorem

∠A + ∠B = 96

i.e., $x + y = 96$ … (i)

Also $x + (108 - 2y) + 96 = 180°$

$\therefore x - 2y + 96 = 0$

$\therefore x - 2y = -96$

Solving (i) and (ii)

$y = 64°$ and $x = 32°$

$\therefore \angle DBC = y = 64°$

64. (b) $ax^2 + bx + 1 = 0$

For real roots

$b^2 - 4ac \geq 0$

$\therefore b^2 - 4a(1) \geq 0$

$\therefore b^2 \geq 4a$

For $a = 1$, $4a = 4$, $\therefore b = 2, 3, 4$

$a = 2$, $4a = 8$, $\therefore b = 3, 4$

$a = 3$, $4a = 12$, $\therefore b = 4$

$a = 4$, $4a = 16$, $\therefore b = 4$

65. (b) $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$

$\log_{10}\left[\frac{x}{\sqrt{n}}\right] = \log_x 100$

$\therefore \log_{10} \sqrt{x} = \frac{\log_{10} 100}{\log_{10} x}$

$\therefore \frac{1}{2} \log_{10} x = \frac{2}{\log_{10} x}$

$\therefore (\log_{10} x)^2 = 4$

$\therefore \log_{10} x = \pm 2$

$\therefore \log_{10} x = 2$ or $\log_{10} x = -2$

$\therefore 10^2 = x$ or $10^{-2} = x$

$\therefore x = 100$ or $x = \frac{1}{100}$

66. (b) Such numbers are 10, 17, ... 94.
These numbers are in an AP. There are 13 numbers.

$$\therefore \text{ Sum} = \frac{10+94}{2}\times 13$$

$$= 52 \times 13 = 676$$

67. (c) The total codes which can be formed = 9 × 9 = 81. (Distinct digit codes)

The digits which can be confusing are 1, 6, 8 and 9 from these digit we can form the codes = $4\times 3 = 12$

Out of these 12 codes two numbers 69 and 96 will not create confusion

Therefore, (12 − 2) = 10 codes will create a confusion.

Therefore the total codes without confusion = 81 − 10 = 71.

68. (a) Consider a square of side x.
Therefore, its area = x^2

Therefore, the area of the largest circle = $\pi\left(\frac{x}{2}\right)^2$

Which can be cut from the square = $\frac{\pi x^2}{4}$

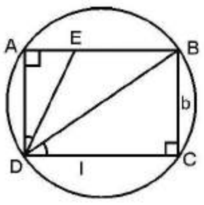
Therefore, the area scrapped

$$= x^2 - \frac{\pi}{4}x^2 = x^2\left(1-\frac{\pi}{4}\right)$$

$$\therefore \frac{\text{Area Scrapped}}{\text{Area Scrapped}} = \frac{x^2\left(1-\frac{\pi}{4}\right)}{x^2} = 1-\frac{\pi}{4} = \text{Constant}$$

As this ratio is constant whether we cut a circle from the smaller square or the larger square, the scrapped area will be a fixed percentage of the square. Therefore, in this problem, as the two squares are of the same size, the ratio will be 1 : 1

69. (a)



BD = 2r

$$\frac{\text{Area of circle}}{\text{Area rectangle}} = \frac{\pi r^r}{lb} = \frac{\pi}{\sqrt{3}}$$

$$\frac{\frac{d^2}{4}}{lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{d^2}{4lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2+b^2}{lb} = \frac{1}{\sqrt{3}}$$

$$\therefore \frac{l^2+b^2}{lb} = \frac{4}{\sqrt{3}}$$

$$\therefore \frac{l}{b}+\frac{b}{l} = \frac{4}{\sqrt{3}} \quad \ldots \text{(i)}$$

Now ΔAEB ~ ΔCBD

$$\therefore \frac{AE}{CB} = \frac{AD}{DC}$$

$$\therefore \frac{AE}{AD} = \frac{BC}{DC}$$

∴ we have to find $\frac{AE}{AD}$, i.e., $\frac{b}{l}$

Let $\frac{b}{l} = x$

Therefore, from (i) we get

$$\frac{1}{x}+x = \frac{4}{\sqrt{3}}$$

$$\frac{1+x^2}{x} = \frac{4}{\sqrt{3}}$$

$$\sqrt{3}+\sqrt{3}x^2 = 4$$

$$\therefore \sqrt{3}x^2 - 4x + \sqrt{3} = 0$$

$$\therefore x = \frac{-(-4)\pm\sqrt{16-4\left(\sqrt{3}\right)\sqrt{3}}}{2\sqrt{3}}$$

$$= \frac{4\pm\sqrt{16-12}}{2\sqrt{3}}$$

$$= \frac{4\pm 2}{2\sqrt{3}}$$

$$= \frac{6}{2\sqrt{3}}$$

Or $\frac{2}{2\sqrt{3}}$

$\frac{\sqrt{3}}{1}$ OR $\frac{1}{\sqrt{3}}$

From options, the answer is $\frac{1}{\sqrt{3}}$ i.e., $1:\sqrt{3}$.

70. (b) $\frac{1}{3}\log_3 M + 3\log_3 N = 1+\log_{0.008} 5$

$$\frac{1}{3}\log_3\left(M^{1/3}N\right) = 1+\frac{(\log 10-\log 2)}{\log 8-\log 1000}$$

$$\frac{1}{3}\log_3\left(M^{1/3}N\right)=1-\frac{(1-\log 2)}{3(1-\log 2)}$$

$$\Rightarrow \frac{1}{3}\log_3\left(M^{1/3}N\right)=1-\frac{1}{3}=\frac{2}{3}$$

Or $M^{1/3}N = 3^{2/3}$

Or $MN^9 = 3^2$

Or $N^9 = 9/M$

71. (a) Let us make the given sum by using minimum number of coins as

Value of coin	*Number of coins*	*Number of coins*	*Number of coins*	*Total number of coins*
50	1	1	1	3
25	–	–	1	1
10	1	2	2	5
5	1	–	–	1
2	2	4	3	9
Total Amount	69	78	101	19

72. (c) It is the standard property among circle, square and triangle. For a given parameter, area of a circle is the highest and area of the triangle is the least whereas area of the square is in-between i.e., c > s > t.

73. (d) Remainder when $\frac{4^{96}}{6}$

Let us come down to basic property of dividing the power of 4 by 6 i.e.,

$$\frac{4^1}{6}=4$$

$$\frac{4^2}{6}=4$$

$$\frac{4^3}{6}=4$$

$$\frac{4^4}{6}=4$$

Hence, any power of 4 when divided by 6 leaves a remainder of 4.

74. (c) $5x + 19y = 64$

We see that if $y = 1$, we get an integer solution for $x = 9$, now if y changes (increases or decreases by $5x$, will the change decrease or increase) by 19. Looking at the options, if $x = 256$, we get $y = 64$. Using these values we see options (a), (b) and (d) are eliminated and also that there exists a solution for $250 < x \le 300$.

75. (d) Sum of $\log m+\log\left(\frac{m^2}{n}\right)+\log\left(\frac{m^3}{n^2}\right)+\ldots n$ terms, such a problem must be solved by taking the value of number of the terms. Let us say 2 and check the given option. If we look at the sum of the 2 terms of the given series it comes out to be log m + log $\frac{m^2}{n} \Rightarrow \log\frac{m\times m^2}{n}=\log\left(\frac{m^3}{n}\right)$. Now look at the option and put number of terms as 2, only option (d) validates the above mentioned answer.

$$\text{As } \log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{\frac{n}{2}} \Rightarrow \log\left[\frac{m^3}{n}\right]^1 \Rightarrow \log\left(\frac{m^3}{n}\right)$$

76. (c) $$\frac{P+\frac{P}{\sqrt{2}}+\ldots\infty}{A+\frac{A}{2}+\ldots\infty}=\frac{\frac{P}{1-\frac{1}{\sqrt{2}}}}{2A}=\frac{p\sqrt{2}}{\left(\sqrt{2}-1\right)}\times\frac{1}{2A}$$

$$=\frac{\sqrt{2}P\left(\sqrt{2}+1\right)}{2A}=\frac{\sqrt{2}\times 4a\left(\sqrt{2}+1\right)}{2\times a^2}$$

$$=\frac{\sqrt{2}\times 2\left(\sqrt{2}+1\right)}{a}=\frac{2\left(2+\sqrt{2}\right)}{a}$$

77. (b) $xyz = 4$

$y - x = z - y$

$2y = x + z$

y is the AM of x, y, z.

Also $\sqrt[3]{xyz}=4^{\frac{2}{3}} \Rightarrow \sqrt[3]{xyz}=2^{\frac{1}{3}}$

AM ≥ GM

$Y \ge 2^{\frac{2}{3}}$

Therefore, the minimum value of y is $2^{\frac{2}{3}}$

78. (a) ∠BAC = ∠ACT + ∠ATC = 50 + 30 = 80°

And ∠ACT = ∠ABC (Angle in alternate segment)

So ∠ABC = 50°

∠BCA = 1870 – (∠ABC + ∠BAC)

= 180 – (50 + 80) = 50°

Since ∠BOA = 2∠BCA = 2 × 50 = 100°

Alternative Method:

Join OC

∠OCT = 90° (TC is tangent to OC)

$\angle OCA = 90° - 50° = 40°$

$\angle OAC = 40°$ (OA = OC being the radius)

$\angle BAC = 50° + 30° = 80°$

$\angle OAB = 80° - 40° = 40° = \angle OBA$ (OA = OB being the radius)

$\angle BOA = 180° - (\angle OBA + \angle OAB) = 100°$

79. Let $S = 1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}$ … (i)

$\therefore \frac{1}{7}S = \frac{1}{7}+\frac{4}{7^2}+\frac{9}{7^3}+\frac{16}{7^4}+\frac{25}{7^5}$ … (ii)

(i) – (ii) gives

$S\left(1-\frac{1}{7}\right) = 1+\frac{3}{7}+\frac{5}{7^2}+\frac{7}{7^3}+\frac{9}{7^4}$ … (iii)

$\frac{1}{7}\times S\left(1-\frac{1}{7}\right) = \frac{1}{7}+\frac{3}{7^2}+\frac{5}{7^3}+\frac{7}{7^4}$ … (iv)

(iii) – (iv) gives

$$S\left(1-\frac{1}{7}\right)-\frac{1}{7}S\left(1-\frac{1}{7}\right) = 1+\frac{2}{7}+\frac{2}{7^2}+\frac{2}{7^3}+\frac{2}{7^4}$$

$$\therefore S\left(1-\frac{1}{7}\right)\left(1-\frac{1}{7}\right) = 1+\frac{2}{7}\left[1+\frac{1}{7}+\frac{1}{7^2}+\ldots\infty\right]$$

$$\therefore S\left(1-\frac{1}{7}\right)^2 = 1+\frac{2}{7}\times\frac{1}{1-\frac{1}{7}}$$

$$\therefore S\left(\frac{6}{7}\right)^2 = 1+\frac{2}{7}\times\frac{7}{6}$$

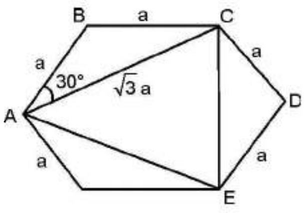
$$\therefore S\times\frac{36}{49} = 1+\frac{1}{3}$$

$$\therefore S = \frac{49}{36}\times\frac{4}{3}$$

$$S = \frac{49}{27}$$

80. (a) By observing, we see 6 will appear in 5 sets T_2, T_3, T_4, T_5 and T_6. Similarly, 12 will also appear in 5 sets and these sets will be distinct from the sets in which 6 appears, i.e. T_8, T_9, T_{10}, T_{11} and T_{12}. Thus, each multiple of 6 will be appear in 5 distinct sets. Till T_{96} there will be 16 multiples which will appear in 16 × 5 = 80 sets.

81. (b)



ΔACE is an equilateral triangle with side $\sqrt{3}a$

Area of hexagon = $\frac{\sqrt{3}}{4}a^2\times 6$

Area as $\Delta ACE = \frac{\sqrt{3}}{4}a^2\times\left(\sqrt{3}a\right)^2$

Therefore, the ratio = $\frac{1}{2}$

82. (a) Let a be the common root.

$\alpha^3 + 3\alpha^2 + 4\alpha + 5 = 0$

$\alpha^3 + 2\alpha^2 + 7\alpha + 3 = 0$

$\alpha^2 - 3\alpha + 2 = 0$

$\alpha = 2, \alpha = 1$

83. (c) $1-\frac{1}{n} < x \le 3+\frac{1}{n}$

Put $n = 1$

$\therefore 0 < x \le 4$

84. (d) $36 \le n \le 72$

$$x = \frac{n^2+2\sqrt{n}(n+4)+16}{n+4\sqrt{n}+4}$$

Put $x = 36$.

$$\therefore x = \frac{(36)^2+2\times 6\times 40+16}{36+24+4}$$

85. (d) $13x + 1 < 2z$ and $z + 3 = 5y^2$

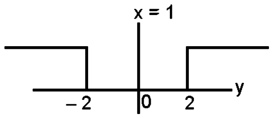
$13x + 1 < 2(5y^2 - 3)$

$13x + 1 < 10y^2 - 6$

$13x + 7 < 10y^2$ put $x = 1$

$20 < 10y^2$ $y^2 > 2$

$y^2 > \sqrt{2}$ $(y^2 - 2) > 0$



86. (d) Let $n = 6$

Therefore, $\sqrt{n} = \sqrt{6} = 2.4$

Therefore, divisors of 6 are 1, 2, 3.

If we take 2 as divisor, then $\sqrt{n} > 2 > 1$

Statement A is true

If we take 3 as divisor, then 6 > 3 > 2.4, i.e., $n > 3 > \sqrt{n}$

Therefore, statement B is true.

87. (b) $x = -|a|b$

Now $a - xb = a - (-|a|b)b$

$= a + |a|b^2$

$\therefore a - xb = a + ab^2 \ldots a \ge 0$ or $a - xb$

$= a - ab^2 \ldots a<0$

$= a(1+b^2) = a(1-b^2)$

Consider the first case:

As $a\geq 0$ and $|b|\geq 1$, therefore $(1+b^2)$ is positive.

$\therefore\ a(1+b^2)\geq 0$

$\therefore\ a - xb \geq 0$

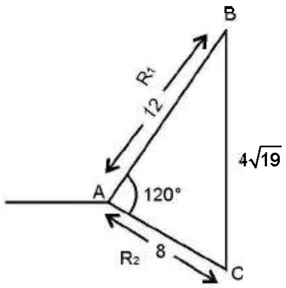
Consider the second case.

As $a<0$ and $\}b\}\geq 1$, therefore $(1-b^2)\leq 0$

$\therefore\ a(1-b^2)\geq 0$ (since –ve ax –ve = +ve and $1-b^2$ can be zero also), i.e., $a - xb \geq 0$.

Therefore, in both cases $a - xb\geq 0$.

88. (d) The required answer is $34\times 0.65\times 0.65 = 14.365$. Because we get two similar triangles and the area is proportional to the square of its side.



$BC^2 = (12)^2 + 8^2 - 2 \times 12 \times \cos 120°$

$\therefore BC = 4\sqrt{19}$

$$t_1 = \frac{4\sqrt{19}+8}{3}$$

$$t_2 = \frac{4\sqrt{19}+12}{2}$$

(Where t_1 and t_2 are the time taken by Ram and Shyam to reach the starting point.)

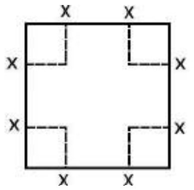
$$t_2 - t_1 = \frac{4\sqrt{19}+12}{2} - \frac{4\sqrt{19}+8}{3}$$

$$t_2 - t_1 = \frac{12\sqrt{19}+36-8\sqrt{19}+10}{6}$$

$$= \frac{4\sqrt{19}+20}{6} = \frac{2\sqrt{19}+10}{3}$$

Therefore, choice (b).

90. (d)



Volume = l × b × h

$V = (12-2x)(12-2x)\times x$

$V' = (12-2x)(12-2x)\,4x$

Where $V' = 4V$

Now sum = $12 - 2x + 12 - 2x + 4x = 24$ (constant)

As sum is constant for the maximum product $12 - 2x = 12 - 2x = 4x$

Therefore, $x = 2$.

91. (a) As any prime number greater than 3 can be expressed in the form $6n\pm 1$, the minimum difference between the three consecutive prime numbers will be 2 and 4. The values that satisfy the given conditions are only 3, 5 and 7 i.e., only one set is possible.

92. (d) $a = 6b = 12c$ and $2b = 9d = 12e$.

Dividing the first equations by 12 and second by 36.

We get, $\frac{a}{12} = \frac{b}{2} = \frac{c}{1}$ and $\frac{b}{18} = \frac{d}{4} = \frac{e}{3}$

$\therefore \frac{a}{108} = \frac{b}{18} = \frac{c}{9} = \frac{d}{4} = \frac{e}{3}$

$\therefore a{:}b{:}c{:}d{:}e = 108:18:19:4:3$

$\therefore \frac{c}{d} = \frac{9}{4}$ is not an integer

93. (c) Let the number of huts before the floods in 2001 be = 32.

∴ Huts destroyed = $\frac{32}{2}$ = 16 and rebuilt = 32.

∴ Total huts after floods in 2001 = 16 + 32 = 48

Similarly, in 2002, the destroyed huts = 24

Rebuilt = 48

Total = 24 + 48 = 72

Similarly, in 2003, the destroyed huts = 36

Rebuilt = 72

Total = 36 + 72 = 108.

∴ Checking with choices the answer is (c).

94. (a) $g^2 = g * g = h$

$g^3 = g^2 * g = h * g = f$

$g^4 = g^3 * g = f * g = e$

∴ n = 4.

95. (d) $f \oplus [f * \{f \oplus (f * F)\}]$

$= f \oplus [\, f * \{f \oplus h\}]$

$= f \oplus [f * e]$

$= f \oplus [f] = h$

96. (a) $e^8 = e^2 * e^2 * e^2$

$= e * e * e$

$= e$

If we observe a * anything = a

$\therefore a^{10} = a$

$\therefore \{a^{10} * (f^{10} \oplus g^{9})\} \oplus e^{8}$

$= a \oplus e$

$= e$

97. (d) **Case I:** ___m___

First place can be selected in five ways and hence the third in four ways

$\therefore 5 \times 4 = 20$ was

Case II: ___n___

First place can be selected in 5 ways and the third in 2 ways

$\therefore 5 \times 2 = 10$ ways

Case III: ___p___

First place can be selected in 5 ways and the last letter will be the same, i.e., one way

$\therefore 5 \times 1 = 5$ way

Total ways = 20 + 10 + 5 = 35 ways

98. (c) As the third letter is e that can be selected in only one way

Case I: 4 m e = 4 ways

Case II: 5 n e = 5 ways

Case III: e p e = 1 ways

= 10 ways

99. (d) It will go by elimination

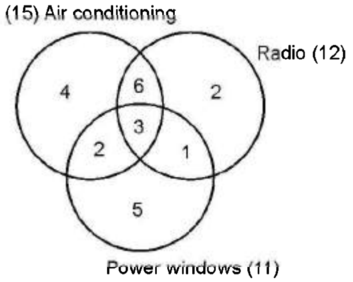
9 – 7 = 2 is even, therefore option (a) is not possible.

2 × 9 = 18 is even, therefore option (c) is not possible.

$\frac{3+9}{3} = \frac{12}{3} = 4$ is even, therefore option (c) is not possible.

$\therefore$ The correct option is (d).

100. (d)



Total = 4 + 6 + 2 + 2 + 3 +1 + 5 = 23

$\therefore$ Cars having none of the option = 25 – 23 = 2.

CAT 2004

This paper contains a total of 35 questions divided into two parts—Part A and Part—B. Part—A contains 20 questions and each question carry one mark each. Part—B contains 15 questions and each question carry two marks each.

There is a negative marking.

PART—A

Directions for questions [illegible]: Answer the questions independently of each other.

Q.39. Two boats, travelling at 5 and 10 km/h, head directly towards each other. They begin at a distance of 20 km from each other. How far apart are they (in km) one min before they collide?

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$ (c) $\frac{1}{4}$ (d) $\frac{1}{3}$

Q.40. A rectangular sheet of paper, when halved by folding it at the mid point of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle?

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\sqrt{2}$ (d) None of these

Q.41. If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms?

(a) 0 (b) -1

(c) 1 (d) Not unique

Q.42. If a man cycles at 10 km/h, then he arrives at a certain place at 1 pm If he cycles at 15 km/h, he will arrive at the same place at 11 am At what speed must he cycle to get there at noon?

(a) 11 km/h (b) 12 km/h

(c) 13 km/h (d) 14 km/h

Q.43. On 1 January 2004, two new societies S_1 and S_2 are formed, each having n members. On the first day of each subsequent month, S_1 adds b members while S_2 multiples its current numbers by a constant factor r. Both the societies have the same number of members on 2 July 2004. If $b = 1\ 0.5n$, what is the value of r?

(a) 2.0 (b) 1.9 (c) 1.8 (d) 1.7

Q.44. If $f(x) = x^3 - 4x + p$, and $f(0)$ and $f(1)$ are of opposite sign, then which of the following is necessarily true?

(a) $-1 < p < 2$ (b) $0 < p < 3$

(c) $-2 < p < 1$ (d) $-3 < p < 0$

Q.45. Suppose n is an integer such that the sum of digits on n is 2, and $10^{10} < n < 10^{11}$. The number of different values of n is

(a) 11 (b) 10 (c) 9 (d) 8

Q.46. A milkman mixes 20 litres of water with 80 litres of milk. After selling one fourth of this mixture he adds water to replenish the quantity that he had sold. What is the current proportion of water to milk?

(a) 2:3 (b) 1:2 (c) 1:3 (d) 3:4

Q.47. If $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$ then r cannot take any value except

(a) 1/2

(b) -1

(c) 1/2 or -1

(d) Cannot be determined

Q.48. Let $y =$ $y = \cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{2+\cfrac{1}{3+\ldots}}}}$

What is the value of y?

(a) $\frac{\sqrt{11}+3}{2}$ (b) $\frac{\sqrt{11}-3}{2}$

(c) $\frac{\sqrt{15}+3}{2}$ (d) $\frac{\sqrt{15}-3}{2}$

Q.49. Karan and Arjun run a 100 m race, where Karan beats Arjun by10 m. To do a favour to Arjun, Karan starts 10 m behind the starting line in a second 100 m race. They both run at their earlier speeds. Which of the following is true in connection with the second race?

(a) Karan and Arjun reach the finishing line simultaneously
(b) Arjun beats Karan by 1 m
(c) Arjun beats Karan by 2 m
(d) Karan beats Arjun by 1 m

Q.50. N persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-min song, one pair after the other. If the total time taken for singing is 28 min, what is N?

(a) 5 (b) 7
(c) 9 (d) None of the above

Q.51. In Nuts and Bolts factory, one machine produces only nuts at the rate of 100 nuts per min and needs to be cleaned for 5 mins after production of every 1,000 nuts. Another machine produces only bolts at the rate of 75 bolts per min and needs to cleaned for 10 mins after production of every 1,500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9,000 pairs of nuts and bolts?

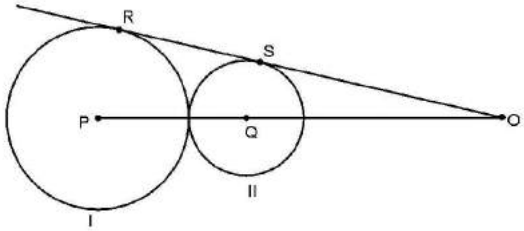
(a) 130 min (b) 135 min
(c) 170 min (d) 180 min

Q.52. A father and his son are waiting at a bus stop in the evening. There is a lamp post behind them. The lamp post, the father and his son stand on the same straight line. The father observes that the shadows of his head and his son's head are incident at the same point on the ground. If the heights of the lamp post, the father and his son are 6 m, 1.8 m and 0.9 m respectively, and the father is standing 2.1 m away from the post then how far (in m) is the son standing from his father?

(a) 0.9 (b) 0.75 (c) 0.6 (d) 0.45

Direction for questions 53 to 55: Answer the questions on the basis of the information given below.

In the adjoining figure, I and II are circles with P and Q as centres respectively. The two circles touch each other and have a common tangent that touches them at points R and S respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio of 4 : 3. It is also known that the length of PQ is 28 cm.



Q.53. What is the ratio of the length of PQ to that of QO?

(a) 1 : 4 (b) 1 : 3 (c) 3 : 8 (d) 3 : 4

Q.54. What is the radius of the circle II?

(a) 2 cm (b) 3 cm (c) 4 cm (d) cm

Q.55. The length of SO is

(a) $8\sqrt{3}$ cm (b) $10\sqrt{3}$ cm
(c) $12\sqrt{3}$ cm (d) $14\sqrt{3}$ cm

[redacted]

Q.56. Let $f(x) = ax^2 - b\,|x|$, where a and b are constants. Then at $x = 0$, $f(x)$ is

(a) maximized whenever $a > 0$, $b > 0$
(b) maximized whenever $a > 0$, $b < 0$
(c) minimized whenever $a > 0$, $b > 0$
(d) minimized whenever $a > 0$, $b < 0$

Q.57. Each family in a locality has at most two adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families, Then the minimum possible number of families in the locality is

(a) 4 (b) 5 (c) 2 (d) 3

Q.58. The total number of integers pairs (x, y) satisfying the equation $x + y = xy$ is

(a) 0
(b) 1
(c) 2
(d) None of the above

PART B—NUMBER OF QUESTIONS = 15

Note: Questions 59 to 73 carry two marks each.

[redacted]

Q.59. Let C be a circle with centre P_0 and AR be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line segment P_1B and so on. Let C_1 C_2 C_3 be circles with diameters P_0P_1, P_1P_2, P_2P_3, respectively. Suppose the circles C_1 C_2 C_3 are all shaded. The ratio of the area of the unshaded portion of C to that of the original circle is
(a) 8 : 9 (b) 9 : 10 (c) 10 : 11 (d) 11 : 12

Q.60. Consider the sequence of numbers a_1, a_2, a_3… to infinity where $a_1 = 81.33$ and $a_2 = -19$ and $a_j = a_{j-1} - a_{j-2}$ for $j \geq 3$. What is the sum of the first 6002 terms of this sequence?
(a) −100.33 (b) −30.00
(c) 0 62.33 (d) 119.33

Q.61. A sprinter starts running on a circular path of radius r metres. Her average speed (in m/min) is πr during the first 30 s, $\frac{\pi r}{2}$ during next one min, $\frac{\pi r}{4}$ during next 2 mins, $\frac{\pi r}{8}$ during next 4 mins, and so on. What is the ratio of the time taken for the nth round to that for the previous round?
(a) 4 (b) 8 (c) 16 (d) 32

Q.62. Let $u = (\log_2 x)^2 - 6\log_2 x + 12$ where x is a real number. Then the equation $x^u = 256$, has
(a) No solution for x
(b) Exactly one solution for x
(c) Exactly two distinct solutions for x
(d) Exactly three distinct solutions for x

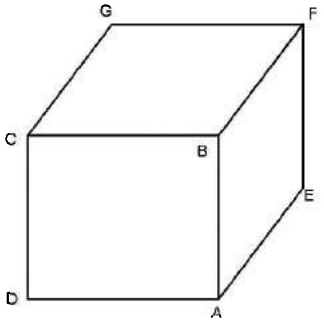
Direction for questions 63 and 64: Answer the questions on the basis of the information given below.

$f_1(x) = x \quad 0 \leq x \leq 1$
$= 1 \quad x \geq 1$
$= 0 \quad$ Otherwise
$f_2(x) = f_1(-x)$ for all x
$f_3(x) = -f_2(x)$ for all x
$f_3(x) = f_3(-x)$ for all x

Q.63. How many of the following products are necessarily zero for every x?
$f_1(x)f_2(x)$, $f_2(x)f_3(x)$, $f_2(x)f_4(x)$?
(a) 0 (b) 1 (c) 2 (d) 3

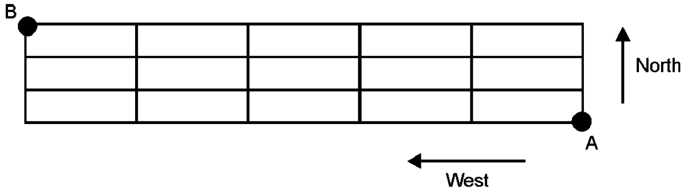
Q.64. Which of the following is necessarily true?
(a) $f_4(x) = f_1(x)$ for all x
(b) $f_1(x) = -f_3(-x)$ for all x
(c) $f_2(-x) = f_4(x)$ for all x
(d) $f_1(x) = f_3(x) = 0$ for all x

Q.65. If the lengths of diagonals DF, AG and CE of the cube shown in the adjoining figure are equal to the three sides of a triangle, then the radius of the circle circumscribing that triangle will be



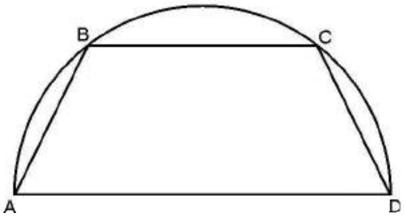
(a) Equal to the side of cube
(b) Times the side of the cube
(c) Times the side of the cube
(d) Impossible to find from the given information.

Q.66. In the adjoining figure shown below, the lines represent one-way roads allowing travel only northwards or only westwards. Along how many distinct routes can a car reach point B from point A?



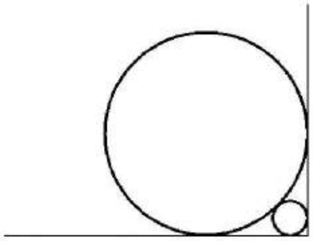
(a) 15 (b) 56 (c) 120 (d) 336

Q.67. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD have length 2, while AD has length 8. What is the length of BC?



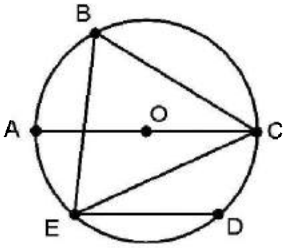
(a) 7.5 (b) 7
(c) 7.75 (d) None of these v

Q.68. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smaller circle?



(a) $3-2\sqrt{2}$ (b) $4-2\sqrt{2}$
(c) $7-4\sqrt{2}$ (d) $6-4\sqrt{2}$

Q.69. In the figure shown below, chord ED is parallel to the diameter AC of the circle. If ∠CBE = 65°, then what is the value of ∠DEC?



(a) 35° (b) 55° (c) 45° (d) 25°

Direction for questions 70 and 71: Answer the questions on the basis of the information given below.

In an examination, there are 100 questions divided into three groups A, B and C, such that each group contains at least one question. Each question in group A carries 1 mark, each question in group B carries 2 marks and each question in group C carries 3 marks. It is known that the questions in group A together carry at least 60% of the total marks.

Q.70. If group B contains 23 questions, then how many questions are there in group C?
(a) 1 (b) 2
(c) 3 (d) Cannot be determined

Q.71. If group C contains 8 questions and group B carries at least 20% of the total marks, which of the following best describes the number of questions in group B?
(a) 11 or 12 (b) 12 or 13
(c) 13 or 14 (d) 14 or 15

[redacted]

Q.72. The reminder, when $(15^{23} + 23^{23})$ is divided by 19, is
(a) 4 (b) 15 (c) 0 (d) 18

Q.73. A new flag is to be designed with six vertical stripes using some or all of the colours yellow, green, blue and red. Then, the number of ways this can be done so that no two adjacent stripes have the same colour, is
(a) 12 × 81 (b) 16 × 192
(c) 20 × 125 (d) 24 × 216

ANSWERS

■■■ CAT 2004

Q. No.	39	40	41	42	43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58
Answer	(c)	(b)	(a)	(b)	(a)	(b)	(a)	(a)	(c)	(d)	(d)	(b)	(c)	(d)	(b)	(b)	(c)	(d)	(d)	(c)
Q. No.	59	60	61	62	63	64	65	66	67	68	69	70	71	72	73					
Answer	(d)	(c)	(c)	(b)	(c)	(b)	(a)	(b)	(b)	(d)	(d)	(a)	(c)	(c)	(a)					

HINTS AND SOLUTIONS

39. (c) The boats will be colliding after a time which is given by;

After this time of 80 min, boat (1) has covered $80\times\frac{5}{60}$ km = $\frac{20}{3}$ km whereas boat (2) has covered $80\times\frac{10}{60}$ km = $\frac{40}{3}$

After 79 min, distance covered by the first boat $= d_1 = \left(\frac{20}{3}-\frac{5}{60}\right)$

After 79 min, distance covered by the second boat $= d_2 = \left(\frac{40}{3}-\frac{10}{60}\right)$

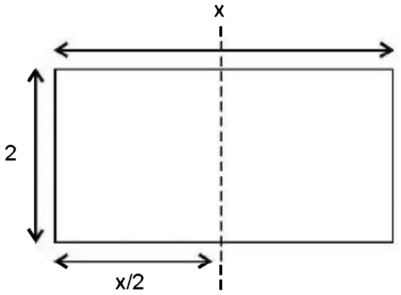
So the separation between the two boats = 20 – $(d_1 + d_2) = \frac{1}{4}$km

Alternative method

Relative speed of two boats = 5 + 10 = 15 km/h i.e., in 60 min they cover (together) = 15 km

$\therefore$ In 1 min they will cover (together) $\frac{15}{60}=\frac{1}{4}$km

40. (b)



In original rectangle ratio = $\frac{x}{2}$

In smaller rectangle ratio = $\frac{2}{\left(\frac{x}{2}\right)}$

Given $\frac{x}{2}=\frac{2}{\frac{x}{2}}\Rightarrow\times 2\sqrt{2}$

Area of smaller rectangle $=\frac{x}{2}\times 2 = x = 2\sqrt{2}$

41. (a) Given

$t_1 + t_2 + \ldots + t_{11} = t_1 + t_2 + \ldots + t_{19}$ (for an AP)

$\Rightarrow \frac{11}{2}\left[2a+(11-1)d\right]=\frac{19}{2}\left[2a+(19-1)d\right]$

$22a + 110d = 28a + 342d$

$16a + 232d = 0$

$\Rightarrow \frac{30}{2}\left[2a+(30-1)d\right]=0$

$\Rightarrow S_{30\text{ terms}} = 0$

42. (b) When speed of the man = 10 km/h = $\frac{d}{t}$ and

When speed of the man = 15 km/h = $\frac{d}{t-2}$

Equating the value of d : $10 \times t = 15 \times (t - 2) \Rightarrow$ $t = 6$ h

Finally desired speed $\frac{d}{t-1}=\frac{10t}{t-1}=\frac{10\times 6}{5}$ = 12 km/h.

43. (a) There will be an increase of 6 times. Number of members s, will be in AP. On 2 July 2004, S_1 will have $n + 6b$ members $= n + 6 \times 10.5n = 64n$

Number of members in s_2 will be in GP

On 2 July 2004, number of members in $s_2 = nr6$

They are equal, hence $64n = nr^6$

$\Rightarrow 64 = r^6 \Rightarrow r = 2$

44. (b) We have

$f(0) = 0^3 - 4(0) + p = p$

$f(1) = 1^3 - 4(1) + p = p - 3$

If P and $p - 3$ are of opposite signs then $p(p - 3) < 0$

Hence, $0 < p < 3$.

45. (a) We have

(1) $10^{10} < n < 10^{11}$

(2) Sum of the digits for 'n' = 2 clearly

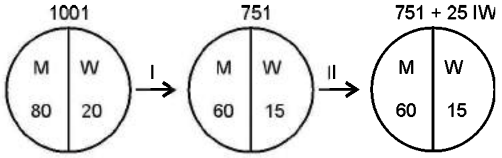
(n) min = 10000000001 (1 followed by 9 zeros and finally 1)

Obviously, we can form 10 such numbers by shifting '1' by one place from right to left again and again. Again, there is another possibility for n

n = 20000000000

So finally: Number of different values for n = 10 + 1 = 11 answer.

46. (a)



The diagram is self explanatory. Removal of 25 litres at stage I will result in the volume of milk being reduced by 80% of 25 litres i.e., 20 litres and the volume of water being reduced by the remaining 5 l. So M = 60 litres and W = 15 litres. Addition of 25 litres water will finally given M = 60 litres and W = 40 m. Hence the ratio of W and M = 40:60 = 2 : 3.

47. (c) If $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}=r$

Then there are only two possibilities.

(i)

If $a+b+c \neq 0$, then

$$\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}=$$

$$\frac{a+b+c}{(b+c)+(c+a)+(a+b)}$$

$$=\frac{a+b+c}{2(a+b+c)}=\frac{1}{2}$$

(ii)

If $a+b+c=0$, then

$b+c=-a$

$c+a=-b$

$a+b=-c$

Hence $\frac{a}{b+c}=\frac{a}{(-a)}=-1$

Similarly, $\frac{b}{c+a}=\frac{c}{a+b}=-1$

Therefore option (c) is the correct one $\frac{1}{2}$ or -1

48. (d) $y=\cfrac{1}{2+\cfrac{1}{3+y}}$

$$\Rightarrow y=\frac{3+y}{7+2y}$$

$$\Rightarrow 2y^2+6y-3=0$$

$$\Rightarrow y=\frac{-6\pm\sqrt{36+24}}{4}$$

$$=\frac{-6\pm\sqrt{60}}{4}=\frac{-3\pm\sqrt{15}}{2}$$

Since 'y' is a +ve number, therefore

$$y=\frac{\sqrt{15}-3}{2}$$

49. (d) Situation (I):

In whatever time Karan covers a distance of 100 m, Arjun covers 90 m in the same time.

Situation (II):

Now Karan is 10 m behind the starting point. Once again to cover 100 m from this new point Karan will be taking the same time as before. In this time Ariun will cover 90 metres only. This means that now both of them will be at the same point, which will be 10 metres away from the finish point. Since both of them are required to cover the same distance of 10 m now and Karan has a higher speed, he will beat Ariun. No need for calculations as option (d) is the only such option.

50. (b) Each person will form a pair with all other persons except the two beside him. Hence, he will form $(n-3)$ pairs.

If we consider each person, total pairs = $n(n-3)$ but here each pair is counted twice.

Hence, the actual number of pairs = $\frac{n(n-3)}{2}$

They will sing for $\frac{n(n-3)}{2}\times 2=n(n-3)$ min

Hence, $n(n-3)=28$

$\Rightarrow n^2-3n-28=0$

$\Rightarrow n=7$ or -4

Discarding the –ve value: $n=7$

51. (c) **Machine I:**

Number of nuts produced in one min = 100

To produce 1000 nuts the time required = 10 min

Cleaning time for nuts = 5 min

Over all time to produce 1000 nuts = 15 min

Over all time to produce 9000 nuts = 138 min – 5 min = 133 min ... (1)

Machine II:

To produce 75 bolts time required = 1 min

To produce 1500 bolts time required = 20 min

Cleaning time for bolts = 10 min

Effective time to produce 1500 bolts = 30 min

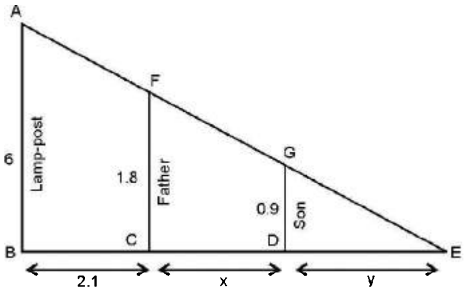
Effective time to produce 9000 bolts = $30\times 6-10$ = 170 min ... (2)

From (1) and (2)

Minimum time = 170 min

52. (d)

52.4



$\Delta ABE \sim \Delta FCE$

$$\therefore \frac{6}{1.8}=\frac{2.1+x+y}{x+y} \quad \ldots \text{(i)}$$

Also $\Delta ABE \sim \Delta GDE$

$$\therefore \frac{6}{0.9}=\frac{2.1+x+y}{x+y} \quad \ldots \text{(ii)}$$

From (i) and (ii) $x=0.45$.

53. (b) $\frac{OP}{OQ}=\frac{PR}{QS}=\frac{4}{3}$

OP = 28

OQ = 21

PQ = OP – OQ = 7

$\frac{PQ}{OQ} = \frac{7}{21} = \frac{1}{3}$

54. (b) PR + QS = PQ = 7

$= \frac{RP}{QS} = \frac{4}{3}$

$\Rightarrow$ QS = 3

55. (c) $SO = \sqrt{OQ^2 - QS^2} = \sqrt{21^2 - 3^2}$

$= \sqrt{24 \times 18} = 12\sqrt{3}$

56. (d) When $a > 0$, $b < 0$,
ax^2 and $-b\,|x|$ are non negative for all x,
i.e., $ax^2 - b|x| \geq 0$ $ax^2 - b\,|x|$ is minimum at x = O when $a > 0$, $b < 0$.

57. (d)

Family	*Adults*	*Children*
I	0, 1, 2	3, 4, 5...
II	0, 1, 2	3, 4, 5...
III	0, 1, 2	3, 4, 5...

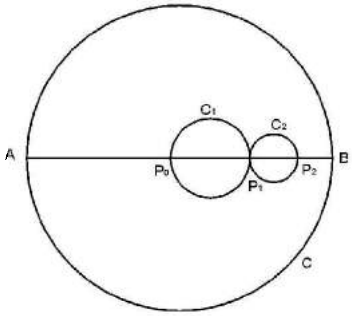
As per the question, we need to satisfy three conditions, namely:

1. Adults (A) > Boys (B)
2. Boys (B) > Girls (G)
3. Girls (G) > Families (F)

Clearly, if the number of families is 2, maximum number of adults can only be 4. Now, for the second condition to be satisfied, every family should have at least two boys and one girl each. This will result in noncompliance with the first condition because adults will be equal to boys. If we consider the same conditions for 3 families, then all three conditions will be satisfied.

58. (c) Given equation is $x + y = xy$
$\Rightarrow xy - x - y + 1 = 1$
$\Rightarrow (x - 1)(y - 1) = 1$
$x - 1 = 1$ & $y - 1 = 1$ or $x - 1 = -1$ & $y - 1 = -1$
Clearly (0, 0) and (2, 2) are the only pairs that will satisfy the equation.

59. (d)



Circle C	Radius r
C_1	$\frac{r}{4}$
C_2	$\frac{r}{8}$
C_3	$\frac{r}{16}$
:	:

$\Rightarrow$ Either $\frac{\text{Area of unshaded portion of } c}{\text{Area of } c}$

$= 1 - \frac{\text{Area of unshaded portion of } c}{\text{Area of } c}$

$= 1 - \frac{\pi\left(\left(\frac{r}{4}\right)^2 + \left(\frac{r}{8}\right)^2 + \ldots\right)}{\pi r^2}$

$= 1 - 1 - \left(\frac{1}{4^2} + \frac{1}{8^2} + \ldots\ldots\right) = 1 - \frac{\frac{1}{16}}{1 - \frac{1}{4}}$

$= \frac{11}{12}$

60. (c) Given $a_1 = 81.33$; $a_2 = -19$
Also:
$aj = a_{j-1} - a_{j-2}$, for $j \geq 3$
$\Rightarrow a_3 = a_2 - a_1 = -100.33$
$a_4 = a_3 - a_2 = -81.33$
$a_5 = a_4 - a_3 = 19$
$a_6 = a_5 - a_4 = +100.33$
$a_7 = a_6 - a_5 = +81.33$
$a_8 = a_7 - a_6 = -19$
Clearly onwards there is a cycle of 6 and the sum of terms in every such cycle = 0. Therefore, when we add a_1, a_2, a_3... upto '2' we will eventually be left with $a_1 + a_2$ only i.e., 81.33 – 19 = 62. 33.

61. (c) As options are independent of n let $n = 2$
Time taken for the first round $\frac{1}{2} + 1 + 2 + 4 =$ 7.5 min
Time taken for the second round = 8 + 16 + 32 + 64 = 120 min
Ratio $= \frac{120}{7.5} = 16$

62. (b) $u = (\log_2 x)^2 - 6\log_2 x + 12$
$x^u = 256$
Let $\log_2 x = y \Rightarrow x = 2^y$
$x^u = 2^8 \Rightarrow uy = 8 \Rightarrow u = \frac{8}{y}$
$\frac{8}{y} = t^2 - 6y + 12 \Rightarrow y^3 - 6y^2 - = 0$
$\Rightarrow (y - 2)^3 = 0 \Rightarrow y = 2$
$\Rightarrow x = 4, u = 4$

63. (c) $f_1 f_2 = f_1(x) f_1(-x)$

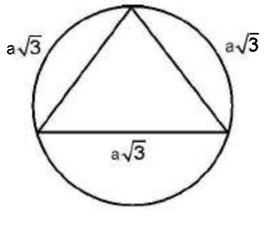
$$f_1(-x) = \begin{cases} -x & -1 \le -x - 1 \\ 1 & -x \ge 1 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} -x & -1 \le -x \le 1 \\ 1 & -x \ge 1 \\ 0 & \text{other wise} \end{cases}$$

$f_1 f_1(-x) = 0 \ \forall x$

64. (b) Check with options
Option (2)
$f_3(-x) = -f_2(-x) = -f_1(x)$
$= f_1(x) = -f_3(-x) \ \forall x$

65. (a) DF, AG and ~ E are body diagonals of cube.
Let the side of cube = a
Therefore, body diagonal is a $\sqrt{3}$

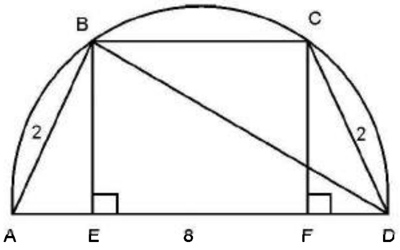


Circum radius for equilateral triangle side = $\frac{side}{\sqrt{3}}$

Therefore, $\frac{a\sqrt{3}}{\sqrt{3}} = a$

66. (b) From A to B, there are 8 on-way roads out of which 3 roads are in northwards and 5 roads are westwards.
Therefore, number of distinct routes is = $\frac{8!}{513!} = 56$

67. (b)



$\frac{1}{2} \times AB \times BD = \frac{1}{2} \times AD \times BE$

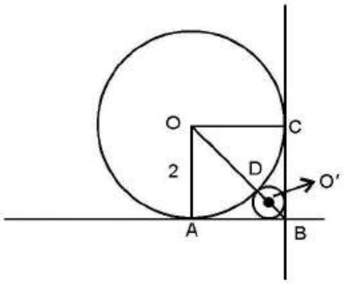
$2\sqrt{8^2 - 2^2} = 8 \times BE$

$BE = \frac{\sqrt{60}}{4} = \frac{\sqrt{15}}{2}$

AE = $AE = \sqrt{2^2 - \left(\frac{\sqrt{15}}{2}\right)^2} = \sqrt{4 - \frac{15}{4}} = \frac{1}{2}$

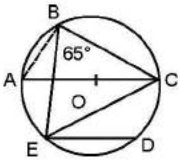
BC = EF = $8 - \left(\frac{1}{2} + \frac{1}{2}\right) = 7$

68. (d)



Let the radius of smaller circle = r
∴ O′B = r $\sqrt{2}$
∴ OB = O′B + O′D + OD
= r $\sqrt{2}$ + r + 2
Also OB = 2 $\sqrt{2}$
= r $\sqrt{2}$ + r + 2 = 2 $\sqrt{2}$
⇒ r = 6 – 4 $\sqrt{2}$

69. (d)



In ΔABC
∠B = 90° (Angles in semicircle)
Therefore ∠ABE = 90 – 65 = 25°
Also ∠ABE = ∠ACE (angel subtended by same arc AE)
Also ∠ACE = ∠CED [AC ||ED]
Therefore ∠CED = 25°

70. (a) Since Group (B) contains 23 questions, the marks associated with this group are 46. Now check for option (1). If Group (C) has one question, then marks associated with this group will be 3. This means that the cumulative marks for these two groups taken together will be 49. Since total number of questions are 100, Group (A) will have 76 questions, the corresponding weightage being 76 marks. This satisfies all conditions and hence is the correct option. It can be easily observed that no other option will fit the bill.

71. (c) Since Group (C) contains 8 questions, the corresponding weightage will be 24 marks. This figure should be less than or equal to 20% of the total marks. Check from the options . Option (c) provides 13 or 14 questions in Group (B), with, a corresponding weightage of 26 or 28 marks. This means that number of questions in Group (A) will either be 79 or 78 and will satisfy the desired requirement.

72. (c) $15^{23} = (19 - 4)^{23} - 19x + (-4)^{23}$ where x is a natural number.

$23^{23} - (19 + 4)^{23} = 19y + (4)^{23}$ where y is a natural number.

$15^{23} + 23^{23} - 19(x + y) + 4^{23} + (-4)^{23}$

$= 19\ (x + y)$

73. (a) The first strip can be of any of the four colours. The 2nd can be of any colour except that of the first (i.e., 3). Similarly, each subsequent strip can be of any colour except that of the preceding strip (= 3)

Hence, the number of ways $= 4 \times 35 = 12 \times 81$

CAT 2005

This paper contains a total of 30 questions divided into two parts—Part A and Part B. Part A contains 10 questions and each question carry one mark each. Part B contains 10 questions and each question carry two marks each.

There is a negative marking of 1/3 of the marks associated with the particular question.

PART—A

[redacted]

Q.1. If $x = (16^3 + 17^{13} + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of
(a) 0 (b) 1 (c) 69 (a) 35

Q.2. A chemical plant has four tanks (A, B, C and D), each containing 1000 litre of a chemical. The chemical is being pumped from one tank to another as follows:
From A to B @ 20 l/min
From C to A @ 90 l/min
From A to D @ 10 l/min
From C to D @ 50 l/min
From B to C @ 100 l/min
From D to B @ 110 l/min
Which tank gets emptied first, and how long does it take (in min) to get empty, after the pumping starts?
(a) A, 16.66 (b) C, 20
(c) D, 20 (d) D, 25

Q.3. Two identical circles intersect so that their centers, and the points at which they intersect, form a square of side 1 cm. The area in cm^2 of the portion that is common to the two circles is

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2}-1$ (c) $\frac{\pi}{5}$ (d) $\sqrt{2}-1$

Q.4. A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, A and B, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track. A jogs along the rectangular track while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting point?
(a) 3.88% (b) 4.22% (c) 4.44% (d) 4.72%

Q.5. In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is
(a) 200 (b) 216 (c) 235 (d) 256

Direction for questions 6 and 7: Answer the questions on the basis of the information given below.

Ram and Shyam run a race between points A and B 5 km apart. Ram starts at 9 am from A at a speed of 5 km/h, reaches B, and returns to A at the same speed. Shyam starts at 9.45 am from A at a speed of 10 km/h, reaches B, and comes back to A at the same speed.

Q.6. At what time do Ram and Shyam first meet each other?
(a) 10 am (b) 10.10 am
(c) 10.20 am (d) 10.30 am

Q.7. At what time does Shyam over take Ram?
(a) 10.20 am (b) 10.30 am
(c) 10.40 am (d) 10.30 am

Directions for questions 8 to 10: Answer the questions independently of each other.

Q.8. If $R = \dfrac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then
(a) $0 < R \le 0.1$ (b) $0.1 < R \le 0.5$
(c) $0.5 < R \le 1.0$ (d) $R > 1$

Q.9. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm?
(a) 1 or 7 (b) 2 or 14
(c) 3 or 21 (d) 4 or 28

Q.10. For which value of k does the following pair of equations yield a unique solution of x such that the solution is positive?
$x^2 - y^2 = 0$ and $(x - k)^2 + y^2 = 1$
(a) 2 (b) 0
(c) $\sqrt{2}$ (d) -2

PART—B

Note: Questions 11 to 30 carry two marks each.

Q.11. Let $n! = 1 \times 2 \times 3 \times ... \times n$ for integer $n \ge 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + ... + (10 \times 10!)$, then $p + 2$ when divided by 11! leaves a remainder of
(a) 10 (b) 0 (c) 7 (d) 1

Q.12. Consider a triangle drawn on the *X*-*Y* plane with its three vertices of (41, 0), (0, 41) and (0, 0), each vertex being represented by its (*X*, *Y*) coordinates. The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is
(a) 780 (b) 800 (c) 820 (d) 741

Q.13. The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If B> A and B-A is perfectly divisible by 7, then which of the following is necessarily true?
(a) 100<A<299 (b) 106<A<305
(c) 112<A<311 (d) 118<A<317

Q.14. If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then a_{100} equals
(a) $3^{99} - 200$ (b) $3^{99} + 200$
(c) $3^{100} - 200$ (d) $3^{100} + 200$

Q.15. Let S be the set of five-digit numbers formed by digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd position are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S?
(a) 228 (b) 216 (c) 294 (d) 192

Q.16. The right most non-zero digits of the number 30^{2720} is
(a) 1 (b) 3 (c) 7 (d) 9

Q.17. Four points A, B, C and D lie on a straight line in the *X*-*Y* plane, such that AB = BC = CD, and the length of AB is 1 m. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one m of any insect repellent. The minimum distance in m the ant must traverse to reach the sugar particle is
(a) $3\sqrt{2}$ (b) $1 + \pi$
(c) $\dfrac{4\pi}{3}$ (d) 5

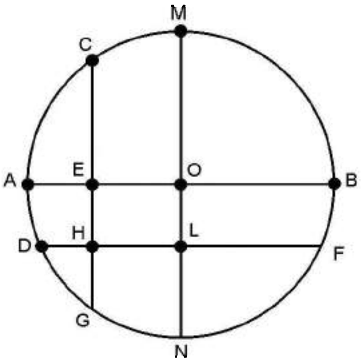
Q.18. If $x \ge y$ and $y > 1$, then the value of the expression
$\log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right)$ can never be
(a) -1 (b) -0.5
(c) 0 (d) 1

Q.19. For a positive integer n, let p_n, denote the product of the digits of n and S_n denote the sum of the digits of n. The number of integers between 10 and 1000 for which $p_n + s_n = n$ is
(a) 81 (b) 16 (c) 18 (d) 9

Q.20. Rectangular tiles, each of the size 70 cm by 30 cm must be laid horizontally on a rectangular floor of size 110 cm by 130 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should overshoot any edge of the floor. The maximum number of tiles that can be accommodated on the floor is
(a) 4 (b) 5 (c) 6 (d) 7

Q.21. In the *X*-*Y* plane, the area of the region bounded by the graph $|x + y| + |x - y| = 4$ is
(a) 8 (b) 12 (c) 16 (d) 20

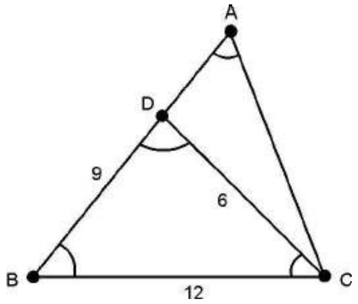
Q.22. In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition, CG is perpendicular to AB such that AE : EB = 1 : 2, and DF is perpendicular to MN such that NL : LM = 1 : 2. The length of DH in cm is



(a) $2\sqrt{2}-1$ (b) $\frac{\left(2\sqrt{2}-1\right)}{2}$

(c) $\frac{\left(3\sqrt{2}-1\right)}{2}$ (d) $\frac{\left(2\sqrt{2}-1\right)}{2}$

Q.23. Consider the triangle ABC shown in the following figure where BC = 12 cm, DB = 9 cm, CD = 6 cm and $\angle BCD = \angle BAC$



What is the ratio of the perimeter of ΔADC to that of the ΔBDC?

Q.24. P, Q, S and R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral PQSR?

(a) $2r\left(1+\sqrt{3}\right)$ (b) $2r\left(2+\sqrt{3}\right)$

(c) $r\left(1+\sqrt{5}\right)$ (d) $2r+\sqrt{3}$

Q.25. Let S be a set of positive integers such that every element n of S satisfies the conditions:

I. $1000 \le n < 1200$

II. Every digit in n is odd

Then how many elements of S are divisible by 3?

(a) 9 (b) 10 (c) 11 (d) 12

Q.26. Let $x=\sqrt{4+\sqrt{4-\sqrt{4+\sqrt{4-\ldots\infty}}}}$, then x equals

(a) 3

(b) $\left(\frac{\sqrt{13}-1}{2}\right)$

(c) $\left(\frac{\sqrt{13}+1}{2}\right)$

(d) $\sqrt{13}$

Q.27. Let $g(x)$ be a function such that $g(x+1)+g(x-1) = g(x)$ for every real x. Then for what value of p is the relation $g(x+p) = g(x)$ necessarily true for every real x?

(a) 3 (b) 3 (c) 2 (d) 6

Q.28. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs 250 and Rs 300 per day respectively. In addition, a male operator gets Rs 15 per call he answers and female operator gets Rs 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job?

(a) 15 (b) 14 (c) 12 (d) 10

Q.29. Three Englishmen and three Frenchmen work for the same company. Each of them knows a secret not known to others. They need to exchange these secrets over person-to-person phone calls so that eventually each person knows all six secrets. None of the Frenchmen knows English, and only one Englishman knows French. What is the minimum number of phone calls needed for the above purpose?

(a) 5 (b) 10 (c) 9 (d) 15

Q.30. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value of the number of tiles along one edge of the floor is

(a) 10 (b) 12 (c) 14 (d) 16

ANSWERS

CAT 2005

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(c)	(b)	(d)	(a)	(b)	(b)	(d)	(d)	(c)	(d)	(a)	(b)	(c)	(b)	(a)	(b)	(d)	(d)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(c)	(b)	(a)	(a)	(a)	(c)	(d)	(d)	(c)	(b)										

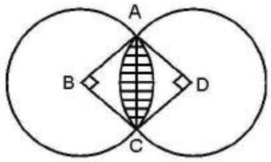
HINTS AND SOLUTIONS

1. (a) $\dfrac{a^n + b^n + c^n + \dots}{a + b + c + \dots}$ is divisible for all the odd values of n.

2. (c)

A	*B*	*C*	*D*
– 20	20		
90		– 90	
– 10			10
		– 50	50
	– 100	100	
	110		– 110
Total + 60	30	– 40	– 50

3. (b)

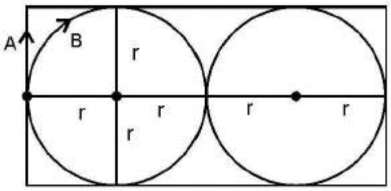


Shaded area = 2 × (area of sector ADC – area of ΔADC)

$= 2\times\left(\dfrac{\pi}{4}\times 1^2 - \dfrac{1}{2}\times 1\times 1\right)$

$= \left(\dfrac{\pi}{2} - 1\right)$

4. (d)



A covers 2r + 2r + 4r + 4r = 12r
B Covers 2πr + 2πr = 4 πr distance

$$\frac{4\pi r}{S_B} = \frac{12r}{S_A} \Rightarrow S_B = \frac{\pi}{3}S_A$$

$$\frac{S_B - S_A}{S_A}\times 100 = \frac{\pi - 3}{3}\times 100 = 4.72\%$$

5. (a) Let there be m boys and n girls

$^nC_2 = 45 = \Rightarrow n\,(n-1) = 90 \Rightarrow n = 10$

$^mC_2 = 190 \Rightarrow \dfrac{m(m-1)}{2} = 190 \Rightarrow m\ (m - 1) = 380 \Rightarrow m = 20$

The number of games between one boy and one girl

$= {}^{10}C_1 \times {}^{20}C_1 = 10 \times 20 = 200$

6. (b) Obviously, Ram and Shyam shall meet each other between C and B, sometime after 10 am. At 10 am, they are moving as shown below:
From now, the time taken to meet

$= \dfrac{2.5}{(10+5)}\times 60\,\text{min} = 10\,\text{min}$

So, they meet each other at 10.10 am.

7. (b) Obvious from the diagram that at 10.30; Shyam overtakes Ram

Alternate: At 10.15 the situation is as shown:

Time taken for Shyam to overtake Ram = $\frac{2.5}{(10+5)}\times 60\,\text{min} = 15\,\text{min}$

Shyam overtakes Ram at 10.30 a.m.

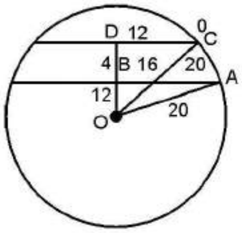
8. (d) $$\frac{30^{65}-(30-1)^{65}}{30^{64}+(30-1)^{64}}=\frac{30^{65}-30^{65}\left(1-\frac{1}{30}\right)^{65}}{30^{64}-30^{64}\left(1-\frac{1}{30}\right)^{64}}$$

Or $R = 30\left\{\frac{1-\left(1-\frac{1}{30}\right)^{65}}{1+\left(1-\frac{1}{30}\right)^{64}}\right\}$

$R = 30\left\{\frac{1-(0.96)^{65}}{1+(0.96)^{64}}\right\}$

The numerator is only slightly less then 1 and denominator is only slightly more than 1. R is slightly less than 1.

9. (d) Case I: chords on same side of the centre.



$OB^2 = OA^2 - AB^2 = 20^2 - 16^2 = 144$

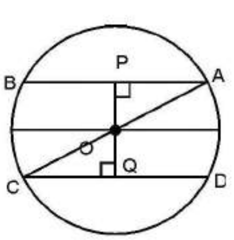
OB = 12

$OD^2 = 20^2 - 12^2 = 400 - 144 = 256$

OD = 16

BD = 4 cm

Case II: Chords on opposite side of the centre



AB = 32 cm

CD = 24 cm

$OP = \sqrt{AO^2 - AP^2}$

$= \sqrt{(20)^2-(16)^2}$

OP = 12 cm

and $OQ = \sqrt{(OC)^2-(CQ)^2}$

$= \sqrt{(20)^2-(12)^2}$

OQ = 16 cm

Distance = PQ = 12 + 16 = 28 cm.

10. (c) $y^2 = x^2$

$2x^2 - 2kx + k^2 - 1 = 0$

D = 0

$\Rightarrow 4k^2 = 8k^2 - 6$

$\Rightarrow 4k^2 = 8$

$k^2 = 2 \Rightarrow 8$

$k^2 = 2 \Rightarrow k = \pm\sqrt{2}$ with $k = \pm\sqrt{2}$

the equation is $2x^2 + 2\sqrt{2}x + 1 = 0$ roots is: $\frac{-b}{2a} = +\frac{1}{\sqrt{2}}$ but with

$k = -\sqrt{2}$, the equation is $-\frac{1}{\sqrt{2}}$ as this root is a –ve it will reject $k = -\sqrt{2}$, only answer is $\Rightarrow k = +\sqrt{2}$ only.

11. (d) If $p = 1! = 1$

Then $p + 2 = 3$ when divided by 2! remainder will be 1.

If $p = 1! + 2 \times 2! = 5$

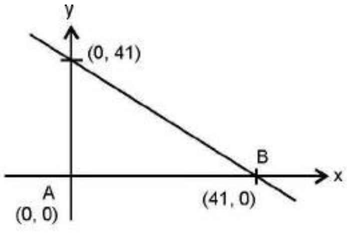
Then $p + 2 = 7$ when divided by 3! remainder is still 1.

Hence, $p = 1! + (2\times 2!) + (3\times 3!) + \ldots + (10\times 10!)$

When divided by 11! leaves remainder 1.

Hence, the remainder is 1.

12. (a)



Equation of line = $x + y = 41$. If the (x, y) co ordinates of the points are integer, their sum shall also be integers so that $x + y = k$ (l, a variable) as we have to exclude points lying on the boundary of the triangle; k can take all values from 1 to 40 only. K = 0 is also rejected as at $k = 0$ will give the point A; which can not be taken. Now, $x + y = k$ ($k = 1, 2, 3, \ldots 40$)

With $k = 40$; $x + y = 40$; taking integral solutions. We get points (1, 39), (2, 38); (3, 37)… (39, 1) i.e., 39 points

1) $x + y = 40$ will be satisfied by 39 point is similarly $x + y = 39$ is satisfied by 38 points.

$x + y = 38$ by 37 point.

$x + y = 3$ by 2 point.

$x + y = 2$ is satisfied by 1 point

$x + y = 1$ by no points.

So, the total number of all such points is: $39 + 38 + 37 + 36 + \ldots 3 + 2 + 1$

$= \dfrac{39 \times 40}{2} = 780$ points

13. (b) Let A = abc, then B = cba

Given, B > A which implies $c > a$ ... (1)

As $B - A = (100c + 10b + a) - (100a + 10b + 1)$

$B - A = 100\,(c - a) + (a - c)$

$B - A = 99\,(c - a)$ and (B−A) is divisible by 7; and 99 is not divisible by 7 (no factor like 7 or 7^2) therefore $(c-a)$ must be divisible by 7 {i.e., $(c-a)$ must be 7, 7^2 etc) as c and a are single digits. $(c-a)$ must be 7 only, the possible values (c, a) {with $c>a$} are (2) and (8,1) with this we can write A as A : $abc = 1b8$ or $2b9$

As b can take values from 0 to 9, the smallest and the largest possible values are:

$A_{min} = 108$

and $A_{max} = 299$

Only (b) satisfies so (b) is the answer.

14. (c) $a_1 = 1, \qquad a_{n+1} - 3a_n + 2 = 4n$

$A_{n+1} = 3a_n + 4n - 2$

When $n = 2$, then $a^2 = 3 + 4 - 2 = 5$

When $n = 3$, then $a_3 = 3 \times 5 + 4 \times 2 - 2 = 21$

From the options, we get an idea that a_n can be expressed in a combination of some power of 3 and some multiple of 100.

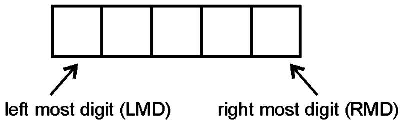
(a) $3^{99} - 200$; tells us that a_n could be: $3^{n-1} - 2 \times n$; but it does not fit a_1 or a_2 or a_3

(b) $3^{99} + 200$; tells us that a_n could be: $3^{n-1} + 2 \times n$; again not valid for a_1, a_2 etc.

(c) $3^{100} + 200$; tells $3^n + 2n$: again not valid.

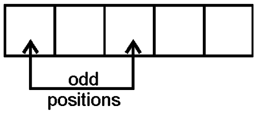
So, (c) is the correct answer.

15. (b)

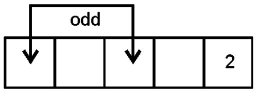


Odd positions can be counted in 2 ways.

(i) Counting form the LMD – end:



We have 1, 2, 3, 4, and 5 to be filled in these blocks. Odd numbers (1, 3, 5) to be be filled at odd positions. Other places are to be filled by even numbers (2 or 4). Let us count, how many such numbers are there with 2 at the unit's digit.



Odd numbers can be filled ${}^3P_2 = 6$ ways

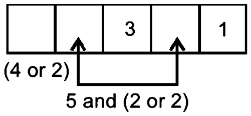
The remaining two places are to be filled by 2 numbers (one odd number left out of 1, 3, 5 and one even i.e., 4) in = 2 ways

S, there are $6 \times 2 = 12$ number with 2 at the right most place. Similarly there are 12 such numbers with 4 at the rightmost digits.

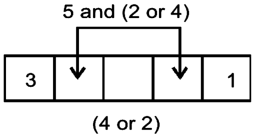
The sum of the rightmost digits in all such number $= 12\,(2 + 4) = 72$

(ii) Now counting from the RMD-end.

Let us place 1 at the unit's place and check, how many numbers are possible with (1, 3) at the odd positions:



Number of such cases $= 2 \times 2 = 4$ ways



Here again number of ways $= 2 \times 2 = 4$ ways

So, there are $4 + 4 = 8$ numbers in which (1, 3) are at odd positions. Similarly there are 8 numbers in which (1, 5) are at odd positions. So, in all there are 16 numbers, where 1 is at the unit's place. Similarly there are 16 numbers with 3 at unit's place and 16 more with 5 at the unit's place.

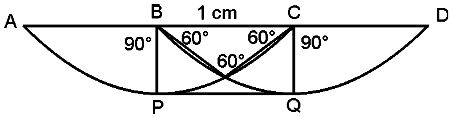
Summing up all the odd unit's digits $= 16\,(1 + 3 + 5) = 144$

From (i) and (ii) we can now sum all (even or odd) numbers at unit place $= 72 + 144 = 216$

Hence, answer is (b)

16. (a) $((30)^4)^{680} = (8100)^{680}$. Hence, the right most non-zero digit is 1.

17. (b)



Draw figure since it could not go within distance of 1 m, so it will go along APQD, which is the path of minimum distance.

$AP = \dfrac{90}{360} \times 2\pi \times 1 = \dfrac{\pi}{2}$

Also AP = QD = $\frac{\pi}{2}$

So the minimum distance = AP + PQ + QD

$= \frac{\pi}{2} + 1 + \frac{\pi}{2} = 1 + p$

18. (d) $P = \log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$

$= \log_x x - \log = x_y + \log_y y - \log_y x$

$= 2 - \log_x y - \log_y x$

Let, $t = \log_x y$

$\Rightarrow p = 2 \ -\frac{1}{t} - t = -\left[\sqrt{t} - \frac{1}{\sqrt{t}}\right]^2$

Which can never be positive, out of the given option it cannot assume a value of +1. So (d) is the answer.

19. (d) $10 < n < 1000$

Let n be two digit number

$n = 10a + b \Rightarrow p_n = ab, s_n = a + b$

Then $ab + a + b = 10a + b$

$\Rightarrow ab = 9a \Rightarrow b = 9$

There are 9 such numbers 19, 29, 33, … 99

Then let n be three digit number

$\Rightarrow n = 100a + 10b + c \Rightarrow P_n = abc, s_n = a + b + c$

Then $ab + a + b + c = 100a + 10b + c$

$\Rightarrow abc = 99a + 9b$

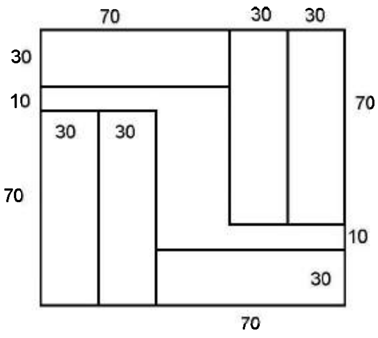
$\Rightarrow bc = 99 + 9\ \frac{b}{a}$

But the minimum value for $bc = 81$ (i.e., when both b and c are 9)

And RHS is more than 99. Hence, no such number is possible.

Hence, option (d).

20. (c)



21. (c) $|x + y| + |x - y| = 4$

Replace "$+x$" by "$-x$" & "$+y$" by "y" everywhere in the curve: we again get the same equation.

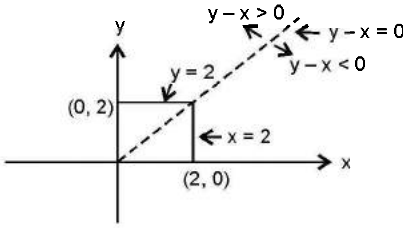
⇒ Curve is symmetric in the 4-quadrants of X–Y plane.

In I-quadrant ($x, y > 0$)

$|x + y| + |x - y| = 4$

$= \begin{cases} (x+y)+(y-x)=4; y>x \\ (x+y)-(y-x)=4; y<x \end{cases}$

Or $\begin{cases} y=2; y>x \\ x=2; y<x \end{cases}$



We can now plot the graph:

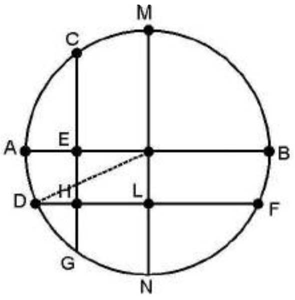
Area in L-quadrant = $(2)^2$ = 4 unit^2

Total area of $|x + y| + |x - y| = 4$ is: 4 x (area of I-quadrant)

= 4 × 4

= 16 sq. unit.

22. (b)



AE = 1 cm

BE = 2 cm and NL = 1 cm

Ml = 2 cm

HL = OE = $\frac{1}{2}$

DL = DH + HL

DL = DH + $\frac{1}{2}$

OB = AO = radius = 1.5

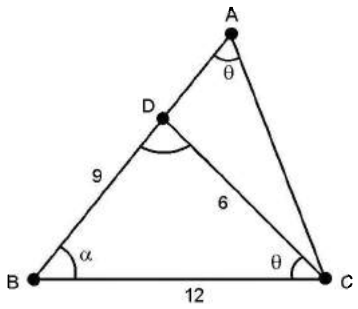
DO^2 = OL^2 + Dl^2

$\left(\frac{3}{2}\right)^2 = \left(\frac{1}{2}\right)^2 + \left(\text{DH} + \frac{1}{2}\right)^2$

$= \left(\text{DH} + \frac{1}{2}\right)^2 = 2 \Rightarrow \text{DH}\sqrt{2} - \frac{1}{2}$

Hence, option (b)

23. (a)



Hence, $\angle ACB = \theta + [180 - (20 + a] = 180 - (\theta + \alpha)$

So, here we can say that the triangle BCD and triangle ABC will be similar $\Delta BCD \sim \Delta BAC$.

Hence, from the property of similarity

$\frac{AB}{12} = \frac{12}{9}$ Hence, AB = 16

$\frac{AC}{6} = \frac{12}{9}$ Hence, AC = 8

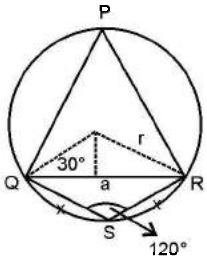
Hence, AD = 7, AC = 8

$S_{ADC} = 8 + 7 + 6 = 21$

$S_{BDC} = 27$

Hence, $r = \frac{21}{27} = \frac{7}{9}$

24. (a)



Here, $\cos 30° = \frac{a}{2r}$

$a = r\sqrt{3}$

From the diagram $\cos 120° = \frac{x^2 + x^2 - a^2}{2x^2}$

$a^2 = 3x^2$

$x = r$

Hence, the circumference will be $2r\left(1+\sqrt{3}\right)$

Hence, answer is (a).

25. (a) The 100th and 1000th position value will be only 1. Now the possibility of unit and tens digits are (1, 3), (1, 9), (3, 1), (5, 5), (7, 3), (7, 9), (9, 1), (9, 7).

26. (c) $x = \sqrt{4 + \sqrt{4 - x}} \Rightarrow x^2 = 4 + \sqrt{4 - x}$

$(X^2 - 4) = \sqrt{4 - x}$

Now put the values from the options. Only the 3rd Option satisfies the condition.

27. (d) $g(x + 1) + g(x - 1) = g(x)$

$g(x + 2) + g(x) = g(x + 1)$

Adding these two equations we get

$g(x + 2) + g(x - 1) = 0$

$\Rightarrow g(x + 3) + g(x) = 0$

$\Rightarrow g(x + 4) + g(x + 1) = 0$

$\Rightarrow g(x + 5) + g(x + 2) = 0$

$\Rightarrow g(x + 6) + g(x + 3) = 0$

$\Rightarrow g(x + 6) - g(x) = 0$

28. (d) Let us form both the equations first:

$40\,m + 50f = 1000$

$250\,m + 300f + 40 \times 15\,m + 50 \times 10 \times f = A$

$850\,m + 8000f = A$

Where m and f are the number of males and females and A is the amount paid by the service provider.

Then the possible values for f are 8, 9, 10, 11 and 12

If $f = 8$, then m = 15

If $f = 9, 10, 11$ then m will not be an integer while $f = 12$ then m will be 10.

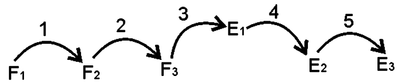
By putting f = 8 and m = 15, A = 18800. When $f = 12$ and m = 10 then A = 18100.

Hence, the number of males will be 10.

29. (c) Frenchmen: F_1, F_2, F_3

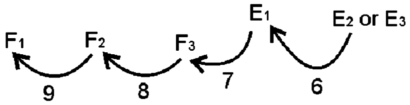
Englishmen: E_1, E_2, E_3

Let E_1 knows fresh 1 round of calls:



Persons	Secrets know after I-round
F_1	$F_{1,} F_2,$
$F_2,$	$F_{1,} F_2, F_3$
$F_3,$	$F_{1,} F_2, F_3, F_4$
$E_1,$	$F_{1,} F_2, F_3, F_2$
$E_2,$	$F_{1,} F_2, F_3, E_{1,} E_2, E_3)$ all known
$E_3,$	$F_{1,} F_2, F_3, E_{1,} E_2, E_3)$ all known

In the 6th call E_1 knows all the secrets. Similarly, after the 9th Call, everybody knows all the secrets.



30. (b) Let the rectangle has m and n tiles along its length and breadth respectively.

The number of white tiles

$W = 2m + 2\,(n - 2) = 2\,(m + n - 2)$

And the number of red tiles = = $R = mn - 2\,(m + n - 2)$ Give $W = R \Rightarrow 4\,(m + n - 2) = mn$

$\Rightarrow mn - 4m - 4n = -8$

$\Rightarrow (m - 4)\,(n - 4) = 8$

As m and n are integers so $(m - 4)$ & $(n - 4)$ are both integers. The possibilities are $(m - 4, n - 4) = (1, 8)$ or $(2, 4)$ giving, (m, n) as $(5, 12)$ or $(6, 8)$ so the edges can have 5, 12, 6 or 8 tiles. Answer is (b) only.

CAT 2006

SECTION III

Each question carries 4 marks with a negative marking of 25%.

Q.51. If $a/b = 1/3$, $b/c = 2$, $c/d = 1/2$, $d/e = 3$ and $e/f = 1/4$, then what is the value of abc/def?
(a) 3/8 (b) 27/8 (c) 3/4 (d) 27/4
(e) 1/4

Q.52. If $x = -0.5$, then which of the following has the smallest value?
(a) $21/x$ (b) $1/x$ (c) $1/x2$
(d) $2X$ (e) $1/\sqrt{-x}$

Q.53. Consider a sequence where the nth term,
$tn = n/(n + 2)$, $n = 1, 2, \ldots.$
The value of $t_3 \times t_4 \times t_5 \times \ldots \times t_{53}$ equals:
(a) 2/495 (b) 2/477
(c) 12/55 (d) 1/1485
(e) 1/2970

Q.54. Which among 21/2, 31/3, 41/4, 61/6 and 121/12 is the largest?
(a) 21/2 (b) 31/3
(c) 41/4 (4) 61/6
(e) 21/12

Q.55. The length, breadth and height of a room are in the ratio 3 : 2 : 1. If the breadth and height are halved and the length is doubled, then the total area of the four walls of the room will
(a) remain the same (b) decrease by 13.64%
(c) decrease by 15% (d) decrease by 18.75%
(e) decrease by 30%

Q.56. A survey of a hundred people was conducted to find out whether they had read the recent issues of *Golmal*, a monthly magazine. A summary of the information regarding the readership in 3 months is given below:

Only September: 18; September but not August: 23; September and July: 28;

September: 28; July: 48; July and August: 10;

None of the three months: 24.

What is the number of surveyed people who have read exactly two consecutive issues (out of the three)?
(a) 7 (b) 9 (c) 12 (d) 14
(e) 17

Q.57. A semi-circle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semi-circle at D. Given that AC = 2 cm and CD = 6 cm, the area of the semi-circle (in cm^2) will be:
(a) 32π (b) 50π (c) 40.5π (d) 81π
(e) undeterminable

Direction for questions 58 and 59: Answer Questions 58 and 59 on the basis of the information given below:

An airline has a certain free luggage allowance and it charges for excess luggage at a fixed rate per kg. Two passengers, Raja and Praja have 60 kg of luggage between them, and are charged Rs 1200 and Rs 2400 respectively for excess luggage. If the entire luggage belonged to one of them, the excess luggage charge would have been Rs 5400.

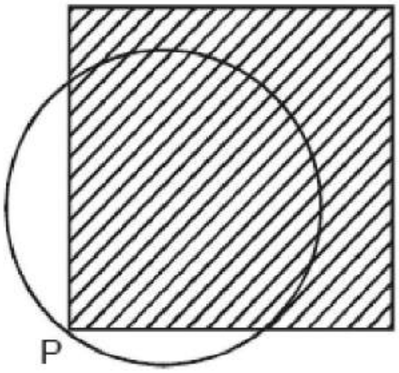
Q.58. What is the weight of Praja's luggage?
(a) 20kg (b) 25 kg (c) 30 kg (d) 35 kg
(e) 40 kg

Q.59. What is the free luggage allowance?
(a) 10kg (b) 5 kg (d) 20 kg (d) 25 kg
(e) 30 kg

Q.60. A group of 630 children is arranged in rows for a group photograph session. Each row contains three fewer children than the row in front of it. Which of the following number of rows is not possible?
(a) 3 (b) 4 (c) 5 (d) 6
(e) 7

Direction for questions 61 to 62: Answer Questions 61 and 62 on the basis of the information given below:

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal of the square.



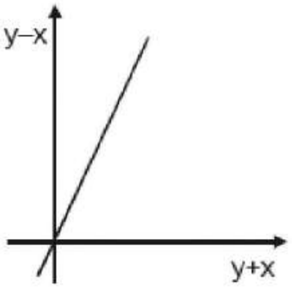
Q.61. The proportion of the sheet area that remains after punching is
(a) $(\pi + 2)/8$ (b) $(6 - \pi)/8$
(c) $(4 - \pi)/4$ (d) $(\pi - 2)/4$
(e) $(14 - 3\pi)/6$

Q.62. Find the area of the part of the circle (round punch) falling outside the square sheet.
(a) $\pi/4$ (b) $(\pi - 1)/2$
(c) $(\pi - 1)/4$ (d) $(\pi - 2)/2$
(e) $(\pi - 2)/4$

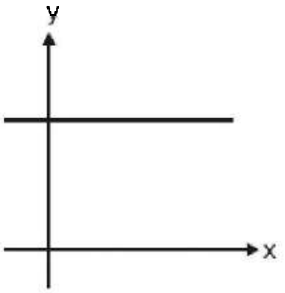
Q.63. What values of x satisfy $x^{2/3} + x^{1/3} - 2 \leq 0$?
(a) $-8 \leq x \leq 1$ (b) $-1 \leq x \leq 8$
(c) $1 < x < 8$ (d) $1 \leq x \leq 8$
(e) $-8 \leq x \leq 8$

Q.64. Consider the set S = {1, 2, 3, . . ., 1000}. How many arithmetic progressions can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements?
(a) 3 (b) 4 (c) 6 (d) 7
(e) 8

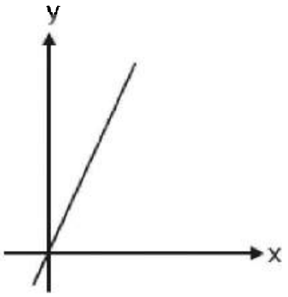
Q.65. The graph of $y - x$ against $y + x$ is as shown below. (All graphs in this question are drawn to scale and the same scale has been used on each axis.)



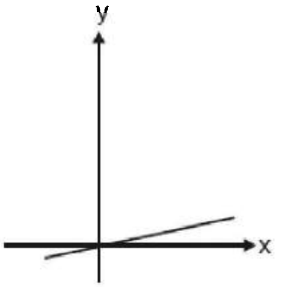
Which of the following shows the graph of y against x?



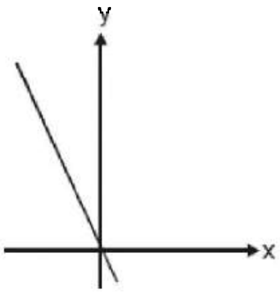
(a)



(b)

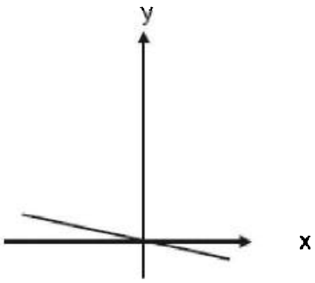


(c)



(d)

(e)



Q.66. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers?

(a) 21 (b) 25 (c) 41 (d) 67
(e) 73

Q.67. The number of solutions of the equation $2x + y = 40$ where both x and y are positive integers and $x \leq y$ is

(a) 7 (b) 13 (c) 14 (d) 18
(e) 20

Q.68. The number of employees in Obelix Menhir Co. is a prime number and is less than 300. The ratio of the number of employees who are graduates and above, to that of employees who are not, can possibly be

(a) 101 : 88 (b) 87 : 100
(c) 110 : 111 (d) 85 : 98
(e) 97 : 84

Q.69. There are 6 tasks and 6 persons. Task 1 cannot be assigned either to person 1 or to person 2, task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done?

(a) 144 (b) 180 (c) 192 (d) 360
(e) 716

Q.70. If $\log yx = (a \, . \log zy) = (b \, . \log xz) = ab$, then which of the following pairs of values for (a, b) is not possible?

(a) (–2, 1/2) (b) (1, 1)
(c) (0.4, 2.5) (d) (π,1/ π)
(e) (2, 2)

Q.71. What are the values of x and y that satisfy both the equations?

$$2^{0.7x}.3^{-1.25y} = 8\sqrt{6}/27$$

$$4^{0.3x}.9^{0.2y} = 8.(81)^{1/5}$$

(a) $x = 2, y = 5$ (b) $x = 2.5, y = 6$
(c) $x = 3, y = 5$ (d) $x = 3, y = 4$
(e) $x = 5, y = 2$

Q.72. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number. Then the minimum possible value of $f(x)$ is

(a) 1/3 (b) 1/2 (c) 2/3 (d) 4/3
(e) 5/3

Q.73. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed?

(a) 5 (b) 6 (c) 7 (d) 8
(e) 10

Q.74. An equilateral triangle BPC is drawn inside a square ABCD. What is the value of the angle APD in degrees?

(a) 75 (b) 90 (c) 120 (d) 135
(e) 150

Q.75. Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km per hour respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours after Arun did Kiranmala start?

(a) 3 (b) 3.5 (c) 4 (d) 4.5
(e) 5

ANSWERS

■■■

■■■ CAT 2006

Q. No.	51	52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70
Answer	(a)	(b)	(a)	(b)	(e)	(b)	(b)	(d)	(*)	(d)	(b)	(d)	(a)	(d)	(d)	(c)	(b)	(e)	(a)	(e)
Q. No.	71	72	73	74	75															
Answer	(e)	(e)	(b)	(e)	(c)															

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* None of the option was correct

HINTS AND SOLUTIONS

51. (a) – $a : b : c = 2 : 6 : 3$
$a : b : c : d : e : f = 6 : 18 : 9 : 18 : 6 : 24$
$abc/def = 3/8$

52. (b) Going through options,

Obviously options (a) and (d) are not possible (Since any power positive or negative raised to any positive number will be positive).

Options (c) and (d) will be positive. Only option (b) is negative.

53. (a) – $t_3 = 3/5$, $t_4 = 4/6$, t_5
$= 5/7$, ……………………..,$t_{53} = 53/55$.
$t_3 \times t_4 \times t_5 \times$ ……………………..$\times\, t_{53}$
$= 3/5 \times 4/6 \times 5/7 \times$……………………..$\times\, 53/55$
$= (3\times4)/(54\times55) = 2/495$.

54. (b) Take LCM of 2, 3, 4, 6, 12 = 12
$2^{1/2} = (2^6)^{1/12} = (64)^{1/12}$
$3^{1/3} = (3^4)^{1/12} = (81)^{1/12}$
$4^{1/4} = (4^3)^{1/12} = (64)^{1/12}$
$6^{1/6} = (6^2)^{1/12} = (36)^{1/12}$
$12^{1/12} = (12)^{1/12} = (12)^{1/12}$

55. (e) Area of 4 walls = 2 H(L + B)

Assume that the length, breadth and height of the room is 30 units, 20 units and 10 units.

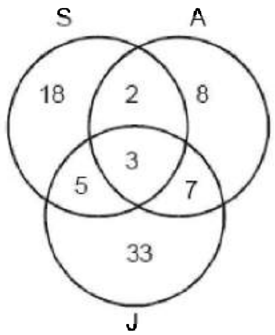
Initial total area of the four walls = 2 × 10 (50) = 1000 sq. units

After the changes, new length = 60 units, new breadth = 10 units and height = 5 units

So, new total area of the four walls = 700 sq. units

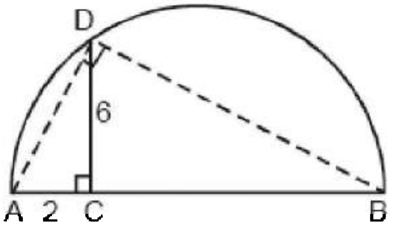
So, area decreases by 30%.

56. (b)



As given in the above diagram, total persons reading in consecutive months i.e., July and August and August and September is 2 + 7 = 9 people.

57. (b)



∠∠ADC = 90° (Anglein a semicircle)

$CD^2 = AC \times CB$

$(6)^2 = 2 \times CB$. So, CB = 18

Hence AB = AC + CB = 20

Area of semicircle = 50 π sq. units

58 and 59.

Let us assume that total allowed luggage for Raja is A and excess luggage is E

Hence, total luggage for Praja should be A + 2E.

E corresponds to Rs 1200.

Hence, A must correspond to (5400 – 3600)

= Rs 1800

If E = 2N; A = 3N

So, total weight = 2(A) + 3E = 12N

Or N = 5

Hence, Praja's luggage weight = 7N = 35 kg

It should be noted here that none of the options given in Question no. 59 was correct.

60. (d) Let the number of students in front row be x.

So, the number of students in next rows be $x - 3$, $x - 6$, $x - 9$…. so on.

If n i.e. number of rows be 3 then no. of students

$x + (x - 3) + (x - 6) = 630$, or, $3x = 639$, so, $x = 213$. Hence, we can conclude that total number of rows can be equal to 3.

Similarly, when n = 4, then

$x + (x - 3) + (x - 6) + (x - 9) = 630$, or, $4x = 648$. So number of rows = 4 is also possible.

Similarly, when $n = 5$, then

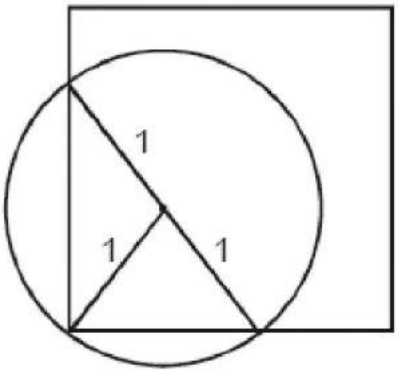
$x + (x - 3) + (x - 6) + (x - 9) + (x - 12) = 630$, or, $5x = 660$. So number of rows = 5 is also possible.

Similarly, when n = 6, then

$x + (x - 3) + (x - 6) + (x - 9) + (x - 12) + (x - 15) = 630$, or, $6x = 675$.

So, number of rows = 6 is not possible.

61. (b)

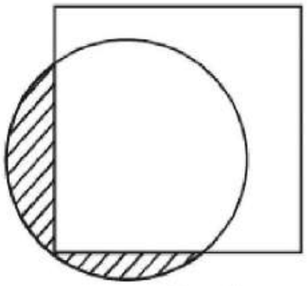


Remaining Area = $(6 - \pi)/2$

So, remaining proportion = $(6 - \pi)/8$

Hence, option (b) is the answer.

62. (d)



Area of the required area = $\pi(1)^2 - [(\pi/2) + 1] = (\pi - 2)/2$

63. (a) $x^{2/3} + x^{1/3} - 2) \leq 0$

Or, $(x^{1/3} - 1)(x^{1/3} + 2) \leq 0$

Or, $-8 \leq x \leq 1$

64. (d) Let number of terms in progression be n, then using $t_n = a + (n - 1)d$

$1000 = 1 + (n - 1)d$, or, $(n - 1)d = 999$

$= 3^3 \times 37$.

Now, making a table of values of n and d gives us the following values of (n-1):

$(n - 1) = 3$ or 9 or 27 or 37 or 111 or 333 or 999

Hence, 7 progressions are possible.

65. (d) From the graph of $(y - x)$ Vs. $(y + x)$, we can conclude

Slope of line = $(y - x) / (y + x) = p$, $0 < p < 1$.

Or, $(y - x)\,p = (y + x)$

Or, $y / x = (1 + p) / (1 - p)$ = slope of the line that shows the graph of y vs. x.

And we know $0 < p < 1$.

$(1 + p) / (1 - p)$ is –ve and is also greater than 1.

The slope of the graph y Vs. x must be negative and greater than 1. Accordingly, only option (d) satisfies.

Option (e) negative slope but absolute value is less than 1.

66. (c) Odd digits are 1, 3, 5, 7 and 9. If sum of four consecutive odd numbers is divisible by 10, then the digits should be 7, 9, 1 and 3.

Now using the option (c) – the numbers are 37, 39, 41 and 43.

67. (b) $-2x + y = 40$, $x \leq y$.

Or, $y = 40 - 2x$

Values of x and y that satisfy the equation can be calculated like:

x	y
1	38
2	36
..	
. ..	.
. ..	.
. ..	.
13 ..	14

These are 13 values.

68. (e) The sum of the numerator and denominator of the ratio should be a prime number.

Note that the number of employees is less then 300.

Consider options.

(a) 101 : 88

101 + 88 = 189

189 × 2 > 300 ⇒ Number of employees who are graduates and above = 101 and the others are 88 in number 189 is not a prime number. Option (a) is eliminated.

(b) 87 : 100

87 + 100 = 187, 187 × 2 > 300

187 is not a prime number.

(c) 110 : 111

110 + 111 = 221, 221 × 2 > 300

221 is not a prime number.

(d) 85 : 98

85 + 98 = 183, 183 × 2 > 300

183 is not a prime number.

69. (a) Task 2 can be given to two persons only viz. (3 and 4), so, Number of ways = 2 ways.

First task can be done in 3 ways by 3 persons.

And finally third task can be done by 4 persons.

So, 4 ways similarly for fourth, five and six tasks, total number of ways of doing is 3, 2 and 1 respectively.

So, total number of ways = 2 × 3 × 4 × 3 × 2 × 1= 144 ways

70. (e) $\log_v^x = a.\log_x^z = a \times b$

$$\Rightarrow a \times b = \frac{\log_y^x}{\log_z^y} \times \left(\frac{\log_y^x}{\log_z^y}\right)$$

$$\left(\frac{\log_k^x}{\log_k^y}\right)^3 = (\log_y^x)^3 = (ab)^3$$

Or, $ab - a^3b^3 = 0$, hence ab = ± 1.
Only option (e) does not satisfy.

71. (e) Equation (ii) can be written as

$$4^{0.3} \times 9^{0.2y} = 8 \times (81)^{1/5}$$

0.6x = 3, so, x = 5 and 0.4y = 4/5, hence y = 2.
If we put the values of x and y in first equation these values satisfy the first equation also.
So the answer is x = 5, y = 2

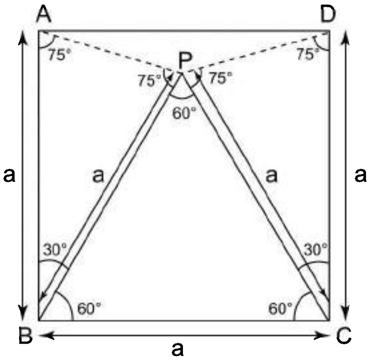
72. (e) To find out min of f(x) = max $(2x + 1, 3 - 4x)$, we should be taking the point of intersection of $(2x + 1)$ and $(3 - 4x)$. [Since one of these equations is increasing and other one is decreasing]

$2x + 1 = 3 - 4x$, or, $x = 1/3$
$2x + 1 = 3 - 4x = 5/3$

Hence option (e) is the answer.

73. (b) We know that $xy - yx = 9\,(x - y)$, where xy is a two digit number and $xy \geq yx$.
So, $x - y = 2$
The possible pairs of (x, y) is (3, 1) (4, 2) (5, 3) (6, 4), (7, 5) (8, 6) (9, 7)
But we want the number other than 13 so, there are 6 possible numbers are there i.e., 24, 35, 46, 57,68, 79.
So, total possible numbers are 6.

74. (e)



∠PBC = ∠CPB = ∠BPC (equilateral triangle) and PC = CD = a.

∠CPD = ∠ PDC = $75^{o}[(180^{o} - 30^{o})/2]$

Similarly, ∠BAP = ∠BPA = 75^{o}

Hence ∠APD = $360^{o} - (75^{o} + 75^{o} + 60^{o)} = 150^{o}$

75. (c) Generate the equations to solve this question. Alternatively, options can also be used to solve this question.

CAT 2007

There are three sections in this paper. Each section has 25 questions. Each section carries 100 marks. All questions carry four marks each. Each wrong answer will attract a penalty of one mark.

PART—A

Q.1. How many pairs of positive integers *m*, *n* satisfy:

$$\frac{1}{m}+\frac{4}{n}=\frac{1}{12}$$

Where *n* is an odd integer less than 60?

(a) 7 (b) 5 (c) 3 (d) 6
(e) 4

Q.2. A confused bank teller transposed the rupees and paise when he cashed a cheque for Shailaja, giving her rupees instead of paise instead of rupees. After buying a toffee for 50 paise, Shailaja noticed that she was left with exactly three times as much as the amount on the cheque. Which of the following is a valid statement about the cheque amount?

1. Over rupees 22 but less than rupees 23
2. Over rupees 18 but less than rupees 19
3. Over rupees 4 but less than rupees 5
4. Over rupees 13 but less than rupees 14
5. Over rupees 7 but less than rupees 8

Q.3. Suppose you have a currency, named Miso, in three denominations: 1 Miso, 10 Misos and 50 Misos. In how many ways can you pay a bill of 107 Misos?

(a) 18 (b) 15 (c) 19 (d) 17
(e) 16

Direction for questions 4 and 5: Answer the questions on the basis of the information given below.

Mr David manufactures and sells a single product at a fixed price in a niche market. The selling price of each unit is Rs 30. On the other hand, the cost, in rupees, or producing x units is $240 + bx + cx^2$, where *b* and *c* are some constants. Mr David noticed that doubling the daily production from 20 to 40 units increases the daily production cost by $66\frac{2}{3}\%$. However, an increase in daily production from 40 to 60 units results in an increase of only 50% in the daily production cost. Assume that demand is unlimited and that Mr David can sell as such as he can produce. His objective is to maximize the profit.

Q.4. How many units should Mr David produce daily?

(a) 70 (b) 150 (c) 130 (d) 100
(e) Cannot be determined

Q.5. What is the maximum daily profit, in rupees, that Mr David can realize from his business?

(a) 840 (b) 760 (c) 620 (d) 920
(e) Cannot be determined

Direction for questions 6 and 7: Answer the questions on the basis of the information given below.

Let $a_1 = p$ and $b_1 = q$, where *p* and *q* are positive quantities. Define

$$a_n = pb_{n-1}, \qquad b_n = qb_{n-1}, \text{ for even } n > 1.$$

$$\text{and} \quad a_n = pa_{n-1}, \qquad b_n = qa_{n-1}, \text{ for even } n > 1.$$

Q.6. Which or the following best describes $a_n + b_n$ for even *n*?

(a) $q^{\frac{1}{2}n}(p+q)$ (b) $q^{\frac{1}{2}n}(p+q)^{\frac{1}{2}n}$

(c) $q(qp)^{\frac{1}{2}n-1}(p+q)^{\frac{1}{2}n}$ (d) $q(oq)^{\frac{1}{2}n-1}(p+q)$

(e) $qp^{\frac{1}{2}n-1}(p+q)$

Q.7. If $p = \frac{1}{3}$ and $q = \frac{2}{3}$, then what is the smallest odd n such that $a_n + b_n < 0.01$?

(a) 11 (b) 9 (c) 15 (d) 7
(e) 13

Q.8. In a tournament, there are n teams $T_1, T_2, ..., T_n$, with $n > 5$. Each team consists of k players, $k > 3$. The following pairs of teams have one player in common: T_1 and T_2, T_2 and T_3, ..., T_{n-1} and T_n and T_n and T_1. No other pair of teams has any player in common. How many players are participating in the tournament, considering all the n teams together?

(a) $n(k-2)$ (b) $k(n-2)$
(c) $(n-1)(k-1)$ (d) $n(k-1)$
(e) $k(n-1)$

Q.9. Consider four digit numbers for which the first two digits are equal and also equal. How many such numbers are perfect squares?

(a) 4 (b) 0 (c) 1 (d) 3
(e) 2

Direction for questions 10 and 11: Answer the questions on the basis of the information given below.

Shabnam is considering three alternatives to invest her surplus cash for a week. She wishes to guarantee maximum returns on her investment. She has three options, each of which can be utilized fully or partially in conjunction with others.

Option A: Invest in a public sector bank. It promises a return of +0.10%

Option B: Invest in mutual funds of ABC Ltd. A rise in the stock market will result in a return of +5%, while a fall will entail a return of –3%.

Option C: Invest in mutual funds of CBA Ltd. A rise in the stock market will result in a return of –2.5%, while a fall will entail of +2%.

Q.10. The maximum guaranteed return to Shabnam is

(a) 0.20% (b) 0.15%
(c) 0.30% (d) 0.25%
(e) 0.10%

Q.11. What strategy will maximize the guaranteed return to Shabnam?

(a) 64% in option B and 36% in option C
(b) 1/3 in each of the three options
(c) 30% in option A, 32% in option B and 38% in option C
(d) 100% in option A
(e) 36% in option B and 64% in option C

Direction for questions 12 and 13: Answer the questions on the basis of the information given below.

Let S be the set of all pairs (I, j) where $1 \leq i < j \leq n$, and $n \geq 4$. Any two distinct members of S are called "friends" if they have one constituent of the pairs in common and "enemies" otherwise. For example, if $n = 4$, then S = {(1, 2), (1, 3), (1, 4), (2, 3), (2, 4), (3, 4)}. Here, (1, 2) and (1, 3) are friends, (1, 2) and (2, 3) are also friends, but (1, 4) and (2, 3) are enemies.

Q.12. For general n, how many enemies will each members of S have?

(a) $2n - 7$ (b) $5 - \frac{5k}{2}$ or $2k - 3$
(c) $\frac{1}{2}(n^2 - 7n + 16)$ (d) $n - 3$
(e) $\frac{1}{2}(n^2 - 3n - 2)$

Q.13. For general n, consider any two members of S that are friends. How many other members of S will be common friends of both these members?

(a) $\frac{191}{8} \approx 24$ (b) $n - 2$
(c) $\frac{1}{2}(n^2 - 7n + 16)$ (d) $\frac{1}{2}(n^2 - 5n + 8)$
(e) $2n - 6$

Q.14. Two circles with centres P and Q cut each other at two distinct points A and B. The circles have the same radii and neither P nor Q falls within the intersection of the circles. What is the smallest range that includes all possible values of the angle AQP in degrees?

(a) Between 0 and 60 (b) Between 0 and 75
(c) Between 0 and 45 (d) Between 0 and 90
(e) Between 0 and 30

Q.15. The price of Darjeeling tea (in rupees per kilogram) is $100 + 0.10n$, on the nth day of 2007 (n = 1, 2, ... 100). And then remains constant. On the other hand, the price of Ooty tea (in rupees per kilogram) is $89 + 0.15n$, on the nth day of 2007 (n = 1, 2, ...,, 365). On which date in 2007 will the prices of these two varieties of tea be equal?

(a) May 20 (b) April 10
(c) June 30 (d) May 21
(d) April 11

Q.16. A quadratic function f(x) attains a maximum of 3 at $x = 1$. The value of the function at $x = 0$ is 1. What is the value of f (x) = 10?

(a) –110 (b) –180 (c) –105 (d) – 119
(e) – 159

Direction for questions 17 and 18: Answer the questions on the basis of the information given below.

Cities A and B are in different time zones. A is located 3000 km east of B. The table below describes the schedules of an airline operating non-stop flights between A and B. All the times indicated are local and on the same day.

Departure		**Arrival**	
City	Time	City	Time
B	8:00 am	A	3:00 pm
A	4:00 pm	B	8:00 pm

Assume that planes cruise at the same speed in both directions. However, the effective speed is influenced by a steady wind blowing from east to west at 50 km per hour.

Q.17. What is the time difference between A and B?
(a) 2 hours and 30 minutes
(b) 1 hour
(c) 1 hour and 30 minutes
(d) 2 hours
(e) Cannot be determined

Q.18. What is the plane's cruising speed in km per hour?
(a) 600 (b) 500 (c) 700 (d) 550
(e) Cannot be determined

Q.19. A function $f(x)$ satisfies $f(1) = 3600$, and $f(1) + f(2) + f(3) + \ldots\ldots + f(n) = n^2 f(n)$, for all positive integers $n > 1$. What is the value of $f(9)$?
(a) 200 (b) 100 (d) 120 (d) 80
(e) 240

Q.20. Consider the set S = {2, 3, 4, ..., 2n + 1}, where n is a positive integer larger than 2007. Define X as the average of the odd integers in S and Y as the average of the even integers in S. What is the value of X – Y?
(a) $\frac{1}{2}n$ (b) $\frac{n+1}{2n}$
(c) 2008 (d) 0
(e) 1

Q.21. Ten years ago, the ages of the members of a joint family of eight people added up to 231 years. Three years later, one member died at the age of 60 years and a child was born during the same year. After another three years, one more member died, again at 60, and a child was born during the same year. The current average age of this eight-member joint family is nearest to
(a) 21 years (b) 25 years
(c) 24 years (d) 23 years
(e) 22 years

Direction for questions 22 and 25: Answer the questions on the basis of the information given below.

Each question is followed by two statements A and B. Indicate your responses based on the following directives:

Mark (a): If the question can be answered using A alone but not using B alone.

Mark (b): If the question can be answered using B alone but not using A alone.

Mark (c): If the question can be answered using A and B together, but not using either A or B alone.

Mark (d): If the question cannot be answered even using A and B together.

Q.22. ABC Corporation is required to maintain at least 400 Kilolitres of water at all times in its factory, in order to meet safety and regulatory requirements. ABC is considering the suitability of a spherical tank with uniform wall thickness for the purpose. The outer diameter of the tank is 10 meters. Is the tank capacity adequate to meet ABC's requirements?
A: The inner diameter of the tank is at least 8 meters.
B: The tank weighs, 30,000 kg when empty, and is made of a material with density of 3 gm/cc.

Q.23. Consider integers x, y and z. What is the minimum possible value of $x^2 + y^2 + z^2$?
A: $x + y + z = 89$
B: Among x, y, z two are equal

Q.24. The average weight of a class of 100 students is 45 kg. The class consists of two sections, I and II, each with 50 students. The average weight, W_I, of Section I is smaller than the average weight, W_{II}, of Section II. If the heaviest student, say Deepak, of Section II is moved to Section I, and the lightest student, say Poonam, of section I is moved to section II then the average weights of the two sections are switched, i.e., the average weight of Section I becomes W_{II} and that of Section II becomes W_I. What is the weight of Poonam?
A: $W_{II} - W_I = 1.0$
B: Moving Deepak from Section II to I (without any move from I to II) makes the average weights of the two sections equal.

Q.25. Rahim plans to draw a square JKLM with a point O on the side JK but is not successful. Why is Rahim unable to draw the square?
A: The length of OM is twice that of OL.
B: The length of OM is 4 cm.

ANSWERS

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Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(b)	(a)	(d)	(d)	(d)	(b)	(d)	(c)	(a)	(a)	(b)	(b)	(a)	(a)	(e)	(b)	(d)	(d)	(e)
Q. No.	21	22	23	24	25															
Answer	(c)	(b)	(a)	(c)	(a)															

HINTS AND SOLUTIONS

1. (c) $\frac{1}{m}=\frac{1}{12}-\frac{4}{n}$

$\Rightarrow m=\frac{12n}{n-48}$

Since m is positive, n must be greater than 48. Possible odd values of n such that $48 < n < 60$ are 49, 51, 53, 55, 57 and 59 and 49, 51 and 57 give integral values of m. Hence (c).

2. (b) Let the original amount be Rs x and y paise. Then interchanged amount = Rs y and x paise. From the given condition:

$3\,(100x + y) = 100y + x - 50$

$\Rightarrow 300x + 3y = 10y + x - 50$

$\Rightarrow 299x = 97y - 50$

$\Rightarrow y=\frac{299x+50}{97}$

Considering the options, only for $x = 18$, we get an integral value for y i.e., $y = 56$. Hence, (b)

3. (a) Possible combinations

Case 1: Bill is paid with only two 50's misos

$2 \times 50 + 1 \times 10 + 7 \times 1$ ... 1 way

Case 2: Bill is paid with only one 50 miso

$$\left.\begin{array}{c}1\times 50+5\times 10+7\times 1\\ 1\times 50+4\times 10+17\times 1\\ 1\times 50+3\times 10+27\times 1\\ . \quad . \quad .\\ . \quad . \quad .\\ . \quad . \quad .\\ 1\times 50+0\times 10+57\times 1\end{array}\right\}\text{i.e., 6 ways}$$

Case 3: Bill is paid with only 50 misos

$$\left.\begin{array}{c}10\times 10+7\times 1\\ 9\times 10+17\times 1\\ . \qquad .\\ . \qquad .\\ . \qquad .\\ 1\times 10+97\times 1\end{array}\right\}\text{i.e., 10 ways}$$

Case 4: Bill is paid with only 10 misos and only 50 misos

1×107 ... 1 way

Therefore, total number of ways = 1 + 6 + 10 + 1 = 18 ways

4. (d)

Quantity produced	CP	SP	Profit
x	$240+bx+cx^2$	$30x$	$30x-240-bx-cx^2$
20	$240+20b+400c$	600	$600-240-20b-400c$
40	$240+40b+1600c$	1200	$1200-240-40b-600c$
60	$240+60b+3600c$	1800	$1800-240-b+3600c$

From the given conditions,

$$(240+40b+1600c)=\frac{5}{3}(240+20b+400c)\left(\frac{\pi}{2}-\theta\right) \quad \text{(i)}$$

Also

$$240+60b+3600c=\frac{3}{2}(240+40b+1600b)$$

$$=\frac{5}{2}(240+20b+400c) \quad \text{(ii)}$$

From (i)

$2800c + 20b - 480 = 0$ (iii)

$200c + 20b - 720 = 0$ (iv)

$2400c = 240$

$$c = \frac{1}{10} \Rightarrow b = 10$$

Profit on x units $= f(x) = 30x - 240 - 10x - \frac{x^2}{10}$

i.e., $f(x) = -\frac{x^2}{10} + 20x - 240$

$f(x)$ is maximum at x if $f'(x) = 0$

i.e., $-\frac{2x}{10} + 20 = 0$

$2x = 200$, or, $x = 100$

5. (d) Maximum daily profit $= f(100)$

$= -1000 + 2000 - 240$

= Rs 760

Answers to 6 and 7:

$a_1 = p, b_1 = q$

$n = 2$: $a_2 = p\,b_1 = pq$; $b_2 = q \quad b_1 = q^2$

$n = 3$: $a_3 = pq_2 = p^2q$; $b_3 = qa_2 = pq^2$

$n = 4$: $a_4 = pb_3 = p^2q^2$; $b_4 = qb_3 = pq^3$

$n = 5$: $a_5 = pa_4 = p^3q^2$; $b_5 = qa_4 = p^2q^3$

$n = 6$: $a_6 = pb_5 = p^3q^3$; $b_6 = qb_2 = p^2q^4$

$n = 7$: $a_7 = pa_6 = p^4q^3$; $b_7 = qa_6 = p^3q^4$

6. (d) $a_2 + b_2 = pq + q^2 = q(p+q)$

$a_4 + b_4 = p^2q^2 + pq^3 = pq^2(p+q) = q(pq)(p+q)$

$a_6 + b_6 = p^3q^3 + p^2q^4 = p^2q^3\,(p+q) = q(pq)^2(p+q)$

Hence in general, $a_n + b_n = q(pq)^{\frac{1}{2}n-1}(p+q)$

7. (b) $p = \frac{1}{3}, q = \frac{2}{3} \Rightarrow p + q = 1 \, and \; pq = \frac{2}{9}$

i.e., $a_1 + b_1 = 1$

Now, $a_3 + b_3 = pq(p+q) = pq$

$$a_5 + b_5 = (pq)^2(p+q) = (pq)^2$$

$$a_7 + b_7 = (pq)^3(p+q) = (pq)^3$$

In general, for odd 'n' and $p = \frac{1}{3}, \quad q = \frac{2}{3}$

$$a_n + b_n = (pq)^{\frac{(n-1)}{2}} = \left(\frac{2}{9}\right)^{\left(\frac{n-1}{2}\right)}$$

Starting from the smallest option

$$a_7 + b_7 = \left(\frac{2}{9}\right)^3 = 0.01$$

$$a_9 + b_9 = \left(\frac{2}{9}\right)^4 = 0.002 < 0.01$$

8. (d) Total number of teams = n and

Number of players in each team = k

Number of players common to two teams = number of teams = n

Hence, total number of players participating in the tournament

$= nk - n = n\,(k - 1)$

9. (c) Let the four-digit number be

$1000a + 100a + 10b + b = 1100a + 11b$

This number will be a perfect square if

$1100a + 11b = k^2$; where k is an integer

$$\Rightarrow 11(100a + b) = k^2$$

$$\Rightarrow 100a + b = \frac{k^2}{11}$$

Therefore, k should be a multiple of 11 such that $100a + b$ is a 3-digit number $k = 44, 55, 66, \ldots 99$. Corresponding values of $100a + b$ will be 176, 275,396,539,704,891.

Therefore, $a, b < 9$ only '704' satisfies this.

Therefore, $1100a + 11b = 7700 + 44 = 7744 = 88^2$

Alternatively, we can solve this question by using elimination and fundas of number system.

Unit digit of a perfect square can be – 0, 1, 4, 5, 6, 9

Only these digits will get repeated at the last two places of the said square in this question.

So, the last two digits can only be – 00, 11, 44, 55, 66, 99

A perfect square cannot have 11, 55, 66, 99 as its last two digits.

(Reason – A perfect square, when divided by 4, should give either of 0 or 1 as the remainder).

So, now we are left with = 00 and 44.

Perfect square lies in between 1000 and 9999 (both including). If the last two digits are 00, then the unit digit should be '0' in the number. Then it is not possible to have other two digits (at the thousands and hundreds place) same.

So the only option left, if possible = 44

Taking a cue from $12^2 = 144$, $88^2 = 7744$

This is the only number satisfying the condition given in the question.

Note:

1. The same question with a different format was asked in CAT 99. The question was – Given $(aa)^2 =$ bbcc, where a, b and c are non-zero digits. Find the value of b.
2. $38^2 = 1444$ is another such interesting number. "444" occurring at the end of this square is the maximum number of times a non-zero digit is occurring at the end of any perfect square.

10. (a) Let the amount invested in option B and C be in the ratio 1 : K

So, depending on whether there is a rise or fall in the stock market, The amount earned will be

$$5-\frac{5k}{2} \text{ or } 2k-3.$$

Therefore, guaranteed return

$$= \min\left\{\frac{5-5k}{2}, 2k-3\right\}$$

Therefore, the maximum guaranteed will be earned when $\frac{5-5k}{2} = 2k-3$ .i.e., $9k = 16$

i.e., $k = \frac{16}{9}$.

Therefore, the maximum guaranteed return is when, the amounts invested are in the ratio 9 : 16 i.e., 36% and 64% respectively. Now, the guaranteed return for this distribution is 0.2% (see Q.11). Since option A gives a return of 0.1% which is lesser than this, no amount should be invested in option A.

Hence, maximum guaranteed return = 0.20%.

11. (a) Let us assume that Shabnam has Rs 100

We calculate her guaranteed return in case of each of the given options:

Option (d): 0.1% of 100 = 0.1

Option (e) : If there is a rise in the stock market, earning = 5% of 36 – 2.5% of 64

$$\frac{-108+128}{100} = 0.2$$

If there is a fall in the stock market, earning = –3% of

$$36 + 2\% \text{ of } +64 = \frac{-108+128}{100} = 0.2$$

Therefore guaranteed return = 0.2

Option (a): Rise in market gives earning = 5% if 64 + 2.5% of 36 = 2.4

Fall in market gives earning = –3% of 64+ 2% of 36 = –1.2 which is negative. Similarly, the guaranteed returns for options (b) and (c) are also negative.

Therefore option (e) offers the highest guaranteed return.

12. (b) A member (a, b) will have an enemy of the form (c, d) where c, d, a, b are all distinct. Therefore c and d can be chosen in ${}^{n-2}C_2$ ways.

So, number of enemies

$$= \frac{(n-2)(n-3)}{2} = \frac{1}{2}(n^2 - 5n + 6)$$

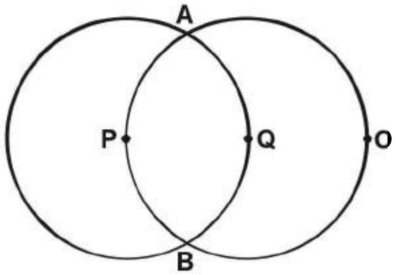
13. (b) Consider friends (a, b) and (a, c).

Their common friend can be either (b, c) or a member of the form (a, d) or (d, a) where d is different from a, b, c.

d can be chosen in $(n - 3)$ ways.

So, number of common friends = $(n - 3) + 1 = n - 2$.

14. (a) P and Q do not lie within intersection of the circles. The extreme case can be that they lie on the circumference of the other circle as shown in the figure.



In this case ΔAPQ will be an equilateral triangle.

Therefore, m $\angle AQP$ is more than 60°, then P and Q will lie within the intersection of the circles.

15. (a) From $100 + 0.10n = 89 + 0.15n$, n will be definitely greater than 100

Price of Darjeeling tea on 100th day and onwards = 100 + 0.10 (100) = Rs 110

Now, price of Ooty tea will be Rs 110 When $89 + 0.15n = 110$

$\Rightarrow 0.15n = 21 \Rightarrow n = 140$

therefore prices will be equal on 140th day i.e., May 20.

16. (e) Let $f(x) = ax^2 + bx + c$

f attains a maximum at $x = 1$

$f (x) = 0$

$\Rightarrow 2ax + b = 0$

$\Rightarrow x = \frac{-b}{2a} = 1$

$\Rightarrow \; -b = 2a$

Also max $f(x) = 3$

$\Rightarrow \; a + b + c = 3$

$\Rightarrow \; a - 2a + c = 3$

$\Rightarrow \; c - a = 3$

$f(0) = 1, \Rightarrow c = 1$ and $a = -2$

Therefore $f(x) = -2x^2 + 4x + 1$

$F(10) = -2(100) + 4(10) + 1 = -200 + 41 = -159$

Answers to questions 17 and 18:

Let the speed of the plane be x km/h

Then its speed from B to A = $(x + 50)$ km/ h

and its speed from A to B = $(x - 50)$ km/ h

The fight starts from city B (8.00 am) and arrives at city B (8.00 am) after taking a halt for 1 hour in city A

Therefore Total time taken = 11 hours

i.e., $\frac{3000}{x+50} + \frac{3000}{x-50} = 11$

$\Rightarrow \frac{2x}{x^2 - 2500} = \frac{11}{3000}$

$\Rightarrow 11x^2 - 6000x - 27500 = 0$

Solving the quadratic equation, $x = 550$

Therefore, time taken from B to A = $\frac{3000}{500}$ =6

Flight reaches A when local time at B is 2.00 p.m. which is same as local time 3.00 p.m. at A. Therefore, required time difference = 1 hour

17. (b)

18. (d)

19. (d) $f(n) = \frac{f(1) + f(2) + ... + f(n-1)}{n^2 - 1}$

$f(1) = 3600$

$f(2) = \frac{f(1)}{3} = \frac{3600}{3} = 1200$

$f(3) = \frac{f(1) + f(2)}{8} = \frac{4800}{8} = 600$

$f(4) = \frac{f(1) + f(2) + f(3)}{15} = \frac{5400}{15} = 360$

$f(5) = \frac{f(1) + ... + f(4)}{24} = \frac{5760}{24} = 240$

$f(6) = \frac{f(1) + ... + f(5)}{35} = \frac{6000}{35} = \frac{1200}{7}$

$f(7) = \frac{f(1) + ... + f(6)}{48} = \frac{6000}{48} + \frac{1200}{7 \times 48}$

125 + 25/7

$f(8) = \frac{f(1) + ... f(7)}{63} = \frac{6125}{63} + \frac{1225}{7 \times 63} = 100$

Similarly $f(9) = 80$

20. (e) S = { 2,3,4,…2n +1}

Total number of elements in S = 2n

$X = \frac{3 + 5 + ... + (2n+1)}{n}$

$Y = \frac{2 + 4 + 6 ... + 2n}{n}$

Therefore X – Y $= \frac{1 + 1 + ... + 1(n \text{ times})}{n} = \frac{n}{n} = 1$

21. (c) 10 years ago, total age of 8 members = 231

Three years later, sum of the ages
= 231 + 8 × 3 – 60 = 195

Three more years later, sum of the ages
= 195 + 8 × 3 – 60 = 159

Sum of current ages = 159 + 8 × 4 = 191

Now, current average age $= \frac{191}{8} \approx 24$ years

22. (b) Using statement A:

For minimum diameter i.e., 8 m, the capacity of the tank $= \frac{4}{3} \times \frac{22}{7} \times 4^3 \text{ m}^3 = 268.19 < 400$

For diameter greater than 8m, for example radius = 4.95 m, capacity of the tank = 508.25.71 > 400

Hence, statement A alone cannot be used to answer the question.

Using statement B:

Volume of material used = $\frac{\text{Mass}}{\text{Density}}$ = V (say), which is given

Therefore Outer volume – Inner Volume = V

Therefore, we can find the inner volume and question can be answered.

23. (a) Using statement A:

$x + y + z = 89$

For $x^2 + y^2 + z^2$ to be minimum, each of x, y, z must take integral value nearest to $\frac{89}{3}$

Let $x = 30, y = 30, z = 29$

Therefore, minimum value of

$x^2 + y^2 + z^2 = (30)^2 + (30)^2 + (29)^2 = 2641$

Thus, statement A alone is sufficient to answer the question.

24. (c) W_I = Average weight of Section I

W_{II} = Average weight of section II

$W_I + W_{II} = 90$ where $W_1 < W_{II}$

Let weight of Deepak and Poonam be D and P kgs respectively

$$\frac{50 \times W_{II} - D + P}{50} = W_I \text{ and } \frac{50 \times W_{II} - D + P}{50} = W_I$$

$\Rightarrow$ $50 (W_{II} - W_I) = D - P$

Using Statement A alone:

$50 \times 1 = D - P$

Thus, D and P can take various values.

So, statement A alone

$50 \times 1 = D - P$ (i)

Thus, D and P can take various values.

Hence, Stat. A alone is not sufficient.

Using Stat. B alone:

$$\frac{50 \times W_I + D}{51} = \frac{50\ W_{II} - D}{49} \quad \text{(ii)}$$

Since values of W_I and W_{II} are not known

We cannot find the value of D.

Combining both the statements,

Values of W_I and W_{II} can be found and hence value of D and P can be found, using (i) and (ii).

25. (a) The farthest point from point M which is on the square is the diagonally opposite point.

Length OM $\le \sqrt{2}$ × side of the square ⇒ Length OM $\le \sqrt{2}$ × length of LK (i)

And length of OL ≥ length of LK (ii)

From (i) and (ii) we can conclude that such a point can not be drawn.

CAT 2008

SECTION I

This section contains 25 questions.

Q.1. The integers 1, 2, ..., 40 are written on a blackboard. The following operation is then repeated 39 times: In each repetition, any two numbers, say a and b, currently on the blackboard erased and a new number a + b – 1 is written. What will be the number left on the board at the end?

(a) 820
(b) 821
(c) 781
(d) 819
(e) 780

Q.2. What are the last two digits of 7^{2008}?

(a) 21
(b) 61
(c) 01
(d) 41
(e) 81

Q.3. If the roots of the equation $x^3 - ax^2 + bx - c = 0$ are three consecutive integers, then what is the smallest possible value of b?

(a) $-\frac{1}{\sqrt{3}}$
(b) -1
(c) 0
(d) 1
(e) $\frac{1}{\sqrt{3}}$

Q.4. A shop stores x kg of rice. The first customer buys half this amount plus half a kg of rice. The second customer buys half the remaining amount plus half a kg of rice. Then the third customer also buys half the remaining amount plus half a kg of rice. Thereafter, no rice is left in the shop. Which of the following best describes the value of x?

(a) $2 \le x \le 6$
(b) $5 \le x \le 8$
(c) $9 \le x \le 12$
(d) $11 \le x \le 14$
(e) $13 \le x \le 18$

Direction for questions 5 and 6: Answer the questions on the basis of the information given below.

Let $f(x) = ax^2 + bx + c$, where a, b and c are certain constants and a $\neq$ 0. It is known that $f(5) = -3f(2)$ and that 3 is a root of $f(x) = 0$.

Q.5. What is the other root of $f(x) = 0$?

(a) –7
(b) – 4
(c) 2
(d) 6
(e) cannot be determined

Q.6. What is the value of a + b + c?

(a) 9
(b) 14
(c) 13
(d) 37
(e) cannot be determined

Q.7. The number of common terms in the two sequences 17, 21, 25, ..., 417 and 16, 21, 26, ..., 466 is

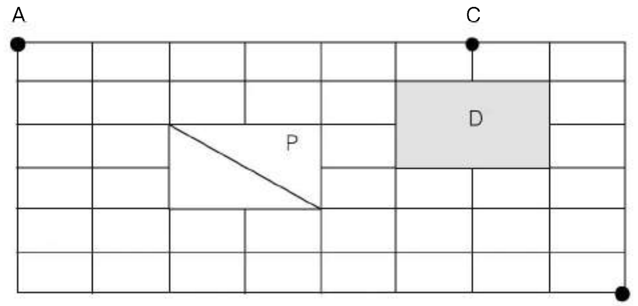
(a) 78
(b) 19
(c) 20
(d) 77
(e) 22

Q.8. How many integers, greater than 999 but not greater than 4000, can be formed with the digits 0, 1, 2, 3 and 4, if repetition of digits is allowed?

(a) 499
(b) 500
(c) 375
(d) 376
(e) 501

Direction for questions 9 and 10: Answer the questions on the basis of the information given below.

The figure below shows the plan of a town. The streets are at tight angles to each other. A rectangular park (P) is situated inside the town with a diagonal road running through it. There is also a prohibited region (D) in the town.



Q.9. Neelam rides her bicycle from her house at A to her office at B, taking the shortest path. Then the number of possible shortest paths that she can choose is

(a) 60 (b) 75
(c) 45 (d) 90
(e) 72

Q.10. Neelam rides her bicycle from her house at A to her club at C, via B taking the shortest path. Then the number of possible shortest paths that she can choose is

(a) 1170 (b) 630
(c) 792 (d) 1200
(e) 936

Q.11. Let $f(x)$ be a function satisfying $f(x)f(y) = f(xy)$ for all real x, y. If $f(2) = 4$ then what is the value of $f(1/2)$?

(a) 0 (b) $\frac{1}{4}$
(c) $\frac{1}{2}$ (d) 1
(e) cannot be determined

Q.12. Suppose, the seed of any positive integer i is defined as follows:

seed(n) = n, if n < 10
= seed(s(n)), otherwise

where s(n) indicates the sum of digits of n. For example, seed (7) = 7, seed (248) = seed (2 + 4 + 8) = seed (14)= (1 + 4) = seed = 5 etc. How many positive integers n, such that n < 500, will have seed(n) = 9?

(a) 39 (b) 72
(c) 81 (d) 108
(e) 55

Q.13. In a triangle ABC, the lengths of the sides AB and AC equal 17.5 cm and 9 cm respectively. Let D be a point on the line segment BC such that AD is perpendicular to BC. If AD=3 cm, then what is the radius (in cm) of the circle circumscribing the triangle ABC?

(a) 17.05 (b) 27.85
(c) 22.45 (d) 32.25
(e) 26.25

Q.14. Consider obtuse-angled triangles with sides 8 cm, 15 cm and x cm. If x is an integer, then how many such triangles exist?

(a) 5 (b) 21
(c) 10 (d) 15
(e) 14

Q.15. Consider a square ABCD with mid points E, F, G, H of AB, BC, CD and DA respectively. Let L denote the line passing through F and H. Consider points P and Q, on L and inside ABCD, such that the angles APD and BQC both equal 120°. What is the ratio of the area of ABQCDP to the remaining area inside ABCD?

(a) $\frac{4\sqrt{2}}{3}$ (b) $2+\sqrt{3}$
(c) $\frac{10-3\sqrt{3}}{9}$ (d) $1+\frac{1}{\sqrt{3}}$
(e) $2\sqrt{3}-1$

Q.16. What is the number of distinct terms in the expansion of $(a + b + c)^{20}$?

(a) 231 (b) 253
(c) 242 (d) 210
(e) 228

Direction for questions 17 and 18: Answer the questions on the basis of the information given below.

Five horses, Red, White, Grey, Black and Spotted participated in a race. As per the rules of the race, the persons betting on the winning horse get four times the bet amount and those betting on the horse that came in second get thrice the bet amount. Moreover, the bet amount is returned to those betting on the horse that came in third, and the rest lose the bet amount. Raju bets Rs.3000, Rs.2000 and Rs.1000 on Red, White and Black horses respectively and ends up with no profit and no loss.

Q.17. Which of the following cannot be true?

(a) At least two horses finished before Spotted
(b) Red finished last
(c) There were three hones between Black and Spotted
(d) There were three horses between White and Red
(e) Grey came in second

Q.18. Suppose, in addition, it is known that Grey came in fourth. Then which of the following cannot be true?

(a) Spotted came in first
(b) Red finished last
(c) White came in second
(d) Black came in second
(e) There was one horse between Black and White

Direction for questions 19 and 20: Answer the questions on the basis of the information given below.

Mark (a): If Question can be answered 1mm A alone but not from B alone.

Mark (b): If Question can be answered from B alone but not from A alone.

Mark (c): If Question can be answered from either of A or B alone.

Mark (d): If Question can be answered from A and B together but not from any of them alone.

Mark (e): If Question cannot be answered even from A and B together.

In a single elimination tournament, any player is eliminated with a single loss. The tournament is played in multiple rounds subject to the following rules:

(a) If the number of players, say n, in any round is even, then the players are grouped in to n/2 pairs. The players in each pair play a match against each other and the winner moves on to the next round.

(b) If the number of players, say it, in any round is odd, then one of them is given a bye, that is, he automatically moves on to the next round. The remaining (n-1) players are grouped into (n-1)/2 pairs. The players in each pair play a match against each other and the winner moves on to the next round. No player gets more than one bye in the entire tournament.

Thus, if n is even, then n/2 players move on to the next round while if n is odd, then (n+1)/2 players move on to the next round. The process is continued till the final round, which obviously is played between two players. The winner in the final round is the champion of the tournament.

Q.19. Question: What is the number of matches played by the champion?

A: The entry list for the tournament consists of 83 players.

B: The champion received one bye.

Q.20. Question: If the number of players, say n, in the first round was between 65 and 128, then what is the exact value of n?

A: Exactly one player received a bye in the entire tournament.

B: One player received a bye while moving on to the fourth round from the third round

Q.21. Two circles, both of radii 1 cm, intersect such that the circumference of each one passes through the centre of the other. What is the area (in sq cm) of the intersecting region?

(a) $\frac{\pi}{3}-\frac{\sqrt{3}}{4}$ (b) $\frac{2\pi}{3}+\frac{\sqrt{3}}{2}$

(c) $\frac{4\pi}{3}-\frac{\sqrt{3}}{2}$ (d) $\frac{4\pi}{3}+\frac{\sqrt{3}}{2}$

(e) $\frac{2\pi}{3}-\frac{\sqrt{3}}{2}$

Q.22. Rahim plans to drive from city A to station C, at the speed of 70 km per hour, to catch a train arriving there from B. He must reach C at least 15 minutes before the arrival of the train. The train leaves B, located 500 km south of A, at 8:00 am and travels at a speed of 50 km per hour. It is known that C is located between west and northwest of B, with BC at 60° to AB. Also, C is located between south and southwest of A with AC at 30° to AB. The latest time by which Rahim must leave A and still catch the train is closest to

(a) 6:15am (b) 6:30 am

(c) 6:45am (d) 7:00am

(e) 7:15am

Q.23. Three consecutive positive integers are raised to the first, second and third powers respectively and then added. The sum so obtained is a perfect square whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers?

(a) $1 \le m \le 3$ (b) $4 \le m \le 6$

(c) $7 \le m \le 9$ (d) $10 \le m \le 12$

(e) $13 \le m \le 15$

Q.24. Find the

$$\sqrt{1+\frac{1}{1^2}+\frac{1}{2^2}}+\sqrt{1+\frac{1}{2^2}+\frac{1}{3^2}}+\ldots..+\sqrt{1+\frac{1}{2007^2}+\frac{1}{2008^2}}$$

(a) $2008-\frac{1}{2008}$ (b) $2007-\frac{1}{2007}$

(c) $2007-\frac{1}{2008}$ (d) $2008-\frac{1}{2007}$

(e) $2008-\frac{1}{2009}$

Q.25. Consider a right circular cone of base radius 4 cm and height 10 cm. A cylinder is to be placed inside the cone with one of the flat surfaces resting on the base of the cone. Find the largest possible total surface area of the cylinder (in sq. cm).

(a) $100\pi/3$ (b) $80\pi/3$

(c) $120\pi/7$ (d) $130\pi/9$

(e) $110\pi/7$

ANSWERS

CAT 2008

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(c)	(c)	(b)	(b)	(b)	(e)	(c)	(d)	(d)	(a)	(b)	(e)	(e)	(c)	(e)	(a)	(d)	(c)	(d)	(d)
Q. No.	21	22	23	24	25															
Answer	(e)	(b)	(a)	(a)	(a)															

HINTS AND SOLUTIONS

1. Let S = 1 + 2 + ... + 40. So 1 + 2 + a + b +... + 40 = S
After 1st step:
1 + 2 + ... + (a + b — 1) + ... + 40 = S — 1
So, after each step the total reduces by 1. Continuing this way after 39 steps numbers left S — 39 = $\frac{40 \times 41}{2} - 39 = 781$
Hence, (c)

2. $7^{2008} = (7^4)502 = (2401)^{502}$
To find tens digit, we need to take care only of last two digits of this number.
Last two digits of $(2401)^{502}$ = Last two digits of $(01)^{502}$
Thus, 7^{2008} end with 01. Hence, [3].

3. Assume that the consecutive roots are $(x - 1)$, x and $(x + 1)$
Now, $b = x(x - 1) + x(x + 1) + (x - 1)(x + 1) = 3x^2 - 1$
Now as $3x^2 \geq 0$ minimum value of $b = 3 \times 0^2 - 1 = -1$
Hence, (b)

4. Let us make a table from the given information:

Quantity of Rice in the shop	Quantity of Rice bought	Quantity Left
x	$\left(\frac{x}{2}+\frac{1}{2}\right)$	$\frac{x}{2}-\frac{1}{2}$
$\frac{x}{2}-\frac{1}{2}$	$\left(\frac{x}{2}-\frac{1}{2}\right)+\frac{1}{2}$	$\frac{x}{4}-\frac{3}{4}$
$\frac{x}{4}-\frac{3}{4}$	$\left(\frac{x}{8}-\frac{3}{8}\right)+\frac{1}{2}$	$\frac{x}{8}-\frac{7}{8}$

$\frac{x}{8}-\frac{7}{8} = 0 \Rightarrow x = 7$
Now, $5 \leq x \leq 8$
Hence, (b)

5. Given $f(x) = ax^2 + bx + c$ (a ≠ 0).
Since 3 is a root of $f(x)$, so, $9a + 3b + c = 0$ ....(i)
Also, $f(5) = 3f(2)$.
So, $25a + 5b + c = -3(4a + 2b + c) = -12a - 6b - 3c$
$\Rightarrow 37a + 11b + 4c = 0$ ....(ii)
From (i) and (ii), $a - b = 0 \Rightarrow a = b$
Thus we get $f(x) = ax^2 + ax + c$
Dividing $f(x)$ by $x - 3$, we get $c = -12a$
$\Rightarrow f(x) = ax^2 + ax - 12a \Rightarrow f(x) = 0 \Rightarrow -4$ is second root.
Hence, (b).

6. a + b + c = a + a - 12a = – 10a
Value of a is not given, so cannot be determined.
Hence, (e).

7. In 1st series, terms of are in the form 4a + 1 (4 < a < 104)
In 2nd series, terms of are in the form 5b + 1 (3 < b < 93)
In order to have same terms, we should get 4a = 5b.
This happens only 20 times.
Thus, we get 21, 41, 61… i.e., 20 common terms. Hence, (c).

8. Numbers are of 4 digits, and less than or equal to 4000.
For numbers less than 4000, following is the box diagram:
$\underline{3} \times \underline{5} \times \underline{5} \times \underline{5} = 375$
Including 4000, total number of numbers = 376
Hence, (d)

9. Neelam has to take path XY
A to X = $^4C_2$ = 6 possibilities
Y to B = $^6C_2$ = 15 possibilities
So total possibilities = 6 × 15 = 90 possibilities
Hence, (d)

10. From A to B = 90 paths
From B to C via N = 6 (and not via M)
From B to C via M = 7 paths
In all 90 V (6 + 7) = 1170 paths
Hence, (a)

11. We have $f(x)f(y) = f(xy)$
So, $f(1)f(1) = f(1 \times 1) = f(1)$
$\Rightarrow f(1)^2 = f(1) \Rightarrow f(1)^2 - f(1) = 0 \Rightarrow f(1)[f(1) - 1] = 0$
Hence $f(1) = 0$ or $f(1) = 1$
If $f(1) = 0$ then $f(x) = 0$ for any x since $x = x \times 1$
$\Rightarrow f(1) = 1$
Now, $f(2) = 4$
So, $1 = f(1) = f\left(\frac{1}{2} \times 2\right) = f\left(\frac{1}{2}\right) f(2) = f\left(\frac{1}{2}\right) \times 4 \Rightarrow f\left(\frac{1}{2}\right) = \frac{1}{4}$
Hence, (b).

12. Our answer would be the number of integers between 1 and 500, which are divisible by 9.
The smallest is 9 and the largest is 495.
In the first 499 natural numbers, we have 495 as the last multiple of 9, and this is 55th multiple of 9.
Hence, (e).

13. Let the sides be a = AB, b = AC, e = BC
$$A(\Delta ABC) = \frac{abc}{4R} \Rightarrow A(\Delta ABC) = \frac{1}{2}$$
$$\text{So, } \frac{abc}{4R} = \frac{1}{2} \times c \times 3$$
$$R = \frac{a \times b}{2 \times 3} = \frac{17.5 \times 9}{2 \times 3} = 26.25 \text{ cm}^2$$
Hence, (e).

14. Let the three sides be a = 8, b = 15 and c = x
Case I:
c is the longest side.
We know that 15 < c < 23. (8 + 15 = 23). For c = 17 we get a right angled triangle. So, c > 17 for an obtuse angled triangle.
Therefore c is 18, 19, 20, 21, 22.
Case II
b is the longest side. In this case 7 < c < 15 (15 — 8 = 7)
For a right angled triangle, $c < \sqrt{15^2 - 8^2} \Rightarrow c < \sqrt{161} \cong 12.7$
So, c can be 8, 9, 10, 11 or 12.
Hence in total there are 10 possibilities.

15. (e).

16. $(a + b + c)^{20}$ has each term of the form $a^x b^y c^z$ where $x + y + z = 20$. This has $^{20+3-1}C_{3-1} = 231$ solutions. (This question is same as distributing 20 apples among three children where any child can get any number of apples. Formula for this is – $^{n+r-1}C_{r-1}$).
Hence, (a).

17. (d) There are 3 possibilities

Horse	**Rank**		
	Case 1	Case 2	Case 3
Red	4/5	4/5	3
White	2	3	4/5
Black	5/4	1	2
Grey	1/3	2/4/5	1/4/5
Spotted	3/1	2/4/5	1/4/5

18. (c)

19. Using statement A alone, if there are 83 if there are 83 players, the number of players in each subsequent round will be as follows:

Round	Players	
1	83	
2	42	
3	21	
4	11	
5	6	
6	3	
7	2	→ Final

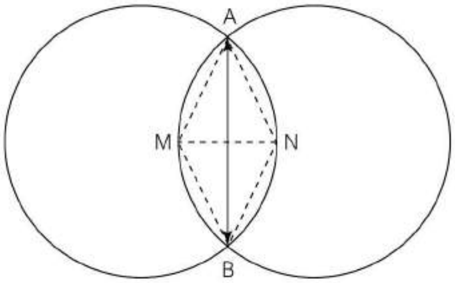
The champion plays in the final and so can play either 6 or 7 matches (depending on whether he gets a bye or not). Hence no unique answer.
Using statement B alone, we can draw no conclusion without knowing the number of rounds. With both the statement together, we can say that the champion plays 6 matches. Hence, (d).

20. Using statement A alone we get various possible cases. We can see that 7 rounds have to be played. The bye could have occurred in any of the first 6. Let us look at the table below now:

Round	**Bye in**					
	Round 1	**Round 2**	**Round 3**	**Round 4**	**Round 5**	**Round 6**
1	127	126	124	120	112	96
2	64	63	62	60	56	47
3	32	32	31	30	28	24
4	16	16	16	15	14	12
5	8	8	8	8	7	6
6	4	4	4	4	4	3
7	2	2	2	2	2	2

Using Statement B alone, we cannot say anything as other players too might have received byes. Using both together we can say that the number of players is 124. Hence, (d).

21. Area required region = 2 × [(Area of sector – AMB) – A (ΔANB)]



$$= 2 \times \frac{120^\circ}{360^\circ} \times \pi r^2 - 2 \times \text{Area}(\Delta ANB)$$

$$= \frac{2}{3}\pi - 2 \times \frac{\sqrt{3}}{4} \times 1 = \frac{2}{3}\pi - \frac{\sqrt{3}}{2} \text{ sq. cm.}$$

Hence (e).

22. (b).

23. $3^1 + 4^2 + 5^3 = 144$

$\sqrt{144} = 3 + 4 + 5$. So, the numbers are 3, 4 and 5.

Hence, (a).

Alternatively, assume the numbers to be n, n+1 and n+2.

So, $n^1 + (n+1)^2 + (n+2)^3 = (n+n+1+n+2)^2 = (3n+3)^2$

Expanding it, we get $n(n^2 - 2n - 3) = 0$

Since $n \neq 0$, so, $(n^2 - 2n - 3) = 0 \Rightarrow (n - 3)(n + 1) = 0$

Hence n = 3

24. Go through the method of induction,

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{\frac{9}{4}} = \frac{3}{2} = 2 - \frac{1}{2}$$

$$\sqrt{1 + \frac{1}{1^2} + \frac{1}{2^2}} + \sqrt{1 + \frac{1}{2^2} + \frac{1}{3^3}}$$

$$= \frac{3}{2} + \frac{7}{6} = \frac{8}{3} = 3 - \frac{1}{3}$$

Similarly,

$$\sqrt{1 = \frac{1}{2^2} + \frac{1}{2^2}} + \ldots\ldots\sqrt{1 + \frac{1}{2007^2} + \frac{1}{2008^2}} = 2008 - \frac{1}{2008}$$

Hence, (a).

25. Consider the figures given below:

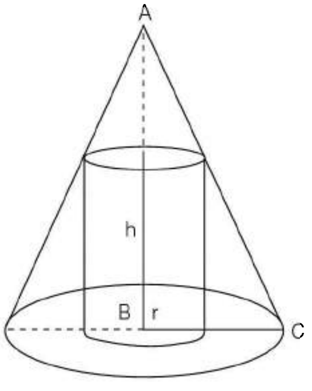
$$\frac{AP}{PB} = \frac{AQ}{QC} = \frac{10 - h}{h}$$

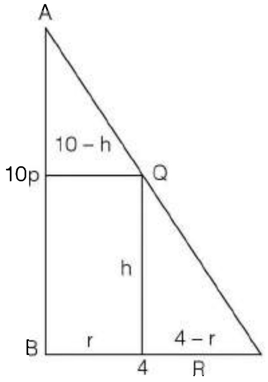
$$\frac{BR}{RC} = \frac{AQ}{QC} = \frac{R}{4 - r}$$

$$\Rightarrow \frac{10 - h}{h} = \frac{r}{4 - r} \Rightarrow 4h = 40 - 10r \Rightarrow 2h = 20 - 5\text{ r}$$

Surface area of cylinder = $2\pi r^2 + 2\pi rh$

$= 2\pi r^2 + \pi r\,(20 - 5r)$





$= 20\,\pi r - 2\,\pi r^2 = \pi\,(20r - 3r^2)$

So, we have to maximize $f(r) = 20r - 3r^2$

We know that that maximum of quadratic equation $ax^2 + bx + c = 0$ arises for $x = \frac{-b}{2a}$ and the maximum value $= \frac{-D}{4a}$

In this case $\frac{-D}{4a} = 100/3$

So the maximum surface area $= \frac{100\pi}{3}$

Hence, (a).

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PART
4

Section Tests

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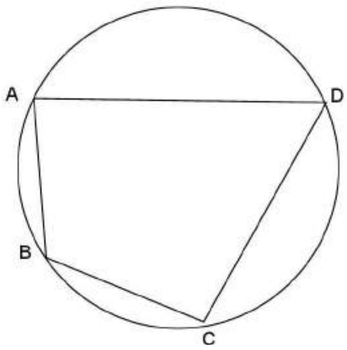
Section Test 1

Q.1. For how many natural number values of N, $N^4 + 4$ will be a prime number?
(a) 0 (b) 1
(c) 2 (d) None of these

Q.2. In the figure given, ABCD is a cyclic quadrilateral and AB = 25 cm, BC = 39 cm, CD = 52 cm and AD = 60 cm. What is the diameter of the circle?



(a) 60 cm (b) 65 cm
(d) 72 cm (d) 78 cm

Q.3. How many natural numbers are there which give a remainder of 41 after dividing 1997?
(a) 2 (b) 4
(c) 6 (d) None of these

Q.4. You are selecting 10 numbers randomly out of the first 100 odd numbers. Sum of these 10 odd numbers is N. How many different values of N are possible?
(a) 900 (b) 1801
(c) 1800 (d) 901

Q.5. How many values of x will satisfy the following equation:

$$\sqrt{x+\sqrt{x+\sqrt{x.............\infty}}} = \sqrt{x.\sqrt{x.\sqrt{x..........\infty}}}$$

(a) 0 (b) 1
(c) 2 (d) None of these

Q.6. ABC is an equilateral triangle with side length 1 unit. P is any point on the circumcircle of this triangle. What is the value of $AP^2 + BP^2 + CP^2$?
(a) $\sqrt{2}$ (b) 2
(c) $2\sqrt{2}$ (d) 3

Q.7. How many integral points are contained inside a triangle with the vertices (0, 0), (21, 0) and (0, 21)?
(a) 190 (b) 231
(c) 210 (d) 171

Q.8. In a race on a circular track, A, B and C start from the same point N in the same direction. Their speeds are n m/s, (n+1) m/s and (n+2) m/s respectively where n is a natural number. How many times will they meet before they meet for the first time at point N(n>0, N>0)?
(a) 1
(b) 1
(c) 2
(d) None of these

Q.9. There are a certain number of students in a class and each of them have at most 4 books. The number of students who have at most 1, 2, 3 and 4 books is 11, 18, 19 and 21 respectively. What is the number of students with at least 3 books?
(a) 0 (b) 1
(c) 2 (d) 3

Direction for questions 10 to 12: Go through the information given below and solve the questions based on it.

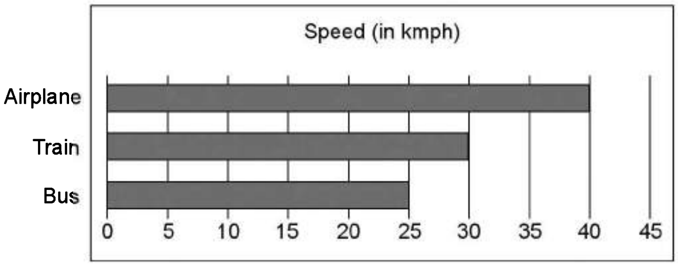
Table 1 – Mode of travel between two cities

	Patna	*Mumbai*	*Calcutta*	*Chennai*	*Bangalore*
Patna	-	Bus	Train	Air plane	Train
Mumbai	Bus	-	Train	Train	Air plane
Calcutta	Train	Train	-	Air plane	Bus

	Patna	*Mumbai*	*Calcutta*	*Chennai*	*Bangalore*
Chennai	Air plane	Train	Air plane	-	Bus
Bangalore	Train	Air plane	Bus	Bus	-

Table 2 - Distance between two cities (In kms)

	Patna	*Mumbai*	*Calcutta*	*Chennai*	*Bangalore*
Patna	-	100	150	150	300
Mumbai	100	-	200	300	500
Calcutta	150	200	-	500	300
Chennai	150	300	500	-	100
Bangalore	300	500	300	100	-



Q.10. Which city can be reached from Patna in the least time?
(a) Mumbai
(b) Calcutta
(c) Chennai
(d) Bangalore

Q.11. For which of the following options, travel time is the least?
(a) Mumbai-Chennai
(b) Patna-Bangalore
(c) Bangalore-Chennai
(d) Mumbai - Calcutta

Q.12. If one can travel by bus or train only, then which two cities will have the least travel time?
(a) Patna-Mumbai
(b) Bangalore-Calcutta
(c) Chennai - Mumbai
(d) None of these

Q.13. In a town 25% families own a phone and 15% own a car. 65% families own neither a phone nor a car. 2000 families own both a phone and a car. Consider the following statements in this regard:
1. 10% families own both a car and a phone
2. 35% families own either a car or a phone
3. 40,000 families live in the town

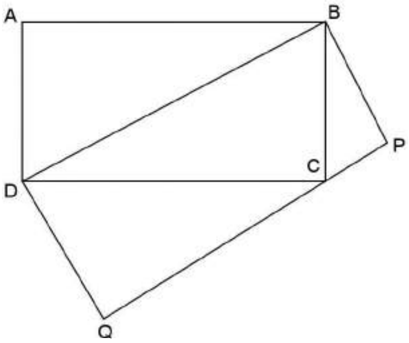
Which of the above statements are correct?
(a) 1 and 2 (b) 1 and 3
(d) 2 and 3 (d) 1, 2 and 3

Q.14. The function $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is any real number, attains a minimum value at
(a) $x = 2.3$ (b) $x = 2.5$
(d) $x = 2.7$ (d) None of these

Q.15. Find the number of natural numbers which are smaller than 2.10^8 written by means of the digits 1 and 2.
(a) 760 (b) 766
(c) 754 (d) 782

Q.16. The quadratic equation $g(x) = (px^2 + qx + r)$, $p \neq 0$, attains its maximum value at $x = 7/2$. Product of the roots of the equation $g(x) = 0$ is equal to 10. What is the value of $p \times q \times r$?
(a) 70 (b) −70
(c) 0 (d) Cannot be determined

Q.17. In the given figure, ABCD and BDQP are two rectangles, in which AB = 12 units and AD = 5 units. Find the area of the rectangle BDQP.



(a) 78 sq. units (b) 60 sq. units
(c) 65 sq. units (d) 52 sq. units

Direction for questions 18 to 20: Read the following instructions and solve the questions based on it.

Choose (a): If the question can be answered by one of the statements alone and not by the other.

Choose (b): If the question can be answered by using either statement alone.

Choose (c): If the question can be answered by using both the statements together, but cannot be using either statement alone.

Choose (d): If the question cannot be answered even by using both the statements together.

Q.18. Is $x > 2$?
Statement A: $|2x - 19| < 7$
Statement B: $x^2 - 4x = 0$

Q.19. Is $x > y$?
Statement A: $3x + 5y = 11$ where x and y are integers
Statement B: $x^5 > y^5$

Q.20. If both m and n are positive integers less than 100 and greater than 10, is the sum m + n a multiple of 11?
Statement A: m – n is a multiple of 22
Statement B: The tens digit and the units digit of m are the same; the tens digit and the units digit of n are the same

Q.21. Rs 180 is divided among 66 boys and girls of a class. The sum of the boy's share and girl's share are in ratio 5 : 4 but their individual shares are 3 : 2 respectively. The number of boys in the class is:
(a) 36 (b) 30
(c) 5 (d) 40

Q.22. If the sum of five consecutive positive integers is A, then the sum of the next five consecutive integers in terms of A is:
(a) A+1 (b) A+5
(c) A+25 (d) 2A

Direction for questions 23 to 26. Go through the table given below and solve the questions based on it.

The following table gives the tariff (in paisa per kilo–watt–hour) levied by the NTPC in 2004–05, in the four sectors and the regions within them. The table also gives the percentage change in the tariff as compared to 2001–02.

	Region 1		*Region 2*		*Region 3*		*Region 4*		*Region 5*	
Sector 1	425	+15	472	+5	420	–4	415	+8	440	+10
Sector 2	430	+12	468	+8	448	+7	423	–3	427	+11
Sector 3	428	+8	478	–4	432	+6	441	+10	439	+8
Sector 4	434	–5	470	+15	456	+10	451	+12	446	–12

Q.23. If the amount of power consumed by the various Regions in Sector 1 is the same, then, as compared to 2001–02, the net tariff in 2004–05...
(a) Increased by 9.5%
(b) Decrease by 1.5%
(c) Decrease by 13.3%
(d) None of these

Q.24. What approximately was the average tariff per sector in Region 3 in 2001–02?
(a) 402 (b) 424
(c) 433 (d) None of these

Q.25. Which of the following has got a maximum percentage increase for the given period?
(a) Region 1 – Sector 1
(b) Region 1 – Sector 2
(c) Region 5 – Sector 2
(d) None of these

Q.26. There are a total of 20 units of NTPC divided into regions and then subdivided into sectors, as given in the above table. For how many units of NTPC in 04 – 05, there is a net increase in the tariff?
(a) 15 (b) 5
(c) 8 (d) None of these

Direction for questions 27 to 30: Go through the table given below and solve the questions based on it.

Table given below gives the Estimated and Actual Sales revenue and cost figures for three years.
Answer the questions based on it.

Sales Revenue	2007	2008	2009	Cost	2007	2008	2009
Estimated	1281	1452	1228	Estimated	986	1051	1024
Actual	1326	1286	1452	Actual	988	1076	1068

Estimated Profit = Estimated Revenue – Estimated Cost
Actual Profit = Actual Revenue – Actual Cost

Q.27. For how many years, actual profit is more than estimated profit?
(a) 0 (b) 1
(c) 2 (d) 3

Q.28. A measure of forecasting done by marketing team is the ratio of Estimated Sales to actual sales. Lower is this value, better is the forecasting. Which year's forecasting is best?
(a) 2007
(b) 2008
(c) 2009
(d) cannot be determined

Q.29. Company plans to use a different mechanism for forecasting of its maximum possible profit = Max(Estimate, Actual(Sales Revenue)) – Min(Estimate, Actual(Cost)). What is the maximum possible profit derived this way?
(a) 340 (b) 401
(c) 428 (d) None of these

Q.30. In a geometric progression, product of 11th term, 18th term and 20th term is equal to the product of 28th term and 5th term. Which of the following is necessarily true about that GP?
(a) One of terms of this GP has to be equal to zero.
(b) One of the terms of this GP has to be equal to one.
(c) This GP will have infinite terms.
(d) Common ratio of this GP will be less than 1.

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■ Section Test 1

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(c)	(d)	(c)	(b)	(a)	(d)	(c)	(c)	(c)	(a)	(c)	(b)	(b)	(d)	(b)	(d)	(b)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(b)	(c)	(d)	(d)	(a)	(a)	(c)	(c)	(c)	(b)										

HINTS AND SOLUTIONS

1. First of all we should understand that we cannot solve this question by taking different values of N and checking if its prime or not. We need to devise some alternative method.

   $N^4 + 4 = (N^2 + 2)^2 - (2N)^2 = (N^2 + 2 + 2N)\,(N^2 + 2 - 2N)$

   Since N is a natural number, each of the above factors will also be a natural number.

   We can conclude that $N^4 + 4 = (N^2 + 2 + 2N)(N^2 + 2 - 2N)$ is product of two natural numbers. Hence it cannot be a prime number except for the value N = 1 for which the values are as follows:

   $(N^2 + 2 + 2N) = 5$ and $(N^2 + 2 - 2N) = 1$, So $(N^2 + 2 + 2N)(N^2 + 2 - 2N) = 5 \times 1 = 5$

   (For N = 1, value can be obtained using $N^4 + 4$. Though for higher values of N, calculating the value and checking if its prime of not is difficult).

2. BC, CD and BD constitute a right-angled triangle (13 × 3, 13 × 4 and 13 × 5). If BD = 65, then it satisfies the other set of values of AB and AD also.

3. Let us assume that the quotient is Q and divisor is D. Using the condition given in question, $1997 = QD + 41 \Rightarrow QD = 1956$. Now we will factorize 1956 in two parts such that D (divisor) is more than 41.

4. The minimum possible value of first 10 consecutive odd numbers = 100 and next value of sum would be obtained as 102(by replacing 19 by 21). And so on all the other even values from 100 to 1900 will be a value of sum. There are 901 even numbers from 100 to 1900.

5. Go through hit and trial.

6. Assume any one vertex of the triangle to be the point of the circum-circle and then solve.

7. Here we need to find out the solution of $2 \leq x + y \leq 20$

   We know that the integral solutions possible of x + y = n, where zero solutions are not possible is ${}^{n-1}C_{r-1}$. So, total points = 19 × 20 /2 = 190

8. Since their speeds are consecutive natural numbers, there will be just one meeting point and that is the starting point. So, answer = 0. Hence option (D) is the answer.

Section Test 2

Let us make a table for the time taken:

	Patna	*Mumbai*	*Calcutta*	*Chennai*	*Bangalore*
Patna		4	5	3.75	10
Mumbai	4	–	6.666667	10	12.5
Calcutta	5	6.666667		12.5	12
Chennai	3.75	10	12.5		4
Bangalore	10	12.5	12	4	–

14. Minima of $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$ at one of the critical points.

    $F(x)_{x=2} = 0 + 0.5 + 1.6 = 2.1$

    $F(x)_{x=2.5} = 0.5 + 0 + 1.1 = 1.6$

    $F(x)_{x=3.6} = 1.6 + 1.1 + 0 = 2.7$

    Obviously, minima is obtained at x = 2.5. Hence option (B) is the answer.

15. 2.10^8 is a nine-digit number. So, all the numbers which are of less than 9 digits will be smaller than 2.10^8.

    Total number of numbers of 1 digit = 2
    Total number of numbers of 2 digit = 2^2
    Total number of numbers of 3 digit = 2^3
    Total number of numbers of 4 digit = 2^4

Total number of numbers of 5 digit = 2^5
Total number of numbers of 6 digit = 2^6
Total number of numbers of 7 digit = 2^7
Total number of numbers of 8 digit = 2^8
Total number of numbers of less than 9 digits = 510
Total number of numbers of 9 digits less than $2.10^8 = 2^8$
Total number of numbers of less than $2\times10^8 = 510 + 256 = 766$
Hence option (B) is the answer.

16. Value of p cannot be calculated, so $p \times q \times r$ cannot be determined.

17. Ar(ΔBCD) = ½ ar(ABCD) = ½ ar (BDQP)
So, ar(ABCD) = ½ ar (BDQP) = 60 sq. units
Hence option (B) is the answer.

18. Statement (A): $|2x-19| < 7$, implies that $-7 < |2x-19| < 7$
Implies that $x > 13$ or $x < 13$
Hence statement (A) alone is not sufficient.
Statement (B):- $x^2 - 4x = 0$ implies that $x = 0$ or $x = 4$
Statement (B) alone is also not sufficient.
Even by using both the statements together, no definite conclusion can be drawn.
Hence option (D) is the answer.

19. Statement (A):- $3x + 5y = 11$
We will assume some values of x to check the in-equality.
If $x = 2$; $y = 1$ implies that $x > y$
If $x = -2$; $y = 7$ implies that $x > y$
If $x = 3$; $y = 4$ implies that $x < y$
Statement (A) alone is not sufficient to give the answer of the question.
Statement (B):- $x^5 > y^5$ implies that $x > y$ (as from the $x^n > y^n$ implies that $x > y$ if n is odd)
Therefore, Statement (B) alone is sufficient.

20. Using statement (A) alone:
Since $m - n$ is a multiple of 22, $m - n$ is a multiple of 11 and of 2 because $22 = 11 \times 2$
If both m and n are multiples of 11, the sum $m + n$ will also be a multiple of 11. Consider the following examples:
$44 - 22 = 22$ {which is a multiple of 11 and of 22}
$44 + 22 = 66$ {which is a multiple of 11 and of 22}
$88 - 66 = 22$ {which is a multiple of 11 and of 22}
$88 + 66 = 154$ {which is a multiple of 11 and of 22}
However, if m and y are not individually divisible by 11, it is possible that $m - n$ is a multiple of 22 (and 11) while $m + n$ is not a multiple of 11. For example:
$78 - 56 = 22$ but $78 + 56 = 134$ is not a multiple of 11.
Hence statement (A) alone is not sufficient to answer the question.
Using statement (B) alone:
Since the tens digit and the units digit of m are the same, the range of possible values for m includes:
11, 22, 33, 44, 55, 66, 77, 88, 99
Since each of these values is a multiple of 11, m must be a multiple of 11.
Since the tens digit and the units digit of n are the same, the range of possible values for n includes:
11, 22, 33, 44, 55, 66, 77, 88, 99
Since each of these values is a multiple of 11, n must be a multiple of 11.
Now if both m and n are multiples of 11, $(m + n)$ and $(m - n)$ will be a multiple of 11.
Statement (B) alone is sufficient.
Hence option (A) is the answer.

21. Boy's share = 180(5/9) = Rs 100
Girl's share = 180(4/9) = Rs 80
Given that $(100/3x) + (80/2x) = 66$ implies that $x = 10/9$
Therefore, number of boys in the class = $(100 \text{ x } 9)/30 = 30$
Hence option (B) is the answer.

22. Best way of solving this question is - Assume values. Assume 1, 2, 3, 4 and 5. Sum of these five numbers = 15 = A. Now next five numbers are 6, 7, 8, 9, 10. Sum of these five numbers = 40.
40 = A + 25. Hence option (C) is the answer.

27. Following is the profit figure:

	2007	2008	2009
Estimated Profit	295	401	204
Actual Profit	338	210	384

Hence option (C) is the answer.

28. Following is the ratio:

2007	2008	2009
0.966063	1.129082	0.84573

Hence option (C) is the answer.

29. Max(Estimate, Actual(Sales Revenue))

2007	2008	2009
1326	1452	1452

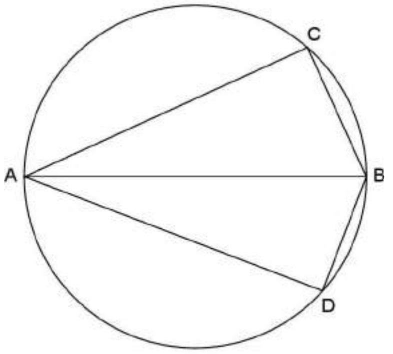
Min(Estimate, Actual(Cost))

2007	2008	2009
986	1051	1024

Maximum possible profit = 428 crores (in 2009)
Hence, option (C) is the answer.

Section Test 2

Q.1. AB is the diameter of the given circle, while points C and D lie on the circumference as shown. If AB is 15 cm. AC is 12 cm and BD is 9 cm, find the area of the quadrilateral ABCD.



(a) 54 sq. cm (b) 216 sq. cm
(c) 162 sq. cm (d) None of these

Q.2. An employee pays 1% of his salary or Rs 75 whichever is higher as professional tax per month. He also pays 20% of the balance per year as income tax where first Rs 30,000 is tax free. If the salary is Rs 6000 per month, how much amount does he pay per year as income tax?
(a) Rs 8220 (b) Rs 9120
(c) Rs 8256 (d) Rs 9156

Q.3. Radha can do 3/4th of a work in 12 days. After working for 8 days Mira joins her and both works together for three days after which Radha leaves. If Mira takes 12 more days to complete the work then how many days Mira will take alone to do it?
(a) 16 (b) 12
(c) 48 (d) 24

[redacted]

Mark (a): If the question can be answered by using one of the statements alone, but cannot be answered by using other statement alone.

Mark (b): If the question can be answered by using either statement alone.

Mark (c): If the question can be answered by using both statements together, but cannot be answered by using either statement alone.

Mark (d): If the question cannot be answered even by using both the statements together.

Q.4. The base of a triangle is 60cm and one of the base angles is 60^0. What is the length of the shortest side of the triangle?
Statement A: The sum of the lengths of the other two sides is 80 cm
Statement B: The other base angle is 45^0

Q.5. A, B, C, D, E & F are six integers such that $E < F$, $B > A$, $A < D < B$. C is the greatest integer. Is A the smallest integer?
Statement A: $E + B < A + D$
Statement B: $D < F$

Q.6. If w, x, y, and z are the digits of the four-digit number N, a positive integer, what is the remainder when N is divided by 9?
Statement A: $w + x + y + z = 13$
Statement B: $N + 5$ is divisible by 9

Q.7. Ram runs 7/4 times as fast as Sham. If Ram gives Sham a start of 300m, how far must the winning post be if both Ram and Sham have to end the race at the same time?
(a) 1400m (b) 700m
(c) 350m (d) 210m

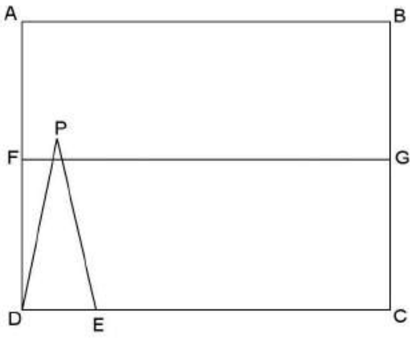
Q.8. A certain sum is lent out at a certain rate of interest for a certain period and the amount is 3.6 times as the principle. Had it been lent out on 60% higher rate of interest for 40% less time period the amount would have been how many times the principle?

(a) 2.75 times (b) 3.5 Times
(c) 3.7 Times (d) 3.4 times

Q.9. The areas of the three adjacent faces of a rectangular box which meet in a point are known. The product of these areas is equal to:

(a) The volume of the box
(b) twice the volume of the box
(c) the square of the volume of the box
(d) the cube root of the volume of the box

Q.10. The area of the square ABCD is 64. Let E, F and G be mid points of DC, AD and BC respectively. If P is any point inside the rectangle ABGF and if X is the area of the triangle DPE, then which one of the following is true?



(a) $8 < \times < 16$ (b) $8 < \times < 32$
(c) $16 < \times < 32$ (d) $16 < \times < 64$

Q.11. Find the number of divisors of 5400 which are exactly divisible by 135?

(a) 12 (b) 8
(c) 16 (d) 14

Q.12. There are 10 types of chocolates in a shop; in how many ways can a person buy 18 chocolates?

(a) $^{28}C_{10}$ (b) $^{27}C_{11}$
(c) $^{27}C_{9}$ (d) $^{28}C_{9}$

Q.13. Let S = $\frac{1}{3}+\frac{1}{3^2}+\frac{1}{3^3}$ + ………. ∞; then $(0.25)\log_2 s$ is?

(a) 2 (b) 4
(c) 8 (d) 1

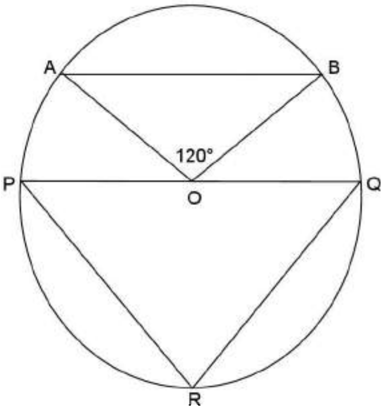
Q.14. In a test, Ram scored 25% of the maximum number of marks and fails by 60 marks, but Sham candidate who scores 42% of the maximum number of marks gets 8 marks more than necessarily passing marks. What is the maximum mark in the test?

(a) 500 (b) 300
(c) 400 (d) 600

Q.15. PQ is the diameter of a circle with center O. Point R on the circumference of the circle is equidistant from P and Q. A and B are two points in the opposite segment such that (arc AB) is 1/3 times the circumference; also AP = BQ. The ration of area of Δ PRQ to Δ AOB is:

(a) 4/√3 (b) √3/4
(c) √3/2 (d) 2/√3

The following diagram could be drawn: ∠AOB = 120°, ∠ PRQ – 90°. PQ = 2r, where is the radius of the circle.



[redacted]

	S1	S2	S3	S4	S5	Total
A1	97	99	98		99	491
A2	92	95		94		
A3	86	89		90	93	443
A4	78		76		85	405
A5		69	64	75		349
A6	49	53	48	60	55	
Total	468		463	502	505	

Q.16. How much did A2 score in S5?

(a) 98 (b) 96
(c) 94 (d) 95

Q.17. What were the marks scored by A4 in S4?

(a) 80 (b) 88
(c) 85 (d) 75

Q.18. What is the average of the total marks scored by all six students?

(a) 304 (b) 436
(c) 336 (d) 404

Q.19. What is the sum of the marks scored by all the six students in S2?

(a) 576 (b) 486
(c) 586 (d) 476

Direction for questions 20 to 22

Table given below provides information about the percentage sales share of four companies A, B, C and D in the four states of Tamilnadu, Karnataka, Kerala and Arunachal Pradesh for four of their products E, F, G, H. Last column also provides the total size of the market in four states for the four products in million Rs.

Product ↓	*Company A*	*Company B*	*Company C*	*Company D*	*Market size Rs. Million*
	TN KT KL AP	TN KT KL AP	TN KT KL AP	TN KT KL AP	TN KT KL AP
E	15 36 20 14	20 30 25 22	35 30 15 36	15 0 30 18	60 48 42 72
F	30 28 16 16	18 32 14 20	24 40 18 40	23 0 42 20	22 18 20 24
G	12 40 15 20	28 44 20 20	30 10 25 20	18 0 35 30	48 50 36 44
H	24 18 10 22	25 24 08 14	28 18 07 24	23 30 55 20	84 96 90 72

TN- Tamilnadu, KT- Karnataka, KL- Kerala and AP- Arunachal Pradesh

Q.20. What is the turnover in Rs million for company C from product H?
(a) 64.38 million
(b) 58.08 million
(c) 43.72 million
(d) 34.68 million

Q.21. In AP what is the sales of product E of companies other than A, B, C, D?
(a) Rs 8 million
(b) Rs 7.2 million
(c) Rs 0.8 million
(d) Cannot be determined

Q.22. The sales of product G for Company A is lowest in which of the four states?
(a) Tamilnadu
(b) Kerala
(c) Arunachal Pradesh
(d) Karnataka

Q.23. If a = 2009, b = 2010 and c = 2011, then find the value of $a^3 + b^3 + c^3 - 3abc$.
(a) 12,030 (b) 18,090
(c) 15,100 (d) 16,060

Q.24. Natural numbers are written as following in the shape of pyramid:

1
2 3
4 5 6
7 8 9 10
11 12 13 14 15

Find the sum of all the numbers in the nth row?

(a) $\frac{n(n+1)}{2}$ (b) $\frac{n^2+1}{2}$
(c) $\frac{n(n^2+1)}{2}$ (d) $\frac{n(n^2+1)}{6}$

Q.25. A quadratic function attains the maximum value 3 at x = –1. The value of the function at x = 0 is 1. What is the value of the function at x = 5?
(a) –159 (b) –29
(c) –19 (d) –119

Q.26. The surface area of cuboids is 24 cm^2. Find the maximum volume of the cuboids?
(a) 4 cm^3
(b) 6 cm^3
(c) 10 cm^3
(d) 8 cm^3

Q.27. If the roots of the equation $x^2 + px + q = 0$ differ from the roots of the equation $x^2 + qx + p = 0$ by the same quantity, then what is the value of (p + q)?
(a) 4 (b) –4
(c) 0 (d) 8

Q.28. If a, b, c and d are real numbers such that b > 0, d > 0 and $\frac{a}{b} < \frac{c}{d}$, then which of the following is true?

(a) $\frac{a}{b} < \frac{a-c}{b+d} < \frac{c}{d}$

(b) $\frac{a}{b} < \frac{a+c}{b+d} < \frac{c}{d}$

(c) $\frac{a}{b} < \frac{a-c}{b-d} < \frac{c}{d}$

(d) $\frac{a}{b} < \frac{a+c}{b-d} < \frac{c}{d}$

Q.29. Tops of two poles of height 20m and 14m of are connected by a wire. If the wire makes an angle 30° with the horizontal, then the length of the wire is

(a) 12m (b) 10 m

(c) 8 m (d) None of these

Q.30. A survey shows that 63% of the Americans like cheese whereas 76% like apples. If x% of the Americans like both cheese and apples then find the range of x?

(a) $0\% \le x \le 23\%$

(b) $0\% \le x \le 39\%$

(c) $4\% \le x \le 35\%$

(d) $39\% \le x \le 63\%$

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■ Section Test 2

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(a)	(c)	(a)	(a)	(b)	(b)	(b)	(c)	(a)	(b)	(c)	(b)	(c)	(a)	(a)	(c)	(d)	(b)	(a)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(b)	(b)	(b)	(c)	(b)	(d)	(b)	(b)	(a)	(d)										

HINTS AND SOLUTIONS

1. Since AB is the diameter of the circle, ∠ ACB would be right angle. In this triangle, we know AB = 15 and AC = 12. So, we can find BC. Since 3 – 4 – 5 forms a triplet, 3 × (3 – 4 – 5) also forms a triplet.

   So, 9 – 12 – 15 forms a triplet. Hence, BC = 9. Since BC = BD, AD – AC (similar triangles).

   Hence, area of ΔABC = Area of ΔABD = 1/2 AC × CB = 1/2 × 12 × 9 = 54.

   So, area of quadrilateral ABCD = 2 × 54 = 108 sq. cm. Hence option (D) is the answer.

2. Total annual salary = 6000 x 12 = 72000

   1% of 6000 = Rs 60

   So, professional tax per month = Rs 75

   Therefore, total professional tax = 75 x 12 = Rs 900

   Salary after professional tax deduction = Rs 71,100

   First Rs 30000 in tax free, so, the taxable income = Rs 71100 – 30000 = 41100

   Therefore, Payable income tax = 20% of 41100 = Rs 8220

   Hence, option (A) is the answer.

3. Radha can do 3/4th of a work in 12 days, she completes the work in 16 days and she does the 11/16th of work in (8 +3) days.

   Mira does 5/16th of work in (3 + 12) days.

   Therefore, she will completed the work alone in (16/5 x 15) = 48 days

   Hence option (C) is the answer.

4. Statement (II):- Since two given angles are 60^0 and 45^0.

   Therefore, Third angle will be 75^0

   So, adjacent side of angle of 60^0 will be the shortest because opposite side of the largest angle will be the shortest.

   So, statement (II) alone is sufficient

   Hence, option (A) is the answer.

5. Statement (II):- $E < F$, $B > A$, $A < D < B$

   So, $A < B$, $A < D < B$

   $A < D < B < C$

   Since, $D < F$

   Therefore, A will be the smallest integer.

   Hence, option (A) is the answer.

6. Question can be solved by using either of the statement alone. Hence, option (B) is the answer.

7.

	Ram	:	Sham
Speed	7	:	4
Time	4	:	7
Distance	4	:	7

Now, $7x - 4x = 300$

Means $x = 100$

Therefore, the wining post is 7 x 100 = 700m away from the starting point

Hence, option (B) is the answer.

Alternatively, question can be done with the help of options too.

9. Let length = l, breadth = b and height = h. Then,

Product of areas of 3 adjacent faces = $(lb \times bh \times lh) = (lbh)^2 = (\text{Volume})^2$.

Hence, option (C) is the answer.

10. Given DE = 4 units. Let PQ be the altitude of the triangle DPE $\therefore 4 < PQ << 8 \rightarrow \frac{1}{2} \times 4 \times 4 <$ Area of triangle DPE $< \frac{1}{2} \times 4 \times 8 \rightarrow 8 <$ Area < 16. Hence option (A) is the answer.

11. As $5400 = 2^3 \times 3^3 \times 5^2$ and $135 = 3^3 \times 5$

Required number of factors will contains at least 3^3 and 5

Therefore, required number of factors is 4 x 2 x 1 = 8

Hence, option (B) is the answer.

12. The required number of ways is the number of non-negative integral solution of

$A_1 + A_2 + \ldots\ldots A_{10} = 18$

Therefore, number of ways is ${}^{18+10-1}C_{10-1} = {}^{27}C_9$

Hence, option (C) is the answer.

13. $S = \dfrac{1/3}{1-1/3} = \frac{1}{2}$

$(0.25)^{\log_2 s} = (0.5)^{-2\log_2 2} = (0.5)^{-2} = 4$

Hence, option (B) is the answer.

14. Let maximum marks is X. So according to the question, 25% of X + 60 = 42% of X – 8. So X = 400

Hence, option (C) is the answer.

15. Area (Δ AOB) = ½ × OA × OB × sin (120).

PR = RQ = $\sqrt{2}$. Area (ΔPRQ) = ½ × PR × RQ × sin (90). So ratio of areas = $(1/2 \times r\sqrt{2} \times 1) / (1/2\sqrt{2} \times 1) / (1/2\sqrt{r} \times \sqrt{3}/2) = 4/\sqrt{3}$. Hence option (A) is the answer.

20. The market sizes for product H in the four states are 84, 96, 90 and 72. the market share of company C for product H in these 4 markets are 28, 18, 7, 24. Therefore, the sales of product H by Company C in the four markets = 28% of 84 + 18% of 96 + 7% of 90 and 24% of 72 = 23.52 + 17.28 + 6.3 + 17.28 = 64.38 million. Hence, option (A) is the answer.

21. The total market share grabbed by companies A, B, C and D in Arunachal Pradesh for product E is 14 + 22 + 36 + 18 = 90%. Therefore, companies other than these 4 companies will have a share of 10% of the total market size of 72 million = 7.2 million. Hence, option (B) is the answer

22. Sales of product G by company A in the four states TN, KT, KL and AP are respectively 12% of 48, 40% of 50, 15% of 36 and 20% of 44 respectively. The sales figures are 5.76, 20, 5.4 and 8.8 respectively. The lowest sales is in the state of Kerala. Hence option (B) is the answer.

23. $a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2)]$

$= \frac{1}{2}(2009 + 2010 + 2011)[(2009 - 2010)^2 + (2010 - 2011)^2 + (2011 - 2009)^2] = \frac{1}{2}(6030)(6) = 18{,}090$. Hence, option (B) is the answer.

Alternatively, since the tens place digit in all the options are different, we can go through only calculating the tens place digit to solve this questions.

Last two digits of $9^3 = 29$ Last two digits of $10^3 = 00$

Last two digits of $11^3 = 31$

Sum of last two digits of $9^3 + 10^3 + 11^3 = 60$

And last two digits of 3abc = 70

Hence, last two digits of the expression should be A60 – B70 = 90. This is given in only option (B).

24. Just put n = 1, find the sum of the first row i.e. 1, option 1, 2 & 3 satisfy the condition.

At n = 2, only option C satisfy the condition

And for all higher values of n, only option C is satisfied.

Hence, option (C) is the answer.

25. Suppose that the quadratic function is $f(x) = ax^2 + bx + c$

Also, f(0) = 1 implies that c = 1

As we know that, f(x) attains maximum value at $\left(\dfrac{\alpha+\beta}{2} = -b/2\alpha\right)$

Or, $\dfrac{\alpha+\beta}{2} = 1$ implies that $\alpha+\beta = -2$ implies that b = –2a (only when a<0)

Or, that f(1) = a + b + c = 3 or a = –2 & b = 4

So, $f(x) = -2x^2 + 4x + 1$

$f(10) = -2(5)^2 + 4(5) + 1 = -50 + 20 + 1 = -29$

Hence, answer option is (B)

26. Given that 2(ab + bc + ca) = 24; if a, b and c are the sides of the cuboids.

Applying AM-GM inequality to ab, bc, ca

$$\frac{ab+bc+ca}{3} \geq \sqrt[3]{ab.bc.ca}$$

abc ≤ 8

Therefore, the maximum volume of the cuboids is 8 cm^3.

Hence, option (D) is the answer.

27. Let x1, x2 be the roots of the equation $x^2 + px + q = 0$ and x3, x4 be the roots of the equation $x^2 + qx + p = 0$.

Hence, x1 + x2 = –p, x1 × x2 = q, x3 + x4 = –q, x3 × x4 = p ---------(1)

According to the question, x1 – x2 = x3 – x4

Or, $(x1 - x2)^2 = (x3 - x4)^2$, or, $(x1 + x2)^2 - 4\,x1 \times x2 = (x3 + x4)^2 - 4\,x3 \times x4$ --------(2)

Putting the values from (1) in (2), we obtain (p – q) (p + q + 4) = 0

Hence, either p = q (not possible otherwise both the equations will become same) or p + q = –4. Hence option (B) is the answer.

28. Option (B) is the answer

30. Maximum will be the lower of 63 and 76 = 63%. To obtain the minimum value, we will use the formula for two sets:

$(\text{Apples} \cap \text{Cheese})_{\text{min}} = \text{Apples} + \text{Cheese} - (\text{Apples} \cup \text{Cheese})_{\text{max}} = 63 + 76 - 100 = 39\%$

Hence option (D) is the answer.

Section Test 3

Q.1. Three persons start walking together and their steps measure 40 cm, 42 cm and 45 cm respectively. What is the minimum distance each should walk so that each can cover the same distance in complete steps?

(a) 25 m 20 cm (b) 50 m 40 cm
(c) 75 m 60 cm (d) 100 m 80 cm

Direction for questions 2 to 4: Read the information given below and answer the questions that follow:

Following is an incomplete table presenting the number of students playing different sports in a school:

	Games played			*Total*
	TT	Football	Basketball	
Male			44	
Female				108
Total		80		

Some more facts are also known:

(a) Half the students played either Football or basketball (but not both).
(b) 40% of the students were female.
(c) One third of the male students played TT.

Q.2. How many female players played TT?

(a) 36
(b) 81
(c) 44
(d) Cannot be determined

Q.3. What percentage of Football players are male?

(a) 20% (b) 80%
(c) 12.5% (d) 4%

Q.4. What proportion of the female players played basketball?

(a) 11/18 (b) 4/27
(c) 11/108 (d) 1/27

Q.5. Given that x,y,z are sides of a triangle, with $x,y,z > 10$. Which of the following is definitely true?

i) $x + 1, y + 1, z + 1$ are also sides of triangles
ii) $x - 1, y - 1, z - 1$ are also sides of triangles
iii) x^2, y^2, z^2 are also sides of triangle

(a) i and iii (b) i and ii
(c) i, ii and iii (d) ii and iii

Direction for questions 6 and 7: Go through the direction given below and solve the questions based on it.

John and Abraham are playing a match stick game. There are 100 match sticks, whosoever burns 100th match stick win the game. Each player can pick up and burn any number of match sticks from 1 to 7.

Q.6. How many match sticks should John burn to ensure his winning after Abraham burns 67th match stick?

(a) 5 (b) 1
(c) 3 (d) he cannot win now

Q.7. How many match sticks should Abraham burn to ensure winning if John starts and burns 7 match sticks in his first chance?

(a) 1 (b) 4
(c) 5 (d) He cannot win now

Q.8. There are three grades of milk: 1%, 2%, and 3% fat by volume. X gallons of 1%, y gallons of 2%, and z gallons of 3% are mixed to give x + y + z gallons of 1.5%. What is x in terms of y and z?

(a) $y + 3z$
(b) $(y + z)/4$
(c) $2y + 3z$
(d) $3y + z$

Q.9. If $M = \sqrt{4} + \sqrt[3]{4} + \sqrt[4]{4}$, the value of M is

(a) < 3 (b) $= 3$
(c) $3 < M < 4$ (d) > 4

Q.10. Company X has 6 regional offices. Each regional office must recommend two candidates, one male and one female, to serve on the corporate auditing committee. If each of the offices must be represented by exactly one member on the auditing committee and if the committee must consist of an equal number of male and female employees, how many different committees can be formed?

(a) 10 (b) 20
(c) 40 (d) 60

Q.11. If x is positive, which of the following could be the correct ordering of $1/x$, $2x$, and x^2?

i. $x^2 < 2x < 1/x$
ii. $x^2 < 1/x < 2x$
iii. $2x < x^2 < 1/x$

(a) None (b) iii
(c) i and ii (d) i, ii, and iii

Q.12. In the arithmetic sequence $t_1, t_2, \ldots t_n$, $t_1 = 23$ and $t_n = t_{n-1} - 3$ for each $n > 1$, what is n when $t_n = -4$?

(a) –1 (b) 7
(c) 10 (d) 14

Q.13. P is the sum of even integers from 40 to 60 (inclusive of both the limits) and Q is the number of even integers from 40 to 60 inclusive. What is the value of P + Q?

(a) 551 (b) 560
(c) 561 (d) 572

Direction for questions 14 to 17: Go through the table given below and solve the questions based on it.

During the last IPL series, there were 7 teams – A, B, C, D, E, F and G, and each team played 3 matches with the other teams. All the matches were having a winner i.e. none of the matches played were a tie.

To decide the winner, if was finally decided to calculate a ratio called as win ratio.

Win ratio = No. of matches won/number of matches played

Following table gives the outcome of some of the matches played in the series:

	A	B	C	D	E	F	G	Win ratio
A	NA	2		3	1			0.61111
B		NA	1	0		0	1	
C	2		NA		3	2		0.7777
D			1	NA	0		2	0.389
E		2			NA	2		
F	1					NA	1	0.5
G			0		3		NA	

So, team C won 2 matches against team A and so on. As it can be seen, some of the data in the above table are missing.

Q.14. How many matches did team C won against team B?

(a) 0 (b) 1
(c) 2 (d) 3

Q.15. Which team won the maximum number of matches?

(a) A (b) B
(c) C (d) E

Q.16. For how many matches, the unique winner team cannot be decided?

(a) 0 (b) 1
(c) 2 (d) 3

Q.17. What is the total number of matches played in the whole series?

(a) 18 (b) 21
(c) 63 (d) None of these

Q.18. ABCD is a trapezium such that AB, DC is parallel and BC is perpendicular to them. If angle (DAB) = 45^0, BC = 2cm and CD = 3cm, then find the length of AB?

(a) 6 cm (b) 4 cm
(c) 3 cm (d) 5 cm

Q.19. What is the number of positive integral solution of the equation $3x + y = 42$?

(a) 12 (b) 13
(c) 14 (d) 15

Q.20. A number lock has three rings. Each ring has digits 0 to 9 written on it. The maximum number of trials needed to open the lock if it is known that correct combination sum is 5?

(a) $10^3 - 745$ (b) 21
(c) 24 (d) 41

Q.21. Let $f(x) = x^3 - 2x^2 - x + k$. If $f(1) = 0$, then the roots of $f(x) = 0$ are

(a) 1, 2, –2 (b) 1, –1, –2
(c) –1, 2, –2 (d) 1, –1, 2

Q.22. The average of 26, 29, n, 35, 43 lies between 25 and 35. If n is always an integer and greater than the average of the given integers then the value of n is :

(a) $33 < n < 47$ (b) $34 < n < 43$
(c) $33 < n < 42$ (d) None of these

Q.23. Three dogs are running in a park in such a way that when dog A takes 5 steps, dog B takes 6 steps and dog C takes 7 steps. But 6 steps of dog A are equal to 7 steps of dog B and 8 steps of dog C. What is the ratio of their speeds?

(a) 140:144:147 (b) 40:44:47
(c) 15:21:28 (d) 252:245:240

Q.24. Find the remainder when $6^{6^{6^{6^{6^{\cdot^{\cdot^{66}}}}}}}$ (100 times) when divided by 10?

(a) 6 (b) 2
(c) 4 (d) 8

Q.25. The smallest possible number which can be written as the sum of the cubes of the two natural number in the two different ways?

(a) 1000 (b) 1728
(c) 1729 (d) 2020

Q.26. Two different sides of a parallelogram are 8cm and 6cm and the ratio of the diagonals is 3 : 4. Find the difference between the lengths of the diagonals?

(a) 5cm (b) 7 cm
(c) 6 cm (d) $\sqrt{8}$ cm

Q.27. Find the unit digit of the expression $199^{2n} + 144^{3n}$, where n is a natural number.

(a) 5 (b) 7
(c) either 5 or 7 (d) 3

Q.28. A certain sum of money triples itself in 8 years. In how many more years will it be five times the original sum (consider Simple Interest)?

(a) 12 years (b) 16 years
(c) 20 years (d) 8 years

Q.29. Find the value of the expression

$$\frac{1}{1+\log_a bc}+\frac{1}{1+\log_b ac}+\frac{1}{1+\log_c ba}?$$

(a) 0 (b) 1
(c) ½ (d) abc

Q.30. A square hole of cross-sectional area 4 cm^2 is drilled across a cube with its length parallel to a side of the cube. If an edge of the cube measures 5 cm, what is the total surface area of the body so formed?

(a) 158 cm^2 (b) 190 cm^2
(c) 166 cm^2 (d) 182cm^2

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■

■■■ Section Test 3

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(a)	(b)	(c)	(c)	(b)	(b)	(c)	(a)	(d)	(b)	(c)	(c)	(c)	(c)	(c)	(a)	(c)	(d)	(b)	(b)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(d)	(c)	(a)	(a)	(c)	(d)	(c)	(d)	(b)	(d)										

HINTS AND SOLUTIONS

1. Answer is LCM of 40, 42, 45 = $2^3 \times 3^2 \times 5^1 \times 7^1 = 2520$ cm = 25.2 m. Hence, option (A) is the answer.

4. Following is the solution:

	Games played			Total
	TT	Football	Basketball	
Male	54	10	44	162
Female	81	70	11	108
Total	135	80	55	270

5. Using x + y > z, statement (i) and statement (ii) satisfies. Option (iii) will not always satisfy this condition. Hence, option (B) is the answer.

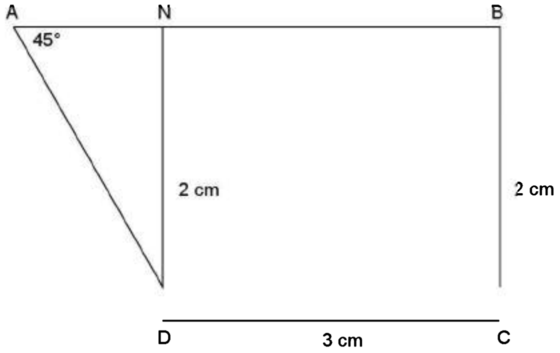
6. John can always make the sum of 8 that is (max picks ie 7 + min picks ie 1). Number of remaining picks = (100 – 67) = 33. Hence he must pick up the number of sticks = remainder when 33 divided by 8.
Hence, option (B) is the answer.

7. John has burned 7 matches. Matches left = (100 – 7) = 93. Hence, John should pick remainder when 93/8 = 5.
Hence, option (C) is the answer.

9. Can be seen that $\sqrt[3]{4}$ is more than 1, similarly $\sqrt[4]{4}$ is more than 1. Hence, net is more than 4.
Hence, option (D) is the answer.

13. P = 40 + 42 + 44 + ….+60 = 550 and Q = 11. Hence P + Q = 561.

18.



$\text{Tan}(45^0) = \dfrac{DN}{AN}$ or $1 = \dfrac{DN}{AN}$

Therefore, AN = DN = 2cm

AB = AN + BN = 2cm + 3cm = 5cm

Hence, answer option is (D)

19. The given linear equation is $3x + y = 42$

X	Y
0	42
1	39
2	36
.	..
.	..
13	3
14	0

By observation number of positive integral solutions are 13.

20. The combination that the sum 5 are (0, 0, 5), (1, 4, 0), (3, 2, 0), (1, 1, 3) and (1, 2, 2)

Maximum number of permutation for each case

(0, 0, 5) → 3!/2! = 3

(1, 4, 0) → 3! = 6

(3, 2, 0) → 3! = 6

(1, 1, 3) → 3!/2! = 3

(1, 2, 2) → 3!/2! = 3

Therefore, total number of permutation is 21

21. Given that $f(x) = x^3 - 2x^2 - x + k$

$f(1) = 0$ means $1^3 - 2(1)^2 - 1 + k = 0$ means $k = 2$

New expression becomes $f(x) = x^3 - 2x^2 - x + 2$

$F(x) = 0$ implies that $x^3 - 2x^2 - x + 2 = 0$ is satisfied only at $x = 1, -1, 2$

Therefore, required roots of the cubic are 1, –1, 2

22. According to the question following equation can be deduced

$$25 < \frac{26 + 29 + n + 35 + 43}{5} < 35$$

Implies that $33 < n < 42$

23. Frequency of steps of A : B : C = 5 : 6 : 7

But in terms of size of steps; 6A = 7B = 8C

Ratio of the speeds of A : B : C = (5/6) : (6/7) : (7/8) = 140 : 144 : 147

24. 6^n (where n is a natural number) will always leaves the remainder 6 when divide by 10.

25. $1729 = 1^3 + 12^3 = 10^3 + 9^3$

26. As for the diagonals X & Y of a parallelogram

$X^2 + Y^2 = 2(A^2 + B^2)$; where A & B are the sides of the parallelogram.

From the information given in the question; sides of a parallelogram are 8cm and 6cm

And ratio of the diagonals is 3 : 4 such that X = 4k and Y = 3k

$16k^2 + 9k^2 = 2(8^2 + 6^2)$, so, $K = \sqrt{8}$

Therefore, the required difference is 4k – 3k = k $= \sqrt{8}$ cm

27. For any n, 199^{2n} has last digit as 1,

But the last digit of 144^{3n} is 4 for odd values of n and 6 for even values of n.

Therefore, last digit of the given expression is either 5 or 7.

Hence, answer option is (C)

28. Kick yourself once if you marked option (B) as the answer. Question is asking for "How many more years" and hence it should be 8 years. Hence, option (D) is the answer.

29. The given expression

$$\frac{1}{1+\log_a bc} + \frac{1}{1+\log_b ac} + \frac{1}{1+\log_c ba}$$ can be written as

$$\frac{1}{\log_a a + \log_a bc} + \frac{1}{\log_b b + \log_b ac} + \frac{1}{\log_c c + \log_c ba}$$

$$= \frac{1}{\log_a abc} + \frac{1}{\log_b bac} + \frac{1}{\log_c cba}$$

$$= \log_{abc} a + \log_{abc} b + \log_{abc} c = \log_{abc} abc = 1$$

30. The resultant figure will be like a square tunnel inside a cube.

Total surface area of the cube = 6 $(\text{Side})^2$ = 150

New surface area added = 4 × (2 × 5) [Surfaces × side of square cross section × depth]

Total old surface area to be subtracted = 4 + 4 = 8

Hence net surface area = 150 + 40 – 8 = 182 cm^2. Hence, option (D) is the answer.

Section Test 4

Q.1. Let T_n denotes the number of triangles which can be formed using the vertices of a regular polygon of n sides. If $T_{n+1} - T_n = 21,$ then n equals:

(a) 5 (b) 7
(c) 6 (d) 4

Q.2. When 96 is added to a N^2, it gives another perfect square. If N is a natural no., how many distinct values of N are possible?

(a) 3 (b) 4
(c) 5 (d) None of these

Q.3. For all x, $x^2 + 2ax + 10 - 3a > 0$, then the interval in which 'a' lies is:

(a) $a < -5$
(b) $-5 < a < 2$
(c) $a > 5$
(d) $2 < a < 5$

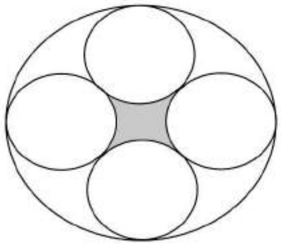
Q.4. A solid cube of side 3 cm, painted red on the outside is cut into 27 equal cubes. When all the 27 cubes are considered, what is the difference between the area that is not red and the area that is red?

(a) 27 (b) 0
(c) 54 (d) 81

Q.5. A perfect square has length n if its last n digits (in base 10) are the same and non-zero. What is the longest possible length in the set of all the natural numbers?

(a) 2 (b) 3
(c) 6 (d) Infinite

Q.6. In the figure below, the radius of the bigger circle is $\left(\sqrt{2}+1\right)$ cm and the radius of all the smaller circles are equal. Each of the smaller circles touches two of the other three smaller circles and the larger circle as shown. Find the area (in cm^2) of the shaded portion.

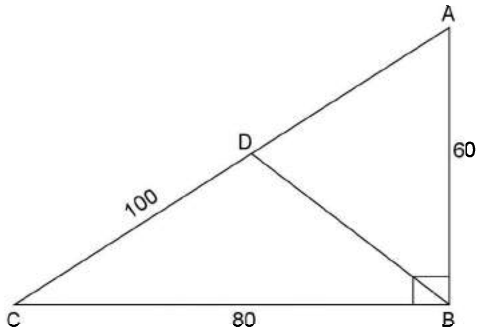


(a) $2 - \frac{\pi}{2}$ (b) 1
(c) $\pi/4$ (d) $(4 - \pi)$

Q.7. P is a series of natural numbers in AP with a common difference of D. If D = 1, what is the least positive integer that must be added to the product of the sixth term, eighth term, tenth term and twelfth term of P to make it a perfect square?

(a) 2 (b) 16
(c) 4 (d) Cannot to determined

Q.8. In the right-angled triangle ABC, BD divides the triangle ABC into two triangles of equal perimeters. Find the length of BD, give that AC=100, BC=80 and AB=60.



(a) 25 (b) $24\sqrt{5}$
(c) $20\sqrt{5}$ (d) None of these

Q.9. What is the greatest area(in sq. units) of a rectangle the sum of whose 3 sides is equal = 100.
(a) 625 (b) 1250
(c) 883.33 (d) 666.66

Q.10. A polygon is said to be concave if at least one of the angles of the polygon is greater than 180^0. A concave polygon is such that each of its internal angles measures either 60^0 or 300^0. If this polygon contains twenty 300^0 angles, find the number of 60^0 angles in it.
(a) 26 (b) 18
(c) 23 (d) 24

Direction for questions 11 to 13: Go through the information given below and solve the questions based on it.

You are the Quality Control manager at the restaurant McDi and facing a dilemma. Go through the case facts given below and solve the questions based on it:

McDi produces chicken nugget box (1,000 boxes at a time in a batch). McDi has to ensure that it does not supply undercooked nuggets (known as defective) to the customers. If a customer receives a defective nugget box, company has to pay a penalty of Rs 150 per nugget box. To check the level of preparedness of nuggets, any of the two tests can be used:

Test 1		*Test 2*	
Cost	Rs. 6 per nugget box	Cost	Rs. 9 per nugget box
Detection Rate	80%	Detection Rate	100%

McDi can re-cook undercooked nuggets at a cost of Rs 75 per nugget box. This box can be now sold to customer as it is now certainly properly cooked.

Let 'x' be the number of defective nugget boxes per batch.

Q.11. Manager should not use any quality control test, if it is expected that
(a) $x < 100$
(b) $100 < x < 200$
(c) $x > 200$
(d) None of these

Q.12. If $200 < x < 400$, then
(a) You should use Test 1
(b) You should use Test 2
(c) You can use Test 1 or Test 2
(d) None of these

Q.13. If $x = 100$, you should use
(a) Test 2
(b) Test 1 and Test 2
(c) Test 1 or No test
(d) None of these

Direction for questions 14 to 16: Go through the information given below and solve the questions based on it.

Following is known about the members of Thumbs-up holidays who might have gone to Goa or Kashmir or both.

		Number of members who went to	
Number of children	Total number of members	Goa	Kashmir
Less than 3 children	150	80	60
3 to 5 children	400	275	225
More than 5 children	50	20	30

Q.14. Minimum number of members who have 3 to 5 children and have gone to atleast one of the destinations is:
(a) 225 (b) 275
(c) 100 (d) Cannot be determined

Q.15. Minimum number of members for all the categories and have gone to atleast one of the destinations is:
(a) 385 (b) 375
(c) 305 (d) 315

Q.16. How many of the members have definitely not gone to any of the two destinations?
(a) 0 (b) 10
(c) 25 (d) Cannot be determined

Q.17. Buy 3 and get 2 free. How much is the effective discount in this case?
(a) 40% (b) 60%
(c) 66.66% (d) None of these

Q.18. In a survey of political preferences, 78% of those asked were in favour of at least one of the proposals: I, II and III. 50% of those asked favoured proposal I, 30% favoured proposal II and 20% favoured proposal III. If 5% of those asked favoured all three of the proposals, what percentage of those asked favoured more than one of the three proposals?
(a) 10 (b) 12
(c) 17 (d) 22

Q.19. Navjivan Express from Ahmedabad to Chennai leaves Ahmedabad at 6.30 am and travels at 50 kmph towards Baroda situated 100 km away. At 7.00 am Howrah-Ahmedabad Express leaves Baroda towards Ahmedabad and travels at 40kmph. At 7.30 am Mr Shah, the traffic controller at Baroda realizes that both the trains are running on the same track. How much time does he have to avert a head-on collision between the two trains?
(a) 15 min (b) 20 min
(c) 25 min (d) 30 min

Q.20. The number of employees in a nationalised bank in a small town is 10, out of which 4 are female and the rest are males. A committee of 5 is to be formed. If m be the number of ways to form such a committee in which there is atleast one female employee and n be the no. of ways to form such a committee which includes at least two male employees, then find the ratio m:n.

(a) 3:2 (b) 5:2

(c) 1:1 (d) 8:9

Q.21. Dilip ate half of a pizza on Monday. He ate half of what was left on Tuesday and so on. He followed this pattern for one week. How much of the pizza would he have eaten during the week?

(a) More than 99% but less than 100%

(b) 100%

(c) Less than 99% but more than 98%

(d) Less than 98%

Q.22. If $a \leq 22$ and $b \geq 12$, then which of the following is always true?

(a) $a > b$ (b) $a - b > 10$

(c) $a - b \leq 10$ (d) $a + b \geq 34$

Directions for questions 23 to 26: Answer the questions on the basis of the information given below.

Chunmun has six different types of fruits - Mangoes, Apples, Oranges, Peaches, Guavas and Litchis – with him. Following table provides information about the minimum and the maximum price of each of the mentioned fruits. The table also provides information about the number of fruits of each type and the average price of each fruit.

Assume that the price of each fruit is an integer. There is only one mango that has the minimum price and there is only one mango that has the maximum price. The same holds true for each type of fruit.

	Minimum Price	*Maximum Price*	*Average Price*	*Number of Fruits*
Mango	20	28	23	16
Apple	40	50	46	24
Orange	4	8	7	40
Peach	16	26	20	34
Guava	3	8	5	28
Litchi	10	18	16	30

Q.23. Find the approximate average price of fruits with Chunmun.

(a) Rs 18

(b) Rs 20

(c) Rs 23

(d) Rs 15

Q.24. The number of mangoes whose price is more than Rs 21 is at least

(a) 3 (b) 4

(c) 5 (d) 6

Q.25. Given that the price of four peaches with Chunmun is Rs 22 each and there are exactly 10 peaches whose price is Rs 18. The number of peaches with Chunmun whose price is at least Rs 25 is at most

(a) 13 (b) 12

(c) 11 (d) 9

Q.26. Total number of guavas with Chunmun whose price is less than the average price of the guavas with Chunmun is at least

(a) 17 (b) 18

(c) 19 (d) 20

Q.27. Consider S = (1,2,3, ……. 10). In how many ways two numbers from S can be selected so that the sum of the numbers selected is a double digit number?

(a) 36 (b) 16

(c) 29 (d) ${}^9C_2 - {}^5C_2$

Q.28. Circumference of a circle and the perimeter of a pentagon and an octagon are same. Their areas are denoted by C, P and O respectively. Which of the following is true their areas?

(a) C > P > O

(b) O > P > C

(c) O > C > P

(d) C > O > P

Q.29. A man walks 10km towards west. Then which of the combinations will take him to the original starting position?

(a) left, left, right

(b) left, right, right

(c) right, left, right

(d) right, right, right

Q.30. A contractor agreeing to finish a work in 150 days, employed 75 men each working 8 hours daily. After 90 days, only 2/7 of the work was completed. Increasing the number of men by …..men will finish the work on time (Fill in the blanks).

(a) 100 men

(b) 75 men

(c) 150 men

(d) None of these

My performance in this Test

Attempts	Right	Wrong	Net Marks	% Accuracy

ANSWERS

■■■

■■■ Section Test 4

Q. No.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Answer	(b)	(b)	(b)	(c)	(b)	(d)	(b)	(b)	(b)	(c)	(a)	(b)	(c)	(b)	(a)	(b)	(a)	(c)	(a)	(c)
Q. No.	21	22	23	24	25	26	27	28	29	30										
Answer	(a)	(c)	(a)	(d)	(d)	(b)	(c)	(d)	(d)	(c)										

HINTS AND SOLUTIONS

1. According to given condition, $T_n = {}^nC_3$ and $T_{n+1} - T_n = 21$
   So, ${}^{n+1}C_3 - {}^nC_3 = 21$
   $$\frac{1}{6}(n+1)(n)(n-1) - \frac{1}{6}n(n-1)(n-2) = 21$$
   So,n =7. Hence option (B) is the answer.
2. $N^2 + 96 = P^2$, or, $96 = P^2 - N^2$. Now factorize 96 and equate it with $(P+N)(P-N)$.
3. $D < 0$, $4a^2 - 4(10 - 3a) < 0$. And hence the answer.
4. Let the side of each small cube be n (n = 1 unit)
   Total red area = $6(9n^2) = 54$
   Total area = $27(6n^2) = 54 \times 3$
   Total area which is not red = 2×54
   Difference of the red and non-red areas = 54
5. Largest possible length is 3, viz. 1444.
6. If the radius of smaller circle is 1 unit, then the radius of the bigger circle is $\sqrt{2} + 1$ units, and converse of this is also true.(It can be proved also)
   So, the answer in this case would be the area of square – 4 quadrants of the smaller circle.
7. Let a, a+d, a+2d and a+3d be the 4 consecutive terms of an AP, as their product is $[a^2+3ad]^2 + 2[(a^2+3ad)]d^2$. If we add d^4 to the product, we get the square of $a^2 + 3ad + d^2$.
8. Let BD = x and CD = y
   So, $(100 - y) + 60 + x = y + x + 80$, Hence y = 40
   Now Cos C = 4/5 and apply cosine rule in ΔCDB to find DB.
9. Given is 2a+b = 100, and area = ab. Assuming 2a = x, we get x+b=100, and area = 1/2xb. Now area will be maximum when x = b, or a = b/2. Thus a = 25, & b = 50, maximum area = 1250.
10. Let the no. of 60° angles be N. So, total number of angles in the triangle = 20 + N. Now, sum of interior angles = $(2n - 4)90^\circ$. Sum of angles in this case = $(20 + N - 2)\times180^\circ$. And $(20 + N - 2)\times180^\circ = N(60^\circ) + 20 \times 300$
11. Only when x is < 100, then only the company should not use any test. Because for x < 100, company has to pay more in the tests than the actual costs spend on it. Hence, option (A) is the answer.
12. For n = 300 (200 < x < 400), you should use Test 2. Hence, option (B) is the answer.
13. For x = 100, you should either use Test 1 or no test. Hence, option (C) is the answer.
14. There are 275 members who have gone to Goa. We are not sure about the 225 members who have gone to Kashmir that if they are different than the members who have gone to Goa. It might be a possibility that these 225 members who have gone to Kashmir have also gone to Goa. Hence minimum number of members who have gone to atleast one of the destinations = 275.
    Hence, option (B) is the answer.
15. Taking a cue from solution to Q14, answer to this question would be = 80 + 275 + 30 = 385
16. We are not sure of the members in the category '3 to 5 children' or 'more than 5 children' that any of these members have not gone to any of the destinations. Although we are sure of the fact that in the category

'Less than 3 children', there are atleast 150 – (80+60) = 10 members who have not gone to any of the destinations.

Hence, option (B) is the answer.

17. Instead of paying for 5, one is supposed to pay only for 3 and 2 are free. Hence percentage discount = $\frac{2}{5}\times 100$ = 40%. Hence, option (A) is the answer.

23. Total price of fruits with Chunmun is 23 x 16 + 46 x 24 + 7 x 40 + 20 x 34 + 5 x 28 + 16 x 30 = Rs 368 + Rs 1104 + Rs 280 + Rs 680 + Rs 140 + Rs 480 = Rs 3052. Total number of fruits with Chunmun is 16 + 24 + 40 + 34 + 28 + 30 = 172. Average price of fruits with Chunmun = (3052/172) = Rs 17.74

24. The total price of mangoes with Chunmun is 16 x 23 = Rs 368.

Total price of 14 mangoes = 368 – (20 + 28) = Rs 320

Out of the given 14 mangoes assume that maximum number of mangoes have price equal to Rs 21.

The maximum price of any mango out of these 14 mangoes is Rs 27

So, 9 x 21 + 1 x 23 + 4 x 27 = Rs 320

Therefore, the number of mangoes whose price is more than Rs 21 is 1 + 4 + 1 = 6.

25. The price of the 18 peaches with Chunmun whose price is not known to us = X

X = 20 x 34 – (16 + 26 + 4 x 22 + 10 x 18) = Rs 370

Assume that maximum number of peaches have price equal to Rs 17

370 = 25 x 8 + 17 x 10. Number of peaches with Chunmun whose price is at least Rs 25 is at most 8 + 1 = 9.

26. The total price of the guavas with Chunmun is 5 x 28 = 140

The total price of the 26 guavas whose price is not known is 140 – (3 + 8) = Rs 129.

Assume maximum number of guavas have price equal to Rs 7

129 = 8 x 7 + 1 x 5 + 17 x 4. Therefore, the number of guavas with Chunmun whose price is less than the average price of the guavas with Chunmun is at least 17 + 1 = 18.

28. For a given perimeter, area of the structure which has more number of sides will be maximum. Hence, area of circle > area of octagon > area of pentagon.

29. Go through the options.